

LIVRES

Pattern formation in biology, vision and dynamics

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World Scientific Publishing Co., 2000. 423 p. \$ 111. ISBN 981-02-3792-8

It is common knowledge that Biology and Mathematics have enjoyed a fruitful interaction for many years with substantive results emerging in both fields. However, a new era has dawned. The depth and importance of mathematics in the biological sciences has reached another order of magnitude, and there is every reason to believe that in the twenty-first century the impact of the life sciences on mathematics will rival or exceed that of the physical sciences.

Many of us as mathematicians were educated in the culture of physics and engineering. In fact over the past two decades the sophistication of mathematicians in theoretical physics has grown impressively. However, relatively few of us are as deeply knowledgeable and cultured in the life sciences. It is time for that to change.

We are not, of course, suggesting that mathematicians should abruptly shift gears and begin to address applied problems in Biology. We simply feel that awareness of the emerging ideas and techniques in life science can, indeed should, have a serious impact on mathematics. For example, the fields of non-linear analysis and dynamics embody a myriad of disparate problems, and it is of great benefit to have means of picking out the important ones. Physics has been particularly effective in this regard — directing us to the Navier-Stokes equations, the KdV-equation, the Yang-Mills equations, etc. Biology too, particularly now with its increasing sophistication is a source of direction in both these fields. Several of the articles in this volume are examples of that effect.

This gorgeously produced book, the result of a meeting: *la Formation des Motifs*, held at I.H.E.S. in December of 1997 gives an important entrée into the emerging world of biological mathematics. The theme is “pattern formation”, but a broad range of topics is covered, from DNA computing and visual perception to the study of pigmentation patterns on the shells of tropical mollusks. The articles vary considerably in their depth of mathematical content. Some are devoted almost exclusively to a presentation of biological (or biochemical) facts; others are quite mathematical and/or speculative. Most lie somewhere in the middle.

However, the editors have carefully organized the material under different rubrics: Growth and form, Reaction-diffusion, Cellular patterns, DNA and genetic control, and Images and Perception. Each topic contains a balance of articles — some scientifically expository and some quite theoretical. Many contributors have made considerable effort to render their work accessible to a wide audience.

This volume contains an array of fascinating articles, and we do not intend to give them an exhaustive review. We shall mention a few of the highlights — enough we hope to intrigue one to go to the source.

The book begins by discussing “growth and form” — self-evolving patterns seen in crystals, leaves and flowers, sponges and corals, etc.. It begins with an article addressing computational aspects of pattern formation. It discusses abstract algorithmic processes which are realized in many physical and biological systems. The next paper studies the impact of the fluid environment on the growth of sponges and

corals, in particular the impact of hydrodynamics on their nutrient-driven growth processes. Stochastic evolutionary growth models are then treated in depth. Cannon, Floyd and Parry develop a mathematical theory of crystalline growth by using discrete approximations to riemannian geometry. In their article they give a very pretty explanation of why self-similarity fractals appear so frequently in nature. Recent results on aperiodic tilings are also discussed.

Some of the fundamental ideas concerning patterns in biology are due to Alan Turing who showed in 1952 that pattern formation requires the interaction of two substances with different diffusion rates. This introduces partial differential equations, the so-called *reaction-diffusion equations*, into the subject. They model “activator-inhibitor” systems and can be thought of as infinite dimensional dynamical systems. This theory is highly developed and is well treated in the book. It has had spectacular success in its application to understanding the color patterns on the shells of tropical mollusks. Some of these patterns are quite complicated and appear to be nearly randomly produced. The accord between the theoretically generated patterns and those which appear in nature is awesome (see pages 126-7).

Closely related to this is the study of phyllotaxis, certain quasi-crystalline structures found everywhere in plants. Pioneering work in this subject was done by the Bravais brothers in 1835-7. It led subsequently to A. Bravais’ classification of the fourteen possible periodic lattices in 3-space. This is an early instance of the impact of life science on mathematics. In the article by Y. Couder and S. Douady we have a good example of very interesting dynamics coming out of a question in phyllotaxis.

One of the fundamental areas of research in biology is the study of cell growth, and the book contains many papers on this subject. Here an extensive knowledge of observed cell patterning is important, and a fair amount of space has been devoted to discussing what is known. For example there is a nicely written and informative article on plant meristems and their patterns. There is then a sequence of papers which address the question of cellular pattern development from wide-ranging perspectives. One is based on finite “cellwork” structures; another develops mechanical stress patterns as an important component of morphogenesis; yet another uses a smooth model based on a “displacement velocity” vector field and its associated covariant derivative called the “growth tensor”. Details aside, it is clear that the mathematics of evolving systems poses geometrically and topologically interesting questions.

One of the most revolutionary and exciting areas discussed in this book is that of DNA computing and DNA nanotechnology. The articles in this section are very well written. We particularly recommend the paper of T. Head which gives a simple, lucid introduction to the subject. DNA computing completely changes ones thinking about combinatorial questions; the challenge now is to find DNA algorithms for solving problems. N. Jonoska in her article discusses a number of NP complete problems, for example the Hamiltonian cycle problem, which can be solved using sophisticated molecules. She also discusses the important role knot theory plays here.

We also recommend N. Seeman’s paper about DNA nanotechnology where knot theory again comes into play. This is an important new field (see for example the New York Times, August 10, 2000 - Science Times section). Ultimately one is interested in manufacturing molecular-sized motors or electronic gadgets: for example miniscule machines that could pass through the human arterial system to repair injuries.

The field also has a theoretical side which is concerned with the possibility and development of programmable DNA computers and also with the type of abstract logical questions common in modern computer science.

The last section of the book is concerned with visual perception – pattern and shape recognition. In a moment’s reflection one can see the rich mathematical content of

this subject. It provides fertile ground for new ideas and structures. We particularly recommend the article on neural coding by S. Thorpe.

We want to point out that the articles in this volume are not of uniform quality and interest. Some are rather routine and will probably bore many readers. This is to be expected. However, many of the articles are quite intriguing. Furthermore, most of the topics touched upon are covered in sufficient depth to give one a serious introduction to the area. Extensive bibliographies have also been carefully compiled.

Mathematicians should find this book a fascinating introduction as well as a useful source-book. At very least it opens a window on the emerging world of biological mathematics and all its possibilities. Many future mathematical models, methods and modes of thought, even techniques of computing will have roots in the exploding world of biological science.

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Reading the Principia. The debate on Newton's mathematical methods for Natural Philosophy from 1687 to 1736

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Cambridge University Press, Cambridge, 1999. 285 p. \$ 80.00. ISBN 0-521-64066-0

Les *Philosophiae Naturalis Principia Mathematica* de Newton sont un livre illustre mais difficile. Difficile surtout en raison de ces « principes mathématiques » beaucoup moins bien connus que la « philosophie naturelle » qu'ils sont censés sous-tendre. Ce sont ces principes mathématiques que N. Guicciardini se propose d'éclairer, en confrontant les lectures qu'en ont faites les contemporains et en démêlant les débats passionnés qu'ils ont provoqués jusqu'à Euler.

Les *Principia* sont un ouvrage de maturité de Newton (1687), bien postérieur aux « anni mirabiles » (1664-70) qui virent éclore sa « méthode analytique des fluxions » (méthode qui resta largement confidentielle). Entre-temps, Newton avait pris ses distances avec ses théories de jeunesse : il lut les Anciens, se convainquit de la supériorité de leur science, en vint même à considérer son œuvre propre comme la redécouverte d'un savoir perdu. Suivant cette évolution philosophique, il délaissa dans ses publications sa méthode analytique au profit d'une « méthode synthétique des fluxions » rappelant Archimède, et s'attacha à donner à la multiplicité des méthodes mathématiques mises en œuvre dans ses *Principia* la façade géométrique unie des traités d'Appolonius et Pappus.

La première partie de *Reading the Principia* est consacrée aux méthodes mathématiques de Newton : exposé de ses théories des séries, fluxions et fluentes, et de leurs avatars géométriques, suivi d'une analyse approfondie des méthodes des *Principia*, au cours de laquelle le lecteur mathématicien moderne, guidé de main de maître, apprend véritablement à lire et apprécier les arguments de Newton dans le texte. C'est fascinant.

Reading the Principia nous relate ensuite la façon dont les démonstrations du grand-œuvre de Newton furent lues et reçues.

L'auteur nous présente trois lecteurs. D'abord Newton lui-même : ses manœuvres compliquées lors de la querelle de priorité entre newtoniens et leibniziens-bernoulliens sur l'invention du « nouveau calcul » — Newton prétendant que les résultats des *Principia* avaient été établis au moyen de « la nouvelle analyse » avant d'être publiés, suivant l'antique tradition, sous forme « synthétique ». Puis Huygens, dont le célèbre *Horologium oscillatorium* avait impressionné Newton : il admira la virtuosité géométrique de Newton tout en critiquant ses écarts de la théorie des proportions d'Eudoxe (et en contestant ses prémisses physiques). Enfin le grand rival, Leibniz. Comme pour