VIOLATIONS OF THE EQUIVALENCE PRINCIPLE IN A DILATON-RUNAWAY SCENARIO
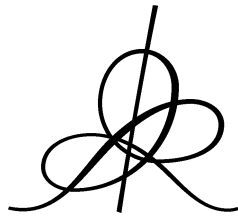
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Violations of the equivalence principle in a dilaton-runaway scenario

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Abstract

We explore a version of the cosmological dilaton-fixing and decoupling mechanism in which the dilaton-dependence of the low-energy effective action is extremized for infinitely large values of the bare string coupling $g_s^2 = e^\phi$. We study the efficiency with which the dilaton ϕ runs away towards its "fixed point" at infinity during a primordial inflationary stage, and thereby approximately decouples from matter. The residual dilaton couplings are found to be related to the amplitude of the density fluctuations generated during inflation. For the simplest inflationary potential, $V(\chi) = \frac{1}{2} m_\chi^2(\phi) \chi^2$, the residual dilaton couplings are shown to predict violations of the universality of gravitational acceleration near the $\Delta a/a \sim 10^{-12}$ level. This suggests that a modest improvement in the precision of equivalence principle tests might be able to detect the effect of such a runaway dilaton. Under some assumptions about the coupling of the dilaton to dark matter and/or dark energy, the expected time-variation of natural "constants" (in particular of the fine-structure constant) might also be large enough to be within reach of improved experimental or observational data.

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I. INTRODUCTION

All string theory models predict the existence of a scalar partner of the spin 2 graviton: the dilaton ϕ, whose vacuum expectation value (VEV) determines the string coupling constant $g_s = e^{\phi/2}$ [1]. At tree level, the dilaton is massless and has gravitational-strength couplings to matter which violate the equivalence principle [2]. This is in violent conflict with present experimental tests of general relativity. It is generally assumed that this conflict is avoided because, after supersymmetry breaking, the dilaton might acquire a (large enough) mass (say $m_\phi \gtrsim 10^{-3}$ eV so that observable deviations from Einstein's gravity are quenched on distances larger than a fraction of a millimeter). However, Ref. [3] (see also [4]) has proposed a mechanism which can naturally reconcile a *massless* dilaton with existing experimental data. The basic idea of Ref. [3] was to exploit the string-loop modifications of the (four dimensional) effective low-energy action (we use the signature $-+++$)

$$S = \int d^4x \sqrt{\widetilde{g}} \left(\frac{B_g(\phi)}{\alpha'} \widetilde{R} + \frac{B_\phi(\phi)}{\alpha'} [2\widetilde{\Box}\phi - (\widetilde{\nabla}\phi)^2] - \frac{1}{4} B_F(\phi) \widetilde{F}^2 - V + \cdots \right), \qquad (1.1)$$

i.e. the ϕ-dependence of the various coefficients $B_i(\phi)$, $i = g, \phi, F, \ldots$, given in the weak-coupling region ($e^\phi \to 0$) by series of the form

$$B_i(\phi) = e^{-\phi} + c_0^{(i)} + c_1^{(i)} e^\phi + c_2^{(i)} e^{2\phi} + \cdots, \qquad (1.2)$$

coming from genus expansion of string theory: $B_i = \Sigma_n g_s^{2(n-1)} c_n^{(i)}$, with $n = 0, 1, 2, \ldots$. It was shown in [3] that, if there exists a special value ϕ_m of ϕ which extremizes all the (relevant) coupling functions $B_i^{-1}(\phi)$, the cosmological evolution of the graviton-dilaton-matter system naturally drives ϕ towards ϕ_m. This provides a mechanism for fixing a massless dilaton at a value where it decouples from matter ("Least Coupling Principle"). A simple situation where the existence of a universally extremizing dilaton value ϕ_m is guaranteed is that of S duality, i.e. a symmetry $g_s \leftrightarrow 1/g_s$, or $\phi \to -\phi$ (so that $\phi_m = 0$).

It has been recently suggested [5] that the infinite-bare-coupling limit $g_s \to \infty$ ($\phi \to +\infty$) might yield smooth *finite* limits for all the coupling functions, namely

$$B_i(\phi) = C_i + \mathcal{O}(e^{-\phi}). \qquad (1.3)$$

Under this assumption, the coupling functions are all extremized at infinity, i.e. $\phi_m = +\infty$. The late-time cosmology of models satisfying (1.3) has been recently explored [6]. In the "large N"-type toy model of [5] it would be natural to expect that the $\mathcal{O}(e^{-\phi})$ term in Eq. (1.3) be *positive*, so that $B_i(\phi)$ be *minimized* at infinity. This would correspond to

couplings $\lambda_i(\phi) \sim B_i^{-1}(\phi) = C_i^{-1} - \mathcal{O}(e^{-\phi})$ which are *maximized* at infinity. Note, however, that the most relevant cosmological coupling for this work, the coupling to the inflaton, $\lambda(\phi)$, contained in V (see Eq. (2.11) below) is closer to a B_i than to its inverse. Thus $\lambda(\phi)$ is naturally *minimized* at infinity (see further discussion of this point below), a crucial property for the attractor mechanism of [3,4].

In this paper* we shall consider in detail the early-time cosmology of models satisfying (1.3). More precisely, our main aims will be: (i) to study the efficiency with which a primordial inflationary stage drives ϕ towards the "fixed point" at infinity $\phi_m = +\infty$ (thereby generalizing the work [8] which considered the inflationary attraction towards a local extremum ϕ_m), and (ii) to give quantitative estimates of the present violations of the equivalence principle (non universality of free fall, and variation of "constants"). Our most important conclusion is that the runaway of the dilaton towards strong-coupling (under the assumption (1.3)) naturally leads to equivalence-principle violations which are rather large, in the sense of not being much smaller than the presently tested level $\sim 10^{-12}$. This gives additional motivation for the currently planned improved tests of the universality of free fall. Within our scenario, most of the other deviations from general relativity ("post-Einsteinian" effects in gravitationally interacting systems: solar system, binary pulsars, ...) are too small to be of phenomenological interest. However, under some assumptions about the coupling of ϕ to dark matter and/or dark energy, the time variation of the natural "constants" (notably the fine-structure constant) predicted by our scenario might be large enough to be within reach of improved experimental and/or observational data. The phenomenologically interesting conclusion that equivalence-principle violations are generically predicted to be rather large after inflation (in sharp contrast with the results of [8]) is due to the fact that the attraction towards an extremum at infinity is much less effective than the attraction towards a (finite) local extremum as originally contemplated in [3]. This reduced effectiveness was already pointed out in Ref. [4] within the context of equivalence-principle-respecting tensor-scalar theories (à la Jordan-Fierz-Brans-Dicke).

II. DILATON RUNAWAY

In this section we study the dilaton's runaway during the various stages of cosmological evolution. We first show (subsection II A) that, like in the case of a local extremum [8], inflation is particularly efficient in pushing ϕ towards the fixed point. We will then argue

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*The main results of this work have been recently summarized in a short note [7].

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(subsection II B) that the order of magnitude of the bare string coupling $e^{\phi} \simeq e^{c\varphi}$ does not suffer further appreciable changes during all the subsequent evolution.

A. The inflationary period

Assuming some primordial inflationary stage driven by the potential energy of an inflaton field $\tilde{\chi}$, and taking into account generic couplings to the dilaton ϕ, we consider an effective action of the form

$$S = \int d^4 x \sqrt{\tilde{g}} \left(\frac{B_g(\phi)}{\alpha'} \tilde{R} + \frac{B_\phi(\phi)}{\alpha'} [2\tilde{\Box}\phi - (\tilde{\nabla}\phi)^2] - \frac{1}{2} B_\chi(\phi)(\tilde{\nabla}\tilde{\chi})^2 - \tilde{V}(\tilde{\chi}, \phi) \right) . \qquad (2.1)$$

In this string-frame action, the dilaton dependence of all the functions $B_i(\phi)$, $\tilde{V}(\tilde{\chi}, \phi)$ is assumed to be of the form (1.2). It is convenient to replace the (σ-model) string metric $\tilde{g}_{\mu\nu}$ by the conformally related Einstein metric $g_{\mu\nu} = C B_g(\phi) \tilde{g}_{\mu\nu}$, and the dilaton field by the variable

$$\varphi = \int d\phi \left[\frac{3}{4} \left(\frac{B'_g}{B_g} \right)^2 + \frac{B'_\phi}{B_g} + \frac{1}{2} \frac{B_\phi}{B_g} \right]^{\frac{1}{2}} , \ B' \equiv \partial B/\partial \phi . \qquad (2.2)$$

The normalization constant C is chosen so that the string units coincide with the Einstein units when $\phi \to +\infty$: $C B_g(+\infty) = 1$. [Note that $C = 1/C_g$ in terms of the general notation of Eq. (1.3).] Introducing the (modified) Planck mass

$$\widetilde{m}_P^2 = \frac{1}{4\pi G} = \frac{4}{C\alpha'} , \qquad (2.3)$$

and replacing also the inflaton by the dimensionless variable $\chi = C^{-1/2} \widetilde{m}_P^{-1} \tilde{\chi}$, we end up with an action of the form

$$S = \int d^4 x \sqrt{g} \left[\frac{\widetilde{m}_P^2}{4} R - \frac{\widetilde{m}_P^2}{2} (\nabla\varphi)^2 - \frac{\widetilde{m}_P^2}{2} F(\varphi)(\nabla\chi)^2 - \widetilde{m}_P^4 V(\chi, \varphi) \right] , \qquad (2.4)$$

where

$$F(\varphi) = B_\chi(\phi)/B_g(\phi) , \ V(\chi, \varphi) = C^{-2} \widetilde{m}_P^{-4} B_g^{-2}(\phi) \tilde{V}(\tilde{\chi}, \phi) . \qquad (2.5)$$

In view of our basic assumption (1.3), note that, in the strong-coupling limit $\phi \to +\infty$, $d\varphi/d\phi$ tends, according to Eq. (2.2), to the constant $(C_\phi/2C_g)^{1/2}$, while the dilaton-dependent factor $F(\varphi)$ in front of the inflaton kinetic term tends to the constant C_χ/C_g. The toy model of

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Ref. [5] suggests that the various (positive) constants C_i in Eq. (1.3) are all largish and comparable to each other. We shall therefore assume that the various ratios C_i/C_j are of order unity. The most important such ratio for the following is $c \equiv (2C_g/C_\phi)^{1/2}$ which gives the asymptotic behaviour of the bare string coupling as

$$g_s^2 = e^\phi \simeq e^{c\varphi} \, . \tag{2.6}$$

In view of the fact that, in the strong-coupling limit we are interested in, the factor $F(\varphi)$ in Eq. (2.4) quickly tends to a constant, we can simplify our analysis (without modifying the essential physics) by replacing it by a constant (which can then be absorbed in a redefinition of χ). Henceforth, we shall simply take $F(\varphi) = 1$. [See, however, the comments below concerning the self-regenerating inflationary regime.]

Following [4,9] it is then useful to combine the Friedmann equations for the scale factor $a(t)$ during inflation ($ds^2 = -dt^2 + a^2(t)\, \delta_{ij}\, dx^i\, dx^j$) with the equations of motion of the two scalar fields $\chi(t)$, $\varphi(t)$, to write an autonomous equation describing the evolution of the two scalars in terms of the parameter

$$p = \int H\,dt = \int \frac{\dot{a}}{a}\,dt = \ln a + \text{const} \tag{2.7}$$

measuring the number of e-folds of the expansion. For any multiplet of scalar fields, $\boldsymbol{\varphi} = (\varphi^a)$, this yields the simple equation [4,9]

$$\frac{2}{3 - \varphi'^2}\,\boldsymbol{\varphi}'' + 2\,\boldsymbol{\varphi}' = -\nabla_{\boldsymbol{\varphi}} \ln |V(\boldsymbol{\varphi})| \, , \tag{2.8}$$

where $\boldsymbol{\varphi}' \equiv d\boldsymbol{\varphi}/dp$, and where all operations on $\boldsymbol{\varphi}$ are covariantly defined in terms of the σ-model metric defining the scalar kinetic terms ($d\sigma^2 = \gamma_{ab}(\varphi)\, d\varphi^a\, d\varphi^b$). In our simple model (with $F(\varphi) = 1$), we have a flat metric $d\sigma^2 = d\varphi^2 + d\chi^2$. [Note that, when $\gamma_{ab}(\varphi)$ is curved the acceleration term $\boldsymbol{\varphi}''$ involves a covariant derivative.]

The generic solution of Eq. (2.8) is easily grasped if one interprets it as a mechanical model: a particle with position $\boldsymbol{\varphi}$, and velocity-dependent mass $m(\boldsymbol{\varphi}') = 2/(3 - \varphi'^2)$, moves, in the "time" $p = \ln a + \text{cst}$, in the manifold $d\sigma^2$ under the influence of an external potential $\ln |V(\boldsymbol{\varphi})|$ and a constant friction force $-2\,\boldsymbol{\varphi}'$. If the curvature of the effective potential $\ln |V(\boldsymbol{\varphi})|$ is sufficiently small the motion of $\boldsymbol{\varphi}$ rapidly becomes slow and friction-dominated:

$$2\,\frac{d\boldsymbol{\varphi}}{dp} \simeq -\nabla_{\boldsymbol{\varphi}} \ln V(\boldsymbol{\varphi}) \, . \tag{2.9}$$

Eq. (2.9) is equivalent to the usual "slow roll" approximation.

Consistently with our general assumption (1.3), we consider potentials allowing a strong-coupling expansion of the form:

$$V(\chi,\varphi) = V_0(\chi) + V_1(\chi)e^{-c\varphi} + O(e^{-2c\varphi}),\qquad(2.10)$$

where $V_0(\chi)$ is a typical chaotic-inflation potential with $V_0(0) = 0$, while $V_1(0) = v_1 \geq 0$ can possibly provide (if $v_1 > 0$) the effective cosmological constant driving today's acceleration in the scenario of [6]. For the sake of simplicity we shall discuss mainly the "factorized" power-law case $V_0(\chi) \sim V_1(\chi) \sim \chi^n$ for which we can conveniently write V in the form:

$$V(\chi,\varphi) = \lambda(\varphi)\,\frac{\chi^n}{n},\qquad(2.11)$$

with a dilaton-dependent coupling constant $\lambda(\varphi)$ of the form

$$\lambda(\varphi) = \lambda_\infty(1 + b_\lambda\, e^{-c\varphi}).\qquad(2.12)$$

This example belongs to the class of the two-field inflationary potentials discussed in [10]. We have checked that our results remain qualitatively the same for the more general potential (2.10) provided that $V_0(\chi)$ and $V_1(\chi)$ are not extremely diffent and given the fact that v_1 is phenomenologically constrained to be very small. [Note that, within the simplified model (2.12), the ratio $V_1(\chi)/V_0(\chi)$ is equal to the constant coefficient b_λ.]

The universal (positive) constant c appearing in the exponential $e^{-c\varphi}$ is the same as in Eq. (2.6) [i.e. $c \equiv (2C_g/C_\phi)^{1/2}$, which is expected to be of order unity]. The coefficient b_λ in (2.12) is such that $b_\lambda\, e^{-c\varphi} \simeq b_\lambda\, e^{-\phi}$ roughly corresponds to a combination of terms $\sim \pm C_i^{-1}\, \mathcal{O}(e^{-\phi})$ coming from the strong-coupling asymptotics of several $B_i(\phi)$, Eq. (1.3) (see Eq. (2.5)). In the toy model of [5] one would therefore expect b_λ to be smallish. Anyway, we shall see that in final results only the ratios of such b_i coefficients enter. More important than the magnitude of b_λ is its sign. It is crucial for the present strong-coupling attractor scenario to assume that $b_\lambda > 0$, i.e that $\lambda(\varphi)$ reaches a *minimum* at strong-coupling, $\varphi \to +\infty$. Note again that this behaviour is consistent with the simple "large N"-type idea of [5] if we assimilate $\lambda(\varphi)$ to one of the inverse couplings B_i appearing in (1.1) (for instance $B_F \sim g_F^{-2}$, where g_F is a gauge coupling), rather than to the coupling itself. If the latter were the case, $\lambda(\varphi)$ would reach a *maximum* as $\phi \to +\infty$, and the attractor mechanism of [3] would drive ϕ towards weak coupling ($\phi \to -\infty$). However, the Einstein-frame ϕ-dependence of $V(\chi)$ gets contributions from several $B_i^{\pm n}(\phi)$, Eq. (2.5), which might conspire to minimize it at strong coupling. This feature is also probably necessary in order to solve the cosmological-constant problem through some argument by which the vacuum at infinity has vanishing energy density.

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Substituting the potential (2.10) into the slow roll equation (2.9) and assuming (for simplicity) that $V_1(\chi)e^{-c\varphi}$ is significantly smaller than $V_0(\chi)$ leads to a decoupled set of evolution equations for χ and φ (where $V' \equiv \partial V/\partial\chi$):

$$\frac{d\chi}{dp} = -\frac{1}{2}\frac{V_0'}{V_0} \, , \tag{2.13}$$

$$\frac{d\varphi}{dp} = \frac{1}{2}\, c\, e^{-c\varphi}\frac{V_1}{V_0} \, . \tag{2.14}$$

Given some "initial" conditions $\chi_{\rm in}$, $\varphi_{\rm in}$ (discussed below) at some starting point, say $p = 0$, the solution of Eqs. (2.13), (2.14) is simply

$$p = 2\int_\chi^{\chi_{\rm in}} d\bar{\chi}\,\bar{\chi}\left(\frac{V_0(\bar{\chi})}{\bar{\chi}\,V_0'(\bar{\chi})}\right) \, , \tag{2.15}$$

$$e^{c\varphi} = e^{c\varphi_{\rm in}} + \frac{c^2}{2}\int dp\,\frac{V_1(\chi(p))}{V_0(\chi(p))} \, , \tag{2.16}$$

which simply become:

$$p = \frac{1}{n}\left(\chi_{\rm in}^2 - \chi^2\right) \, , \quad e^{c\varphi} + \frac{b_\lambda\,c^2}{2n}\,\chi^2 = {\rm const.} = e^{c\varphi_{\rm in}} + \frac{b_\lambda\,c^2}{2n}\,\chi_{\rm in}^2 \, , \tag{2.17}$$

in the simplified case of eqs. (2.11), (2.12).

Equations (2.17) show that, in order for the string coupling $g_s^2 \simeq e^{c\varphi}$ to have reached large values at the end of inflation, a large total number of e-folds must have occurred while the (dimensionless) inflaton field χ decreases from a large initial value, to a value of order unity (in Planck units). To get a quantitative estimate of the string coupling at the end of inflation we need to choose the initial conditions $\chi_{\rm in}$, $\varphi_{\rm in}$. A physically reasonable way (which is further discussed below) of choosing $\chi_{\rm in}$ is to start the classical evolution (2.13)-(2.17) at the exit of the era of self-regenerating inflation (see [10] and references therein). We will now show how to relate the exit from self-regenerating inflation to the size of density fluctuations generated by inflation.

Let us recall (see [10] and references therein) that the density fluctuation $\delta \equiv \delta\rho/\rho$ on large scales (estimated in the one-field approximation where the inflaton χ is the main contributor) is obtained by evaluating the expression

$$\delta(\chi) \simeq \frac{4}{3}\frac{1}{\pi}\left(\frac{2}{3}\right)^{\frac{1}{2}}\frac{V^{3/2}}{\partial_\chi V} \tag{2.18}$$

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at the value $\chi = \chi_\times$, at which the physical scale we are considering crossed the horizon outwards during inflation. For the scale corresponding to our present horizon this usually corresponds to a value $\chi_\times(H_0)$ (χ_H for short) reached some 60 e-folds before the end of slow-roll. From Ref. [10], $\chi_H \simeq 5\sqrt{n}$ for the model (2.11) (and with our modified definition of χ). The numerical value of $\delta_H \equiv \delta(\chi_H)$ which is compatible with cosmological data (structure formation and cosmic microwave background) is $\delta_H \simeq 5 \times 10^{-5}$. In the model (2.11) the function $\delta(\chi)$ defined by (2.18) scales with χ as $\chi^{\frac{n+2}{2}}$. Putting together this information we obtain a relation between $\chi_{\rm in}$ and $\delta(\chi_{\rm in})$, which involves the value of the observable horizon-size fluctuations $\delta_H \equiv \delta(\chi_H)$:

$$\frac{\delta(\chi_{\rm in})}{\delta(\chi_H)} = \left(\frac{\chi_{\rm in}}{\chi_H}\right)^{\frac{n+2}{2}} , \qquad (2.19)$$

i.e.

$$\chi_{\rm in} \simeq \chi_H \left(\frac{\delta_{\rm in}}{\delta_H}\right)^{\frac{2}{n+2}} \simeq 5\sqrt{n} \left(\frac{\delta_{\rm in}}{\delta_H}\right)^{\frac{2}{n+2}} , \qquad (2.20)$$

where we introduced the short-hand notation $\delta_{\rm in} \equiv \delta(\chi_{\rm in})$.

Inserting Eq. (2.20) into Eq. (2.16) we then obtain the following estimate of the string coupling constant after inflation as a function of $\varphi_{\rm in}$ and $\delta(\chi_{\rm in})$

$$e^{c\varphi_{\rm end}} - e^{c\varphi_{\rm in}} \simeq \frac{c^2}{2}\langle V_1/V_0\rangle p \sim \frac{c^2}{2n}\langle V_1/V_0\rangle \chi_{\rm in}^2 \sim \frac{25c^2}{2}\langle V_1/V_0\rangle \left(\frac{\delta_{\rm in}}{\delta_H}\right)^{\frac{4}{n+2}} , \qquad (2.21)$$

where $\langle V_1/V_0\rangle$ denotes the average value of V_1/V_0: $\langle V_1/V_0\rangle \equiv \int dp(V_1/V_0)/\int dp$ [note that this average ratio is equal to b_λ in the simplified model (2.12)].

To get a quantitative estimate of $e^{c\varphi_{\rm end}}$ we still need to estimate the value of $\delta(\chi_{\rm in})$ corresponding to the chosen "initial" value of the inflaton. As we will now check, taking for $\chi_{\rm in}$ the value corresponding to the exit from self-regenerating inflation corresponds simply to taking $\delta(\chi_{\rm in}) \sim 1$. Indeed, let us first recall that, during inflation, each (canonically normalized) scalar field (of mass smaller than the expansion rate H) undergoes typical quantum fluctuations of order $H/(2\pi)$, per Hubble time (see, e.g., [10]). This implies (for our dimensionless fields) that the value of χ at the exit from self-regeneration, say $\chi_{\rm ex}$, is characterized by $\widehat{H}_{\rm ex}/(2\pi) \approx [\partial_\chi V/(2V)]_{\rm ex}$, where $\widehat{H} \equiv H/\widetilde{m}_P$ is the dimensionless Hubble expansion rate and where the right-hand side (RHS) is the classical change of χ per Hubble time (corresponding to the RHS of Eq.(2.13)). Using Friedmann's equation (in the slow-roll approximation) $\widehat{H}_{\rm ex}^2 \approx (2/3)V(\chi_{\rm ex})$, it is easily seen that that the exit from self-regeneration corresponds to $\delta(\chi_{\rm ex}) \approx 4/3 \sim 1$. It is, a posteriori, physically quite reasonable to start using

the classical evolution system only when the (formal extrapolation) of the density fluctuation $\delta(\chi)$ becomes smaller than one.

Within some approximation (see [10]), one can implement the effect of the combined quantum fluctuations of (φ, χ) by adding random terms with r.m.s. values $\widehat{H}/2\pi$ on the right hand side of equations (2.13) and (2.14), $d\chi/dp$ and $d\varphi/dp$ being precisely the shifts of the fields in a Hubble time. The system of equations becomes thus of the Langevin-type

$$\frac{d\chi}{dp} = -\frac{1}{2} \frac{V_0'}{V_0} + \frac{\widehat{H}}{2\pi} \xi_1 , \tag{2.22}$$

$$\frac{d\varphi}{dp} = \frac{1}{2} c\, e^{-c\varphi} \frac{V_1}{V_0} + \frac{\widehat{H}}{2\pi} \xi_2 , \tag{2.23}$$

where $\widehat{H} \approx [(2/3)V(\chi, \phi)]^{1/2}$ (in the slow-roll approximation) is the dimensionless expansion rate, and where ξ_1 and ξ_2 are (independent) normalized random white noises:

$$\langle \xi_i(p_1)\, \xi_j(p_2) \rangle = \delta_{ij} \delta(p_1 - p_2), \qquad i, j = 1, 2 . \tag{2.24}$$

When the random force terms dominate the evolution in either equation (2.22) or equation (2.23) the quasi-classical description (2.13) , (2.14) breaks down. The phase space of the sytem can thus be roughly divided into four regions according to whether the evolution of none, one or both of the two fields is dominated by quantum fluctuations. This is depicted in Figure 1 where such regions are delimited by dashed, thick curves in the case of a power-law potential (2.11).

Apart from factors of order one, the evolution of the inflaton χ is quasi-classical in the region under the line $\chi = \lambda_\infty^{-1/(n+2)}$. In the chaotic inflationary models [10] such an inflaton's value corresponds to the exit from the self-regenerating regime and to the beginning of the quasi-classical slow-roll inflation. As mentioned above it also corresponds to a perturbation $\delta(\chi) \sim 1$. Somewhat surprisingly, in the model at hand, however, the quasi-classical region for the inflaton evolution $\chi \lesssim \lambda_\infty^{-1/(n+2)}$ is affected by quantum fluctuations that still dominate the evolution of φ in the region above the hyperbola-like curve $\chi = b_\lambda^{2/n} \lambda_\infty^{-1/n} e^{-2c\varphi/n}$. We must therefore study, in some detail, the evolution of the system in the presence of the noise term for φ as in eq. (2.23), assuming a quasi classical evolution for χ. This is done in the appendix. The final result is that, if we start at $\chi \lesssim \lambda_\infty^{-1/(n+2)}$, (classical evolution for χ) the average value of $e^{c\varphi}$ is *multiplicatively renormalized*, by a factor of order unity, with respect to the classical trajectory $e^{c\varphi_{cl}}$, given by solving equations (2.13) - (2.14), i.e. $\langle e^{c\varphi} \rangle = \mathcal{O}(1) e^{c\varphi_{cl}}$. One also finds that the dispersion of $e^{c\varphi}$ around its average value is comparable to its average value.
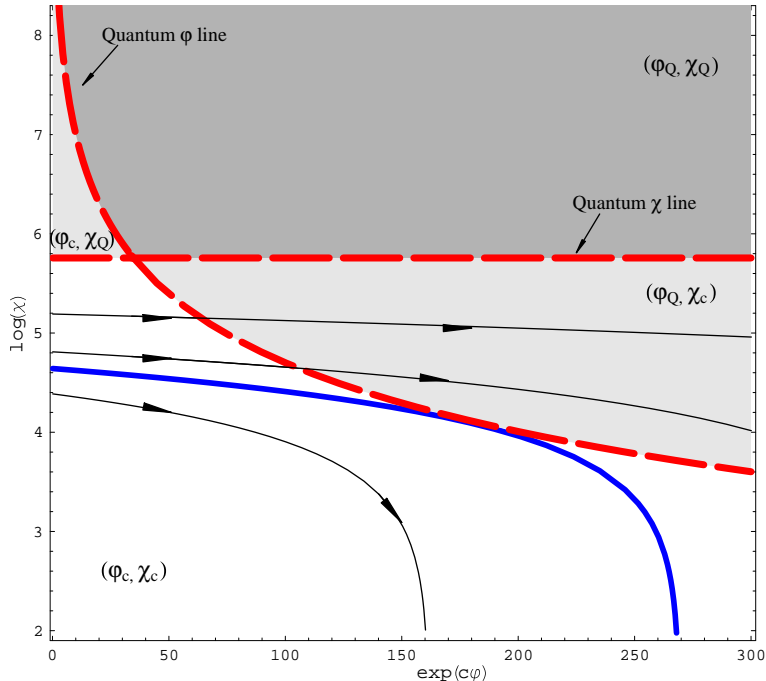
FIG. 1. *The phase space of the system is represented in the case of a power–law potential (2.11) with $n = 2$, $b_\lambda = 0.1$ and $\lambda_\infty = 10^{-10}$. The thick–dashed (red) curves delimitate the quantum behaviour of the two fields, the horizontal curve $\chi = \lambda_\infty^{-1/(n+2)}$ and the hyperbola-like curve $\chi = b_\lambda^{2/n} \lambda_\infty^{-1/n} e^{-2c\varphi/n}$ being the limit of the quantum behaviour for χ and φ respectively. In the white region both fields have a classical behaviour. The last "fully classical" trajectory has been represented by a thick (blue) curve. The bright–gray regions are those where either the φ or the χ evolution are dominated by quantum fluctuations. The fully–quantum region is the dark–gray region on the top right.*

We shall not try to discuss here what happens in the self-regenerating region $\chi \gtrsim \chi_{\rm in} \sim \lambda_\infty^{-1/(n+2)}$. Let us recall that the simple decoupled system (2.13) , (2.14) was obtained by neglecting the kinetic coupling term $F(\varphi)$ in Eq. (2.4). If we were to consider a more general model, we would have more coupling between χ and φ and we would expect that (contrary to Fig. 1 which exhibits a "classical φ region" above the "quantum χ line") the evolution in the self-regenerating region involves a strongly coupled system of Langevin equations. Then, as discussed in [10], solving such a system necessitates to give boundary conditions on all the boundaries of the problem: notably for $\chi \to \infty$, but also for $\varphi \to +\infty$ and $\varphi \to -\infty$. We leave to future work such an investigation (and a discussion of what are reasonable boundary conditions). In this work we shall content ourselves with "starting" the evolution on the quantum χ boundary line $\chi_{\rm in}$ with some value $\varphi = \varphi_{\rm in}$, assuming that $e^{c\varphi_{\rm in}}$ is smaller

than the driving effect due to inflation, i.e. than the RHS of Eq. (2.21). [This assumption is most natural in a work aimed at studying the "attracting" effect due to primordial inflation.]

Going back to our result (2.21), we can now insert, according to the preceding discussion, the values $\delta_{\text{in}} = 1$ and $e^{c\varphi_{\text{in}}} \ll e^{c\varphi_{\text{end}}}$. Finally, in the simplified model (2.11), (2.12), we get the estimate:

$$e^{c\varphi_{\text{end}}} = \mathcal{O}(1) \cdot e^{c\varphi_{\text{cl, end}}} \sim \mathcal{O}(1) \cdot \frac{25c^2}{2} \, b_\lambda \, (\delta_H)^{-\frac{4}{n+2}} \ . \tag{2.25}$$

A more general analysis, based on the potential (2.10) leads to the same final result but with n replaced by some average value of $\frac{\chi V_{0,\chi}}{V_0}$, and with b_λ replaced by some average of the ratio V_1/V_0. Note that smaller values of the exponent n lead to larger values of $e^{c\varphi_{\text{end}}}$, i.e. to a more effective attraction towards the "fixed point at infinity". The same is true if we take different exponents n_0 and n_1 (for V_0 and V_1 respectively) and assume $V_1(\chi_{\text{in}}) \gg V_0(\chi_{\text{in}})$ to hold as a result of $\chi_{\text{in}} \gg 1$ and $n_1 > n_0$. Also note that, numerically, if we consider $n = 2$, i.e. the simplest chaotic-inflation potential $V = \frac{1}{2} m_\chi^2(\varphi) \chi^2$, Eq. (2.25) involves the large number $12.5 \times \delta_H^{-1} \sim 2.5 \times 10^5$. In the case where $n = 4$, i.e. $V = \frac{1}{4} \lambda(\varphi) \chi^4$, we have instead the number $12.5 \times \delta_H^{-2/3} \sim 0.92 \times 10^4$. To understand the phenomenological meaning of these numbers we need to relate $e^{c\varphi_{\text{end}}}$ to the present, observable deviations from general relativity. This issue is addressed in Section III after having argued that the post-inflationary evolution of φ is sub-dominant.

B. Attraction of φ by the subsequent cosmological evolution

We have discussed above the efficiency with which inflation drives the dilaton towards a fixed point at infinity. We need to complete this discussion by estimating the effect of the many e-folds of expansion that took place between the end of inflation and the present time. To address this question, we need to study in more detail the coupling of a runaway dilaton to various types of matter, say a multi-component distribution of (relativistic or non-relativistic) particles. Following [2,3], one introduces the crucial dimensionless quantity

$$\alpha_A(\varphi) \equiv \frac{\partial \ln m_A(\varphi)}{\partial \varphi} \ , \tag{2.26}$$

measuring the coupling of φ to a particle of type A. [For consistency with previous work, we keep the notation α_A but warn the reader that this should not be confused with the various gauge coupling constants, often denoted $\alpha_i = g_i^2/4\pi$.] The quantity α_A determines the effect of cosmological matter on the evolution of φ through the general equation [4,3]

$$\frac{2}{3 - \varphi'^2} \varphi'' + \left(1 - \frac{P}{\rho}\right) \varphi' = -\sum_A \alpha_A(\varphi) \frac{\rho_A - 3P_A}{\rho}, \qquad (2.27)$$

where the primes denote derivatives with respect to $p = \ln a + \text{const}$ and where $\rho = \Sigma_A \rho_A$ and $P = \Sigma_A P_A$ are the total "material" energy density and pressure respectively, both obtained as sums over the various components filling the universe *at the exception of* the kinetic energy density and pressure of φ, $\rho_k = (\widetilde{m}_P^2/2)(d\varphi/dt)^2 = (\widetilde{m}_P^2/2)H^2\varphi'^2$ and $P_k = \rho_k$. Accordingly, the Friedmann equation reads

$$3H^2 = \frac{2}{\widetilde{m}_P^2} \rho_{\text{tot}} = \frac{2\rho}{\widetilde{m}_P^2} + H^2\varphi'^2. \qquad (2.28)$$

Note that ρ and P may also account for the potential energy density and pressure of the scalar field, $\rho_V = V(\varphi)$, $P_V = -\rho_V$ and that one can formally extend Eq. (2.27) to the "vacuum energy" component $V(\varphi)$ by associating to the potential $V(\varphi)$ the mass scale $m_V(\varphi) \equiv V(\varphi)^{\frac{1}{4}}$ which gives $\alpha_V = \partial \ln m_V(\varphi)/\partial\varphi = \frac{1}{4} \partial \ln V(\varphi)/\partial\varphi$. Equation (2.8) is then recovered in the limit where the scalar field is the dominant component.

In the simple cases (which are quite frequent; at least as approximate cases) where one "matter" component, with known "equation of state" $P_A/\rho_A = w_A = \text{const.}$, dominates the cosmological density and pressure, Eq. (2.27) yields an autonomous equation for the evolution (with redshift) of φ. Using (2.28) one finds that the "equation of state parameter" $w_{\text{tot}} \equiv P_{\text{tot}}/\rho_{\text{tot}}$ corresponding to the *total* energy and pressure (including now the kinetic contributions of φ; i.e. $\rho_{\text{tot}} = \rho + (\widetilde{m}_P^2/2)(d\varphi/dt)^2$, $P_{\text{tot}} = P + (\widetilde{m}_P^2/2)(d\varphi/dt)^2$) is given in terms of the "matter" equation-of-state parameter $w \equiv P/\rho$ by

$$w_{\text{tot}} = w + \frac{1 - w}{3}(\varphi')^2. \qquad (2.29)$$

The knowledge of w_{tot} then allows one to write explicitly the energy-balance equation $d\rho_{\text{tot}} + 3(\rho_{\text{tot}} + P_{\text{tot}})d\ln a = 0$, which is easily solved in the simple cases where w_{tot} is (approximately) constant.

We see from Eq. (2.27) that, during the radiation era (starting, say, immediately after the end of inflation), i.e. when the universe is dominated by an ultra-relativistic gas ($\rho_A - 3P_A = 0$), the "driving force" on the right-hand side of (2.27) vanishes, so that φ is not driven further away towards infinity. Actually, one should take into account both the "inertial" effect of the "velocity" φ' acquired during the preceding inflationary driving of φ, and the integrated effect of the many "mass thresholds", $T_A \sim m_A$, when some component becomes non-relativistic (so that $\rho_A - 3P_A \neq 0$). Using the results of [4,3] one sees that, in our case, both these effects have only a small impact on the value of φ. Therefore, to a good approximation $\varphi \simeq \varphi_{\text{end}}$ until the end of radiation era.

On the other hand, when the universe gets dominated by non-relativistic matter, one gets a non-zero driving force in Eq. (2.27). In the slow roll approximation, as the transient behaviour has died out, since $w = P/\rho$ gets negligible, we have simply

$$\varphi'_m = -\alpha_m(\varphi),\qquad(2.30)$$

where φ'_m stands the φ-velocity during matter domination. and $\alpha_m(\varphi)$ denotes the coupling (2.26) to dark matter.

The coupling to dark matter, $\alpha_m(\varphi)$, depends on the assumption one makes about the asymptotic behaviour, at strong bare string coupling, of the mass of the WIMPs constituting the dark matter. One natural looking, minimal assumption is that dark matter, like all visible types of matter, is coupled in a way which levels off at strong-bare-coupling, as in Eq. (1.3). In other words, one generally expects that $m_m(\varphi) \simeq m_m(+\infty)(1 + b_m\, e^{-c\varphi})$ so that $\alpha_m(\varphi) \simeq -b_m\, c\, e^{-c\varphi}$. It is then easy to solve Eq. (2.30), with initial conditions $\varphi_0 = \varphi_{\text{end}}$, $\varphi'_0 = 0$ (inherited from radiation era) at the beginning of the matter era. But we shall not bother to write the explicit solution because it is easily seen that the smallness of $e^{-c\varphi_{\text{end}}}$ guarantees that the "driving force" $\propto \alpha_m(\varphi)$ remains always so small that the $\mathcal{O}(10)$ e-folds of matter era until vacuum-energy domination (or until the present) have only a fractionally negligible effect on φ.

A more significant evolution of φ during the matter era is provided if, as first proposed in [11], dark matter couples much more strongly to φ than "ordinary" matter. Such a stronger coupling to dark matter, which is not constrained by usual equivalence principle experiments, follows assuming more general quantum corrections in the dark matter sector of the theory, i.e. corrections such that the dark matter mass $m_m(\varphi)$, instead of levelling off, either vanishes or keeps increasing at strong bare coupling: $m_m(\varphi) \propto e^{c_m\varphi}$, so that $\alpha_m = c_m$ is a (negative or positive) constant. In [6] (but see also [12]) it has been shown that under the latter assumption (i.e. with a positive coupling parameter $\alpha_m > 0$) the dilaton can play the role of quintessence, leading to a late-time cosmology of accelerated expansion. By Eq. (2.30) we have $\varphi = \varphi_{\text{end}} - \alpha_m p$, where p is now counted from the end of the radiation era. Given that about nine e-folds separate us from the end of the radiation era, we see that such an evolution might (if $|\alpha_m|$ is really of order unity) have a significant effect on the present value of φ (when compared with the value at the end of inflation, i.e. $c\varphi_{\text{end}} \sim \ln(1/\delta_H) \sim 10$). However, the running of φ during the matter era changes the standard recent cosmological picture and is therefore constrained by observations. In fact, by equation (2.29), the total matter-era equation of state parameter w_{tot} in the presence of the dilaton reads $w_{\text{tot}} = (\varphi'_m)^2/3$. Accordingly, the matter density varies as $\rho \propto a^{-3(1+w_{\text{tot}})} = a^{-(3+(\varphi'_m)^2)}$, possibly affecting the standard scenario of structure formation as well as the global temporal picture between now

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and the epoch of matter-radiation equality. The compatibility with phenomenology therefore puts constraints on the magnitude of $\varphi_m'^2 = \alpha_m(\varphi)^2$. In [6] $w_{\text{tot}} < 0.1$, i.e. $v \equiv \varphi_m'^2/0.3 < 1$, was suggested to be the maximal deviation one can roughly tolerate during the matter era, the establishment of a more precise bound being presently under study [13]. We shall therefore assume either that we are in the "normal" case where the dilaton does not couple more strongly to dark matter than to ordinary matter (so that $\alpha_m(\varphi) \simeq -b_m\, c\, e^{-c\varphi} \ll 1$), or that, if it does, $\alpha_m^2 < 0.3$. This leads to a displacement of φ during matter era smaller than the dispersion $\varphi_{\text{end}} - \varphi_{\text{cl,end}} \sim \varphi_{\text{cl,end}}$ produced by quantum fluctuations during inflation, Eq. (2.25).

In this context one should also consider the attraction effect of a negative pressure component, either in the form of a φ-dependent vacuum energy (dilatonic quintessence) or in the form of any other, φ-independent component (such as a "genuine" cosmological constant). Of course, the present recent ($z \lesssim 1$) accelerated expansion phase is very short (in "p-time") and sensible changes of the dilaton value since the end of matter domination are not expected. Still, it is crucial to estimate the present dilaton velocity φ_0' since it is related to the cosmological variations of the coupling constants (see next section). In the general case where both non-relativistic matter and (possibly φ-dependent) vacuum energy density $V(\varphi)$ are present, the value of φ_0' predicted by our model is obtained by applying Eq. (2.27) (in the slow-roll approximation):

$$(\Omega_m + \Omega_V)(1 - w_0)\varphi_0' = (\Omega_m + 2\Omega_V)\varphi_0' = -\Omega_m \alpha_m - 4\Omega_V \alpha_V. \tag{2.31}$$

In the above expression Ω_m and Ω_V are, respectively, the non-relativistic (dark) matter- and the vacuum-fraction of critical energy density ($\rho_c \equiv (3/2)\widetilde{m}_P^2 H^2$), and the already mentioned prescriptions $\alpha_V = \frac{1}{4}\partial \ln V(\varphi)/\partial \varphi$, $P_V = -\rho_V = -V(\varphi)$ have been used.

The value of φ_0' is therefore some combination of the values of α_m and α_V. We can have two classes of contrasting situations: In the first class, the dilaton couples "normally" (i.e. weakly) both to dark matter and to dark energy, i.e. both $\alpha_m \simeq -b_m\, c\, e^{-c\varphi} \ll 1$ and $\alpha_V \ll 1$ and Eq. (2.31) implies $\varphi_0' \ll 1$. In the second class, the dilaton couples more strongly to some type of dark matter or energy, i.e. either (or both) α_m or/and α_V is of order unity so that $\varphi_0' = \mathcal{O}(1)$. The second case is realized in the scenario of [6]. In the context of this scenario we have an exponential dependence of the potential on φ, $V(\varphi) \simeq V_1 e^{-c\varphi}$ so that $\alpha_V \simeq -(c/4)$ and

$$\varphi_0' = \frac{c\,\Omega_V - \alpha_m \Omega_m}{2\Omega_V + \Omega_m} \lesssim \frac{c}{3}. \tag{2.32}$$

The last inequality follows from the bound $\alpha_m > c/2$ (which is a necessary condition to have positive acceleration in the model [6]) and the reasonable bound $\Omega_m > 0.25$.

In the present work, we wish, however, to be as independent as possible from specific assumptions (as the ones used in [6]). Therefore, rather than insisting on specific (model-dependent) predictions for the present value of φ_0' we wish to find the (model-independent) upper bounds on the possible values of φ_0' set by current observational data. There are several ways of getting such phenomenological bounds, because the existence of a kinetic energy (and pressure) associated to $d\varphi/dt = H\varphi'$ has several observable consequences. A rather secure bound can be obtained by relating the value of φ' to the deceleration parameter $q \equiv -\ddot{a}a/\dot{a}^2$. In the general class of models that we consider, the cosmological energy density and pressure have (currently) three significant contributions: dark matter ($\Omega_m = \rho_m/\rho_c$), dark energy (Ω_V) and the kinetic effect of a scalar field ($\Omega_k = \rho_k/\rho_c$ with $\rho_k = (\widetilde{m}_P^2/2)(d\varphi/dt)^2 = (\widetilde{m}_P^2/2)H^2\varphi'^2$ so that $\Omega_k = \varphi'^2/3$). We assume (consistently with recent cosmic background data) that the space curvature is zero. Therefore we have the first relation

$$\Omega_m + \Omega_V + \Omega_k = 1 = \Omega_m + \Omega_V + \varphi'^2/3 \,. \tag{2.33}$$

The deceleration parameter is given by the general expression $2q = \Sigma_A \Omega_A(1 + 3w_A)$. Using $w_m = 0$, $w_V = -1$ and $w_k = +1$, we get

$$2q = \Omega_m - 2\Omega_V + \frac{4}{3}\varphi'^2. \tag{2.34}$$

Using the relation (2.33) above to eliminate Ω_V we get the following expression for φ'^2 in terms of the observable quantities q and Ω_m

$$\varphi'^2 = 1 + q - \frac{3}{2}\Omega_m. \tag{2.35}$$

The supernovae Ia data [14] give as strict upper bound on the present value q_0 that it is *negative*. A generous lower bound on the present value of Ω_m is $\Omega_{m0} > 0.2$ [15]. Inserting these two constraints in Eq.(2.35) finally yields the safe upper bound

$$\varphi_0'^2 < 0.7 \,, \text{ i.e. } |\varphi_0'| < 0.84 \,. \tag{2.36}$$

To summarize, quite different rates of evolution for the dilaton are possible. A very slow variation is expected whenever dilaton couplings to both dark energy and dark matter follow the "normal" behaviour (1.3). Otherwise, dilaton variations on the Hubble scale are expected. However, cosmological observations set the strict upper bound (2.36) on the present time variation of φ. For the purpose of the present section (evaluating the current location of the dilaton) these two alternatives do not make much difference because the vacuum-dominance era has started less than about 0.7 e-folds away ($\ln(1 + z_*)$ with $z_* < 1$).

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Therefore, φ did not have enough "p-time", during vacuum dominance, to move much, even if it is coupled to vacuum energy with $\alpha_V \simeq -(c/4) \sim 1$.

Finally, we conclude from this analysis that, to a good approximation (and using the fact that the phenomenology of the matter-era constrains the dark-matter couplings of the dilaton to be rather small), the value of φ now is essentially given by the value φ_{end} at the end of inflation, i.e. by Eq. (2.21).

III. DEVIATIONS FROM GENERAL RELATIVITY INDUCED BY A RUNAWAY DILATON

A. Composition-independent deviations from general relativity

The previous section has reached the conclusion that present deviations from general relativity are given, to a good approximation, by the values of the matter-coupling coefficients $\alpha_A(\varphi)$ given by Eq. (2.26) calculated at $\varphi \simeq \varphi_{\text{end}}$ as given by Eq. (2.25). Let us now see the meaning of this result in terms of observable quantities.

Let us first consider the (approximately) composition-independent deviations from general relativity, i.e. those that do not essentially depend on violations of the equivalence principle. Most composition-independent gravitational experiments (in the solar system or in binary pulsars) consider the long-range interaction between objects whose masses are essentially baryonic (the Sun, planets, neutron stars). As argued in [2,3] the relevant coupling coefficient α_A is then approximately universal and given by the logarithmic derivative of the QCD confinement scale $\Lambda_{\text{QCD}}(\varphi)$, because the mass of hadrons is essentially given by a pure number times $\Lambda_{\text{QCD}}(\varphi)$. [We shall consider below the small, non-universal, corrections to $m_A(\varphi)$ and $\alpha_A(\varphi)$ linked to QED effects and quark masses.] Remembering from Eq. (1.1) the fact that, in the string frame (where there is a fixed cut-off linked to the string mass $\widetilde{M}_s \sim (\alpha')^{-1/2}$) the gauge coupling is dilaton-dependent ($g_F^{-2} = B_F(\varphi)$), we see that (after conformal transformation) the Einstein-frame confinement scale has a dilaton-dependence of the form

$$\Lambda_{\text{QCD}}(\varphi) \sim C^{-1/2} \, B_g^{-1/2}(\varphi) \exp[-8\pi^2 \, b_3^{-1} \, B_F(\varphi)] \, \widetilde{M}_s \,, \tag{3.1}$$

where b_3 denotes the one-loop (rational) coefficient entering the Renormalization Group running of g_F. Here $B_F(\varphi)$ denotes the coupling to the SU(3) gauge fields. For simplicity, we shall assume that (modulo rational coefficients) all gauge fields couple (near the string cut

off) to the same $B_F(\varphi)$. This yields the following approximately universal dilaton coupling to hadronic matter

$$\alpha_{\text{had}}(\varphi) \simeq \left(\ln \left(\frac{\widetilde{M}_s}{\Lambda_{\text{QCD}}} \right) + \frac{1}{2} \right) \frac{\partial \ln B_F^{-1}(\varphi)}{\partial \varphi} \, . \tag{3.2}$$

Numerically, the coefficient in front of the R.H.S. of (3.2) is of order 40. Consistently with our basic assumption (1.3), we parametrize the φ dependence of the gauge coupling $g_F^2 = B_F^{-1}$ as

$$B_F^{-1}(\varphi) = B_F^{-1}(+\infty) \left[1 - b_F \, e^{-c\varphi} \right] . \tag{3.3}$$

Note that, like b_λ (see section II A), also b_F is expected to be smallish [$\sim B_F^{-1}(+\infty)$ or, equivalently, $\sim C_F^{-1}$ in the notations of (1.3)] and typically the ratio b_F/b_λ is of order unity. We finally obtain

$$\alpha_{\text{had}}(\varphi) \simeq 40 \, b_F \, c \, e^{-c\varphi} \, . \tag{3.4}$$

We can now insert the estimate (2.25) of the value of φ reached because of the cosmological evolution. Neglecting the $\mathcal{O}(1)$ renormalization factor due to quantum noise, we get the estimate

$$\alpha_{\text{had}}(\varphi_{\text{end}}) \simeq 3.2 \, \frac{b_F}{b_\lambda \, c} \, \delta_H^{\frac{4}{n+2}} \, , \tag{3.5}$$

$$\alpha_{\text{had}}^2(\varphi_{\text{end}}) \simeq 10 \left(\frac{b_F}{b_\lambda \, c} \right)^2 \delta_H^{\frac{8}{n+2}} \, . \tag{3.6}$$

As said above, it is plausible to expect that the quantity c (which is a ratio) and the ratio b_F/b_λ are both of order unity. This then leads to the numerical estimate $\alpha_{\text{had}}^2 \sim 10 \, \delta_H^{\frac{8}{n+2}}$, with $\delta_H \simeq 5 \times 10^{-5}$. An interesting aspect of this result is that the expected present value of α_{had}^2 depends rather strongly on the value of the exponent n (which entered the inflaton potential $V(\chi) \propto \chi^n$). In the case $n = 2$ (i.e. $V(\chi) = \frac{1}{2} m_\chi^2 \, \chi^2$) we have $\alpha_{\text{had}}^2 \sim 2.5 \times 10^{-8}$, while if $n = 4$ ($V(\chi) = \frac{1}{4} \lambda \, \chi^4$) we have $\alpha_{\text{had}}^2 \sim 1.8 \times 10^{-5}$.

How do these numbers compare to present (composition-independent) experimental limits on deviations from Einstein's theory [16]? This question has been addressed in the literature. Concerning solar-system (post-Newtonian) tests it was shown (see, e.g., [17]) that the two main "Eddington" parameters $\gamma - 1$ and $\beta - 1$ measuring post-Newtonian deviations from general relativity are linked as follows to the dilaton coupling $\alpha_{\text{had}}(\varphi)$:

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$$\gamma - 1 = -2 \frac{\alpha_{\text{had}}^2}{1 + \alpha_{\text{had}}^2} \simeq -2 \, \alpha_{\text{had}}^2 \,, \tag{3.7}$$

$$\beta - 1 = \frac{1}{2} \frac{\alpha_{\text{had}}' \, \alpha_{\text{had}}^2}{(1 + \alpha_{\text{had}}^2)^2} \simeq \frac{1}{2} \, \alpha_{\text{had}}' \, \alpha_{\text{had}}^2 \,, \tag{3.8}$$

where $\alpha_{\text{had}}' \equiv \partial \alpha_{\text{had}}(\varphi)/\partial \varphi$.

From Eq. (3.4) we see that $\alpha_{\text{had}}' \simeq -c \, \alpha_{\text{had}}$, so that the deviation $\beta - 1$ is $\mathcal{O}(\alpha_{\text{had}}^3)$ and thereby predicted to be too small to be phenomenologically interesting. This leaves $\gamma - 1 \simeq -2 \, \alpha_{\text{had}}^2$ as the leading observable deviation. The best current solar-system limit on $\gamma - 1$ comes from Very Long Baseline Interferometry measurements of the deflection of radio waves by the Sun and is (approximately) $|\gamma - 1| \lesssim 2 \times 10^{-4}$, corresponding to $\alpha_{\text{had}}^2 \lesssim 10^{-4}$ (see [16] for reviews and references). In addition to solar-system tests, we should also consider binary-pulsar tests which provide another high-precision window on possible deviations from general relativity. They have been analyzed in terms of the two quantities α_{had} (denoted α) and α_{had}' (denoted β) in [18]. The final conclusion is that the binary-pulsar limit on α_{had} is of order $\alpha_{\text{had}}^2 \lesssim 10^{-3}$.

At this stage it seems that the runaway scenario explored here is leading to deviations from general relativity which are much smaller than present experimental limits. However, we must turn our attention to *composition-dependent* effects which turn out to be much more sensitive tests.

B. Composition-dependent deviations from general relativity

Let us then consider situations where the non-universal couplings of the dilaton induce (apparent) violations of the equivalence principle. Let us start by considering the composition-dependence of the dilaton coupling α_A, Eq. (2.26), i.e. the dependence of α_A on the type of matter we consider. The definition of α_A is such that, at the Newtonian approximation, the interaction potential between particle A and particle B is $-G_{AB} \, m_A \, m_B / r_{AB}$ where [3]

$$G_{AB} = G(1 + \alpha_A \, \alpha_B) \,. \tag{3.9}$$

Here, G is the bare gravitational coupling constant entering the Einstein-frame action (2.4), and $\alpha_A = \alpha_A(\varphi)$ is the strength of the dilaton coupling to A-particles, taken at the present (cosmologically determined) VEV of φ. The term $\alpha_A \, \alpha_B$ comes from the additional attractive

effect of dilaton exchange. Two test masses, made respectively of A- and B-type particles will then fall in the gravitational field generated by an external mass m_E with accelerations differing by

$$\left(\frac{\Delta a}{a}\right)_{AB} \equiv 2\,\frac{a_A - a_B}{a_A + a_B} \simeq \left(\alpha_A - \alpha_B\right)\alpha_E\,. \tag{3.10}$$

We have seen above that in lowest approximation $\alpha_A \simeq \alpha_{\text{had}}$ does not depend on the composition of A. We need, however, now to retain the small composition-dependent effects to α_A linked to the φ-dependence of QED and quark contributions to m_A. This has been investigated in [3] with the result

$$\left(\frac{\Delta a}{a}\right)_{AB} = \left(\frac{\alpha_{\text{had}}}{40}\right)^2 \left[C_B\,\Delta\left(\frac{B}{M}\right) + C_D\,\Delta\left(\frac{D}{M}\right) + C_E\,\Delta\left(\frac{E}{M}\right)\right]_{AB}, \tag{3.11}$$

where $(\Delta X)_{AB} \equiv X_A - X_B$, where $B \equiv N + Z$ is the baryon number, $D \equiv N - Z$ the neutron excess, $E \equiv Z(Z-1)/(N+Z)^{1/3}$ a quantity linked to nuclear Coulomb effects, and where $M \equiv m/u$ denotes the mass in atomic mass unit, $u = 931.49432$ MeV. It is difficult (and model-dependent) to try to estimate the coefficients C_B and C_D. It was argued in [3] that their contributions to (3.11) is generically expected to be sub-dominant with respect to the last contribution, $\propto C_E$, which can be better estimated because it is linked to the φ-dependence of the fine-structure constant $e^2 \propto B_F^{-1}(\varphi)$. This then leads to the numerical estimate $C_E \simeq 3.14 \times 10^{-2}$ and a violation of the universality of free fall approximately given by

$$\left(\frac{\Delta a}{a}\right)_{AB} \simeq 2 \times 10^{-5}\,\alpha_{\text{had}}^2 \left[\left(\frac{E}{M}\right)_A - \left(\frac{E}{M}\right)_B\right]. \tag{3.12}$$

The values of B/M, D/M and E/M have been computed in [19]. For mass-pairs that have been actually used in recent experiments (such as Beryllium and Copper), as well as for mass-pairs that are planned to be used in forthcoming experiments (such as Platinum and Titanium) one finds: $(E/M)_{\text{Cu}} - (E/M)_{\text{Be}} = 2.56$, $(E/M)_{\text{Pt}} - (E/M)_{\text{Ti}} = 2.65$. Using the average estimate $\Delta(E/M) \simeq 2.6$, we get from (3.12) and (3.6) the estimate

$$\left(\frac{\Delta a}{a}\right) \simeq 5.2 \times 10^{-5}\,\alpha_{\text{had}}^2 \simeq 5.2 \times 10^{-4} \left(\frac{b_F}{b_\lambda\,c}\right)^2 \delta_H^{\frac{8}{n+2}}\,. \tag{3.13}$$

Note also (from (3.7)) the link between composition-dependent effects and post-Newtonian ones

$$\left(\frac{\Delta a}{a}\right) \simeq -2.6 \times 10^{-5}(\gamma - 1)\,. \tag{3.14}$$

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As current tests of the universality of free fall (UFF) have put limits in the 10^{-12} range (e.g. $(\Delta a/a)_{\text{Be Cu}} = (-1.9 \pm 2.5) \times 10^{-12}$ from [20]), we see from Eq. (3.14) that this corresponds to limits on $\gamma - 1$ or α_{had}^2 in the 10^{-7} range. Therefore tests of the UFF put much more stringent limits on dilaton models than solar-system or binary-pulsar tests.

If we insert the estimate $\delta_H \sim 5 \times 10^{-5}$ in (3.13) we obtain a level of violation of UFF due to a runaway dilaton which is

$$\frac{\Delta a}{a} \simeq 1.3 \left(\frac{b_F}{b_\lambda c} \right)^2 \times 10^{-12} \quad \text{for } n = 2 \,, \tag{3.15}$$

$$\frac{\Delta a}{a} \simeq 0.98 \left(\frac{b_F}{b_\lambda c} \right)^2 \times 10^{-9} \quad \text{for } n = 4 \,. \tag{3.16}$$

At face value, one is tempted to conclude that a scenario with $n = 4$ (i.e. $V(\chi) \propto \chi^4$) tends to be too weak an attractor towards $\varphi = +\infty$ to be naturally compatible with equivalence-principle tests. [See, however, the discussion below.] On the other hand, the simple scenario $n = 2$ ($V(\chi) = \frac{1}{2} m_\chi^2 \chi^2$) is quite appealing in that it naturally provides enough attraction towards $\varphi = +\infty$ to be compatible with all existing experimental tests. At the same time it suggests that a modest improvement in the precision of UFF experiments might discover a violation caused by a runaway dilaton.

C. Cosmological variation of "constants"

Let us now consider another possible deviation from General Relativity and the standard model: a possible variation of the coupling constants, most notably of the fine structure constant $e^2/\hbar c$ on which the strongest limits are available. We will discuss first the effects due to the cosmological time-variation of the homogeneous component of φ and, in the next subsection, the possible spatial (and time) variations due to quantum fluctuations of φ as they got amplified during inflation.

Consistently with our previous assumptions we expect $e^2 \propto B_F^{-1}(\varphi)$ so that, from (3.3),

$$e^2(\varphi) = e^2(+\infty) \left[1 - b_F \, e^{-c\varphi} \right]. \tag{3.17}$$

The present logarithmic variation of e^2 (using again $dp = H \, dt; \varphi' = d\varphi/dp$) is thus given by

$$\frac{d \ln e^2}{H \, dt} = \frac{d \ln e^2}{dp} \simeq b_F \, c \, e^{-c\varphi} \, \varphi_0' \,, \tag{3.18}$$

where the current value of φ', φ'_0, is given in general by Eq. (2.31). Using Eq. (3.4), we can rewrite the result (3.18) in terms of the hadronic coupling:

$$\frac{d\ln e^2}{H\,dt} \simeq \frac{1}{40}\alpha_{\text{had}}\varphi'_0\,. \tag{3.19}$$

As said in section II B, we have basically two alternatives concerning the current coupling of the dilaton to the dominant energy sources in the universe. These two alternatives lead to drastically different predictions for the current value of the rate of variation of the fine-structure constant. We shall consider these two alternatives in turn.

In the conservative case where the dilaton does not play any special role in the present accelerated phase of the universe ($\alpha_V \simeq 0$) nor does it have any stronger coupling to dark matter than to visible matter ($\alpha_m \simeq -b_m\,c\,e^{-c\varphi}$) the dilaton "velocity" φ' is exponentially suppressed (so that, from (2.33), $\Omega_V \simeq 1 - \Omega_m$) and by Eq. (2.31) one obtains

$$\frac{d\ln e^2}{H\,dt} \simeq -\frac{\Omega_m}{\Omega_m + 2\Omega_V}\,b_F\,c\,e^{-c\varphi}\alpha_m(\varphi) \simeq \frac{\Omega_m}{2-\Omega_m}\,b_F\,b_m\,c^2\,e^{-2c\varphi}. \tag{3.20}$$

An indicative value for the ratio $\Omega_m/(\Omega_m + 2\Omega_V) \simeq \Omega_m/(2 - \Omega_m)$, by taking for instance $\Omega_m = 0.3$, is 0.18. As above, it is useful to relate (3.20) to the estimate (3.4) for α_{had}. This yields

$$\frac{d\ln e^2}{H\,dt} \simeq \frac{1}{(40)^2}\frac{\Omega_m}{2-\Omega_m}\frac{b_m}{b_F}\alpha_{\text{had}}^2\,. \tag{3.21}$$

In terms of the UFF level $\Delta a/a$ predicted by our model in (3.13) we see also that

$$\frac{d\ln e^2}{H\,dt} \simeq 12\,\frac{\Omega_m}{2-\Omega_m}\frac{b_m}{b_F}\frac{\Delta a}{a}\,. \tag{3.22}$$

Even if the universe were completely dominated by dark matter ($\Omega_m = 1$) we see, assuming that b_m/b_F is of order unity, that current experimental limits on UFF ($\Delta a/a \lesssim 10^{-12}$) imply (within dilaton models) that $|d\ln e^2/dt| \lesssim 10^{-11}\,H \sim 10^{-21}\,\text{yr}^{-1}$ (the sign of $d\ln e^2/dt$ being given by the sign of b_m/b_F). This level of variation is much smaller than the current best limit on the time variation of e^2, namely $|d\ln e^2/dt| \lesssim 5 \times 10^{-17}\,\text{yr}^{-1} \sim 5 \times 10^{-7}\,H$, as obtained from an analysis of Oklo data [21].

The situation, however, is drastically different if we consider the alternative case where the dilaton coupling to the current dominant energy sources does not tend to triviality, as in the case of a φ-dependent vacuum energy $V(\varphi) = V_0 + V_1 e^{-c\varphi}$ when the first term is zero or negligible. In such a case the dilaton shares a relevant part of the total energy density

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and more significant (though still quite constrained by UFF data) variations of the coupling constants are generally expected. A general expression for the dilaton "velocity" is given in eq. (2.35) in terms of observable quantities. Using Eqs. (2.35) and (3.19) one can relate the expected variation of the electromagnetic coupling constant to the hadronic coupling:

$$\frac{d \ln e^2}{H \, dt} \simeq \pm \frac{\alpha_{\text{had}}}{40} \sqrt{1 + q_0 - 3\Omega_m/2} \, . \tag{3.23}$$

We can also use the estimate (3.5) relating α_{had} to the density fluctuations generated during inflation. We obtain

$$\frac{d \ln e^2}{H \, dt} \simeq \pm 8 \times 10^{-2} \sqrt{1 + q_0 - 3\Omega_m/2} \, \frac{b_F}{b_\lambda c} \, \delta_H^{\frac{4}{n+2}} \, . \tag{3.24}$$

However, in view of the theoretical uncertainties attached to the initial conditions χ_{in} and φ_{in} used in the estimate (3.5), as well as the ones associated to the order unity ratio $b_F/(b_\lambda c)$, it is more interesting to rewrite our prediction in terms of $observable$ quantities. Using again the link Eq. (3.13) between α_{had} and the observable violation of the universality of free fall (UFF) the above result can be written in the form

$$\frac{d \ln e^2}{H \, dt} \simeq \pm 3.5 \times 10^{-6} \sqrt{1 + q_0 - 3\Omega_m/2} \sqrt{10^{12} \frac{\Delta a}{a}} \, . \tag{3.25}$$

Note that the sign of the variation of e^2 is in general model-dependent (as it depends both on the sign of b_F and the sign of φ_0'). Specific classes of models might, however, favour particular signs of de^2/dt. For instance, from the point of view of [5] one would expect the $\mathcal{O}(e^{-\phi})$ terms in Eq. (1.3) to be positive, which would then imply that b_F is positive. If we combine this information with the prediction Eq. (2.32) of the model [6] implying that φ' is also positive, we would reach the conclusion that e^2 must be currently $increasing$.

Independently of this question of the sign, we see that Eq. (3.25) predicts an interesting link between the observational violation of the UFF (constrained to $\Delta a/a \lesssim 10^{-12}$), and the current time-variation of the fine-structure constant. Contrary to the relation (3.22), obtained above under the alternative assumption about the dilaton dependence of the dominant cosmological energy, which predicted a relation linear in $\Delta a/a$, we have here a relation involving the square root of the UFF violation (such a relation is similar to the result of [11] which concerned the time-variation of the Newton constant).

The phenomenologically interesting consequence of Eq. (3.25) is to predict a time-variation of constants which may be large enough to be detected by high-precision laboratory experiments. Indeed, using $H_0 \simeq 66$ km/s/Mpc, and the plausible estimates $\Omega_m = 0.3$,

$q_0 = -0.4$, Eq. (3.25) yields the numerical estimate $d \ln e^2/dt \sim \pm 0.9 \times 10^{-16} \sqrt{10^{12} \Delta a/a}$ yr^{-1}. Therefore, the current bound on UFF violations ($\Delta a/a \sim 10^{-12}$) corresponds to the level 10^{-16}yr^{-1}, which is comparable to the planned sensitivity of currently developed cold-atom clocks [26]. [Present laboratory bounds are at the 10^{-14}yr^{-1} level [27,26].] Note that if we insert in Eq. (3.25) the secure bounds $\Omega_m > 0.2$ and $q_0 < 0$ (leading to the limit Eq. (2.36)), we get as maximal estimate of the time variation of the fine-structure constant $d \ln e^2/dt \sim \pm 2.0 \times 10^{-16} \sqrt{10^{12} \Delta a/a}$ yr^{-1}. We note also that the upper limit on the variation of e^2 given by the Oklo data, i.e. $|d \ln e^2/dt| \lesssim 5 \times 10^{-17}$ yr^{-1} [21], "corresponds" to a violation of the UFF at the level $\sim 10^{-13}$.

In this respect, it is interesting to consider not only the *present* variation of e^2 (the only one relevant for laboratory experiments), but also its variation over several billions of years. (We recall that the Oklo phenomenon took place about two billion years ago, and that astronomical observations constrain the variation of e^2 over the last ten billion years or so). In particular, an interesting question is to see whether our model could reconcile the Oklo limit with the recent claim [23] of a variation $\Delta e^2/e^2 = (-0.72 \pm 0.18) \times 10^{-5}$ around redshifts $z \approx 0.5 - 3.5$ as proposed in [24]. The only hope of reconciling the two results would be to allow for a faster variation of e^2 for redshifts $z > 0.5$. Such recent redshifts have (apparently) been connected to a transition from matter dominance to vacuum dominance. Let us see whether taking into account this transition might allow for a large enough change of e^2 around redshifts $z \approx 0.5 - 3.5$. We must clearly assume the "strong coupling" scenario $\alpha_m = \mathcal{O}(1)$. In this scenario, the variation of φ during the matter era is given by Eq.(2.30). Neglecting, for simplicity, the transient evolution effects localized around the matter-vacuum transition (and treating both $\varphi'_m = -\alpha_m$ and $\varphi'_V = \varphi'_0$ as constants), the solution giving the recent cosmological evolution of φ reads $\varphi - \varphi_0 = -\varphi'_0 \ln(1+z)$ during the vacuum era, and $\varphi - \varphi_0 = -\varphi'_0 \ln(1+z_*) - \varphi'_m \ln[(1+z)/(1+z_*)]$ during the matter era (the index 0 refers to the present epoch, i.e. $z = 0$; z_* denotes the transition redshift). Inserting this change in Eq.(3.17) leads to the following expression for the cosmological change of the fine-structure constant:

$$\frac{e^2 - e_0^2}{e_0^2} = -\text{sign}(b_F) \, 3.5 \times 10^{-6} \, [\varphi'_0 \ln(1+z_*) + \varphi'_m \ln \frac{1+z}{1+z_*}] \sqrt{10^{12} \frac{\Delta a}{a}}. \qquad (3.26)$$

Here, we have written the result for the matter era. During the vacuum era the bracket is simply $[\varphi'_0 \ln(1+z)]$. Remembering that the absolute value of φ'_m is (like that of φ'_V) observationally constrained to be smaller than $\sqrt{0.3} \simeq 0.55$ (and that φ'_0 is also constrained by $|\varphi'_V| < 0.84$), we see that there is no way, within our model, to explain a variation of e^2 as large as $\Delta e^2/e^2 = (-0.72 \pm 0.18) \times 10^{-5}$ around redshifts $z \approx 0.5 - 3.5$ [23]. In our model, even under the assumption that UFF is violated just below the currently tested

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level, such a change would have to correspond to a value $|\varphi_m'| > 2$, entailing observationally unacceptable modifications of standard cosmology. [For instance, in the model [6] a value as large as $\alpha_m > 1$ already leads to a pathological behaviour ("total dragging") where all the components scale like radiation. This difficulty with reconciling the Oklo limit and the claims of [23] does not seem to have been noticed in [24]]. On the other hand, the maximal variation predicted by Eq.(3.26) for redshifts corresponding to the matter era (obtained when $\Delta a/a = 10^{-12}$ and $\varphi_m' = \pm\sqrt{0.3}$; and assuming a smaller value of φ_0' to be compatible with the Oklo constraint), is of order $\Delta e^2/e^2 = \pm 1.9 \times 10^{-6}$. This is only a factor ~ 4 below the claim [23] and is at the level of their one sigma error bar. Therefore a modest improvement in the observational precision (accompanied by an improved control of systematics) will start to probe a domain of variation of constants which, according to our scenario, corresponds to an UFF violation smaller than the 10^{-12} level.

D. Spatio-temporal fluctuations of the "constants"

We now turn to the second possible source of spatial-temporal variations for e^2 in our model, the quantum fluctuations of the dilaton generated during inflation. Within linear perturbation theory, the relevant calculation may be summarized as follows.

Consider a flat FRW universe $ds^2 = -dt^2 + a(t)^2 \Sigma_i dx_i^2$. The dilaton fluctuations can be expanded in Fourier components $\delta\varphi_{\mathbf{k}}$ of given comoving momentum $\mathbf{k}$ as follows:

$$\delta\varphi(\mathbf{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\, \delta\varphi_{\mathbf{k}}(t) e^{i\mathbf{k}\mathbf{x}}, \qquad (3.27)$$

where t is the cosmological time. Each Fourier mode $\delta\varphi_{\mathbf{k}}$ "leaves" the horizon during inflation with an amplitude $\sim \widehat{H}_{\mathrm{ex}}(k)/\sqrt{2k^3}$ [25] where, by definition, $\widehat{H}_{\mathrm{ex}}(k)$ is the value of the dimensionless Hubble expansion rate as ka^{-1} equals H during inflation (note that we denote here $\widehat{H}_{\mathrm{ex}}$ what was denoted $\widehat{H}_{\times}$ above). Well after the exit ($k \ll aH$) the amplitude of each mode "freezes out", i.e. remains roughly constant, until it reenters the horizon during the post-inflationary epoch ($ka_{re}^{-1} \simeq H_{re}$). After re-entry the amplitude starts to damp out as a^{-1}. For a given Fourier mode $\delta\varphi_{\mathbf{k}}(t)$, the latter damping effect is described by the piecewise function

$$f_z(k) \equiv \begin{cases} 1 & \text{if} & a_0^{-1}H_0^{-1}\,k\ <\ (z+1)^{1/2} \\ a_0^2 H_0^2\,(z+1)k^{-2} & \text{if} & (z+1)^{1/2}\ <\ a_0^{-1}H_0^{-1}\,k\ <\ 10^2 \qquad (3.28) \\ 10^{-2} a_0 H_0\,(z+1)k^{-1} & \text{if} & a_0^{-1}H_0^{-1}\,k\ >\ 10^2 \end{cases}$$

Here the cosmological redshift $z = a_0/a(t) - 1$ has been introduced in replacement of the cosmological time t. The first case refers to Fourier modes that have not reentered yet at redshift z and whose amplitudes are still frozen. The second and third cases refer to modes that reenter during matter and radiation domination respectively. Putting all together, and assuming a gaussian probability distribution for the perturbations, we have:

$$\langle \delta\varphi_{\mathbf{k}}(t)^* \, \delta\varphi_{\mathbf{k'}}(t') \rangle = \frac{\widehat{H}_{\text{ex}}^2(k)}{2k^3} \, f_z(k) \, f_{z'}(k) \, \delta^3(\mathbf{k} - \mathbf{k'}) \,. \tag{3.29}$$

Possible spatial/temporal variations of e^2 induced by the fluctuations of the dilaton will be given by

$$\left. \frac{\Delta^{\text{fluc}} e^2}{e^2} \right|_{(\mathbf{x},t;\,\mathbf{x'},t')} = \frac{d\ln e^2}{d\varphi} \, \Delta^{\text{fluc}}\varphi|_{(\mathbf{x},t;\,\mathbf{x'},t')} \,, \tag{3.30}$$

where the r.m.s $\Delta^{\text{fluc}}\varphi$ between two events $(\mathbf{x}, t)$ and $(\mathbf{x'}, t')$ is defined as follows

$$\Delta^{\text{fluc}}\varphi|^2_{(\mathbf{x},t;\,\mathbf{x'},t')} \equiv \langle \, [\delta\varphi(\mathbf{x}, t) - \delta\varphi(\mathbf{x'}, t')]^2 \rangle$$

$$= \frac{\widehat{H}_{\text{ex}}^2}{(2\pi)^3} \int \frac{d^3k}{2\,k^3} \left[f_z(k)^2 + f_{z'}(k)^2 - 2f_z(k)f_{z'}(k) \, e^{i\mathbf{k}(\mathbf{x}-\mathbf{x'})} \right] \tag{3.31}$$

$$= \frac{\widehat{H}_{\text{ex}}^2}{(2\pi)^2} \int_0^\infty \frac{dk}{k} \left\{ [f_z(k) - f_{z'}(k)]^2 + 2f_z(k)f_{z'}(k) \left[1 - \frac{\sin kx}{kx} \right] \right\} \,. \tag{3.32}$$

Here, $x \equiv |\mathbf{x} - \mathbf{x'}|$ is the coordinate distance between the two events and, consistently with the slow-roll approximation, the Hubble expansion rate at exit has been assumed to be scale-invariant: $\widehat{H}_{\text{ex}}(k) \simeq \widehat{H}_{\text{ex}} \simeq 3 \times 10^{-5}$.

If one considers spatial fluctuations over terrestrial or solar system proper length scales $l = a_0 k^{-1} \ll H_0^{-1}$ at the present time $t = t' = t_0$, the first square brackets in (3.32) vanishes and one can expand the sine function at small kx obtaining

$$\Delta^{\text{fluc}}\varphi|_{l;\,z=0} \simeq \frac{\widehat{H}_{\text{ex}}}{2\pi} \frac{H_0 \, l}{\sqrt{3}} \qquad ; \qquad \left. \frac{\Delta^{\text{fluc}} e^2}{e^2} \right|_{l;\,z=0} \simeq 10^{-2} \, \alpha_{\text{had}} \, \widehat{H}_{\text{ex}} \, H_0 \, l \,. \tag{3.33}$$

As expected, these variations are extremely small, $\Delta^{\text{fluc}} e^2/e^2|_{l;\,z=0} \simeq 10^{-33} \, l/\text{km}$. It is also interesting to compare dilaton fluctuations at different redshifts along a comoving observer wordline. By putting $x \simeq 0$ in (3.32) the second term in the square brackets vanishes and one has:

$$\Delta^{\text{fluc}}\varphi|_{z;\,x=0} \simeq \frac{\widehat{H}_{\text{ex}}}{2\pi} \frac{1}{\sqrt{2}} \left[\log(1+z) - \frac{z}{1+z} + \frac{10^{-8}}{2}z^2 \right]^{1/2} \simeq \frac{\widehat{H}_{\text{ex}}}{2\pi} \left[\frac{z}{2} - \frac{z^2}{3} + \ldots \right] \,. \tag{3.34}$$

It is slightly more complicated to compare dilaton fluctuations between "now" and events at redshift z along a null ray. Expanding in powers of z around $z = 0$ one gets from (3.32), after a straightforward calculation:

$$\Delta^{\text{fluc}}\varphi|_z \simeq \frac{\widehat{H}_{\text{ex}}}{2\pi} \left[\frac{1}{2}\sqrt{\frac{7}{3}}\, z - \frac{2}{\sqrt{21}}\, z^2 + \dots \right]. \tag{3.35}$$

Numerically, at redshift $z \sim 1$, the effects of dilatonic fluctuations give $\Delta^{\text{fluc}}\varphi|_{z=1} \sim \widehat{H}_{\text{ex}}/(2\pi) \sim 5 \times 10^{-6}$. This is to be contrasted with the effects of the cosmic, homogeneous evolution which yields $\Delta\varphi|_{z=1} \simeq \alpha_m$. In the "normal" case where $\alpha_m \sim e^{-c\varphi} \sim \delta_H^{4/(n+2)}$, the two effects, though a priori unrelated, are related in our scenario , when $n = 2$. Indeed, if $n = 2$, $\delta_H^{4/(n+2)} = \delta_H \sim 5 \times 10^{-5}$ is linked to $\widehat{H}_{\text{ex}}/(2\pi)$ via $\delta(\chi_{\text{ex}}) = A\widehat{H}_{\text{ex}}/(2\pi)$ with $A = (8/3)V/\partial_\chi V = (8/3)(\chi/n) \simeq 40/(3\sqrt{n}) \sim 10$. On the other hand, in the case where φ is strongly coupled to dark matter, the homogeneous evolution $\Delta\varphi|_{z=1} \simeq \alpha_m \sim 1$ is parametrically larger than the fluctuations $\Delta^{\text{fluc}}\varphi|_{z=1} \sim \widehat{H}_{\text{ex}}/(2\pi)$.

To conclude on this subsection, we see that the inhomogeneous space-time fluctuations of the fine-structure constant are typically too small to be observable (if the limits from UFF are already satisfied), being suppressed, relative to their natural values $H_0 l$, $H_0 t$, by the small factor $\alpha_{\text{had}}\widehat{H}_{\text{ex}}$.

IV. SUMMARY AND CONCLUSION

We have studied the dilaton-fixing mechanism of [3] within the context where the dilaton-dependent low-energy couplings are extremized at $\varphi = +\infty$, i.e. for infinitely large values of the bare string coupling $g_s^2 = e^\phi \simeq e^{c\varphi}$. [The crucial coupling to the inflaton, say $\lambda(\varphi)$ in Eq.(2.11), must be *minimized* at $\varphi \to +\infty$; the other couplings can be either minimized or maximized there.] This possibility of a fixed point at infinity (in bare string coupling space) has been recently suggested [5], and its late-cosmological consequences have been explored in [6]. We found that a primordial inflationary stage, with inflaton potential $V(\chi) = \lambda(\varphi)\chi^n/n$, was much less efficient in decoupling a dilaton with least couplings at infinity than in the case where the least couplings are reached at a finite value of φ (as in [3,8]). This reduced efficiency has interesting phenomenological consequences. Indeed, it predicts much larger observable deviations from general relativity. In the case of the simplest chaotic potential [10] $V(\chi) = \frac{1}{2}m_\chi^2(\varphi)\chi^2$, we find that, under the simplest assumptions about the pre-inflationary state, this scenario predicts violations of the universality of free fall (UFF) of order $\Delta a/a \sim 5 \times 10^{-4}\delta_H^2$ where δ_H is the density fluctuation generated by inflation on horizon scales. The observed level of large-scale density (and cosmic microwave background

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temperature) fluctuations fixes δ_H to be around 5×10^{-5} which finally leads to a prediction for a violation of the UFF near the $\Delta a/a \sim 10^{-12}$ level. This is naturally compatible with present experimental tests of the equivalence principle, and suggests that a modest improvement in the precision of UFF tests might be able to detect a deviation linked to dilaton exchange with a coupling reduced by the attraction towards the fixed point at infinity. Because of the presence of unknown dimensionless ratios $(c, b_F/b_\lambda)$ in our estimates, and of quantum noise in the evolution of the dilaton, we cannot give sharp quantitative estimates of $\Delta a/a$. However, we note that dilaton-induced violations of the UFF have a rather precise signature with a composition-dependence of the form (3.11), with probable domination by the last (Coulomb energy) term [3]. As explored in [19] this signature is quite distinct from UFF violations induced by other fields, such as a vector field. We note that the approved Centre National d'Etudes Spatiales (CNES) mission MICROSCOPE [28] (to fly in 2004) will explore the level $\Delta a/a \sim 10^{-15}$, while the planned National Aeronautics and Space Agency (NASA) and European Space Agency (ESA) mission STEP (Satellite Test of the Equivalence Principle) [29] could explore the $\Delta a/a \sim 10^{-18}$ level. Our scenario gives additional motivation for such experiments and suggests that they might find a rather strong violation signal, whose composition-dependence might then be studied in detail to compare it with Eq. (3.11).

In the case of inflationary potentials $V(\chi) \propto \chi^n$ with $n > 2$ our simplest estimates predict a violation of the UFF of order $\Delta a/a \sim 5 \times 10^{-4} \, \delta_H^{\frac{8}{n+2}}$ which is larger than 10^{-12}. At face value this suggests that existing UFF experimental data can be interpreted as favouring $n \leq 2$ over $n > 2$. However, we must remember that our estimates have made several simplifying assumptions. It is possible that the large quantum fluctuations of the inflaton in the self-regenerating regime $\chi > \chi_{\text{in}}$, with χ_{in} defined by Eq. (2.18), can give more time for φ to run away towards large values, so that the effective value of $e^{c\varphi_{\text{in}}}$ to be used in Eq. (2.21) turn out to dominate the first term in the R.H.S. that we have used for our estimates. We leave to future work a study of the system of Langevin equations describing the coupled fluctuations of ϕ and χ during the self-regenerating regime.

Finally let us note some other conclusions of our work.

We recover the conclusion of previous works on dilaton models that the most interesting experimental probes of a massless weakly coupled dilaton are tests of the UFF. The composition-independent gravitational tests (solar-system, binary-pulsar) tend to be much less sensitive probes (as highlighted by the relations (3.14), (3.21) and (3.22)).

However, a possible exception concerns the time-variation of the coupling constants. Here the conclusion depends crucially on the assumptions made about the couplings of the dilaton to the cosmologically dominant forms of energy (dark matter and/or dark energy).

If these couplings are of order unity (and as large as is phenomenologically acceptable, i.e. so that $(\varphi_0')^2 = 0.7$), the present time variation of the fine-structure constant is linked to the violation of the UFF by the relation $d\ln e^2/dt \sim \pm 2.0 \times 10^{-16} \sqrt{10^{12}\Delta a/a}\,\mathrm{yr}^{-1}$. [The most natural sign here being $+$, i.e. $b_F > 0$, which corresponds to *smaller* e^2 in the past, just as suggested by the claim [23].] Such a time variation might be observable (if $\Delta a/a$ is not very much below its present upper bound $\sim 10^{-12}$) through the comparison of high-accuracy cold-atom clocks and/or via improved measurements of astronomical spectra.

More theoretical work is needed to justify the basic assumption (1.3) of our scenario. In particular, it is crucial to investigate whether it is natural to expect that the sign of the crucial coefficient b_λ in Eq. (2.12) be indeed *positive*. [Recall that the general mechanism of [3] is an attraction towards "Least Couplings" while Eq. (1.3) with $\mathcal{O}(e^{-\phi}) > 0$ leads to largest couplings at infinity.] Note in this respect that the sign of the other b_i's is not important as, once inflation has pushed $e^{c\varphi}$ to very large values $e^{c\varphi_{\mathrm{end}}}$, the subsequent cosmological evolutions tend to be ineffective in further displacing φ.

ACKNOWLEDGMENTS

APPENDIX A: THE STOCHASTIC EVOLUTION OF THE DILATON

In this appendix we study the stochastic evolution of the dilaton φ during inflation as described by the Langevin-type equation (2.23). We restrict our attention to the region of phase space where the evolution of the inflaton χ is classical, and to a power-law potential of the form (2.11). It follows that the inflaton evolves according to the classical slow-roll equation (2.14) whose solution reads

$$\chi^2 = \chi_{\mathrm{in}}^2 - np, \tag{A1}$$

where p, the parameter defined in (2.7), is shifted in such a way that $p_{\mathrm{in}} \equiv 0$. Equation (2.23) takes the form

$$\frac{d\varphi}{dp} = \frac{1}{2}\, b_\lambda\, c\, e^{-c\varphi} + \xi(p)\,, \tag{A2}$$

where $\xi(p)$ is a gaussian stochastic variable (GSV), with a "time-dependent" r.m.s. amplitude $\widehat{H}(p)/2\pi$:

$$\langle \xi(p_1)\,\xi(p_2)\rangle \;=\; \frac{\widehat{H}^2}{(2\pi)^2}\,\delta(p_1 - p_2), \tag{A3}$$

[the relation with the normalized random white noise term of (2.23) is $\xi(p) = \xi_2(p)\widehat{H}/2\pi$]. For any given source term $\xi(p)$, the formal solution of (A2) reads

$$e^{c\,\varphi(p)} \;=\; e^{c\,\varphi_{\rm in}}\,e^{c\,\eta(p)} + \frac{b_\lambda c^2}{2}\int_0^p dp'\, e^{c\,[\eta(p)-\eta(p')]}, \qquad \eta(p) \equiv \int_0^p dp'\,\xi(p')\,. \tag{A4}$$

Note that the classical solution in (2.17), $e^{c\,\varphi_{\rm cl}(p)} = e^{c\,\varphi_{\rm in}} + (b_\lambda c^2/2)\,p$, can be easily recovered in the small noise limit $\xi(p) \to 0$, $\eta(p) \to 0$.

It proves convenient to compare the true solution to the classical one by studying the statistical behaviour of the ratio $A(p) \equiv e^{c\,\varphi(p)}/e^{c\,\varphi_{\rm cl}(p)}$. As we will show below, $\langle e^{c\,\eta(p)}\rangle = \mathcal{O}(1)$. Moreover, we are also assuming $e^{c\,\varphi_{\rm in}} = \mathcal{O}(1)$ or, at least, $e^{c\,\varphi_{\rm in}} \ll (b_\lambda c^2/2)p$ (see Section II for details) so that the leading contribution to the first equation in (A4) is given by the integral, and we have

$$A(p) \;\equiv\; e^{c\,\varphi(p)}/e^{c\,\varphi_{\rm cl}(p)} \;\simeq\; \frac{1}{p}\int_0^p dp'\, e^{c\,[\eta(p)-\eta(p')]}\,. \tag{A5}$$

Since $\xi(p)$ is a GSV, also its integral $\eta(p)$ is a (centered) GSV. Moreover, if x is a GSV with $\sigma_x^2 \equiv \langle x^2\rangle - \langle x\rangle^2 = \langle x^2\rangle$, by Bloch's theorem $y = e^x$ is a new stochastic variable with $\langle y\rangle = \langle e^x\rangle = e^{\langle x^2\rangle/2}$ and $\sigma_y^2 = e^{2\langle x^2\rangle} - e^{\langle x^2\rangle}$. The average value of $A(p)$ thus reads

$$\langle A(p)\rangle \;\simeq\; \frac{1}{p}\int_0^p dp'\, e^{(c^2/2)\,\langle[\eta(p)-\eta(p')]^2\rangle}\,. \tag{A6}$$

The exponent on the right hand side of the above equation can be estimated by using (A3) and the slow-roll approximation $\widehat{H}^2 \simeq 2V(\chi,\varphi)/3 = 2\lambda(\varphi)\chi^n/3n \simeq 2\lambda_\infty\chi^n/3n$. One gets

$$\langle[\eta(p)-\eta(p')]^2\rangle \;=\; \frac{1}{(2\pi)^2}\int_{p'}^p \widehat{H}^2 dp'' \;\simeq\; \frac{n}{2(n+2)}\left[\left(\frac{\chi(p')}{\chi_{\rm in}}\right)^{n+2} - \left(\frac{\chi(p)}{\chi_{\rm in}}\right)^{n+2}\right], \tag{A7}$$

where $\chi_{\rm in}$ is the value at exit from self-regenerating inflation: $\widehat{H}(\chi_{\rm in})/2\pi = n/(2\chi_{\rm in})$ (see Section II for more details). Since we are interested in evaluating (A6) at the end of inflation, $p = p_{\rm end} \simeq \chi_{\rm in}^2/n$, we can thus write

$$\langle[\eta(p)-\eta(p')]^2\rangle \;\simeq\; \frac{n}{2(n+2)}\left[1 - \frac{p'}{p_{\rm end}}\right]^{(n+2)/2}\,. \tag{A8}$$

When evaluated at $p' = 0$, the above formula gives $\langle \eta(p)^2 \rangle = n/(2(n+2))$. Thus the normalization factor to the initial condition in (A4) is of order one, as anticipated: $\langle e^{c\eta(p)} \rangle = e^{\frac{1}{2}c\langle \eta(p)^2 \rangle} = \mathcal{O}(1)$. From (A6) and (A8) we have:

$$
\begin{aligned}
\langle A(p_{\mathrm{end}}) \rangle &\simeq \frac{1}{p_{\mathrm{end}}} \int_0^{p_{\mathrm{end}}} dp' \exp\left[\frac{c^2 n}{4(n+2)} \left(1 - \frac{p'}{p_{\mathrm{end}}} \right)^{(n+2)/2} \right] \\
&= \int_0^1 \exp\left[\frac{c^2 n}{4(n+2)} x^{(n+2)/2} \right] dx \\
&= \exp\left(\frac{c^2 n \theta}{4(n+2)} \right) = \mathcal{O}(1),
\end{aligned}
\tag{A9}
$$

with $0 < \theta < 1$.

We can estimate the dispersion of the same quantity by expanding the exponential inside the integral (A6) in powers of $\xi(p)$:

$$
A(p) \simeq 1 + \frac{c}{p} \int_0^p dp' \int_{p'}^p dp'' \xi(p'') + \frac{1}{2} \frac{c^2}{p} \int_0^p dp' \left(\int_{p'}^p dp'' \xi(p'') \right)^2 + \ldots
\tag{A10}
$$

At lowest order in $\xi(p)$ the variance of the above quantity calculated at $p = p_{\mathrm{end}}$ reads

$$
\begin{aligned}
\sigma_{A(p_{\mathrm{end}})}^2 &= \left\langle \left(\frac{c}{p} \int_0^p dp' \int_{p'}^p dp'' \xi(p'') \right)^2 \right\rangle \\
&= \frac{c^2}{p^2} \int_0^p dp' \int_0^p dp'' \int_{\max(p',p'')}^p dp''' \frac{\widehat{H}^2(p''')}{(2\pi)^2} \\
&= \frac{2c^2}{p^2} \int_0^p dp' p' \int_{p'}^p dp'' \frac{\widehat{H}^2(p'')}{(2\pi)^2}.
\end{aligned}
\tag{A11}
$$

As in equation (A7) we can use the slow-roll approximation and obtain

$$
\sigma_{A(p_{\mathrm{end}})}^2 = \frac{c^2 n}{n+2} \int_0^1 x\,(1-x)^{(n+2)/2} dx = c^2 \mathcal{O}(1).
\tag{A12}
$$

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