MINI-COURSE: GEOMETRY AND TOPOLOGY OF STANDARD GROUP EMBEDDINGS

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Wednesdays, 1.30 - 3.30 pm. Mimar Sinan University. Mathematics Department.

1. Aims and objectives:

The purpose of this mini-course is to study normal projective compactifications of a reductive group (standard group embeddings). These embeddings are obtained as projectivizations of a reductive monoid and, in general, have singularities. During these lectures, we focus on the case when a standard embedding has rationally smooth singularities. We show that any rationally smooth standard embedding comes equipped with a nice cellular decomposition. Moreover, we describe the equivariant cohomology of such embeddings in terms of finite combinatorial data.

2. Outline:

(1) Equivariant Cohomology and GKM theory.
   (a) Localization in equivariant cohomology.
   (b) Equivariantly formal spaces and T-skeletal actions.
   (c) Examples: Schubert varieties, flag varieties, toric varieties.

(2) Rational smoothness, cellular decompositions and GKM theory.
   (a) Rational smoothness and Poincaré duality.
   (b) The Bialynicki-Birula decomposition.
   (c) Rational cells and $\mathbb{Q}$-filtrable varieties.
   (d) Equivariant Euler classes and local indices.

(3) Standard group embeddings.
   (a) Reductive Monoids.
   (b) Finite combinatorial invariant associated to a reductive monoid.
   (c) Standard group embeddings.
   (d) Main differences between standard embeddings and regular embeddings.
   (e) Rationally smooth standard embeddings. Classification.

(4) Equivariant Cohomology of Rationally Smooth Standard Embeddings.
   (a) GKM data of a rationally smooth standard embedding $\mathbb{P}_\epsilon(M)$.
   (b) Associated characters and the GKM graph.
   (c) GKM description of $H^*_{T\times T}(\mathbb{P}_\epsilon(M))$ and $H^*_{G\times G}(\mathbb{P}_\epsilon(M))$.
   (d) Quasi-regular embeddings and comparison with the associated toric variety.

(5) Extensions to Equivariant Intersection Cohomology and K-theory (Vista).

Duration: 6 lectures.

Start date: November 30th, 2011.

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