Gravity and running coupling constants

1) Motivation and history

2) Brief review of running couplings

3) Gravity as an effective field theory

4) Running couplings in effective field theory

5) Summary

6) If time – incomplete comments on pure gravity and asymptotic safety

John Donoghue
IHES 3/10/11

The motivation for the subfield:

PRL **96,** 231601 (2006) PHYSICAL REVIEW LETTERS week ending 16 JUNE 2006

Gravitational Correction to Running of Gauge Couplings

Sean P. Robinson* and Frank Wilczek†
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(Received 30 March 2006; published 15 June 2006)

We calculate the contribution of graviton exchange to the running of gauge couplings at lowest nontrivial order in perturbation theory. Including this contribution in a theory that features coupling constant unification does not upset this unification, but rather shifts the unification scale. When extrapolated formally, the gravitational correction renders all gauge couplings asymptotically free.

$$\beta(g, E) \equiv \frac{dg}{d\ln E} = -\frac{b_0}{(4\pi)^2} g^3 + a_0 \frac{E^2}{M_{\mathrm{P}}^2} g,$$

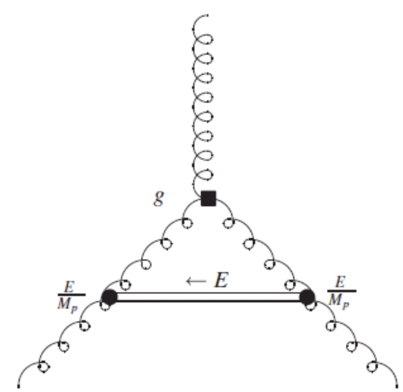

FIG. 1. A typical Feynman diagram for a gravitational process contributing to the renormalization of a gauge coupling at one loop. Curly lines represent gluons. Double lines represent gravitons. The three-gluon vertex ■ is proportional to g, while the gluon-graviton vertex ● is proportional to E/M_{P}.

A hint of asymptotic freedom for all couplings

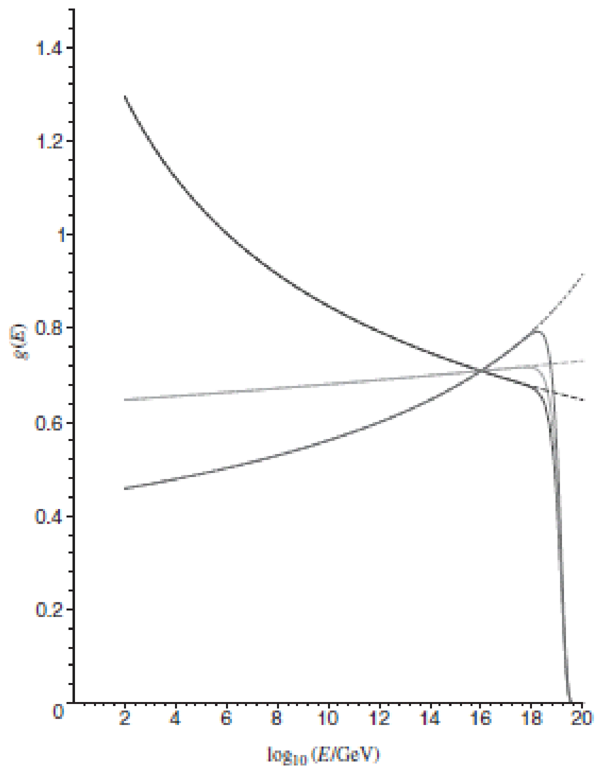

FIG. 2. When gravity is ignored, the three gauge couplings of a model theory evolve as the inverse logarithm of E at one-loop order (dashed curves). Initial values at 100 GeV were set so that the curves exactly intersect at approximately 10^{16} GeV. When gravity is included at one loop (solid curves), the couplings remain unified near 10^{16} GeV, but evolve rapidly towards weaker coupling at high E.

when the energy approaches the Planck scale, and soon after that one loses the right to neglect higher-order graviton exchanges. Though neglect of additional corrections is not justified beyond $E \ll M_{\rm P}$, it is entertaining to consider some consequences of extrapolating Eq. (2) as it stands to these energies, taking into account $a_0 < 0$. The integral on the right-hand side converges as $E \to \infty$, and so Eq. (20) arises as an asymptotic relation. Thus, the effective coupling vanishes rapidly beyond the Planck scale, rendering the gauge sector approximately free at these energies. In

A Rough History:

Prehistory: Fradkin, Vilkovisky, Tseytlin, Diennes, Kiritsis, Kounnas…

Start of "modern era":

S. P. Robinson and F. Wilczek, "Gravitational correction to running of gauge couplings," Phys. Rev. Lett. **96**, 231601 (2006) [arXiv:hep-th/0509050].

Claims that RW are wrong

-analysis in dimensional regularization

-couplings **do not** run

A. R. Pietrykowski, "Gauge dependence of gravitational correction to running of gauge couplings," Phys. Rev. Lett. **98**, 061801 (2007) [arXiv:hep-th/0606208].

D. J. Toms, "Quantum gravity and charge renormalization," Phys. Rev. D **76**, 045015 (2007) [arXiv:0708.2990 [hep-th]].

D. Ebert, J. Plefka and A. Rodigast, "Absence of gravitational contributions to the running Yang-Mills coupling," Phys. Lett. B **660**, 579 (2008) [arXiv:0710.1002 [hep-th]].

Y. Tang and Y. L. Wu, "Gravitational Contributions to the Running of Gauge Couplings," arXiv:0807.0331 [hep-ph].

Claims that couplings do run:

- analysis using cutoff regularization

D. J. Toms, "Quantum gravitational contributions to quantum electrodynamics," Nature **468**, 56-59 (2010). [arXiv:1010.0793 [hep-th]].

H. -J. He, X. -F. Wang, Z. -Z. Xianyu, "Gauge-Invariant Quantum Gravity Corrections to Gauge Couplings via Vilkovisky-DeWitt Method and Gravity Assisted Gauge Unification," [arXiv:1008.1839 [hep-th]].

Y. Tang, Y. -L. Wu, "Quantum Gravitational Contributions to Gauge Field Theories,"

[arXiv:1012.0626 [hep-ph]].

S. Folkerts, D. F. Litim, J. M. Pawlowski, "Asymptotic freedom of Yang-Mills theory with gravity," [arXiv:1101.5552 [hep-th]].

Claims that running couplings do not make sense

M. M. Anber, J. F. Donoghue, M. El-Houssieny, "Running couplings and operator mixing in the gravitational corrections to coupling constants," [arXiv:1011.3229 [hep-th]].

J. Ellis, N. E. Mavromatos, "On the Interpretation of Gravitational Corrections to Gauge Couplings,"

[arXiv:1012.4353 [hep-th]].

Then the press picks it up:

naturenews

nature news home | news archive | specials | opinion | features | news blog

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- Physics
- Mathematics

Stories by keywords

- Gravity
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Published online 3 November 2010 | Nature | doi:10.1038/news.2010.580

News

Gravity shows its helpful side

Theoretical study shows that the force can ease quantum calculations.

Geoff Brumfiel

Gravity is usually an obstacle to a theory of everything

MEHAU KULYK/SCIENCE PHOTO LIBRARY

Gravity is unruly. It can throw theorists' equations into chaos, and has proved a stumbling block to the creation of a single 'theory of everything'. But an analysis now shows that gravity may at least make some fundamental calculations more manageable.

David Toms, a theoretical physicist at Newcastle University, UK, has found that gravity seems to calm the electromagnetic force at high energies. The finding could make some calculations easier, and is a rare case in which gravity seems to work in harmony with quantum mechanics, the theory of small particles. His paper is published today in *Nature*[1].

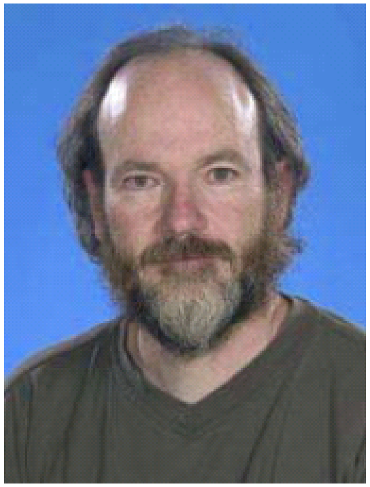
And many theorists are sceptical about whether Toms's calculations will bear close examination. "His mathematics could well be right, but I don't think his interpretation is," says John Donoghue, a theoretical physicist at the University of Massachusetts Amherst. Donoghue is concerned that when the method is applied to other interactions, involving different particles, it might yield a different answer. "The effects are not universal," he says. That would be a big problem for theorists, who want their methods to apply to everything equally.

Toms concedes that he "can't say for certain" whether his method will be universal. He now plans to take a second look at what happens to the strength of gravity at high energies, using the new approach. If gravity weakens like the other forces, theorists really might be closer to a theory of everything. Toms says that the calculations will be harder to do. But, he adds, "I think I know how to do it". ■

Quantum gravity corrects QED

Nov 3, 2010 10 comments

This week's issue of *Nature* includes a paper that's remarkable for two reasons: it is about quantum gravity – a topic usually not covered in the journal – and it is written by just one person. Now, after a little digging, *physicsworld.com* can answer all of the important questions about this paper.


David Toms

So, whose citation index ranking is about to go into the stratosphere?

The paper was written by David Toms, a Canadian mathematical physicist and lecturer at Newcastle University in the UK.

What has Toms done?

He has shown that interactions between quantum gravity and quantum electrodynamics (QED) cause electric charge to vanish at very high energies (above about 10^{15} GeV). He told *physicsworld.com* that his technique can be generalized to apply to the two other "gauge couplings", which define the strong and weak forces.

See Inside

Forces to Reckon With

Does gravity muck up electromagnetism?

By George Musser | January 30, 2011 | 0

A magazine news story on the unification of physics usually begins by saying that Einstein's general theory of relativity and quantum theory are irreconcilable. The one handles the force of gravity, the other takes care of electromagnetic and nuclear forces, but neither covers all, so physicists are left with a big jagged crack running down the middle of their theoretical world. It's a nice story line, except it's not true. "Everyone says quantum mechanics and gravity don't get along—they're incompatible," says John F. Donoghue of the University of Massachusetts at Amherst. "And you still hear that, but it's wrong."

Last November, David J. Toms of Newcastle University in England argued that even if gravity does not bring all the forces into line, it at least qualitatively reconciles electromagnetism with the nuclear forces. Neglecting gravity, electromagnetism intensifies as you go down in size, whereas the nuclear forces weaken. But gravity emasculates electromagnetism, causing it to behave like the nuclear forces on the very smallest scales.

Wilczek calls Toms's paper "impressive." Around the same time, however, Donoghue and his graduate students Mohamed M. Anber and Mohamed El-Houssieny cast doubt on the whole approach. Although gravity surely interferes with the other forces in some way or other, they question whether the effect is so straightforward as a tweak to the force strength. The rocococity of gravity should infect the other forces.

What is going on?

1) Dim-reg vs cutoff regularization – why the difference?

2) Running with $(\text{Energy})^2$
 - dimensional coupling constant

3) Why don't other effective field theories use running couplings?

4) **Application in a physical process**
 - does the running coupling work?

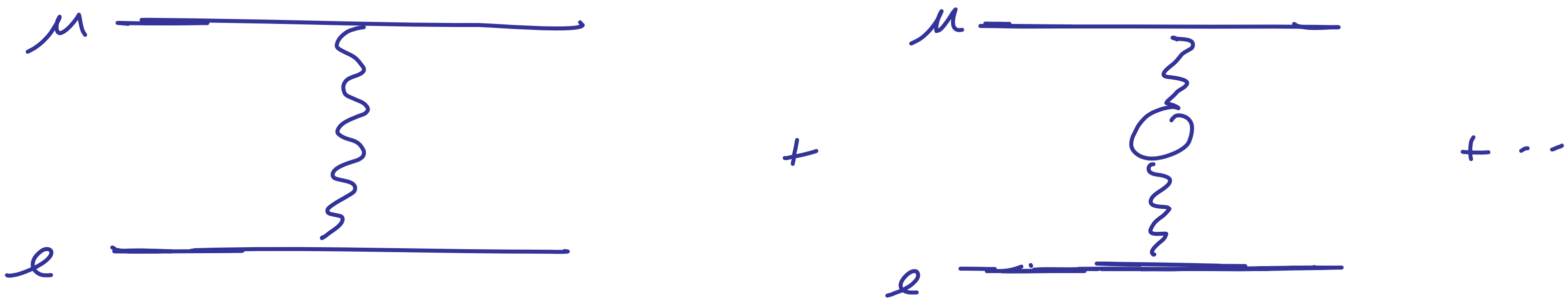
Quick review – running couplings

1) Physical processes - **useful**

2) Renormalization of the charge - **universal**

3) Wilsonian (only later, if time and interest suggest)

1) Physical processes – “useful”



Processes modified by vacuum polarization

$$\frac{e_0^2}{1+\Pi(q^2)} = \frac{e^2}{1+\hat{\Pi}(q^2)} = e^2(q^2)$$

with

$$\begin{aligned}
\Pi(q) &= \frac{e_0^2}{6\pi^2}\Bigg[\frac{1}{\epsilon} + \ln\sqrt{4\pi} - \frac{\gamma}{2} \\
&\quad -3\int_0^1 dx\; x(1-x)\ln\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right) + \mathcal{O}(\epsilon)\Bigg] \\
&= \frac{e_0^2}{6\pi^2}\begin{cases} \dfrac{1}{\epsilon} + \ln\sqrt{4\pi} - \dfrac{\gamma}{2} + \dfrac{5}{6} - \dfrac{1}{2}\ln\dfrac{-q^2}{\mu^2} + \ldots & (|q^2| \gg m^2), \\ \dfrac{1}{\epsilon} + \ln\sqrt{4\pi} - \dfrac{\gamma}{2} - \dfrac{1}{2}\ln\dfrac{m^2}{\mu^2} + \dfrac{q^2}{10m^2} + \ldots & (m^2 \gg |q^2|). \end{cases}
\end{aligned}$$

Renormalization of the charge:

$$e^2 = \frac{e_0^2}{1+\Pi(0)} \simeq e_0^2[1-\Pi(0)]$$

Residual effect gives running coupling:

$$\frac{e^2}{1+\hat{\Pi}(q^2)} = e^2(q^2)$$ with $$\hat{\Pi}(q^2) = -\frac{\alpha}{3\pi}\ln(-q^2/m^2)$$

Beta function:

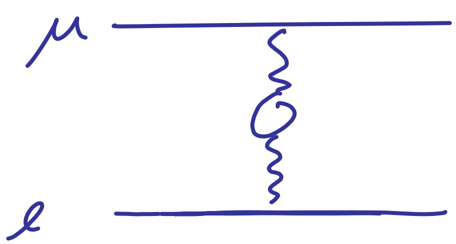
$$\beta(e) = q^2\frac{\partial e}{\partial q^2} = \frac{e^3}{12\pi^2}$$

Integrating the beta function:

$$\alpha(q^2) = \frac{3\pi}{\ln(\Lambda^2/q^2)}$$

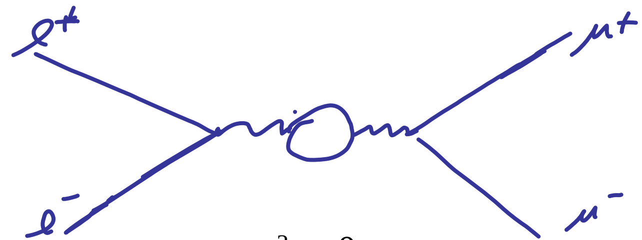
Note for later applications:

Space-like vs time-like processes:



$q^2 < 0$

vs



$q^2 > 0$

Imaginary part gives unitarity via physical intermediate states;

$\ln(-s) = \ln(|s|) - i\pi\theta(s)$ yields $\mathrm{Im}\, t_l^I(s) = |t_l^I(s)|^2$

Running coupling is the same for both space-like and time-like reactions

$$\alpha(q^2) = \frac{3\pi}{\ln(\Lambda^2/|q^2|)}$$

2) Renormalization of the charge – "universal"

Dimensional regularization:

$$\Pi(q^2) = \frac{e_0^2}{12\pi^2}\left(\frac{\mu^2}{-q^2}\right)^{\epsilon/2}\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma + \frac{5}{3} + \mathcal{O}(\epsilon)\right]$$
$$= \frac{e_0^2}{12\pi^2}\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma + \frac{5}{3} - \ln\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\epsilon)\right]$$

One can read off the logarithms just knowing the divergences

Explains the **universality** of the running coupling constant
- tied uniquely to the renormalization of the charge

Cutoff regularization:

$$\Pi(q^2) = -\frac{e_0^3}{12\pi}\ln(-q^2/\Lambda^2)$$

The cutoff dependence must trace the q^2 dependence

General Relativity as an Effective Field Theory

Effective Field Theory

- general and practical technique
- separates known low energy physics from high energy phyiscs
- I will present only EFT with dimensionful coupling (like gravity)

What to watch for:

- presence of new operators in Lagraingian of higher order in energy expansion
- loops generate higher powers of the energy
- what gets renormalized (hint: the higher order operators)

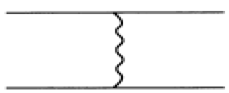
Important fact used in power counting:

$$R \sim \partial g \partial g + ... \sim E^2$$

Key Steps

1) High energy effects are local (when viewed at low E)

Example = W exchange



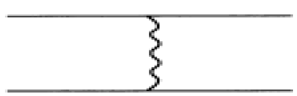
=> local 4 Fermi interaction

Even loops

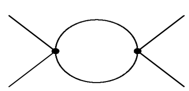
=> local mass counterterm

Low energy particle propagate long distances:

Photon:



Not local $\quad V \sim \frac{1}{q^2} \sim \frac{1}{r}$



Even in loops – cuts, imag. parts….

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots.$$

Even if you don’t know the true effect, you know that it is local

-use most general local Lagrangian

2) Energy Expansion

Order lagrangians by powers of (low scale/high scale)^N

Only a finite number needed to a given accuracy

Then:

Quantization: use lowest order Lagrangian

Renormalization: **

-U.V. divergences are **local**

- can be absorbed into couplings of local Lagrangian

Remaining effects are predictions

General Procedure

1) Identify Lagrangian

-- most general (given symmetries)

-- order by energy expansion

2) **Calculate and renormalize**

-- start with lowest order

-- renormalize parameters

3) **Phenomenology**

-- measure parameters

-- residual relations are predictions

Note: Two differences from textbook renormalizable field theory:

1) no restriction to renormalizable terms only

2) energy expansion

Parameters

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

1) Λ = cosmological constant

$$\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$$

$$M_P = 1.22 \times 10^{19} \text{ GeV}$$

-this is observable only on cosmological scales
-neglect for rest of talk
-interesting aspects

2) Newton's constant

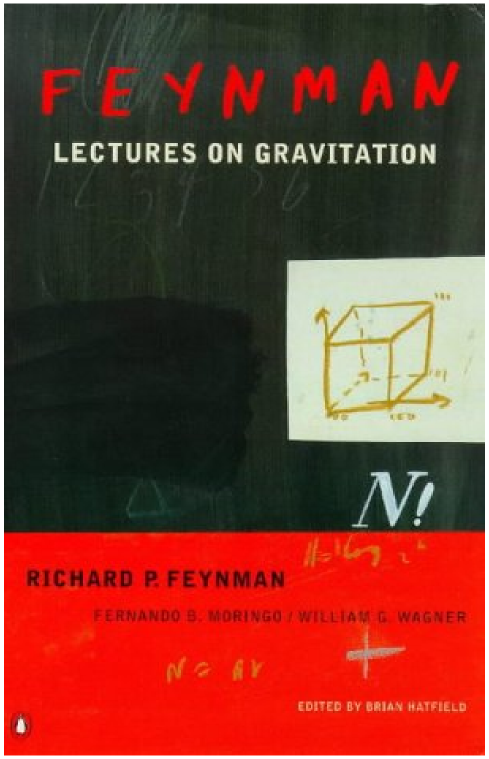
$$\kappa^2 = 32\pi G$$

3) Curvature –squared terms c_1, c_2

- studied by Stelle
- modify gravity at very small scales
- essentially unconstrained by experiment

$$c_1, c_2 \leq 10^{74}$$

Quantizing general relativity



Feynman quantized gravity in the 1960's

Quanta = gravitons (massless, spin 2)

Rules for Feynman diagrams given

Subtle features:

$h_{\mu\nu}$ has 4x4 components – only 2 are physical DOF

-need to remove effects of unphysical ones

Gauge invariance (general coordinate invariance)

- calculations done in some gauge
- need to maintain symmetry

In the end, the techniques used are very similar to other gauge theories

Quantization

"Easy" to quantize gravity:

-Covariant quantization   Feynman deWitt

-gauge fixing

-ghosts fields

-Background field method   't Hooft Veltman

-retains symmetries of GR

-path integral

Background field:

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\ g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu_\lambda h^{\lambda\nu} + \ldots \end{aligned}$$

Expand around this background:

$$S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \ldots \right]$$

$$\begin{aligned} \mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[\bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\ \mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\ &\quad + \bar{R} \left(\frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left(2h^\lambda_\mu h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu} \end{aligned}$$

Linear term vanishes by Einstein Eq.

$$\bar{R}^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$$

Performing quantum calculations

Quantization was straightforward, but what do you do next?
- calculations are not as simple

Next step: Renormalization

- divergences arise at high energies
- not of the form of the basic lagrangian
- key role of dimensionful coupling constant

$$\mathcal{M} = \mathcal{M}_0 + b\frac{q^2}{M_P^2}$$

Solution:

- renormalize divergences into parameters of the most general lagrangian ($c_1, c_2 \ldots$)

Power counting theorem:

- each graviton loop $\Longrightarrow$ 2 more powers in energy expansion
- 1 loop $\Longrightarrow$ Order $(\partial g)^4$
- 2 loop $\Longrightarrow$ Order $(\partial g)^6$

Renormalization

One loop calculation: ‘t Hooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

dim. reg. preserves symmetry

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity
“one loop finite”
since $R_{\mu\nu}$=0

Note: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein, Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses.

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q))\\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}$$

Potential found using from

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int\frac{d^3q}{(2\pi)^3}e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura
Ohta et al

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

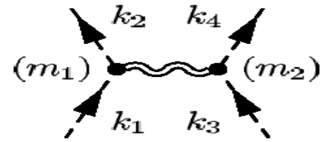
short range

Non-analytic

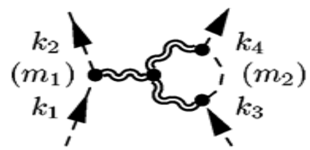
analytic

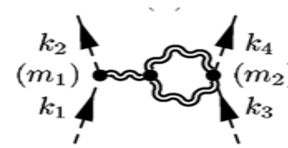
The calculation:

Lowest order:

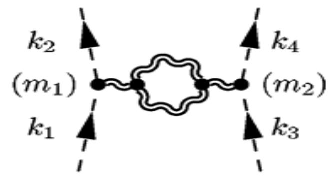


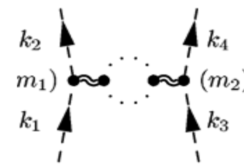
Vertex corrections:



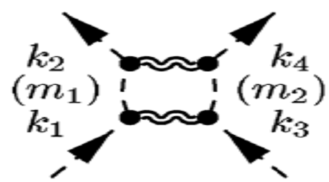


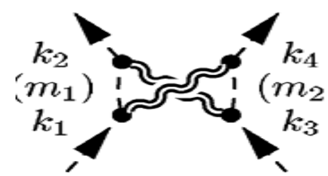
Vacuum polarization:
(Duff 1974)



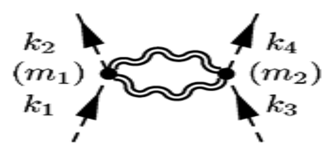


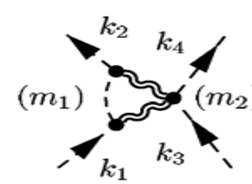
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$
\begin{aligned}
M_{5(a)+5(b)}(\vec{q}) &= 2G^2 m_1 m_2 \left(\frac{\pi^2 (m_1 + m_2)}{|\vec{q}|} + \frac{5}{3} \log \vec{q}^2 \right) \\
M_{5(c)+5(d)}(\vec{q}) &= -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2
\end{aligned}
$$

Sum diagrams:

$$
\boxed{V(r) = -\frac{G m_1 m_2}{r} \left[1 + 3 \frac{G(m_1 + m_2)}{r} + \frac{41}{10\pi} \frac{G\hbar}{r^2} \right]}
$$

Gives precession
of Mercury, etc
(Iwasaki ;
Gupta + Radford)

Quantum
correction

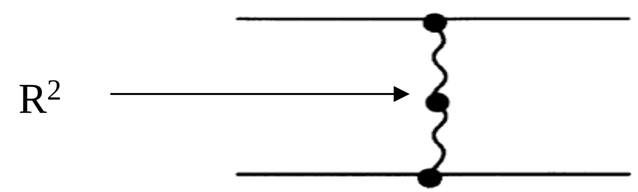
Where did the divergences go?

Recall: divergences like local Lagrangian ~R^2

Also unknown parameters in local Lagrangian ~c_1,c_2

But this generates only “short distance term”

Note: R^2 has 4 derivatives $\quad R^2 \sim q^4$



Then: Treating R^2 as perturbation

$$V_{R^2} \sim G^2 Mm \;\frac{1}{q^2}\; q^4 \;\frac{1}{q^2} \sim \text{const.} \to G^2 Mm\delta^3(x)$$

Local lagrangian gives only short range terms – renormalized couplings here

Equivalently could use equations of motion to generate contact operator:

$R_{\mu\nu} \to T_{\mu\nu}$ generates local operator $c_i R^2 \to T_{\mu\nu}T^{\mu\nu}$

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Duff, JFD; Muzinich and Vokos; Hamber and Liu;
   Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

5) Why not done 30 years ago?
   - power of effective field theory reasoning

Summary for purpose of this talk:

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

$$V(r) = -\frac{GMm}{r} \left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2 c^3} \right] + cG^2 Mm\delta^3(r)$$

1) Loops do not modify the original coupling

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\}$$

2) **Loops involved in renormalization of higher order coupling**

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

3) **Matrix elements expanded in powers of the momentum**

$$V(q^2) = \frac{GMm}{q^2} \left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2 \right]$$

4) **Corrections to lowest order have two features**

- higher order operators and power dependence
- loops also generate logarithms **at higher order**

Running couplings and gravity:

1) Usual RGE in EFT

2) Direct calculation of matrix elements

3) Critique of cut-off renormalization interpretation

4) Is the idea of a gravitationally corrected running coupling useful?

<u>Standard EFT practice and Renormalization Group</u>

$$U = exp[i\frac{\tau\cdot\phi}{F_\pi}]$$

Closest analogy is chiral perturbation theory:

- also carries dimensionful coupling and similar energy expansion

$$\mathcal{L} = F^2 Tr(\partial_\mu U \partial^\mu U^\dagger) + \ell_1 [Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 + \ell_2 Tr(\partial_\mu U \partial_\nu U^\dagger)\, Tr(\partial^\mu U \partial^\nu U^\dagger) + \ldots\ldots$$

- renormalization and general behavior is analogous to GR

$$\Delta\mathcal{L} = \frac{1}{192\pi^2(d-4)}\left[[Tr(D_\mu U D^\mu U^\dagger)]^2 + 2Tr(D_\mu U D_\nu U^\dagger)\, Tr(D^\mu U D^\nu U^\dagger)\right]$$

<u>**RGE**</u>: (Weinberg 1979, Colangelo, Buchler, Bijnens et al, M. Polyakov et al)

- Physics is independent of scale μ in dim. reg
- One loop – $1/\epsilon$ goes into renormalizaton of ℓ_i
    - comes along with specified $\ln \mu$ and $\ln q^2$ dependence
- Even better at two loops
    - two loops (hard) gives q^4/ϵ^2 terms – correlated with $q^4 \ln^2 q^2/\mu^2$
    - cancelled by one loop (easy) calculation using ℓ_i
    - RGE fixes leading $(q^2 \ln q^2)^n$ behavior

<u>This has been explored in depth</u>:

TABLE I: Table of $I=0$ LL coefficients for the $4D$ σ-model, $\omega_{nl}^{I=0} \cdot (N-1)^{-1}$

$n \setminus l$	0	2	4
1	1		
2	$\frac{N}{2} - \frac{1}{9}$	$\frac{5}{18}$	
3	$\frac{N^2}{4} - \frac{61N}{144} + \frac{59}{144}$	$-\frac{13N}{144} + \frac{13}{48}$	
4	$\frac{N^3}{8} - \frac{631N^2}{2700} + \frac{46279N}{194400} - \frac{13309}{194400}$	$\frac{173N^2}{2160} - \frac{4313N}{38880} + \frac{5333}{38880}$	$\frac{N^2}{200} - \frac{49N}{5400} + \frac{8}{675}$
5	$\frac{N^4}{16} - \frac{136N^3}{675} + \frac{2498743N^2}{7776000} - \frac{3083771N}{11664000} + \frac{619889}{4665600}$	$-\frac{1417N^3}{40320} + \frac{481367N^2}{3628800} - \frac{727373N}{4082400} + \frac{1071107}{6531840}$	$-\frac{N^3}{280} + \frac{9787N^2}{756000} - \frac{449681N}{27216000} + \frac{81007}{5443200}$

<u>For our purposes:</u>

- Lowest order operator does not run
- Higher order operator gets renormalized
- With renormalization comes ln μ dependence
- Can exploit for leading high power x leading log
- Tracks higher order log dependence ($q^2 \ln q^2$)
- Multiple higher order operators – different processes have different effects

Also – process dependence

Wide variety of processes are described by ℓ_i
- different combinations of s, t, u, … and ℓ_i enter into each process
- the single and double logs are also process dependent

Again a reason for not using a universal running coupling in EFT

Now consider gravity corrections to gauge interactions:

(Anber,
El Houssienny,
JFD)

-we have done this in great detail for Yukawa
- I will be schematic for gauge interactions in order to highlight key points

Lowest order operator:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu\psi A^\mu \qquad \mathcal{L}_{l.o.} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Higher order operator

$$\mathcal{L}_{h.o} = c_2\bar{\psi}\gamma_\mu\psi\partial^2 A^\mu \qquad \mathcal{L}_{h.o.} = -k\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$$

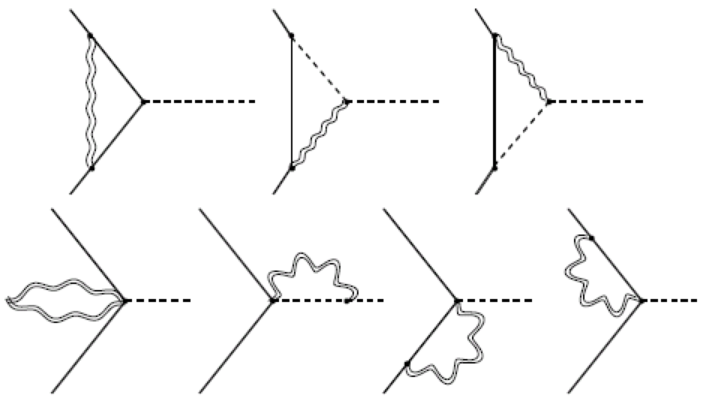
Equations of motion

$$\partial^2 A^\mu = J^\mu$$

Equivalent contact operator:

$$\mathcal{L}_{h.o} = c_2\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi = c_2 J_\mu J^\mu$$

Direct calculation



Vertex (fermions on shell) found to be:

$$\mathcal{V} = \bar{u}\left[e\gamma^{\mu} + a(q^2)e\kappa^2 q^2 \gamma^{\mu}\right]u$$

with

$$a(q^2) = a_0\left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{\gamma}{2} - \frac{1}{2}\ln(-q^2/\mu^2)\right]$$

Physical process:

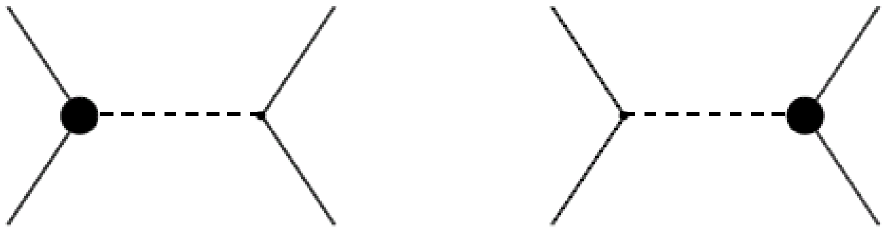


FIG. 4: Tree diagram for the on-shell scattering processes involving fermion.The filled circle denotes the set of vertex renormalization diagrams.

Overall matrix element:

$$\begin{aligned}\mathcal{M} &= \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right]u\,\frac{1}{q^2}\,\bar{u}\gamma_\mu u + h.c. + c_2\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u \\ &= \bar{u}\gamma^\mu u\;\bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2\ln(-q^2/\mu^2)\right)\right]\end{aligned}$$

Think of the Lamb shift

Describes the two reactions:

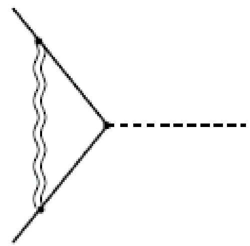
$$\begin{aligned} q^2 &> 0 \;\; \text{for}\, f + \bar{f} \to f + \bar{f} \\ &< 0 \;\; \text{for}\, f + f \to f + f \end{aligned}$$

Renormalization of higher order operator:

$$c_2^r = c_2 - a_0\left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{\gamma}{2}\right]$$

Lamb shift analogy:

- Corrections to vertex diagram gives q^2 dependent terms



$$\mathcal{V}^\mu = \bar{u}\left[e\gamma^\mu\left(1 + a\frac{q^2}{m_e^2}\right)\right]u$$

$$\mathcal{M} = \bar{u}\left[e^2\gamma^\mu\left(1 + a\frac{q^2}{m_e^2}\right)\right]u\;\frac{1}{q^2}\;\bar{u}\gamma_\mu u$$

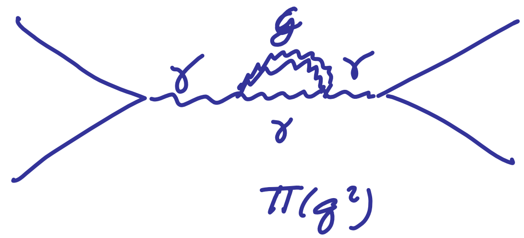
Leads to a contact interaction:

$$\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi$$

Influences S states only

- Not counted as a running coupling

Similar in the modification of photon properties



Photon propagator correction:

$$\Pi = c\kappa^2 q^4$$

Like $\mathcal{L}_{h.o.} = -k\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$

Again looks like contact interaction:

$$\mathcal{M} = \bar{u}\gamma^\mu u \, \frac{1}{q^2} \, \left[e^2 + e^2 c\kappa^2 q^2\right] \, \frac{1}{q^2} \, \bar{u}\gamma_\mu u + c_2 \bar{u}\gamma^\mu u \bar{u}\gamma_\mu u$$

Can this be packaged as a running coupling?

Propose:

$$e^2(M^2) = e^2\left[1 + b_0\kappa^2 M^2\right]$$

Is the amplitude equal to?

$$\begin{aligned}\mathcal{M} \quad ?=? \quad & \bar{u}\gamma^\mu u \; \bar{u}\gamma_\mu u \left[\frac{e^2(M^2)}{q^2} + (c_2')\right] \\ ?=? \quad & \bar{u}\gamma^\mu u \; \bar{u}\gamma_\mu u \left[\frac{e^2}{q^2} + \frac{e^2 2b_0\kappa^2 M^2}{q^2} + (c_2')\right]\end{aligned}$$

$$\begin{aligned} q^2 \quad &> \quad 0 \quad \text{for}\, f + \bar{f} \to f + \bar{f} \\ &< \quad 0 \quad \text{for}\, f + f \to f + f \end{aligned}$$

Recall

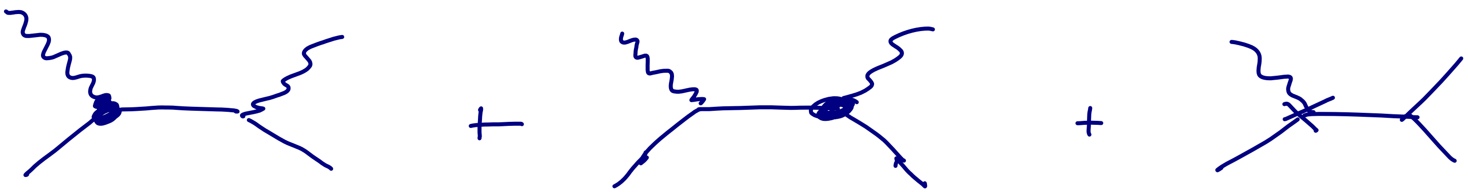
$$\begin{aligned}\mathcal{M} \quad &= \quad \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right] u \,\frac{1}{q^2}\, \bar{u}\gamma_\mu u + h.c. + c_2 \bar{u}\gamma^\mu u \bar{u}\gamma_\mu u \\ &= \quad \bar{u}\gamma^\mu u \; \bar{u}\gamma_\mu u \left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2 \ln(-q^2/\mu^2)\right)\right]\end{aligned}$$

You can make the definition work for either process but not for both

- No universal definition

Other forms of non-universality:

Other processes have other divergences and other operators:



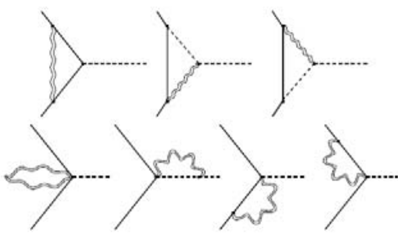
Lowest order:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu\psi A^\mu$$

Different higher order operator is relevant

$$\mathcal{L}_{h.o.} = c_3 A^\mu \bar{\psi}\gamma_\mu \partial^2 \psi$$

Calculation of the vertex corrections:



$$\mathcal{V} = \bar{u}\left[e\gamma^\mu + b(p^2)e\kappa^2 p^2 \gamma^\mu\right] u$$

Different value for the correction (verified in Yukawa case)

$$b(q^2) \neq a(q^2)$$

Different correction to matrix element

$$\mathcal{M} \;=\; e^2\epsilon_\mu\epsilon_\nu \left(\bar{u}\gamma^\mu\left[1 + b((q+p_1)^2)\kappa^2(q+p_1)^2\right]\frac{1}{\not{q}+\not{p}_1}\gamma^\nu u + h.c. + c_3\bar{u}\gamma^\mu(\not{q}+\not{p}_1)\gamma^\nu\gamma_\mu u\right)$$

What about calculations with dimensionful cutoff?

- above agrees with EFT logic and dim-reg conclusions
- new papers with cutoff make very different claim

Work with:

$$A_\mu \to \frac{1}{e_0} A_\mu \quad , \quad D_\mu \to \partial_\mu + i A_\mu \quad , \quad \mathcal{L} \to \frac{1}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Quadradic dependence on the cutoff:

- different methods but find effective action

$$\mathcal{L} = \frac{1 + c\kappa^2 \Lambda^2}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + b \ln(\Lambda^2) F_{\mu\nu} \partial^2 F^{\mu\nu}$$

Toms and others interpret this as a running coupling constant

$$e^2(\Lambda) = e^2(1 - c\kappa^2 \Lambda^2)$$

$$\beta(e^2) = \Lambda \frac{\partial e^2}{\partial \Lambda} = -c\kappa^2 e^2 \Lambda^2$$

But this cutoff dependence is unphysical artifact

- wavefuntion/charge renormalization
- disappears from physical processes

$$\frac{e_0}{4\pi(1+c\kappa^2\Lambda^2)} = \frac{e^2}{4\pi} = \frac{1}{137}$$

The quadratic cutoff dependence disappears in physical processes

$$\mathcal{M} = \frac{e_0^2(1-c\kappa^2\Lambda^2)}{q^2} + a\kappa^2 e^2 \frac{1}{q^2} q^4 \frac{1}{q^2}\left[\ln\frac{-q^2}{\Lambda^2} + ...\right] + c_2$$

After renormalization, obtain exactly the dim-reg result:

$$\mathcal{M} = \frac{e^2}{q^2} + \left(c_2^r(\mu) + a\kappa^2 e^2 \ln\frac{-q^2}{\mu}\right)$$

1) **Quadratic cutoff dependence is NOT running of charge**
2) **Agreement of different schemes**

<u>Summary of gauge coupling section:</u>

-We have addressed renormalization of effective field theories

-Organized as a series of operators

- **Running coupling is NOT an accurate description of quantum loops in the EFT regime**

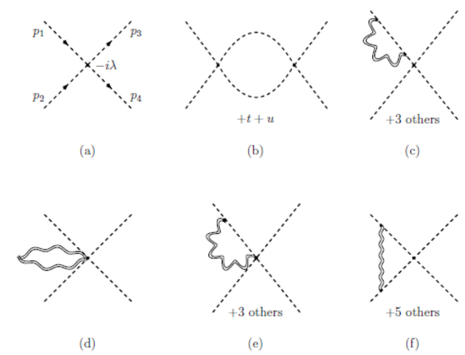
- Confusion in the literature is understood as misunderstanding of results calculated with a dimensionful cutoff

-There is no scheme dependence to physical processes

Could gravity influenced running couplings **eventually** play a role?
 - **Maybe after EFT regime**

How a running coupling <u>could</u> work with gravity- $\lambda\varphi^4$:

$\lambda\phi^4$ is special



Direct and crossed channels both occur in every amplitude and in every loop

Higher order operator vanishes on-shell

$\mathcal{L}_{\lambda_1} \equiv -\lambda_1 \phi^2 \partial_\mu \phi \partial^\mu \phi$ renormalized at one loop $\mathcal{M}_{div} \sim \lambda\kappa^2 \frac{1}{\epsilon}(s+t+u)$

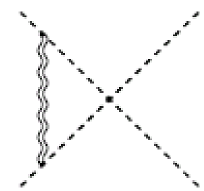
-but vanishes since $s+t+u=0$

- mixing when renormalized at high renormalization scale
  either on shell $s = 2E^2 \quad t = u = -E^2$
  or off shell $p_1^2 = p_2^2 = p_3^2 = p_4^2 = -M^2$

Definition has no obvious flaws at one loop

$$\mathcal{A}_{(f)} = -\frac{i\kappa^2\lambda}{2\,(4\pi)^2}\left[s\log\left(\frac{-s}{\mu^2}\right)+t\log\left(\frac{-t}{\mu^2}\right)\right.$$
$$\left.+u\log\left(\frac{-u}{\mu^2}\right)\right]$$
$$+\frac{i\kappa^2\lambda}{2\,(4\pi)^2}\left[s^2C(s)+t^2C(t)+u^2C(u)\right] \leftarrow \text{IR}$$

from



+5 others

(f)

Can define renormalized coupling at $s = 2E^2 \quad t = u = -E^2$

$$-i\lambda(E) = -i\lambda - \frac{3i\lambda^2}{2\,(4\pi)^2}\log\left(\frac{E^2}{\mu^2}\right) - \frac{i\log(2)\kappa^2\lambda}{(4\pi)^2}E^2$$

Because of s,t,u symmetry, and vanishing next order operator amplitude does not have the problems of gauge theory amplitudes

$$\mathcal{A}_{(f)} = -i\lambda(E) - \frac{i\lambda^2(E)}{2\,(4\pi)^2}\left[\log\left(\frac{s}{2E^2}\right)+\log\left(\frac{-t}{E^2}\right)\right.$$
$$\left.+\log\left(\frac{-u}{E^2}\right)+i\pi\right]$$
$$-\frac{i\kappa^2\lambda(E)}{2\,(4\pi)^2}\left[s\log\left(\frac{-s}{2E^2}\right)+t\log\left(\frac{-t}{E^2}\right)\right.$$
$$\left.+u\log\left(\frac{-u}{E^2}\right)\right] \quad +\frac{i\kappa^2\lambda(E)}{2\,(4\pi)^2}\left[s^2C(s)+t^2C(t)+u^2C(u)\right]$$

Under consideration: Anber, JFD

Gravity itself and asymptotic safety

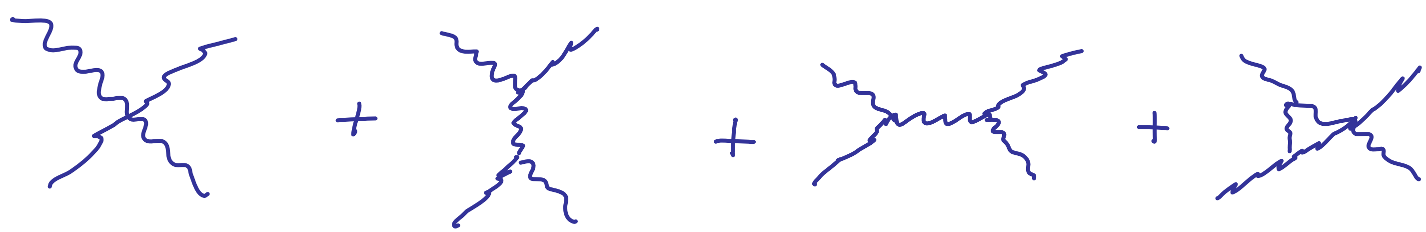
Generally – **can we define a running $G(q^2)$ in perturbative region?**

,   ,

A.S. = Hypothesis of Eudlidean UV fixed point

$$g = G(k_E)k_E^2 \to g_* \qquad G(k_E^2) \to \frac{g_*}{k^2} \qquad G(k_E^2) \sim \frac{G}{1 + Gk_E^2/g*}$$

Pure gravity may be more like: $\lambda\phi^4$



- s, t u symmetry
- next order operator vanishes R^2
- polarization variables may spoil perfect symmetry

Lets look at graviton –graviton scattering

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\,\frac{\kappa^2 s^3}{tu}$$

One loop: Dunbar and Norridge

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\,\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\,\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\qquad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Infrared divergences are not issue:

JFD +
Torma

-soft graviton radiation
-made finite in usual way
1/ϵ -> ln(1/resolution) (gives scale to loops)
-cross section finite

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \qquad (29)$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

<u>Correction is positive in physical region:</u>

- **increases** strength of interaction

$$s = 2E^2 \quad t = u = -E^2$$

$$\text{“}G(E)\text{”} = G\left[1 + \frac{4GE^2}{\pi}\left(\ln^2 2 + \frac{2297}{1440}\right)\right]$$

Gravity matter coupling again has kinematic problem:

Recall: $$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2\right]$$

Including all diagrams:

$$\text{“}G(q^2)\text{”} = G\left[1 + \frac{41}{20\pi}Gq^2 \ln q^2\right]$$

Excluding box plus crossed box:

$$\text{“}G(q^2)\text{”} = G\left[1 - \frac{347}{60\pi}Gq^2 \ln q^2\right]$$

Either way

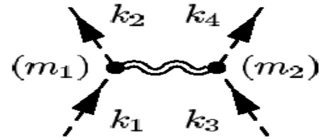
– kinematic problem, plus result seems disconnected from pure gravity
- useful and universal?

A.S. community has not yet addressed addressed matter couplings:

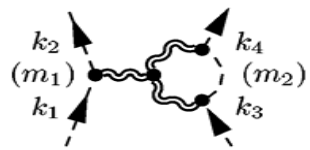
- do matter couplings track that of pure gravity?

Components of log in matter coupling

Lowest order:



Vertex corrections:



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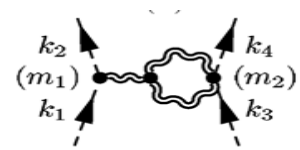
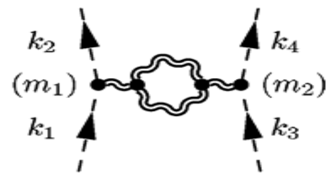


Vacuum polarization:
(Duff 1974)



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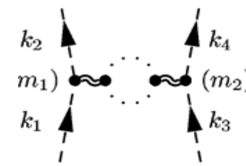
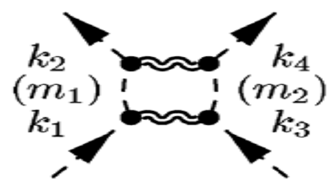


Box and crossed box



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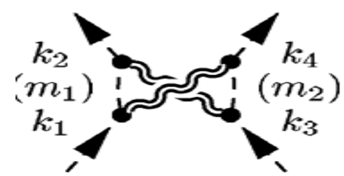
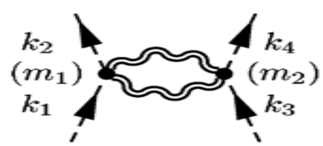


Others:



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