Boundary layer theory for the Navier-Stokes equation

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Objectives. The aim of this course is to give mathematical insights into a central problem of fluid mechanics: the understanding of fluid flows around obstacles. This problem appears in many situations of practical interest, for instance the spreading of air around the wings of an airplane. The main difficulty comes from high speed, or low viscosity fluid flows. Mathematically, one needs to describe the asymptotics, as ν goes to zero, of the Navier-Stokes equation

$$(NS_\nu) \qquad \left\{ \begin{aligned} \partial_t u + u\nabla u + \nabla p - \nu\Delta u &= 0,\ t > 0,\ x \in \Omega\ , \\ \nabla \cdot u &= 0,\ t > 0,\ x \in \Omega\ , \\ u_{|t=0} = u_0\ ,\ \ u_{|\partial\Omega} &= 0\ . \end{aligned} \right.$$

in a domain Ω with boundary. As ν goes to zero, it is known from experiments that the velocity u_ν concentrates near $\partial\Omega$ in a thin zone near the boundary, called a *boundary layer*. The mathematical description of this layer, and its impact on the asymptotics $\nu \to 0$ is still poorly understood. In particular, it is not known in general whether or not a sequence of smooth solutions (u_ν) of (NS_ν) converges to a solution of the Euler equation.

During the course we shall describe the main mathematical results on this convergence problem, namely:

- 1. The convergence criteria of Kato [3];
- 2. The Prandtl approach for proving convergence, and the well-posedness results of Oleinik on the Prandtl model for the boundary layer [4];
- 3. The justification of the Prandtl approach in the analytic setting [5];
- 4. Instability problems in the Sobolev setting [1, 2].

Prerequisites. Acquaintance with some basic notions of mathematical fluid mechanics (local existence of strong solutions for Navier-Stokes and Euler, or global existence of weak solutions for Navier-Stokes) is recommended, but not necessary.

References

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[3] T. K , *Remarks on Zero Viscosity Limit for Nonstationary Navier-Stokes Flows with Boundary,* S.S. Chern (ed.), Seminar on nonlinear PDE, MSRI, 1984.

[4] O.A. O , V.N. S , *Mathematical Models in Boundary Layer Theory,* Applied Math. Math. Computation, **15** Chapman & Hall/CRC, Boca Raton, FL, 1999.

[5] M. S , R.E. C , *Zero Viscosity Limit for Analytic Solutions of the Navier-Stokes Equation on a Half-space,* Commun. Math. Phys. **192** (1998), 433–491.