LARGE TIME DYNAMICS FOR THE ONE DIMENSIONAL SCHRÖDINGER EQUATION IHES, JULY 2010

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1. Introduction, presentation of the problem

In this course I will present some recent results with L. Thomann and N. Tzvetkov on the behaviour of solutions to Schrödinger equations with random initial data. The main question I want to address is the following: Are solutions to Schrödinger equations better behaved when one consider initial data randomly chosen (in some sense) than what would be predicted by the deterministic theory? To my knowledge the first result known in this direction is due to Rademacher/Kolmogorov/Paley-Zygmund, and states that random series on the torus enjoy better L^p bounds that the deterministic bounds. These lectures are somehow a natural extension on the partial differential equations field of these harmonic analysis results. We shall use some basic results from probability theory. The non linear Schrödinger I will be interested in, is the following one dimensional non linear harmonic oscillator

$$
\begin{cases} i\partial_t u + \Delta u - |x|^2 u = |u|^{r-1} u, & (t,x) \in \mathbb{R} \times \mathbb{R}, \\ u(0,x) = f(x), \end{cases} \tag{1.1}
$$

where $r > 1$ is the order of the non linearity. On a deterministic point of view, this equation is well posed in $L^2(\mathbb{R})$ as soon as $p \leq 5$, and the assumption $p \leq 5$ is known to be optimal in some sense (see the works by Christ-Colliander-Tao and Burq-Gérard-Tzvetkov in slightly different contexts). However, we shall prove, that for all non linearities $|u|^{p-1}u$, not only is the equation well posed for a large set of initial data whose Sobolev regularity is below L^2, but also that the flows enjoys very nice large time behaviour (in a probabilistic sense).

References

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