Hidden Convexity in Nonlinear PDEs from Geometry and Physics

Yann B (CNRS-Université de Nice)

The purpose of the course is to analyze several examples of nonlinear PDEs -with both strong geometric and physical features- which enjoy a hidden con- vex structure. Robust existence and uniqueness results can be unexpectedly obtained for very general data. Of course, as usual, regularity issues are left over as a hard post-process, but, at least, existence, uniqueness and stability results are obtained in a large, global, framework.

We will discuss:

- 1. The real Monge-Ampère Equation (we will show how the convex structure is related to *Optimal Transport Theory*);
- 2. The Euler Equations of Fluid Mechanics (that describe the motion of inviscid, incompressible fluids and provide the most famous example of a geodesic flow in infinite dimension) and their *hydrostatic* and *semi-geostrophic* limits;
- 3. The Born-Infeld System (a non-linear electromagnetic model introduced in 1934, playing an important role in high energy Physics since the 1990's).

References

[1] V.I. A , B. K , *Topological Methods in Hydrodynamics,* Applied Math. Sciences **125**, Springer-Verlag, 1998.

[2] C. D , *Hyperbolic Conservation Laws in Continuum Physics,* Springer-Verlag, 2005.

[3] C. V , *Topics in Optimal Transportation*, Grad. Studies in Math **58**, Amer. Math. Soc,, 2003.

[4] G. B , C. D , P. L , T.P. L , *Recent Mathematical Methods in Nonlinear Wave Propagation,* Lect. Notes Math. **1640** C.I.M.E., Springer-Verlag, 1994.

[5] Y. B , *Topics on Hydrodynamics and Volume Preserving Maps,* Handbook of mathematical fluid dynamics, **II**, 55–86, North-Holland, 2003.