David Ruelle: Chaos Theory and Historical Probabilities.

Nature presents us with a bewildering variety of complex systems, from the Earth's atmosphere to the stock market, and to the anatomy of the common fly. These systems are complex in different ways and for different reasons. One cannot hope that a general theory of complexity will explain them all at the same time. But some theoretical ideas may give us an understanding of some classes of complex systems. So the idea of biological evolution helps us understand the complexity of living beings. And the idea of chaos helps us make sense of some complicated, irregular, time evolutions like the flow of a turbulent fluid.

How can you obtain oscillations with a complicated, irregular, time dependence? One way (but not the most interesting) is to put together a large number of oscillators with different frequencies. Another way is to use just a few oscillators (say three), and throw in a nonlinear interaction, resulting in a deterministic time evolution with *sensitive dependence on initial condition.* What is that? In a deterministic time evolution, if you know the state $x(0)$ of the system at time 0, you can predict its state $x(t)$ at time t with complete precision. But if there is an imprecision $\epsilon(0)$ on the initial condition $x(0)$ the imprecision $\epsilon(t)$ on $x(t)$ may grow *exponentially* with t, at least for a while. This is sensitive dependence on initial condition, and you can see it if the initial condition is that of a pencil that you try to balance on its tip: depending on details of the initial condition it falls on one side or the other. The surprise is that, for some systems, there is sensitive dependence on initial condition whatever the initial condition. This is called *chaos*: a chaotic system is everywhere unstable. Obviously, the evolution in time of a chaotic system has limited predictability. Also, the system exhibits irregular oscillations (because if they were regular they would be predictable).

The mathematical possibility of chaos was understood a hundred years ago by J. Hadamard and H. Poincaré, but theory and applications were developed more recently. Apart from an isolated and remarkable paper of the meteorologist Ed Lorenz in 1963, the flow of papers starts in the 70's (D. Ruelle and F. Takens on the nature of turbulence, M. Feigenbaum on period doubling cascades, and many, many more). Computer studies soon made it clear that chaos is a rather common phenomenon, We now know that hydrodynamical turbulence is chaotic. This does not constitute a full theory of turbulence, but certainly puts the subject in a new perspective. We also know that the motion of the Earth's atmosphere has sensitive dependence on initial condition, and this sets fundamental limits on weather predictions. By the way note that, while we have good control of short term weather evolution, understanding the longer term climate evolution still largely escapes us. Among more recent successful applications of chaos, the most spectacular (due to J. Wisdom and J. Laskar) deal with the astronomy of the solar system. It is now established that parameters of the orbit of the Earth vary chaotically with a characteristic time of about five million years (geologically not a very long time) and this has implications for paleoclimates.

The basic concepts and methods of chaos have become widely accessible. And they have been successfully applied to various interesting, relatively simple, precisely controlled systems. Other attempted applications (to the stock market for example) have not yielded convincing results. Beyond the technically precise applications, there are however also philosophically important consequences. Indeed, as noted already by Poincaré, the uncertainty of our predictions justifies a probabilistic description of the world. But we can go further: it has become feasible to compute the probabilities of some historical events, such as the chance of rain on Piccadilly circus or Times Square, given what is known of the state of the Earth's atmosphere 48 hours earlier. Similarly, Laskar has computed probabilities associated with the history of the solar system.

This leads me to digress on a tantalizing topic, that of *historical probabilities.* Philosophers and scientists have speculated about the probability for life to arise on Earth, or elsewhere. Can one construct a probabilistic history of the world: the formation of the solar system, life on Earth, or human history? In principle one can always speak about the *conditional probability* of a certain event, given what is known about the state of the universe at some prescribed earlier time. Two things are needed for such a probability to be of interest: first that it can be estimated at all, and second that the estimate doesn't depend too much on insignificant details of the assumed earlier state of the universe. The estimate may involve not just chaos theory, but other ideas of smooth dynamics, quantum uncertainty, and further concepts yet to be developed. In this respect it is encouraging that some experts can make useful guesses on the probable evolution of the stock market, or the outcome of a political election.

Let us look at a concrete example. Instead of asking when biological evolution has decided that terrestrial vertebrates shall have four limbs, we might ask what the probability was at some prescribed earlier time, say the end of the Cambrian, that they would develop six limbs. More precisely, the question is this: can one hope for a reasonable description of life at the end of the Cambrian, such that the probability of the number of limbs of future terrestrial vertebrates can be estimated in a stable manner? Animals with six limbs would be interesting because they could adapt two limbs for manipulative purposes (centaurs) or for flying (dragons). But since centaurs and dragons do not exist, it is tempting to say that they must have had negligible probability to come into being, and dismiss the question. One cannot so easily dismiss the problem of how life on Earth would have evolved if the great cataclysm and extinction at the end of the Cretaceous had not taken place. Are such problems outside of the scope of scientific investigation? At the very least, their solution would require a considerable increase in depth of our understanding of biological evolution.

We must admit that questions of hypothetical history, or historical probabilities, are in many cases unanswerable, at least in our present state of knowledge. Yet, these questions are not *a priori* meaningless. It is possible that one can make sense of some probabilities that arise in the history of life on Earth. And we may hope to evaluate historical probabilities in other areas of interest. After all, such probabilities can already be estimated in the simple situations of weather prediction and astronomy of the solar system.