Indsheaves, temperate holomorphic functions and irregular Riemann-Hilbert correspondence

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Aim of the course

The aim of the course is to describe the Riemann-Hilbert correspondence for holonomic $\mathscr{D}$-modules in the irregular case following [DK13], and its applications to integral transforms with irregular kernels, following [KS14]. On a complex manifold X, this approach makes an essential use of the indsheaf of temperate holomorphic functions $\mathcal{O}_X^{\mathrm{t}}$ (this indsheaf can also be viewed as a sheaf on the subanalytic site X_{sa} associated with X). Unfortunately, the sheaf $\mathcal{O}_X^{\mathrm{t}}$ still does not contain enough informations to recover a holonomic $\mathscr{D}$-modules from the complex $\mathrm{R}\mathscr{H}om_{\mathscr{D}}(\mathscr{M}, \mathcal{O}_X^{\mathrm{t}})$ and one has to work with the enhanced ind-sheaf $\mathscr{O}_X^{\mathsf{E}}$, roughly speaking, the sheaf of solutions in $\mathcal{O}_{X\times\mathbb{C}_s}^{\mathrm{t}}$ of the equation $(\partial_s - 1)u = 0$. As an application, we show that the De Rham functor (with values in $\mathscr{O}_X^{\mathsf{E}}$) commutes with irregular kernels and we treat explicitly the Laplace transform.

Organization

The course will cover 12 hours along 6 weeks, from Thursday 12/02/2015 to Thursday 19/03/2015.

Contents

1. Indsheaves and subanalytic sheaves (after [KS01])
2. The indsheaf $\mathcal{O}_X^{\mathrm{t}}$ of temperate holomorphic functions (after [KS01, KS03])
3. Enhanced indsheaves (after [DK13], inspired by [Ta08])
4. The enhanced indsheaf $\mathscr{O}_X^{\mathsf{E}}$ (after [DK13])
5. The Riemann-Hilbert correspondence (after [DK13], using [Sa00, Mo09, Mo11, Ke10, Ke11])

6. Integral transform with irregular kernels and Laplace transform (after [KS14])

Prerequisites

The audience is supposed to be familiar with the basic language of derived categories and sheaf theory (see [SGA4, KS06]) and $\mathscr{D}$-module theory (see [Ka03]).

References

[DK13] A. D'Agnolo and M. Kashiwara, *Riemann-Hilbert correspondence for holonomic D-modules*, `arXiv:1311.2374v1`

[Ka03] M. Kashiwara, *D-modules and Microlocal Calculus,* Translations of Mathematical Monographs **217** American Math. Soc. (2003) xvi+254 pp.

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[KS14] ———, *Irregular holonomic kernels and Laplace transform,* arXiv:math.:1402.3642

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[Ke11] ———, *Good formal structures for flat meromorphic connections, II: Excellent schemes*, J. Amer. Math. Soc. **24** (2011) 183–229.

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[Mo11] ———, *Wild Harmonic Bundles and Wild Pure Twistor $\mathscr{D}$-modules*, Astérisque **340** (2011) x+607 pp.

[Sa00] C. Sabbah, *Équations différentielles à points singuliers irréguliers et phénomène de Stokes en dimension* 2, Astérisque **263** (2000) viii+190 pp.

[SGA4] S-G-A 4, Sém. Géom. Alg. (1963–64) by M. Artin, A. Grothendieck and J-L. Verdier, *Théorie des topos et cohomologie étale des schémas,* Lecture Notes in Math. **269, 270, 305** (1972/73).

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