Spreading-out for families of rigid analytic spaces
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(joint work with Brian Conrad)

Let K be a complete rank 1 valued field with ring of integers $\mathcal{O}_K$, A an adic nœtherian ring and $\varphi : A \to \mathcal{O}_K$ an adic morphism. We show that if $g : X \to Y$ is a proper flat morphism between rigid analytic spaces over K then locally on Y a flat formal model of g is the pullback of a proper flat morphism between formal schemes topologically of finite type over A. For this, if S is an affine nœtherian scheme, $T_0 \to S$ affine of finite type and $X_0 \to T_0$ proper flat, we construct a compatible system of versal n-th order deformations of $X_0 \to T_0$ over S. As an application, one can prove that for a proper smooth g and K of characteristic 0, the Hodge to de Rham spectral sequence for g degenerates and the $R^q g_* \, \Omega^p_{X/Y}$ are locally free. This is reduced to the case where K is a finite extension of $\mathbb{Q}_p$ and Y is a nilpotent thickening of $\operatorname{Sp} K$, where the result over K was proved by Scholze and follows for Y by imitating the proof of Deligne over $\mathbb{C}$ using a construction of crystalline cohomology in this case.