

Séminaire de Géométrie Arithmétique Paris-Tokyo

Vidéo-Séminaire organisé conjointement par
l'Institut des Hautes Études Scientifiques et
le Département des Sciences Mathématiques de l'Université de Tokyo

Organisateurs scientifiques :

Ahmed ABBES (C.N.R.S.), Christophe BREUIL (C.N.R.S.-I.H.É.S.),
Takeshi SAITO (Université de Tokyo), Atsushi SHIHO (Université de Tokyo),
Takeshi TSUJI (Université de Tokyo)

Centre de conférences Marilyn et James Simons

Mercredi 12 mai 2010 : M. MATSUMOTO (Université de Tokyo)

10h30 – 11h30 :

Differences between Galois representations in outer-automorphisms of the fundamental group and those in automorphisms, implied by topology of moduli spaces

Abstract: Fix a prime l . Let C be a proper smooth geometrically connected curve over a number field K , and x be a closed point. Let Π denote the pro- l completion of the geometric fundamental group of C with geometric base point x . We have two non-abelian Galois representations:

$$\rho_A : \text{Gal}_{K(x)} \rightarrow \text{Aut}(\Pi), \quad \rho_O : \text{Gal}_K \rightarrow \text{Out}(\Pi).$$

Our question is: in the natural inclusion $\text{Ker}(\rho_A) \subset \text{Ker}(\rho_O) \cap \text{Gal}_{K(x)}$, whether the equality holds or not.

Theorem: Assume that $g \geq 3$, l divides $2g - 2$. Then, there are infinitely many pairs (C, K) with the following property. If l does not divide the extension degree $[k(x) : K]$, then $\text{Ker}(\rho_A) \neq (\text{Ker}(\rho_O) \cap \text{Gal}_{K(x)})$ holds.

This is in contrast to the case of the projective line minus three points and its canonical tangential base points, where the equality holds (a result of Deligne and Ihara).

There are two ingredients in the proof: (1) Galois representations often contain the image of the geometric monodromy (namely, the mapping class group) [M-Tamagawa 2000] (2) A topological result [S. Morita 98] [Hain-Reed 2000] on the cohomological obstruction of lifting the outer action of the mapping class group to automorphisms.

LE BOIS-MARIE, 35, ROUTE DE CHARTRES, F-91440 BURES-SUR-YVETTE, FRANCE
téléphone : 01 60 92 66 00 • télécopie : 01 60 92 66 09 • cour. élect. : cecile@ihes.fr • site internet : www.ihes.fr

RER ligne B, direction Saint-Rémy-lès-Chevreuse, arrêt Bures-sur-Yvette