

Rigid analytic K -theory and p -adic Chern character

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I will explain a joint work with Moritz Kerz and Georg Tamme on a newly developed theory of analytic K -theory $K^{\text{an}}(\mathcal{X})$ for rigid spaces \mathcal{X} over a complete discrete valuation field κ . The plan of the talk is as follows.

Part I: (i) the construction of a pro-spectrum $K^{\text{an}}(\mathcal{X})$, first for affinoids via “pro-homotopization” and “analytic Bass delooping” of the connective algebraic K -theory, and then its globalization using descent for admissible coverings. (ii) If X is a scheme separated of finite type over the integer ring $\mathcal{O} \subset \kappa$ and $\mathcal{X} := \widehat{X}^{\text{rig}}$ is the rigid space associated to the formal completion \widehat{X} of X along the special fiber, $K^{\text{an}}(\mathcal{X})$ is compared with the continuous K -theory pro-spectrum

$$K^{\text{cont}}(X) := \varprojlim_m K(X \otimes_{\mathcal{O}} \mathcal{O}/(\pi^m)),$$

where $\pi \in \mathcal{O}$ is a prime element. Thus the algebrization problem for $K^{\text{cont}}(X)$ is rephrased by the same sort of a problem for $K^{\text{an}}(\mathcal{X})$.

Part II: Assume $\text{ch}(\kappa) = 0$ and the residue field of \mathcal{O} is perfect of characteristic $p > 0$. Let X be a smooth scheme of relative dimension d over \mathcal{O} and $\mathcal{X} := \widehat{X}^{\text{rig}}$. For integers $0 \leq i < p - 2 - d$ and $n > 0$, we construct the p -adic Chern character isomorphism:

$$K_i^{\text{an}}(\mathcal{X}, \mathbb{Z}/p^n\mathbb{Z}) \simeq \bigoplus_{r \leq d+i} H_{\text{Nis}}^{2r-r}(X_n, S_n(r)_{\text{Nis}}),$$

where $S_n(r)_{\text{Nis}} = \tau_{\leq r} R\epsilon_* S_n(r)$ with $S_n(r)$ the log syntomic complex on the syntomic site $(X_n)_{\text{syn}}$ with $X_n = X \otimes_{\mathcal{O}} \mathcal{O}/(p^n)$, introduced by Fontaine-Messing, Kato and Tsuji, and $\epsilon : (X_n)_{\text{syn}} \rightarrow (X_n)_{\text{Nis}}$ is the natural map of sites. As a consequence, the algebrization problem is related to a Bloch-Kato conjecture on the image of p -adic regulator maps for motivic cohomology of the generic fiber of X .