

The Nekrasov Partition Function

Winter 2015

Course Title: The Nekrasov Partition Function

Goal: The partition function introduced in the paper of Nekrasov [Nek03] is a fundamental player in the physics of $N = 2$ four-dimensional gauge theory. It admits two distinct mathematical interpretations, as a generating function for certain equivariant cohomology classes over instanton moduli space, and as the exponential of the Seiberg-Witten prepotential; Nekrasov and Okounkov [NO06] proved that these two expressions coincide, but their theorem has a natural interpretation as the scale invariance of certain correlation functions in $N = 2$ gauge theory. The primary goal of the course this quarter is to understand this object in its own right. Later on, we would like to understand some aspects of the AGT conjecture [AGT10] which relates it to a generating function for a dual two-dimensional conformal field theory.

Topics:

- (Chris) Goals of the seminar. $N = 2$ SUSY gauge theory with matter. Topological twists. [Tac13b]
- The geometry of the instanton moduli space. The ADHM construction. [AHDm78]
- Uhlenbeck compactification.
- Integration on the moduli space = equivariant localization. Define the partition function. [Nek03, Oko06]
- Seiberg-Witten solutions of the $N = 2$ gauge theories. [SW94, Don97]
- The SUSY prepotential, relating to the Nekrasov partition function. [NO06, NY04]
- Relation to two-dimensional CFT and the AGT conjecture. [AGT10, Rod13, Tac13a]

References

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- [Rod13] Robert Rodger. A pedagogical introduction to the AGT conjecture. Master's thesis, Universiteit Utrecht, 2013.
- [SW94] Nathan Seiberg and Edward Witten. Monopoles, duality and chiral symmetry breaking in $N = 2$ supersymmetric QCD. *Nuclear Physics B*, 431(3):484–550, 1994.
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