M. Gromov

ASYMPTOTIC GEOMETRY OF HOMOGENEOUS SPACES.

Let X be a homogeneous Riemannian manifold. If X is non-compact, then "asymptotic" refers to properties of X "near infinity". For example, if X is a symmetric space of non-compact type, then essential asymptotic features of X can be described in terms of the *ideal boundary* ∂X of X, whose points represent asymptotic classes of geodesic rays in X. This boundary plays a crucial role in the work of Mostow on rigidity of symmetric spaces. Mostow's starting points consist of a study of quasi-isometric maps $f: X_1 \to X_2$, which means distortion $< \infty$, where

$$\operatorname{distor} f \overset{\text{def}}{=} \sup_{x \neq y} |\log(\operatorname{dist}_1(x, y)/\operatorname{dist}_2(f(x), f(y)))| .$$

He shows, at least for $\operatorname{rank}_{\mathbb{R}} X_i = 1$, $i = 1, 2$, that such an f extends to a continuous map $\partial X_1 \to \partial X_2$, and that this extension is *quasi-conformal* in a suitable sense. Thus he proves (among many other results) that two rank one spaces are quasi-isometric if and only if they are isometric up to the choice of normalizing constants.

This approach applies also to more general homogeneous spaces, but one does not know yet if quasi-isometric homogeneous spaces are necessarily isomorphic. Furthermore, one can sharpen Mostow's result for rank one symmetric spaces by estimating from below a natural distance between different spaces. For example, if a (non-homogeneous) manifold Y has sectional curvature pinched between -1 and $-1 + (2n)^{-1}$, $n = \dim Y$, and if Y is quasi-isometric to a symmetric space X, then X has a constant negative curvature.

Now let X be compact homogeneous space. To be specific, let X be

ant Riemannian metrics on X are gi-
forms on $I\!R^n \approx T_e(X)$, $n = \dim X$.
$C_+ \subset I\!R^{\frac{n(n+1)}{2}}$, and asymptotic
rics $g_t \in C_+$ on X approaches the
ample, an appropriate family of inva-
non-Riemannian (*Carnot's,* see Pansu's
$SU(2)$ whose Hausdorff's dimension
fact, this $SU(2)$ with the limit me-
he complex hyperbolic plane CH^2.
n C_+ one seeks a map $f : (X, g_1) \to$
nd then defines a metric on C_+ by
led to the study of the asymptotic of

$SU(2)$ one does not know kow this
trivial estimates can be obtained either
(see these proceedings) or with the

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written) paper by M. Gromov: "Asymptotic

ymmetric Spaces", Ann. Math, Studies, 78

ic Objects", to appear in the Proc. of the

a compact Lie group. Then, left invari
ven by positive definite symmetric
These constitute a convex cone, say (
phenomena appear if a family of met
boundary ∂C_+ for $t \to \infty$. For ex
riant metrics on $SU(2)$ approaches a
talk in these proceedings) metric on
equals $4 = 1 + \dim_{top}$ $SU(2)$. In
tric appears as the ideal boundary of
Given two metrics g_1 and g_2 i
$\to (X, g_2)$ with a minimal distortion
DIST $(g_1, g_2) = \text{distor} f$. Thus, one i
the metric DIST on C_+.

Even in the simplest case of $X =$
DIST looks like. However, some non-
with Pansu's isoperimetric inequality
L_p-cohomology of CH^2.

REFER

A detailed exposition will appear in a (yet un
Geometry of Riemannian Manifolds".

For the background, see:

G. MOSTOW: "Strong Rigidity of Locally S
(1973), Princeton.

M. GROMOV: "Infinite Groups as Geomet
ICM Warsaw 1983.