Statistics of renormalized hoppings for Anderson localization transitions

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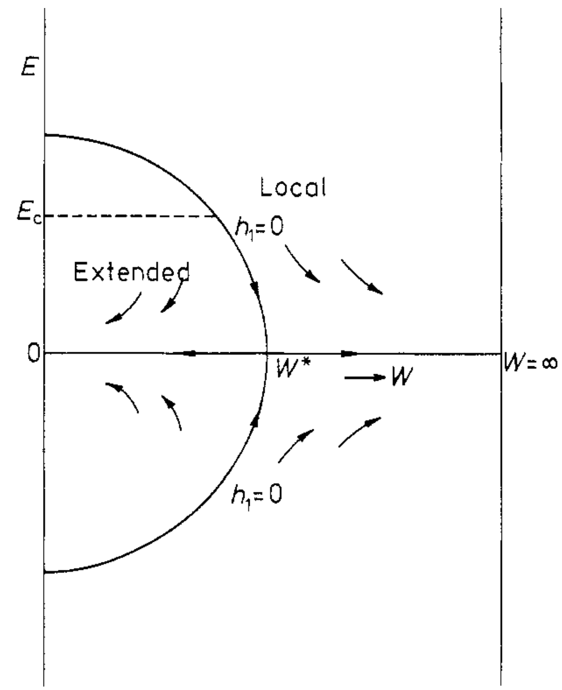
One electron in a disordered potential (Anderson 1958)

$$H = \sum_i \epsilon_i |i\rangle\langle i| + \sum_{i,j} V_{i,j} |i\rangle\langle j|,$$

$$H|\Psi\rangle = E|\Psi\rangle$$

Phase transition between localized and extended states, d>2



$$p(\epsilon_i) = \frac{1}{W}\theta\left(-\frac{W}{2} \le \epsilon_i \le \frac{W}{2}\right).$$

Localization phase transition

order parameter? fluctuations ? renormalization?....
multifractality of the critical wave function....

$*$Presence of disorder => one needs to renormalize probability distributions

$*$ No translational invariance => real space RG procedure

review: F. Evers and A.D. Mirlin, Rev. Mod. Phys. 80, 1355 (2008)

$$H = \sum_i \epsilon_i |i\rangle\langle i| + \sum_{i,j} V_{i,j} |i\rangle\langle j|, \qquad p(\epsilon_i) = \frac{1}{W}\theta\left(-\frac{W}{2} \leq \epsilon_i \leq \frac{W}{2}\right).$$

RG rules for the <u>decimation</u> of one site i_0 :Aoki (1980)

$$\psi(i_0) = \frac{1}{E - \epsilon_{i_0}} \sum_j V_{i_0,j} \psi(j)$$

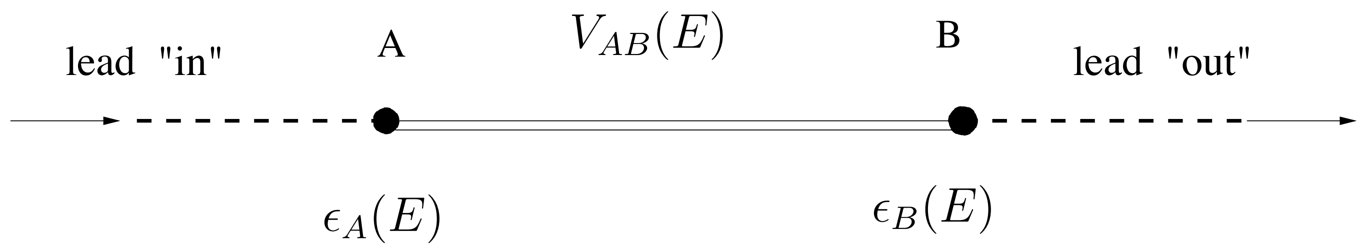
if (i, j) neighbors of i_0

$$\epsilon_i^{\mathrm{new}} = \epsilon_i + \frac{V_{i,i_0} V_{i_0,i}}{E - \epsilon_{i_0}}.$$

$$V_{i,j}^{\mathrm{new}} = V_{i,j} + \frac{V_{i,i_0} V_{i_0,j}}{E - \epsilon_{i_0}}$$

fixed energy E

Physical meaning of the renormalized parameters (I)



$$\psi_{in}(x \leq x_A) = e^{ik(x-x_A)} + re^{-ik(x-x_A)},$$

$E = 2\cos\, k$

$$\psi_{out}(x \geq x_B) = te^{ik(x-x_B)}.$$

Landauer transmission

$$T \equiv |t|^2 = 1 - |r|^2.$$

$$T(E=0) = \frac{4V_{AB}^2(\epsilon_B^2 + 1)}{[\epsilon_A(\epsilon_B^2 + 1) - V_{AB}^2\epsilon_B]^2 + [\epsilon_B^2 + 1 + V_{AB}^2]^2}.$$

Physical meaning of the renormalized parameters (II)

*let the spectrum of the closed system be (E_n, ϕ_n)

$$H_{\text{system}} = \sum_n E_n |\phi_n\rangle\langle\phi_n| .$$

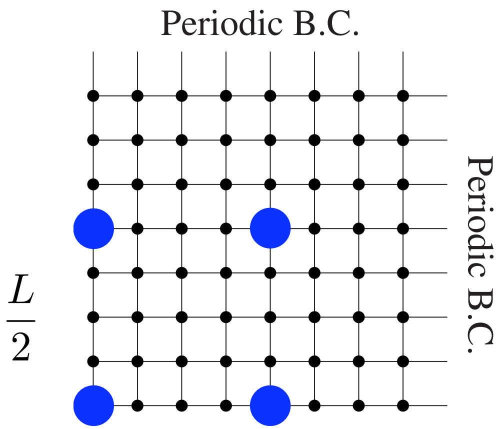
$$H|\Psi\rangle = E|\Psi\rangle \qquad \text{et} \qquad |\Psi\rangle = \sum_n \alpha_n |\phi_n\rangle$$

$$V_{AB} = \frac{G_{AB}}{G_{AA}G_{BB} - G_{AB}^2}$$

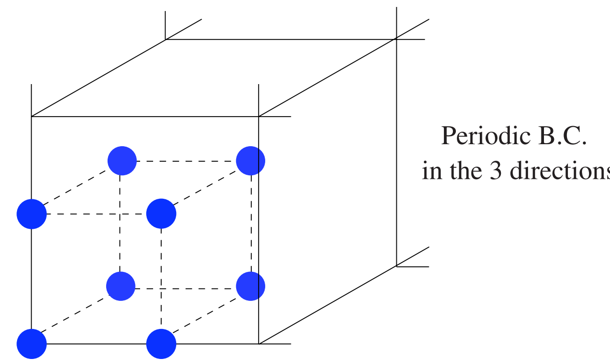
$$G_{IJ} = \sum_n \frac{\phi_n(I)^* \phi_n(J)}{E - E_n}$$

Numerical study of the localized phase in d=2,3

$$(E = 0)$$



$L = 120$

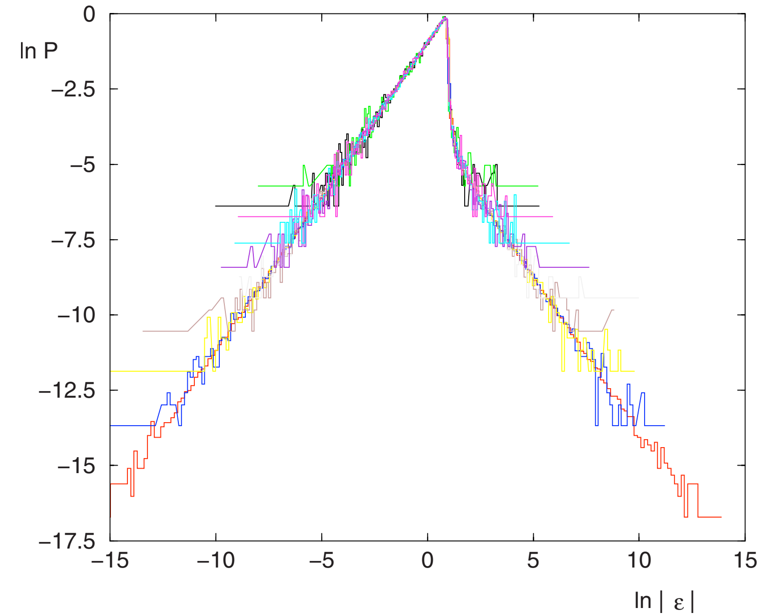


$L = 30$

One keeps the points ● and <u>decimates</u> all others

=> histograms of ϵ and $V_{L/2}$

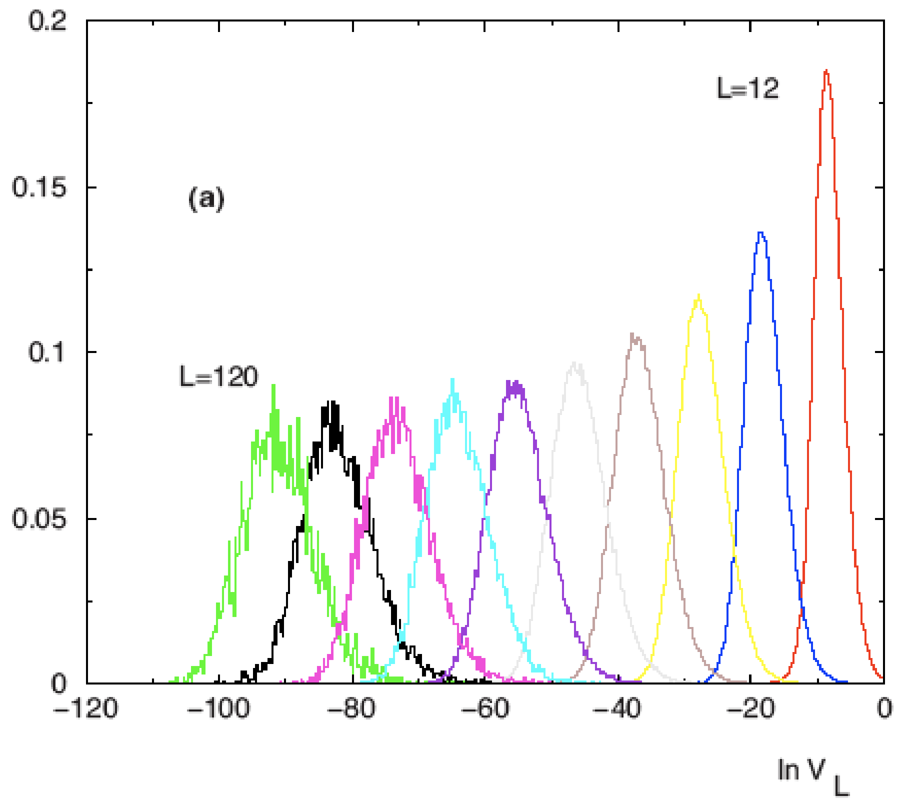
The renormalized on-site energies remain finite random variables in all phases

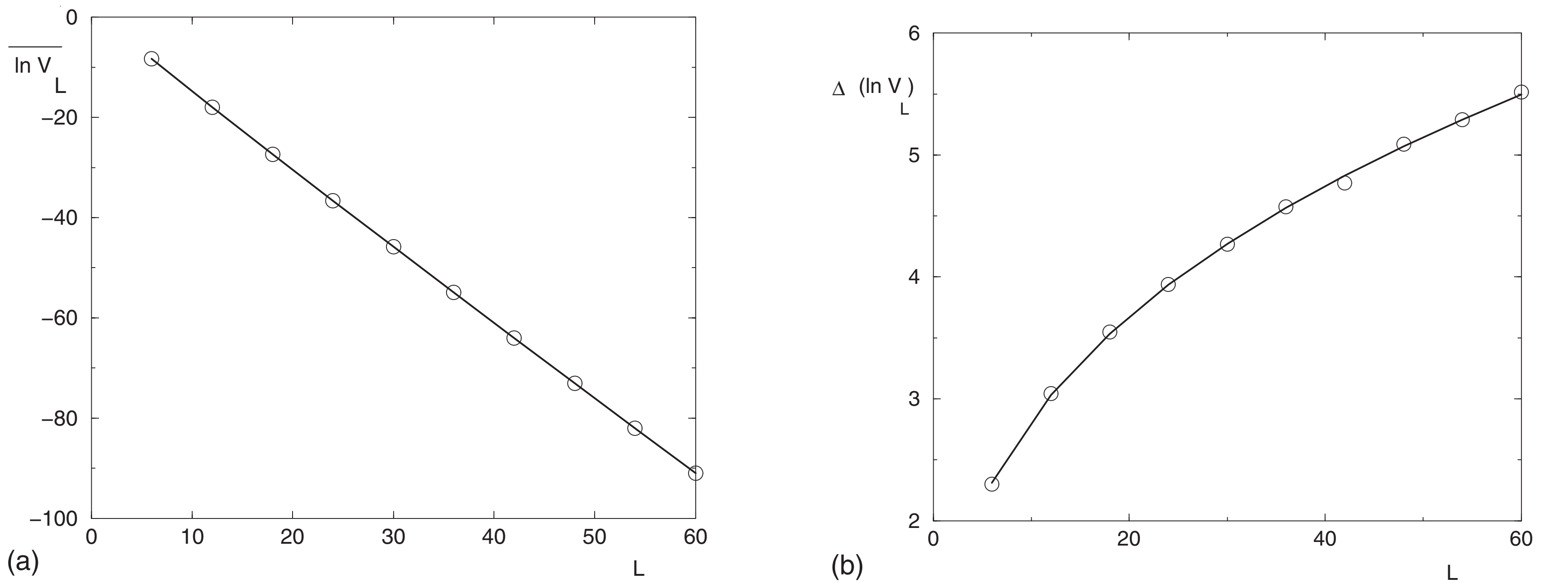


$$\mathcal{P}_\infty(\epsilon = 0) = \text{finite}$$

$$\mathcal{P}_\infty(\epsilon) \underset{\epsilon \to \pm\infty}{\propto} \frac{1}{\epsilon^2}.$$

$$\ln T_{L/2}(E = 0) = \ln V_{L/2}^2 + \text{finite}$$





$$\ln V_L \simeq -\frac{L}{\xi_{loc}} + L^{\omega(d)} u + \cdots .$$

$$\omega(2) = 0.33..$$
$$\omega(3) = 0.24..$$

*universality class of the directed polymer in a random medium (Nguyen-Spivak-Shlovskii 85, Medina-Kardar 92, Prior-Somoza-Ortuno 05)

* in d=2, u has Tracy-Widom distribution

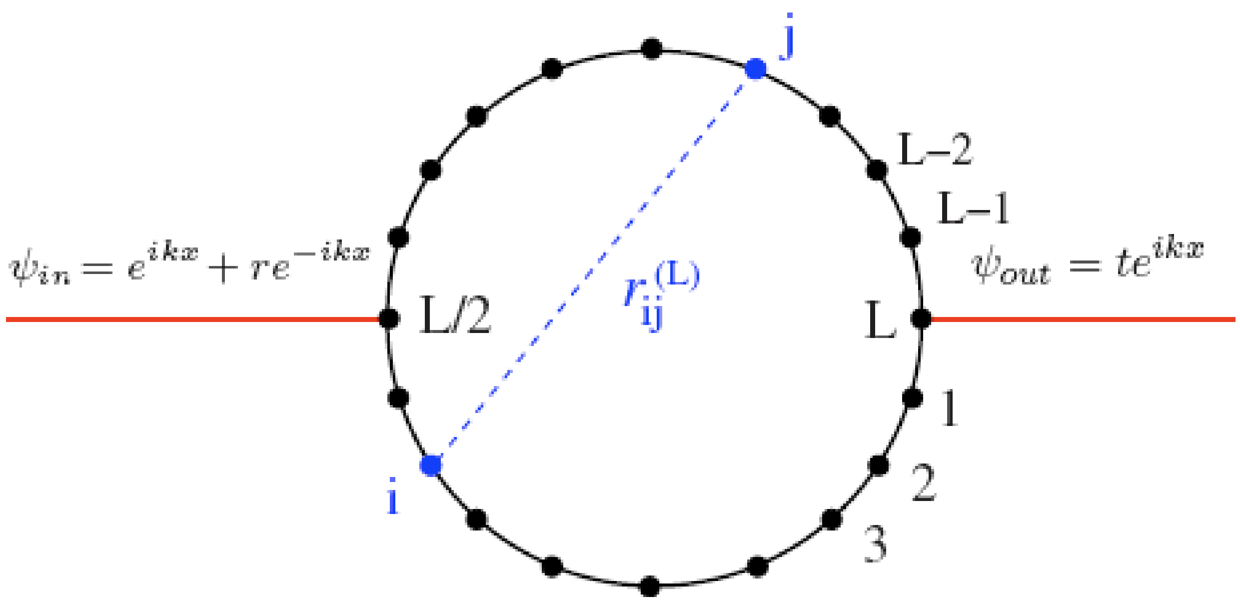
* in d=3, at the transition: multifractality of V_L

Multifractality of the critical Landauer transmission

$$H = \sum H_{ij} |i\rangle\langle j|$$

PRBM model : independent Gaussian variables

$$\begin{cases} \overline{H_{i,j}} = 0 \\ \overline{H_{i,j}^2} = \dfrac{1}{1 + \left(\frac{r_{i,j}}{b}\right)^{2a}}. \end{cases}$$

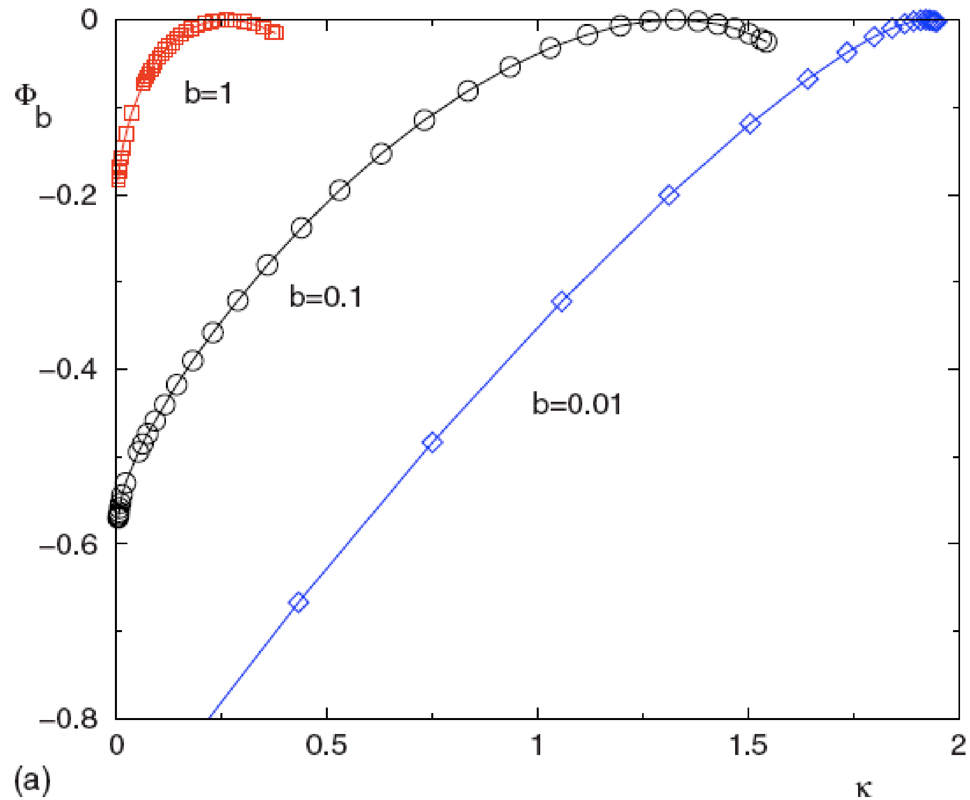


$$r_{i,j}^{(L)} = \frac{L}{\pi} \sin\left(\frac{\pi(i-j)}{L} \right)$$

Aoki RG at E=0

*criticality: a=1

$$\mathrm{Prob}(T_L \sim L^{-\kappa})dT \underset{L\to\infty}{\propto} L^{\Phi_b(\kappa)}d\kappa$$



$$\Phi_b(\kappa) = 2\ (f_b(\alpha(\kappa)) - 1)$$

$$\alpha(\kappa) = 1 + \frac{\kappa}{2}$$

Interactions between electrons and many-body localization

(Altshuler et al. 97, Gornyi et al 05, Basko et al 06..)

$$H = \sum_{i=1}^{L} \left[w_i n_i + V \left(n_i - \frac{1}{2} \right) \left(n_{i+1} - \frac{1}{2} \right) + c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + c_i^\dagger c_{i+2} + c_{i+2}^\dagger c_i \right]$$

(Oganesyan et Huse 07)

$w_i,$ i.i.d Gaussian

$$H = \sum_{C_i, C_j} V_{C_i, C_j} |C_i\rangle\langle C_j|$$

$$\left\{ \begin{array}{l} n_i = c_i^\dagger c_i = 0, 1 \\ 2^L \text{ configurations } C_i \\ W^2 = \text{ variance of } w_i \end{array} \right.$$

decimation of configuration C_{i_0}

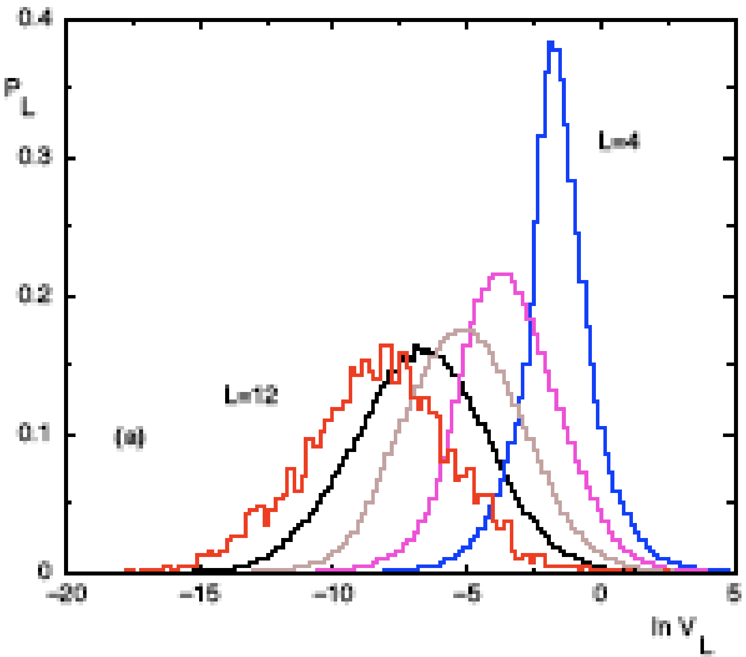
$$E\psi(\mathcal{C}_{i_0}) = \sum_{\mathcal{C}_j} V_{\mathcal{C}_{i_0}, \mathcal{C}_j} \psi(\mathcal{C}_j) \quad \Rightarrow \quad V^{new}_{\mathcal{C}_i, \mathcal{C}_j} = V_{\mathcal{C}_i, \mathcal{C}_j} + \frac{V_{\mathcal{C}_i, \mathcal{C}_{i_0}} V_{\mathcal{C}_{i_0}, \mathcal{C}_j}}{E - V_{\mathcal{C}_{i_0}, \mathcal{C}_{i_0}}}$$

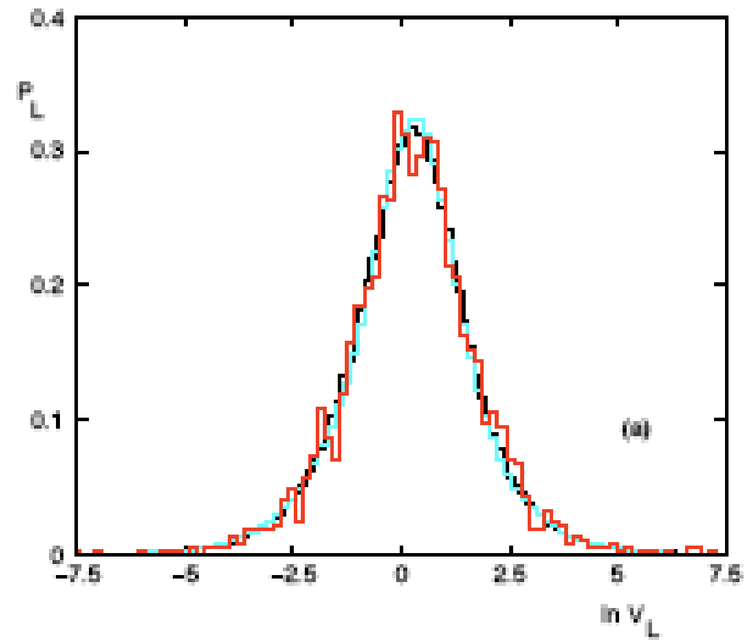
* half-filled case $\dfrac{2^L}{\sqrt{L}}$ configurations => $L \leq 14$

* $C_A = 10101010101010$
$C_B = 01010101010101$ $\quad V_{C_A,C_B}(E=0)$

distance in configuration space=L/2

$W = 20$ $\quad W_c \sim 5.6$ $\quad W = 2$





$$\ln(V_L^{typ}) \equiv \overline{\ln V_L(W > W_c)} \underset{L\to\infty}{\simeq} -\frac{L}{\xi_{loc}(W)}$$

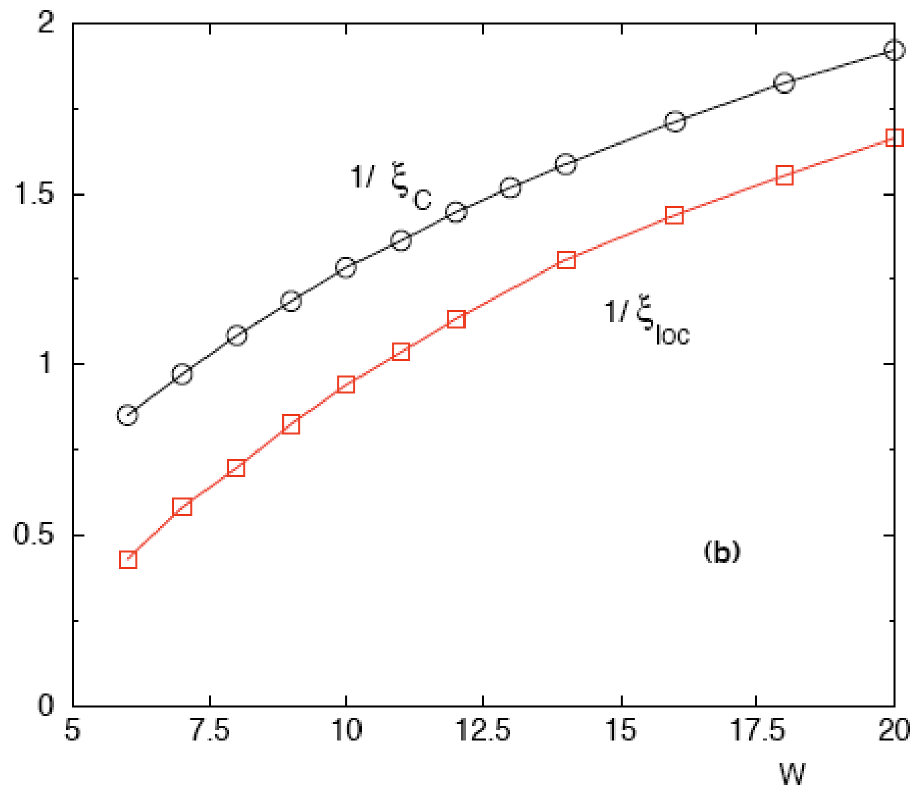
$$\overline{\ln V_L(W < W_c, L)} \underset{L\to\infty}{\simeq} \overline{\ln V_\infty(W < W_c)} \quad \text{finite}$$

What happens in real space ?

$$C(L) \equiv | < \psi_{mid} | c_1^\dagger c_L | \psi_{mid} > |^2$$

$|\psi_{mid}\rangle$ eigenstate of H in the middle of the energy spectrum $\quad E \sim 0$

$$\ln(C_L^{typ}) \equiv \overline{\ln C_L(W > W_c)} \underset{L\to\infty}{\simeq} -\frac{L}{\xi_C(W)}$$



*link with localization on the Cayley tree ?

*link with infinite randomness fixed point (Pal-Huse 2010)?

*consequences of the transition in real space ?

Conclusion

*applications of Aoki's RG rules to localization transitions

*Anderson localization: localized phase and directed polymer
multifractal behavior at criticality (PRBM)

*many-body localization: more questions than answers