Non-linear structure formation with massive neutrinos

Yacine Ali-Haïmoud and Simeon Bird (IAS)

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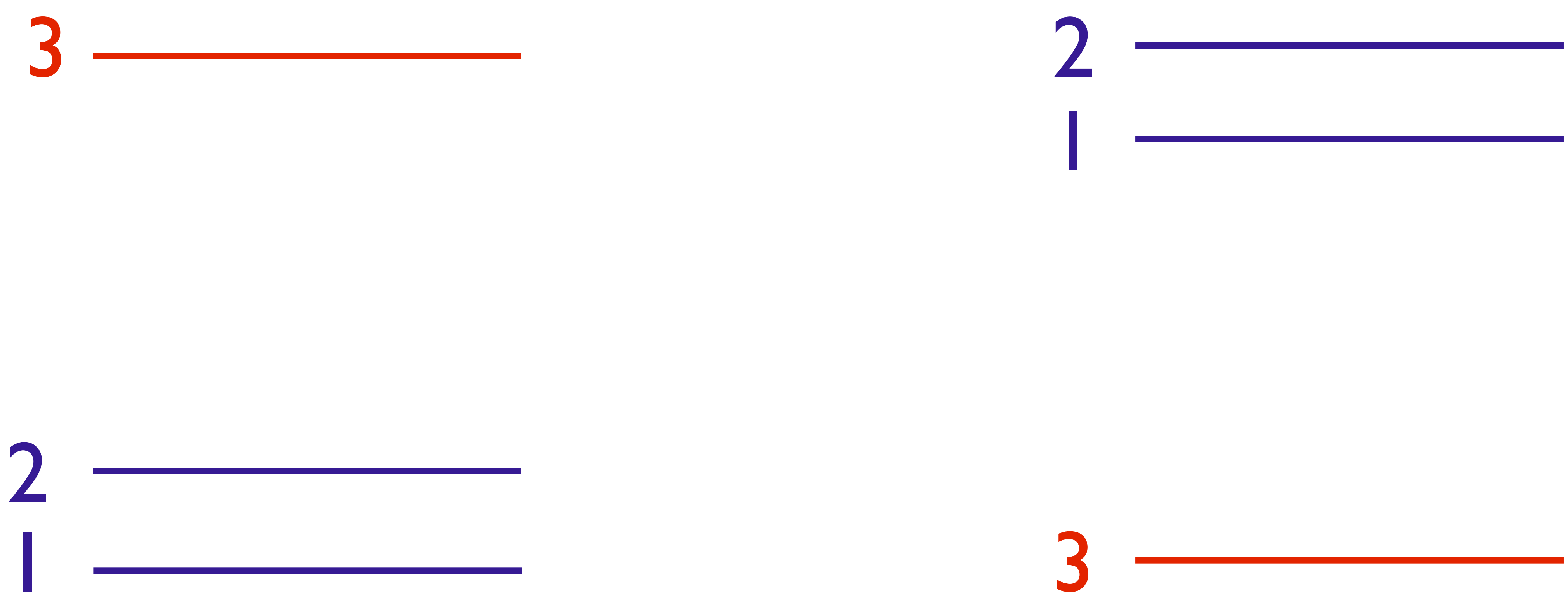
Neutrinos masses

From oscillation experiments:

$m_2^2 - m_1^2 \approx (0.009 \text{ eV})^2$, $|m_3^2 - m_1^2| \approx (0.05 \text{ eV})^2$

Either "normal" or "inverted" hierarchies



Neutrinos masses

- ***Total mass*** not (yet) measured by particle physics experiments, but must be at least $\sum m_\nu \gtrsim 0.06$ eV (normal hierarchy) or $\sum m_\nu \gtrsim 0.1$ eV (inverted hierarchy)
- Cosmological observations mostly probe the total mass. If sensitive enough can eventually lead to the absolute neutrino masses.

Current constraint: $\sum m_\nu \lesssim 0.2 - 0.3$ eV

Cosmological neutrinos

- Decouple at T ~ 1 MeV, while ultra-relativistic.
- Keep a relativistic Fermi-Dirac distribution

$$f(p,z) = \frac{g}{h^3}\left(\exp[\frac{p}{T_\nu(z)}]+1\right)^{-1}, \quad T_\nu(z) = (1+z)T_\nu(0)$$

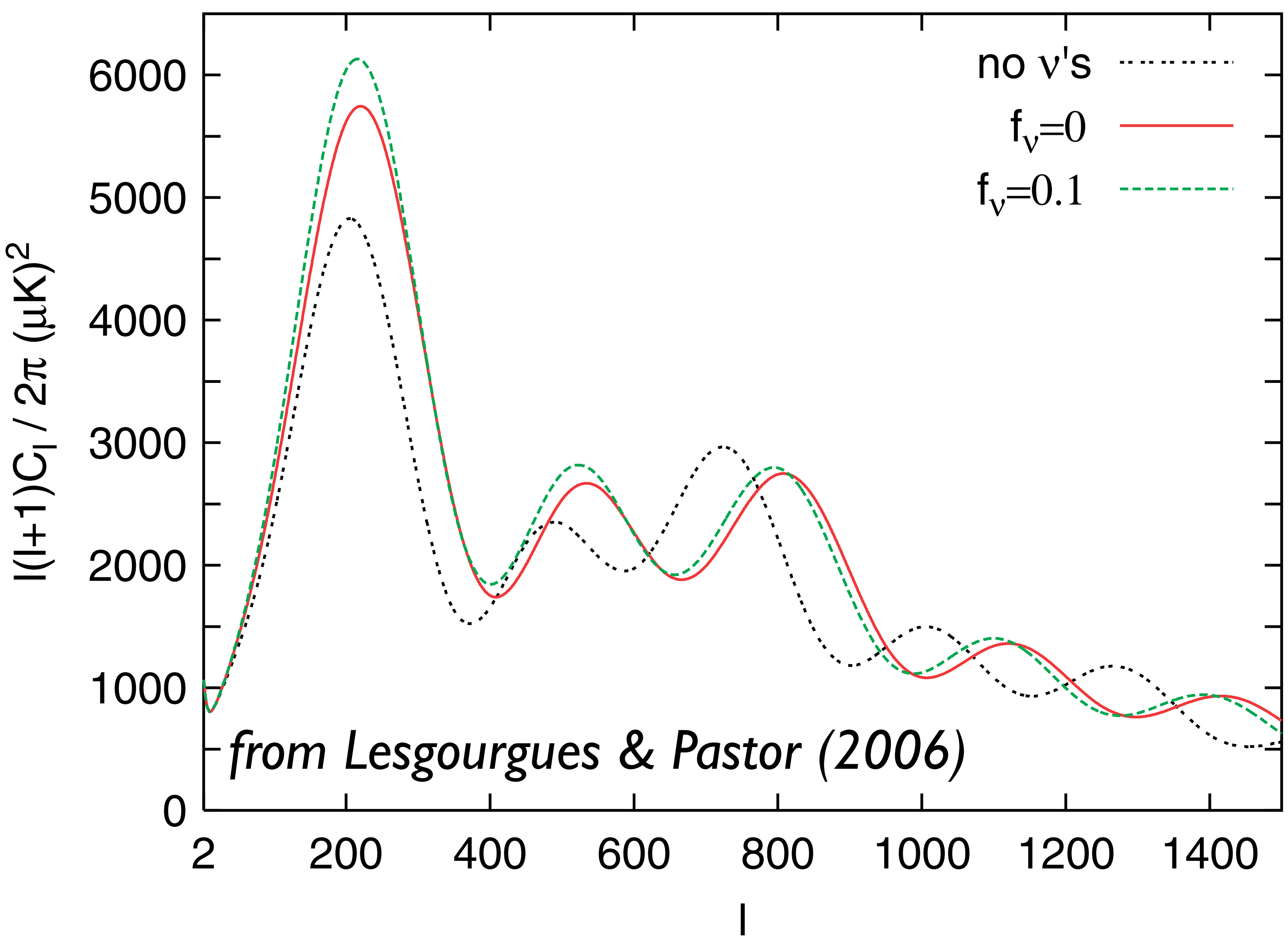
$$T_\nu(0) = 1.95 \text{ K} = 1.68 \times 10^{-4} \text{ eV}$$

- Become non-relativistic at $z_{\rm nr} \approx 200 \, \frac{\sum m_\nu}{0.3 \text{ eV}}$
- Contribute a fraction of the total DM

$$f_\nu = \frac{1}{\Omega_m h^2}\frac{\sum m_\nu}{94\text{eV}} \approx 0.02 \, \frac{\sum m_\nu}{0.3\text{eV}}$$

Cosmological effects

- Affect the *background expansion* (in particular time of matter-radiation equality), hence CMB.
  WMAP + H0 + BAO: $\sum m_\nu \lesssim 0.6$ eV
  Planck +WMAP +SPT+ACT+BAO: $\sum m_\nu \lesssim 0.23$ eV

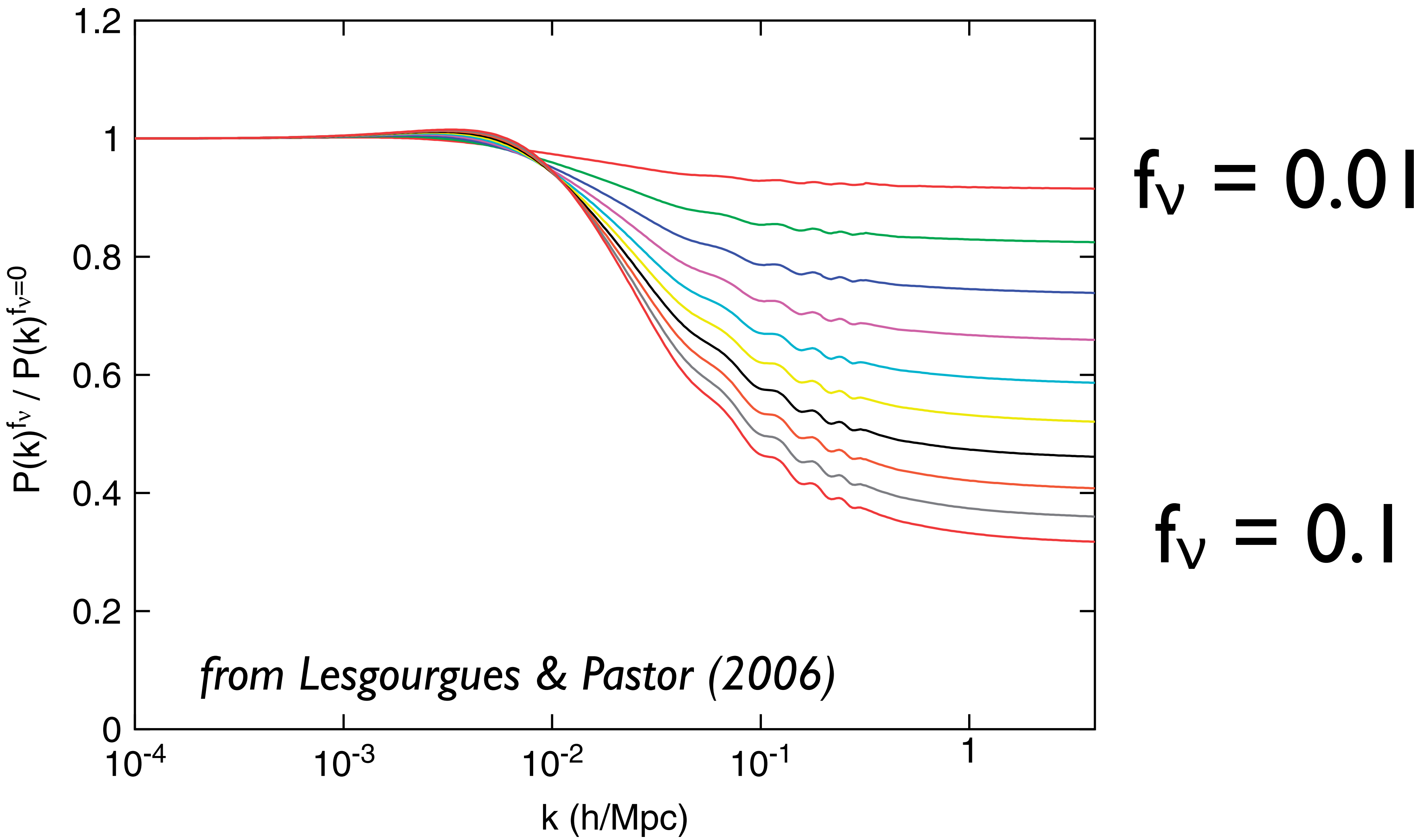


Cosmological effects

- Slow down the *growth of structure* on scales smaller than the *free-streaming* scale.

$$k_{fs} \approx 0.08\ (1+z)^{1/2}\ (\sum m_\nu/0.3\ \text{eV})$$

In linear regime:



Cosmological effects

- Most LSS probes are sensitive to mildly non-linear modes (Ly α, galaxy distribution) or to full non-linear evolution (clusters).
- Current constraints: $\sum m_\nu \lesssim 0.2\text{-}0.3\ eV$. Could get much better in future, provided we model their effect accurately enough.
- Neutrinos are "simple" (gravity only!), so we should be able to model their effect very precisely.

Nonlinear regime: I) higher-order perturbations

Third Order Solutions

For $n = 3$, the continuity and Euler equations are given by

$$
\begin{aligned}
&3\dot{a}(\tau)a^2(\tau)g_3(\mathbf{k},\tau)\delta_{3,c}(\mathbf{k}) + a^3(\tau)\dot{g}_3(\mathbf{k},\tau)\delta_{3,c}(\mathbf{k}) + \dot{a}(\tau)a^2(\tau)h_3(\mathbf{k},\tau)\theta_{3,c}(\mathbf{k}) \\
&= \dot{a}(\tau)a^2(\tau)\frac{1}{(2\pi)^6}\iiint d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 \delta_D(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3-\mathbf{k})\delta_{1,c}(\mathbf{q}_1)\delta_{1,c}(\mathbf{q}_2)\delta_{1,c}(\mathbf{q}_3) \\
&\times \left[\frac{\mathbf{k}\cdot\mathbf{q}_1}{q_1^2}g_1(\mathbf{q}_1)g_2(\mathbf{q}_{23})F_2^{(s)}(\mathbf{q}_2,\mathbf{q}_3) + \frac{\mathbf{k}\cdot\mathbf{q}_{12}}{q_{12}^2}h_2(\mathbf{q}_{12})g_1(\mathbf{q}_3)G_2^{(s)}(\mathbf{q}_1,\mathbf{q}_2)\right] \\
&\equiv \dot{a}(\tau)a^2(\tau)A_3(\mathbf{k}),
\end{aligned} \tag{B18}
$$

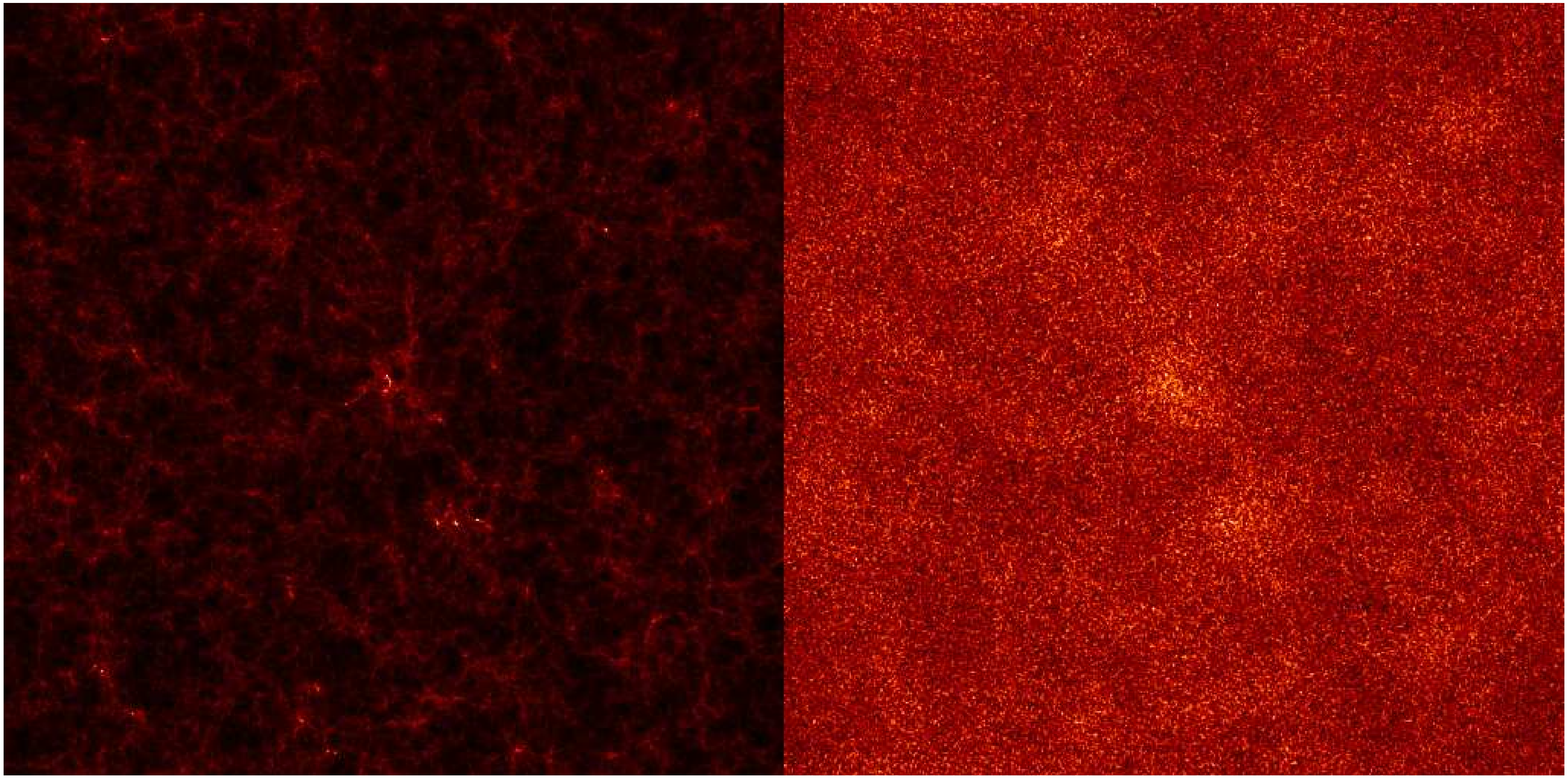
$$
\begin{aligned}
&\left[\ddot{a}(\tau)a^2(\tau) + 2\dot{a}^2(\tau)a(\tau)\right] h_3(\mathbf{k},\tau)\theta_{3,c}(\mathbf{k}) + \dot{a}(\tau)a^2(\tau)\dot{h}_3(\mathbf{k},\tau)\theta_{3,c}(\mathbf{k}) + \frac{2}{\tau}\dot{a}(\tau)a^2(\tau)h_3(\mathbf{k},\tau)\theta_{3,c}(\mathbf{k}) \\
&+\frac{6}{\tau^2}a^3(\tau)\delta_{3,c}(\mathbf{k}) - \frac{6}{\tau^2}\frac{k^2}{k_J^2}a^3(\tau)\delta_{3,c}(\mathbf{k}) \\
&= \dot{a}^2(\tau)a(\tau)\frac{1}{(2\pi)^6}\iiint d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 \delta_D(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3-\mathbf{k})\delta_{1,c}(\mathbf{q}_1)\delta_{1,c}(\mathbf{q}_2)\delta_{1,c}(\mathbf{q}_3) \\
&\times \left[-\frac{k^2(\mathbf{q}_1\cdot\mathbf{q}_{23})}{2q_1^2 q_{23}^2}g_1(\mathbf{q}_1)h_2(\mathbf{q}_{23})G_2^{(s)}(\mathbf{q}_2,\mathbf{q}_3) - \frac{k^2(\mathbf{q}_{12}\cdot\mathbf{q}_3)}{2q_{12}^2 q_3^2}h_2(\mathbf{q}_{12})g_1(\mathbf{q}_3)G_2^{(s)}(\mathbf{q}_1,\mathbf{q}_2)\right. \\
&\left. -\frac{3}{4}\frac{k^2}{k_J^2}g_1(\mathbf{q}_1)g_2(\mathbf{q}_{23})F_2^{(s)}(\mathbf{q}_2,\mathbf{q}_3) - \frac{3}{4}\frac{k^2}{k_J^2}g_2(\mathbf{q}_{12})g_1(\mathbf{q}_3)F_2^{(s)}(\mathbf{q}_1,\mathbf{q}_2) + \frac{1}{2}\frac{k^2}{k_J^2}g_1(\mathbf{q}_1)g_1(\mathbf{q}_2)g_1(\mathbf{q}_3)\right] \\
&\equiv \dot{a}^2(\tau)a(\tau)B_3(\mathbf{k}).
\end{aligned} \tag{B19}
$$

Shoji & Komatsu 2009

See also Lesgourgues et al 2009

Still, simplifying assumptions for neutrinos (either described with simple pressure term or assumed linear)

II) Particle-based simulations

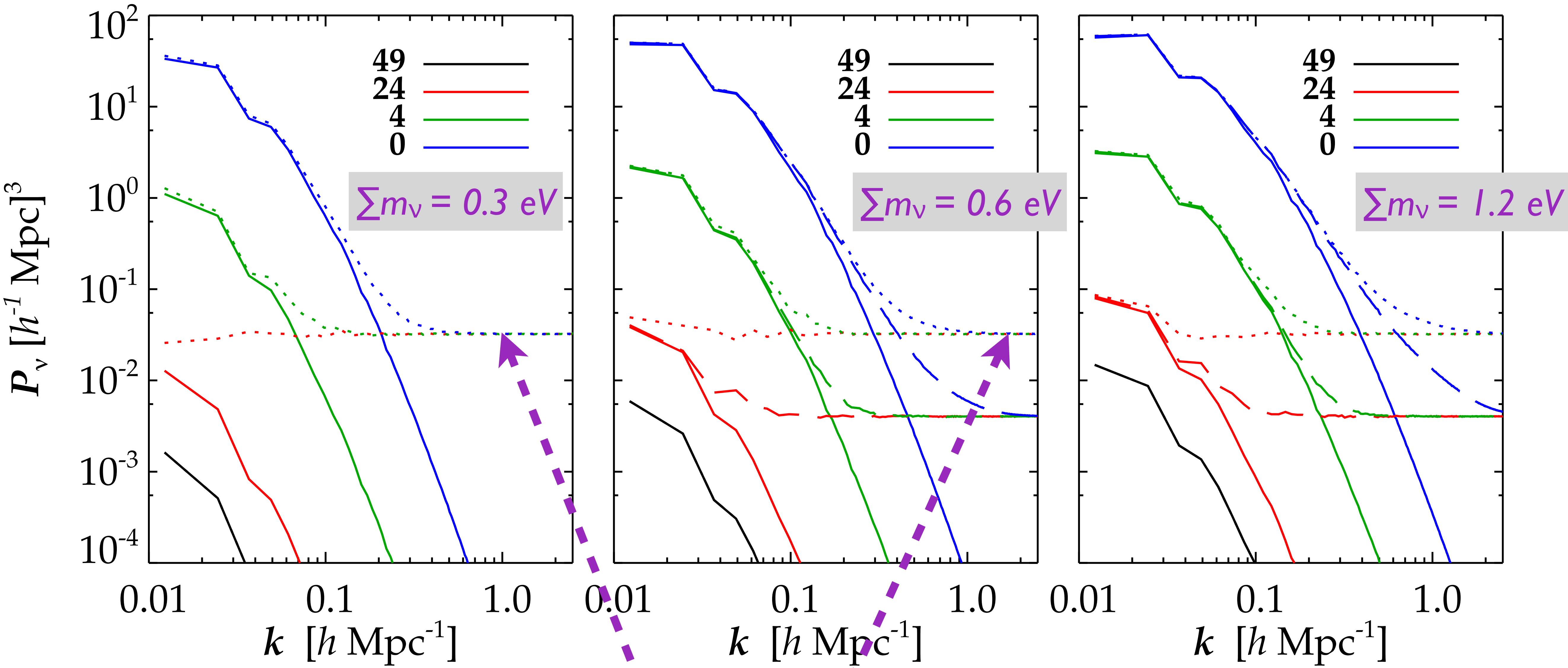


CDM Neutrinos

Simulation from Brandyge & Hannestad (2009).
$\sum m_\nu = 0.6$ *eV, z=4*

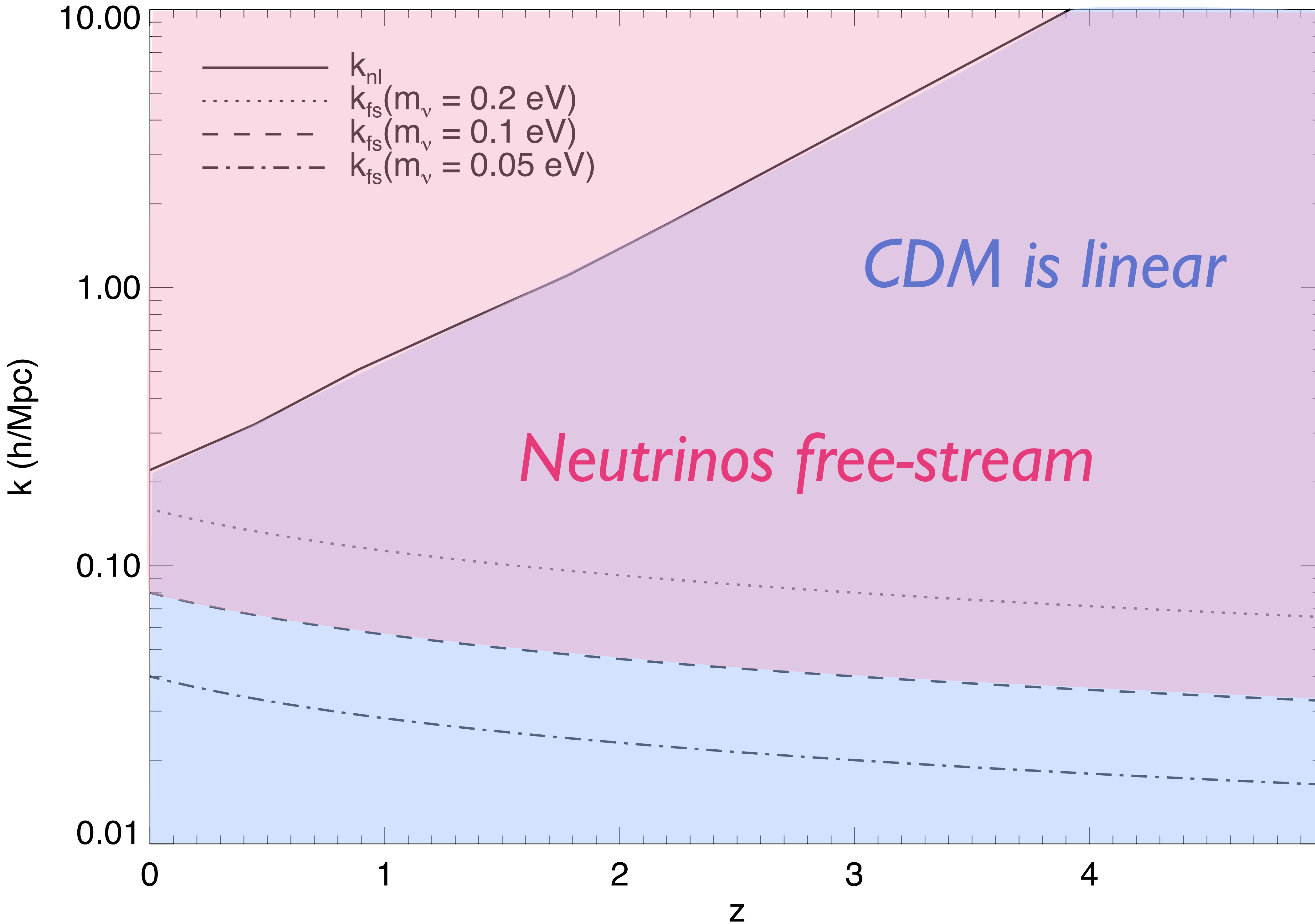
Particle-based simulations

Neutrino power-spectrum from Brandyge & Hannestad (2009).

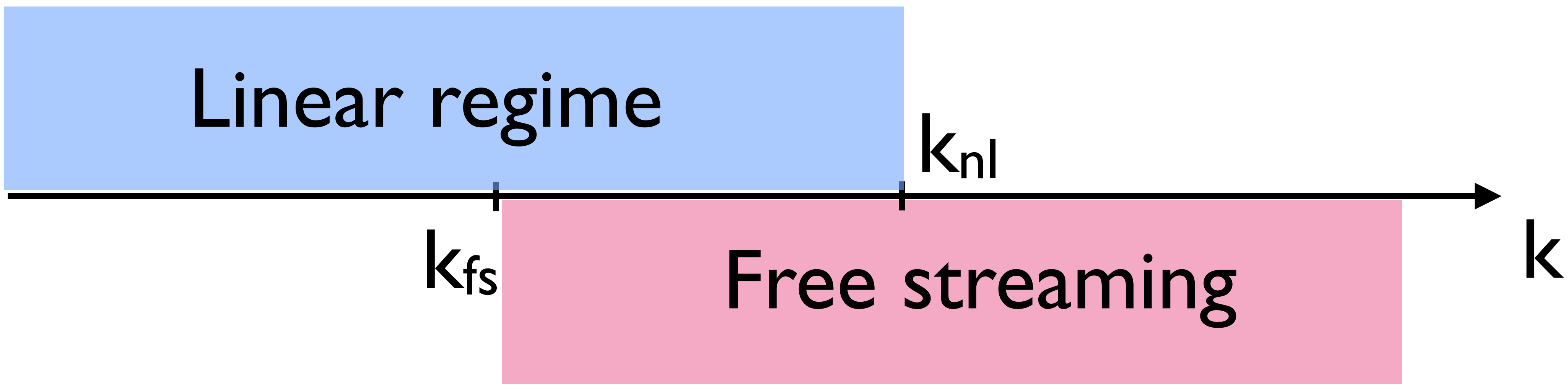


Shot noise P(k) = 1/n

Characteristic scales



Characteristic scales



➡ *Neutrinos should be nearly linear at all scales*

Linear evolution of neutrino perturbations with non-linear CDM :

- Vlasov equation for neutrino distribution function $f(\tau, \vec{x}, \vec{q} \equiv am\vec{v})$:

$$\partial_\tau f + \frac{\vec{q}}{ma} \cdot \partial_{\vec{x}} f - ma\, \partial_{\vec{x}}\phi \cdot \partial_{\vec{q}} f = 0$$

- Linearize around $f_0(q)$ then Fourier transform :

$$\partial_\tau(\delta f) + i\frac{\vec{q}\cdot\vec{k}}{ma}\delta f = i\frac{ma}{q}(\vec{q}\cdot k)\frac{df_0}{dq}\phi$$

- Write down explicit integral solution

- Integrate over momenta to get δ_ν:

$$\delta_\nu(\tau,\vec{k}) = F[\delta f(\tau_i,\vec{k})] + \int_{\tau_i}^{\tau} G(k,\tau',\tau)\phi(\vec{k},\tau')d\tau',$$
$$G(k,\tau' \to \tau) \to 0,$$
$$G(\tau - \tau' \gg \tau_{\rm cross}(k)) \to 0.$$

$\rightarrow$ We have a prescription for δ_ν given previous ϕ

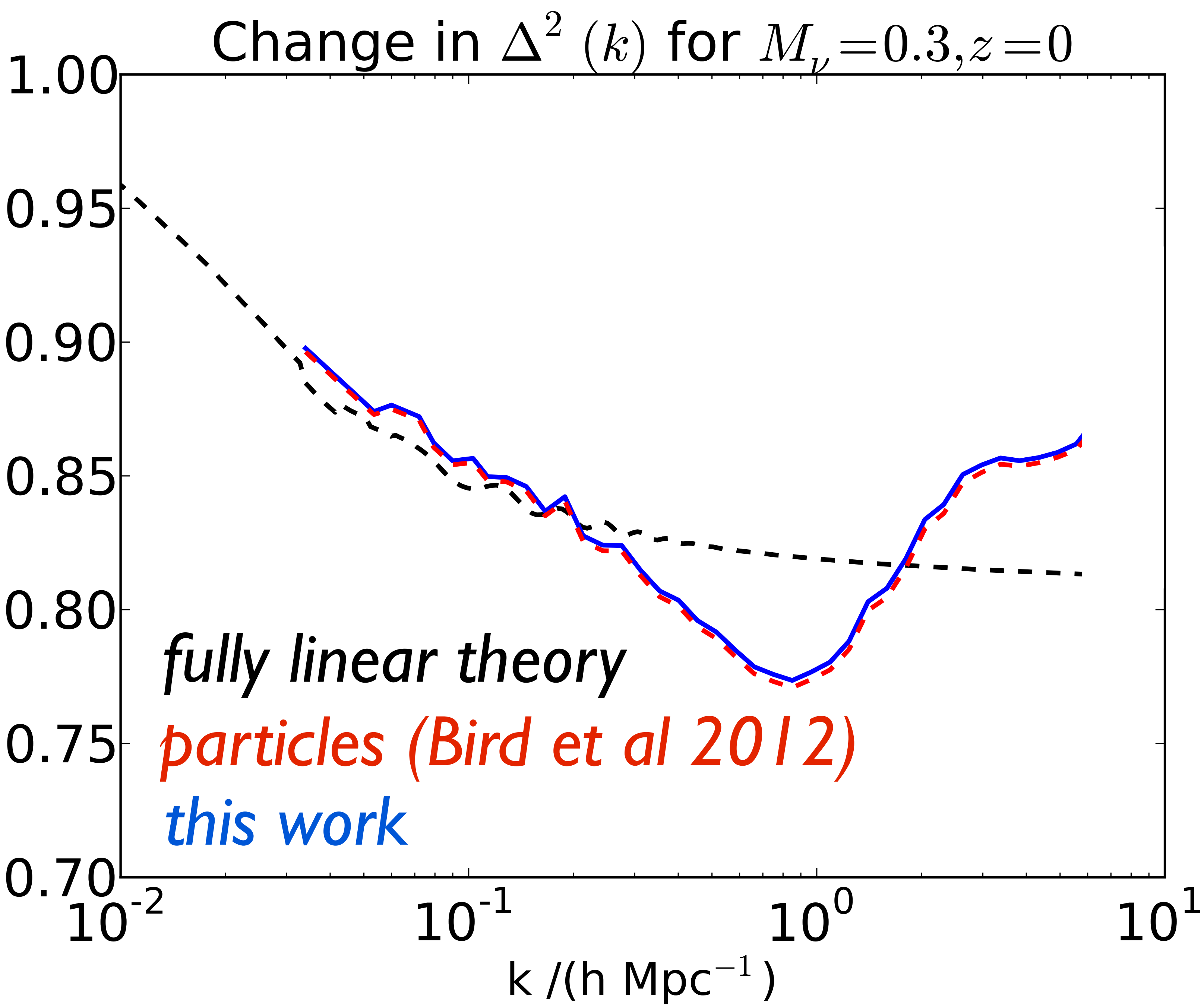
- Given ϕ, update δ_c with N-body code

- Close the system with Poisson equation:
$k^2\phi = -4\pi a^2(\overline{\rho}_c\delta_c + \overline{\rho}_\nu\delta_\nu)$

☛ We have replaced following 10^9 neutrinos by performing a simple integral

Results

Effect on the total matter power spectrum

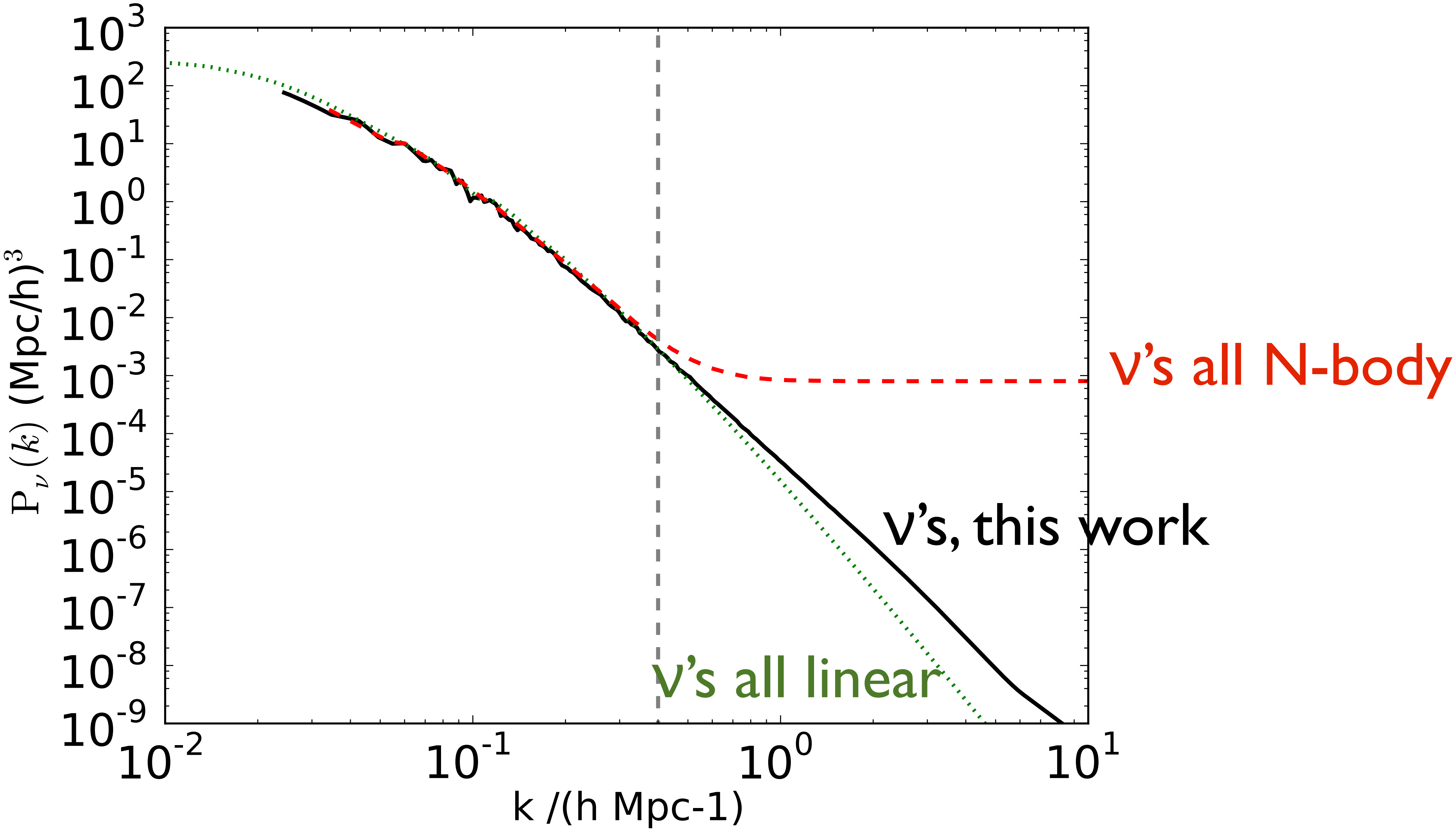


Agreement with particle method:
at z = 0,
0.2% for $\sum m_\nu$ = 0.3 eV
1% for $\sum m_\nu$ = 0.6 eV
4% for $\sum m_\nu$ = 1.2 eV

at z > 1, all agree to better than 1%

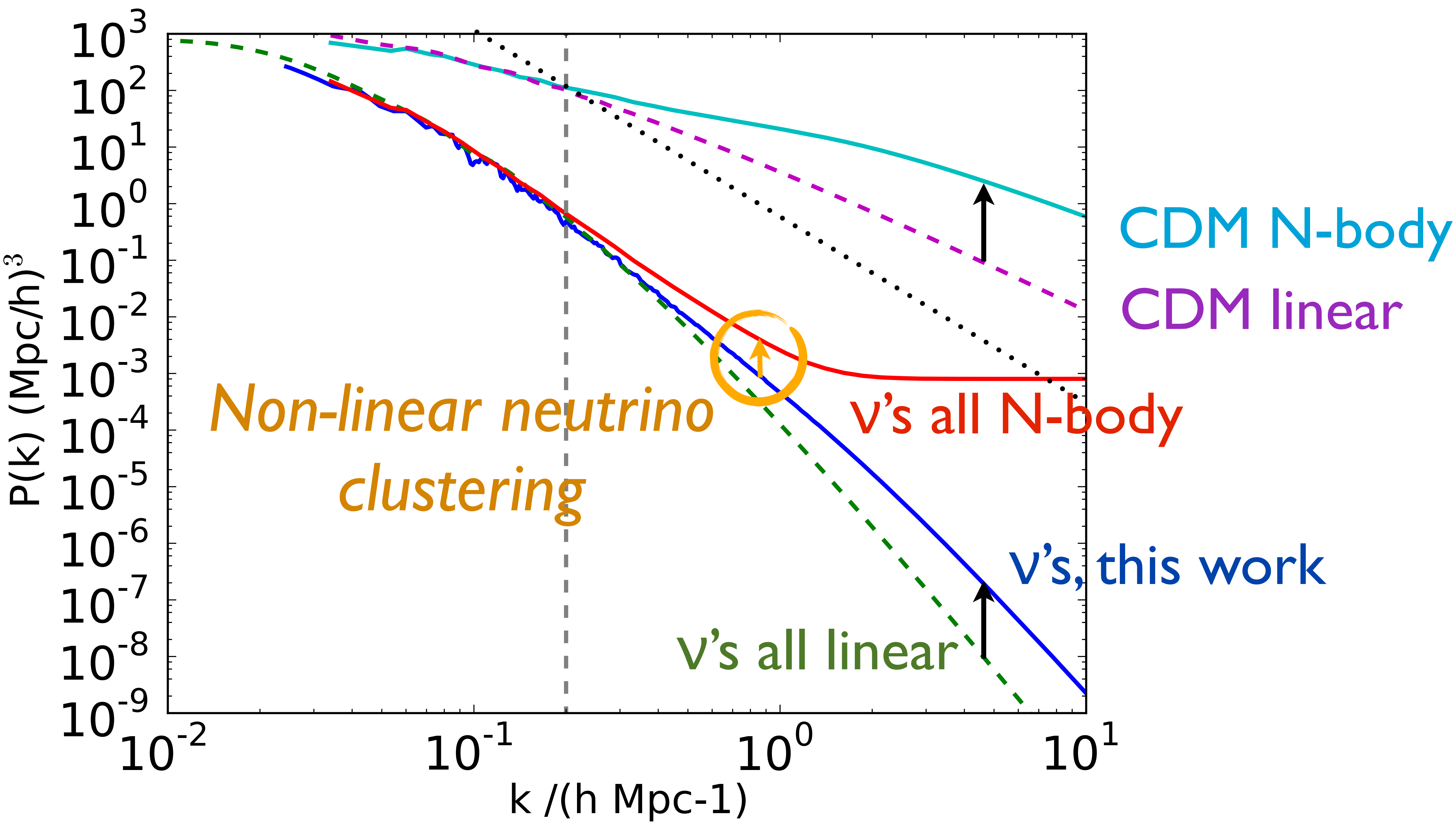
Results

Neutrino power spectrum at $z = 1, \sum m_\nu = 0.3$ eV



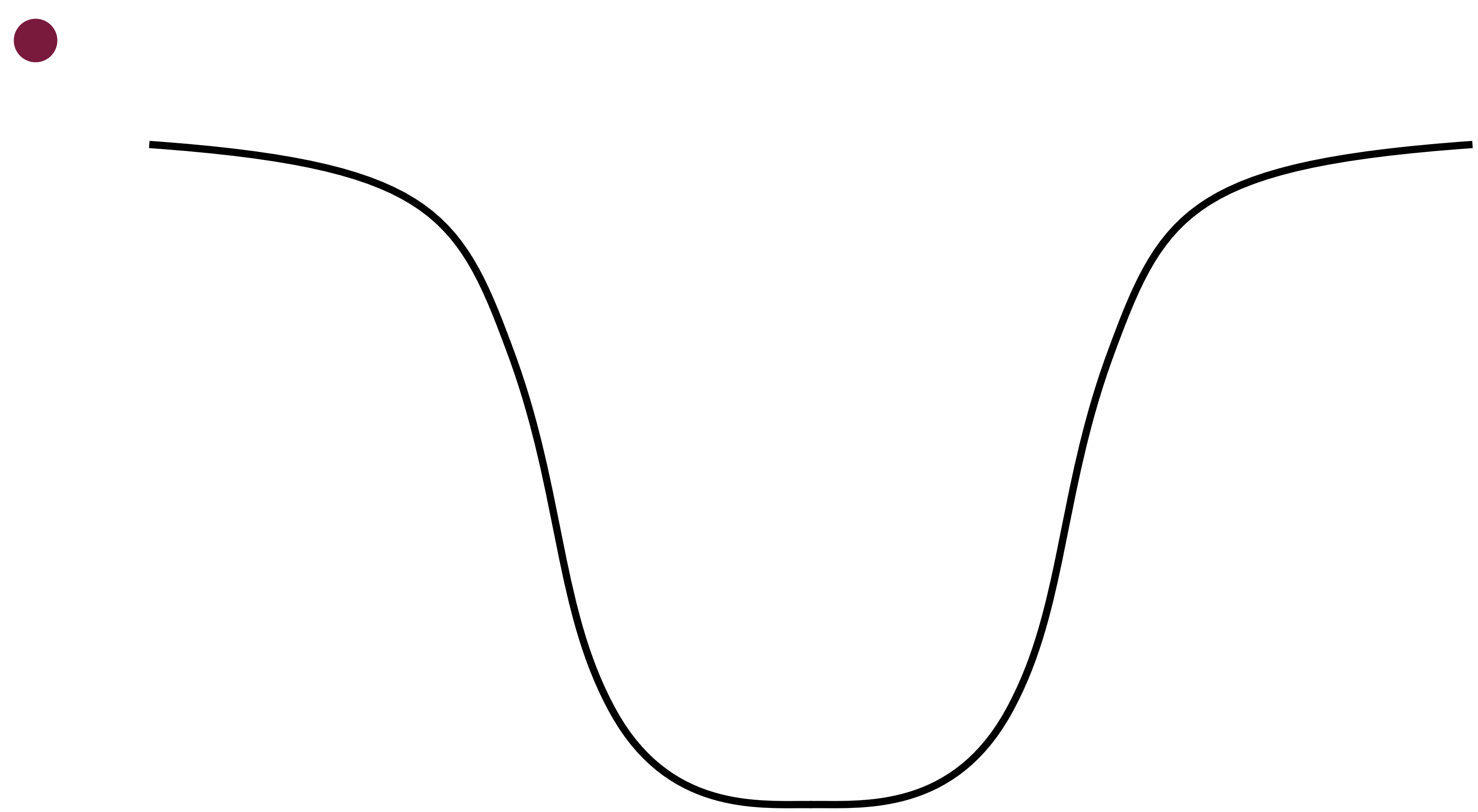
Limitation

This method does not account for the non-linear clustering of neutrinos in massive clusters at z = 0



Non-linear neutrino clustering in massive haloes

- At $z = 0$, characteristic $r_{halo} \lesssim 1$ Mpc << L_{fs}
- $v_\nu \approx 500$ km/s $(0.1 \text{ eV}/m_\nu)$ << $|\phi|^{1/2}$ ~800-3000 km/s



Non-linear neutrino clustering in massive haloes

If halo grows on ~ Hubble timescale, neutrinos may be captured. Escape condition:

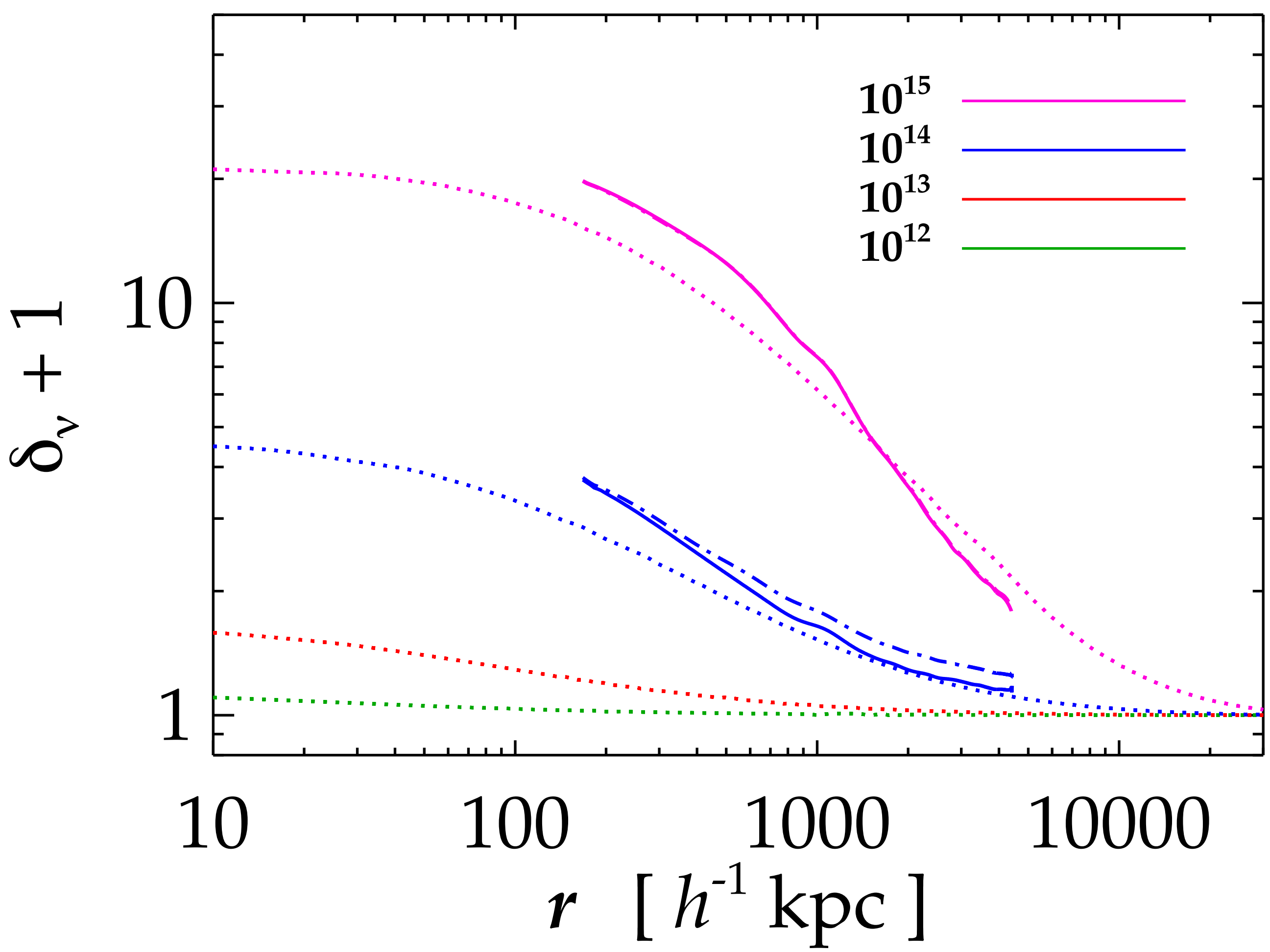
$$\frac{p}{T_\nu} \gtrsim \frac{m_\nu}{T_{\nu,0}} \frac{1}{\sqrt{H_0 \Delta t_\phi}} \left(2H_0 \; r_0 \sqrt{|2\phi_0|}\right)^{1/2}$$

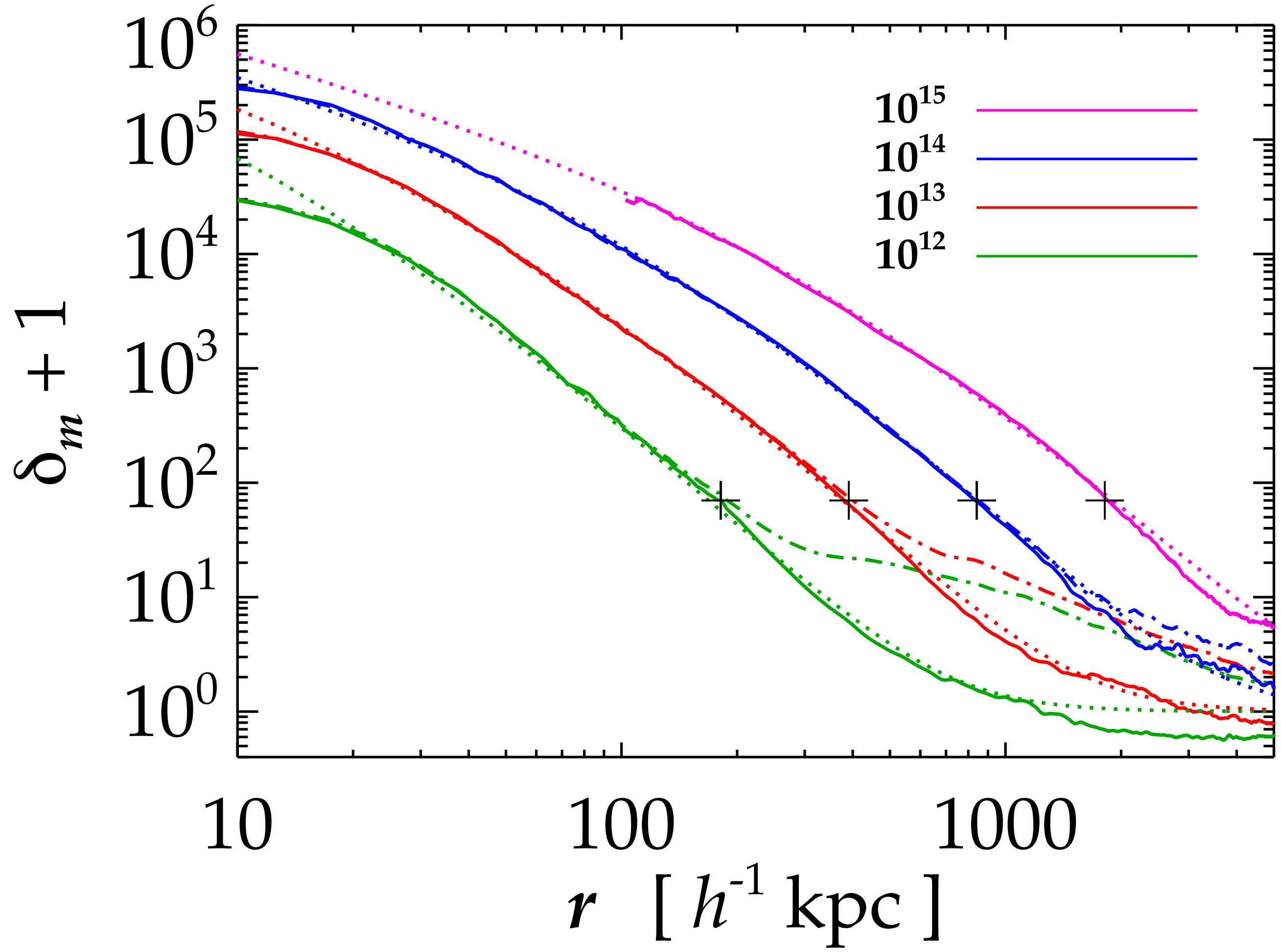
$$\approx (H_0 \Delta t_\phi)^{-1/2} \frac{m_\nu}{0.1 \text{ eV}} \left(\frac{r_0}{0.5 \; h^{-1}\text{Mpc}}\right)^{1/2} \left(\frac{\sqrt{|\phi_0|}}{3000 \text{ km/s}}\right)^{1/2}$$

About 94 % of neutrinos have p > T for Fermi-Dirac distribution. ☛ Most remain linear, a small fraction get captured and become very non-linear

Non-linear neutrino clustering in massive haloes

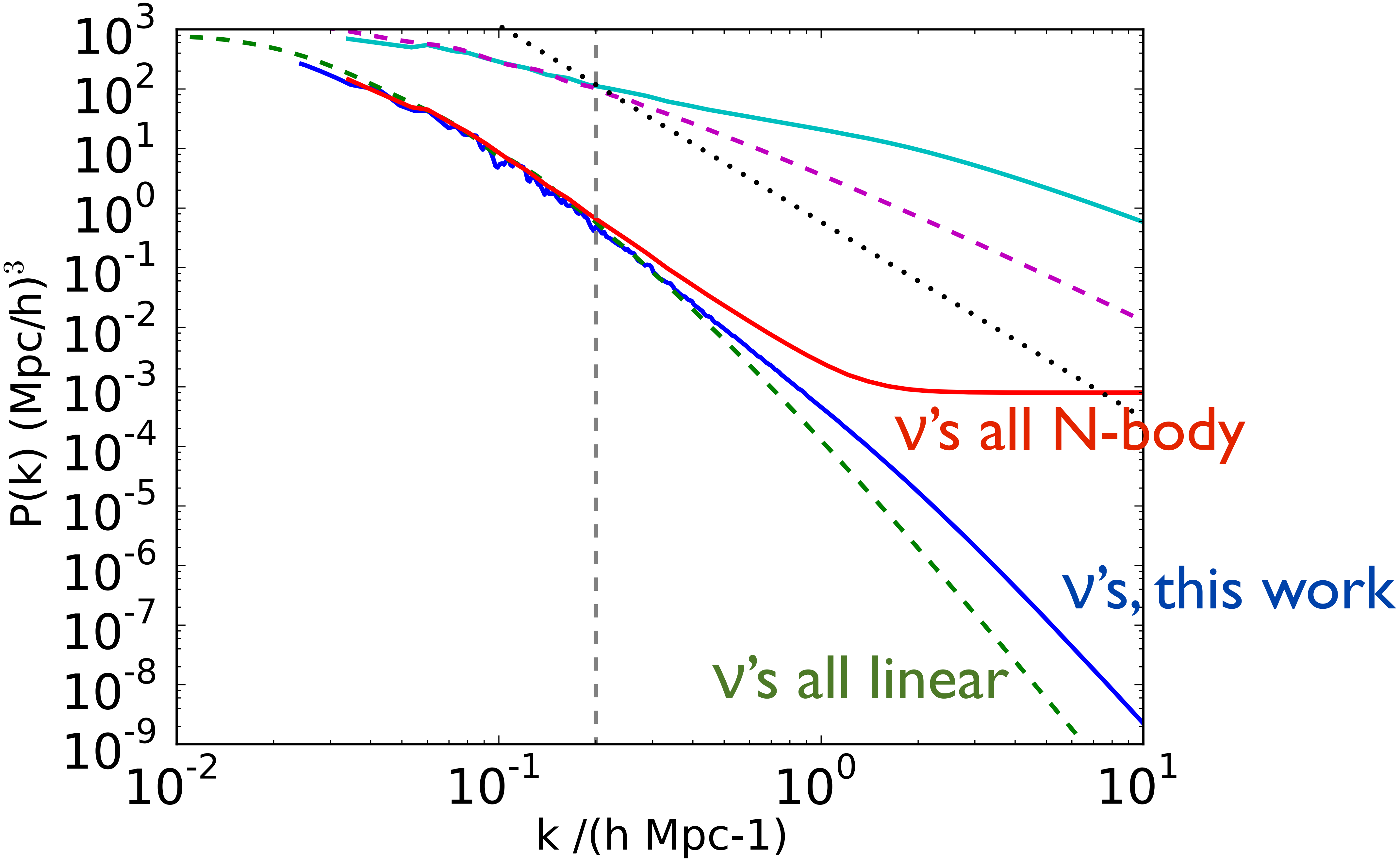
$\sum m_\nu = 0.3$ eV





Brandbyge et al. 2010

Still have < δ_ν^2 > << 1 on all scales, because haloes make a small fraction of the total volume.



Conclusions

- Total power spectrum is not very sensitive to exact clustering of neutrinos on small scales. In practice, $\delta_\nu(k>>k_{fs}) << \delta_{CDM}$ is what really counts. May as well use a simple method!
- It is accurate to better than 0.2% for the matter power spectrum at $z = 0$ for $\sum m_\nu \lesssim 0.3$ eV and nearly exact at $z > 1$.
- Our method is about 20% faster than particle-based method. Patch for GADGET publicly available (S. Bird webpage)
- Future work: accurately modeling the clustering of neutrinos themselves / use hybrid methods.