Gravity localization & AdS4 / CFT3

C. Bachas, ENS - Paris

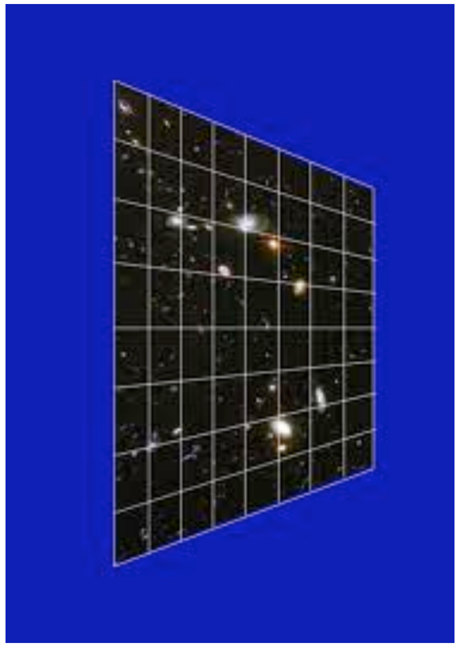
IHES - IHP, november 2011

CB, J. Estes, *arXiv:1103.2800 [hep-th]*

B. Assel, CB, J. Estes, J. Gomis, *1106.4253 [hep-th]*

& *in preparation*

Can there be non-compact extra dimensions ?



Requires that the observed particles and their interactions be confined to a **3-brane** world.

Rubakov, Shaposhnikov '83

D-branes allow us to achieve this within string theory, for all particles with **spin = 1 or less**.

Polchinski '95

But how about gravity ?

The question has received much attention, starting with
Randall & Sundrum ‘99

Closely related to the following:

Can the graviton have mass ?

Could it be a resonance ?

Are sectors “hidden” from gravity possible ?

Other IR modifications of Einstein equations ?

The subject has a long history, to which I will not do justice here .....

The short answer is: **we still don’t know .**

In our work we adopted a conservative attitude:

* Restrict to 2-derivative (super)gravity

* Assume negative-curvature brane world

(to avoid problems of horizon, vDVZ discontinuity etc)

... but we demand supersymmetry and string-theory embedding.

(to guarantee stability against quantum corrections)

$\Lambda < 0$ not the "real" problem, but let's push on !

Summary

1. KK reduction of spin 2
2. *AdS*-brane world model
3. Defect CFTs & Holography

4. The UCLA solutions
5. Page charges & parameters
6. New AdS_4 / $SCFT_3$
7. Limiting geometries

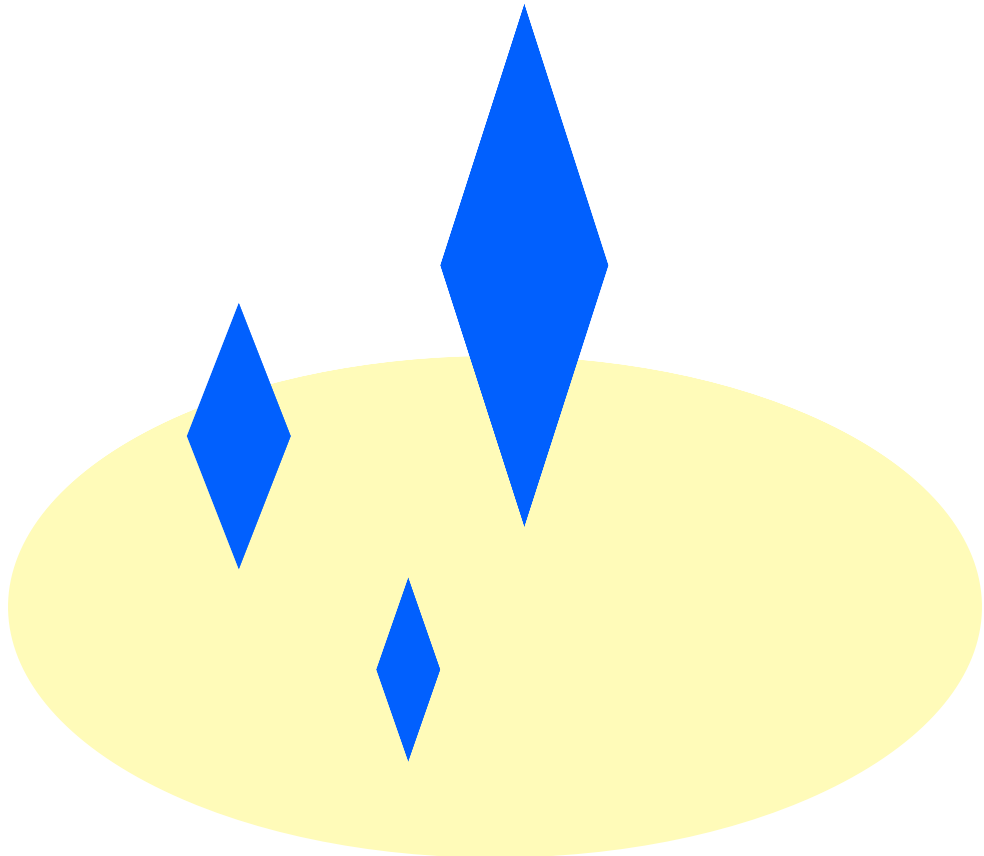
1. KK reduction for spin 2

Interested in *warped*-(A)dS background geometries,

$$\widehat{ds^2} = e^{2A(y)}\,\bar{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + \hat{g}_{ab}(y)\,dy^a dy^b$$

$$\bar{\mathcal{M}}_4 = \mathrm{AdS}_4,\ \mathbb{M}_4,\ \mathrm{dS}_4$$

$$k = -1, 0, 1$$



Consider (consistent reduction of) metric perturbations:

$$ds^2 = e^{2A}\,(\bar{g}_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu + \hat{g}_{ab}\,dy^a dy^b \ ,$$

with $$h_{\mu\nu}(x,y) \ = \ h^{[\mathrm{tt}]}_{\mu\nu}(x)\,\psi(y)$$

where $$(\bar{\Box}^{(2)}_x - \lambda)\,h^{[\mathrm{tt}]}_{\mu\nu} = 0 \qquad \text{and} \qquad \bar{\nabla}^\mu h^{[\mathrm{tt}]}_{\mu\nu} = \bar{g}^{\mu\nu} h^{[\mathrm{tt}]}_{\mu\nu} = 0 \ .$$

Pauli-Fierz equations $(\lambda = m^2 + 2k)$

Linearizing the Einstein equations $R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}$

leads to a Schrödinger problem in the 6D transverse space:

$$-\frac{e^{-2A}}{\sqrt{[\hat{g}]}} \left(\partial_a \sqrt{[\hat{g}]} \, \hat{g}^{ab} e^{4A} \partial_b \right) \psi = m^2 \psi$$

PS. This is also equivalent to the scalar-Laplace equation in 10 dimensions :

$$\frac{1}{\sqrt{\hat{g}}} \left(\partial_M \sqrt{\hat{g}} \, \hat{g}^{MN} \partial_N \right) h_{\mu\nu}(x, y) \; = \; 0 \; .$$

Important remark:

The linearized equation is **universal**, i.e. it depends only on the geometry - not on details of the “matter” fields.

Brandhuber, Sfetsos ‘99
Csaki, Erlich, Hollowood, Shirman ‘00
CB, JE ‘11

NB: this is specific to 2-derivative gravity, and it is not true for the non-linear terms.

Hence: **localization of spin-2 can only come from geometry**

Second remark:

The wavefunction norm is $$\|\psi\|^2 = \int [dy] e^{2A} \psi^* \psi$$

Then $$\langle \psi, \mathcal{M}^2 \psi \rangle = \int [dy] e^{4A} \partial_a \psi^* \partial^a \psi$$ from which we conclude:

$\mathcal{M}^2 \geq 0$ and $\mathcal{M}^2 = 0 \longrightarrow \psi_0 = constant$

For a 4D massless graviton we thus need $$\int [dy]\, e^{2A} < \infty$$

Hence, if the transverse space is non-compact

- either there is a continuum

- or the 4D graviton is massive

Wait a minute : **Can't the warp factor help?**

Well, when it does "infinity" is an apparent horizon

(which can be reached in finite proper time).

Let us see why in the case of flat-4D slicing, and one extra dimension:

For a particle moving in the transverse dimension y , $e^{2A} = C\sqrt{e^{2A} - \dot{y}^2}$

As y goes to infinity, we need $e^A \longrightarrow 0$, so $\dot{y} \simeq e^A \longrightarrow 0$

The total proper time $\int d\tau = \int dt\, C^{-1} e^{2A} \simeq \int dy\, e^A$ should be infinite, for **geodesic completeness**.

If we request $\int dy\, e^{2A}$ = finite, for a normalizable zero mode, then

$$A \simeq -\nu \log y \qquad \text{with} \qquad 1 > \nu > 1/2$$

This is ruled out by the weak energy conditions which imply $A'' \leq 0$ ("**holographic c-theorem**")

Girardello et al '98, Freedman et al '99

One can of course cutoff space before the horizon, with an **“IR brane”**;

this is compactification .

Randall, Sundrum I

Else, one needs to supplement the quantum theory with **boundary conditions**;

there exists then a continuum (and open issues of stability).

Randall, Sundrum II

For an AdS-brane world: no horizon, no “holographic c-theorem”,

the 4D graviton is massive.

Karch, Randall ’00 , ‘01

2. Model of AdS brane world

Starting point is 5D Einstein action plus a **thin 3-brane**

$$I_{\rm KR} = -\frac{1}{2\kappa_5^2}\int d^4x\,dy\,\sqrt{g}\left(R+\frac{12}{L^2}\right)+\lambda\int d^4x\,\sqrt{[g]_4}\ ,$$

The solution is:

AdS4

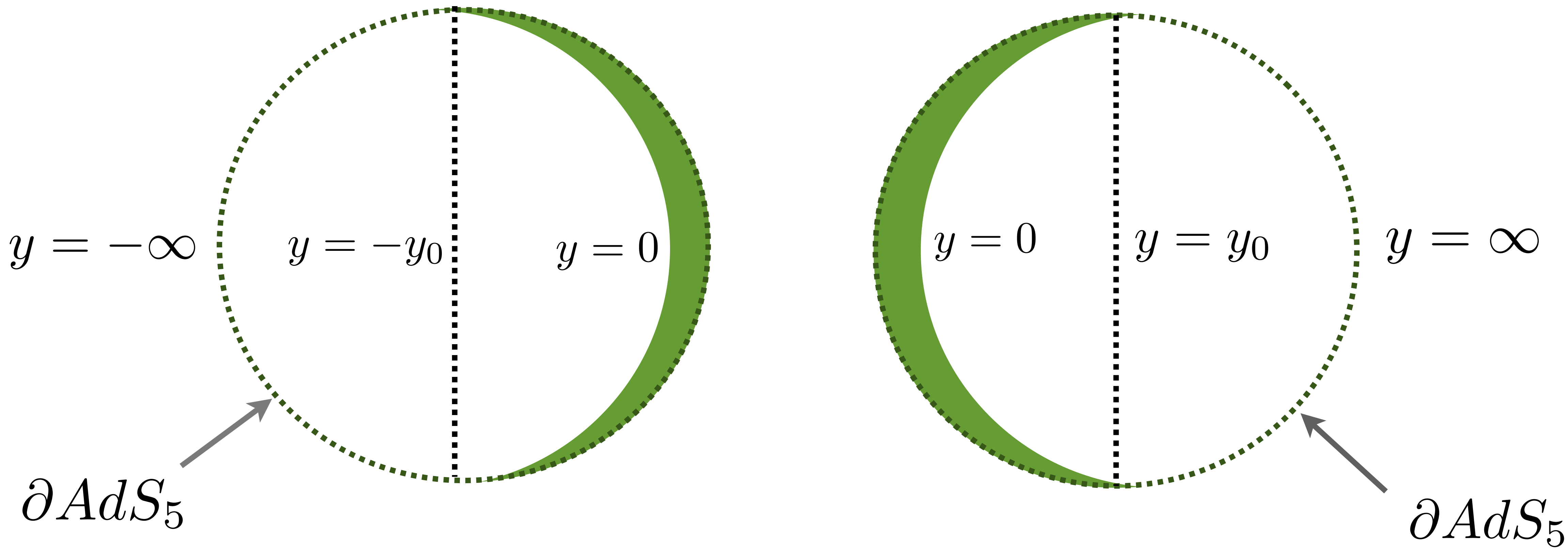
$$ds^2 = L^2\cosh^2\left(\frac{y_0-|y|}{L}\right)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \quad \textit{where} \quad y_0 = L\,\mathrm{arctanh}\left(\frac{{\kappa_5}^2\lambda L}{6}\right)$$

It describes two (large) pieces of AdS_5 glued along a AdS_4 brane of radius

$$\ell^2 = e^{2A(0)} = L^2\cosh^2\left(\frac{y_0}{L}\right)\ .$$

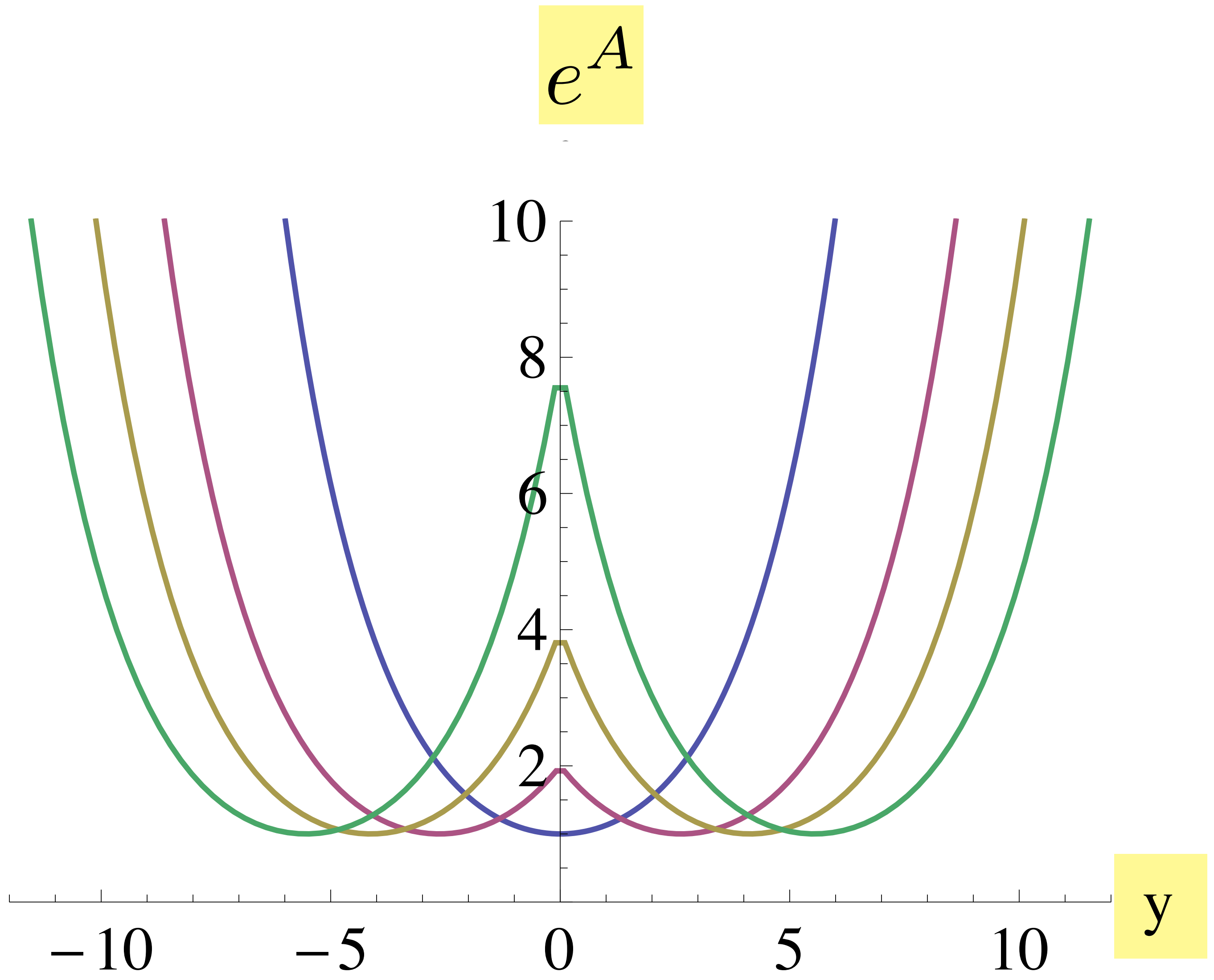
One can tune $\lambda \to \frac{3\pi}{{\kappa_5}^2 L}$ **so that** $\ell \gg L$

The AdS_5 metric in AdS_4 foliation is $L^2\cosh^2(\frac{\tilde{y}}{L})\bar{g}_{\mu\nu}dx^\mu dx^\nu + d\tilde{y}^2$



So, cut away green slices and glue.

For $\ell \gg L$ the brane is near the boundary of either AdS_5 part.



Warp factor $e^{2A} \equiv {f_4}^2 = L^2 \cosh^2\left(\frac{y_0 - |y|}{L}\right)$

as ℓ/L is gradually tuned up

vanishes near
bottom of warp
factor wells

- a nearly-constant, **nearly massless** mode $\ell^2 m_0^2 \simeq \frac{3L^2}{2\ell^2}$

Spectrum :

for an observer
on the brane

- two towers of AdS_5 modes

$$\ell^2 m^2 \simeq (2n+1)(2n+4) \qquad n = 0, 1, \cdots$$

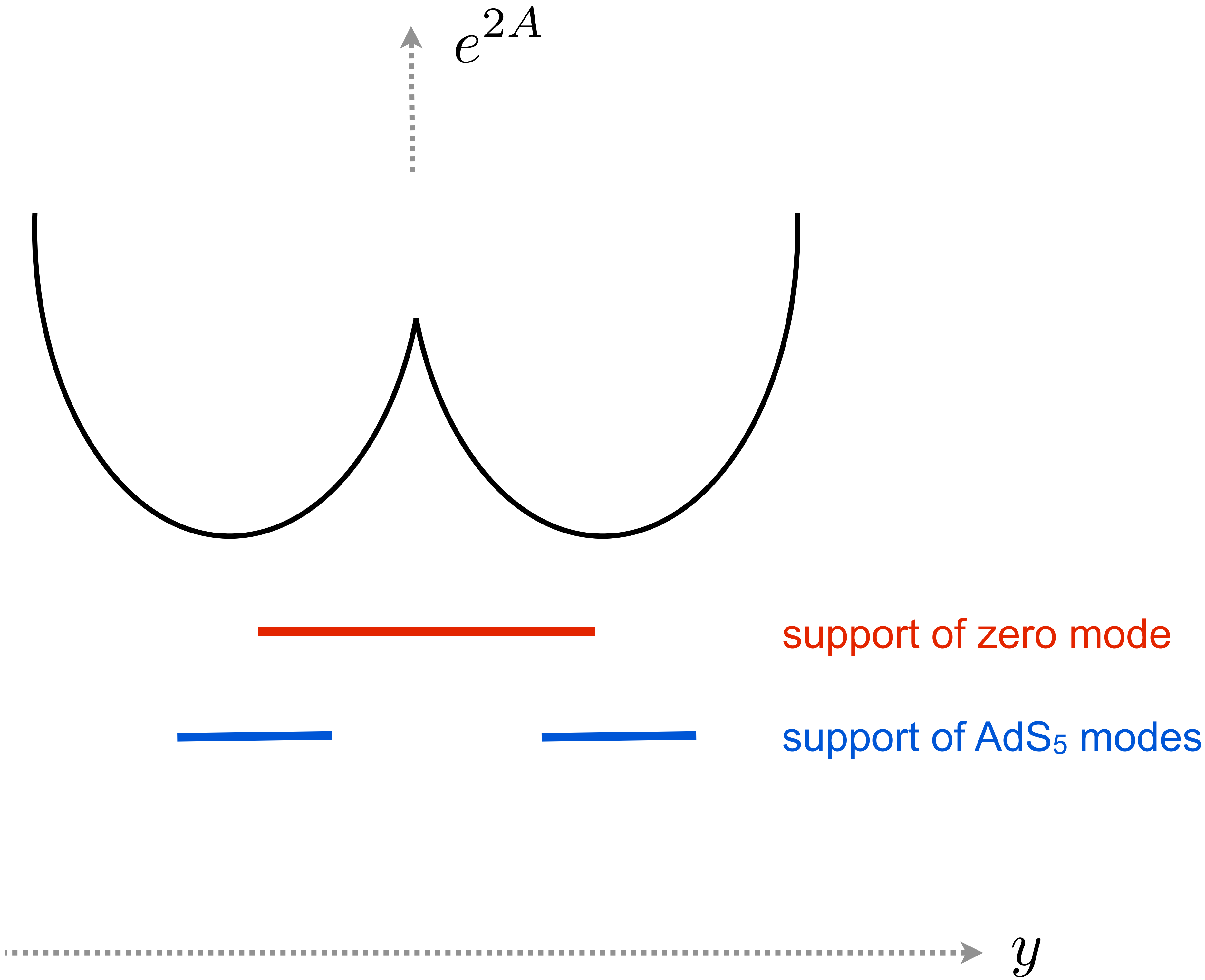
solutions of the AdS5 eigenvalue problem

= Legendre equation & gluing conditions

The AdS_5 states live at the bottom of the warp-factor wells .

Their wavefunctions are **exponentially suppressed** at the brane position.

Schematically:



effective 4D parameters:

$$8\pi G_N \simeq \kappa_5^2/L$$

as in usual KK

$$V_{\rm Newton} + \Delta V \simeq -\frac{G_N m_1 m_2}{r}\left(1 + \gamma \frac{L^2}{r^2} + \cdots\right)$$

unlike standard KK

so $$\frac{\ell}{L} \sim 10^{31} - 10^{62}$$

Also $\int \psi_0 \psi^\dagger \psi \neq \text{universal}$ because $\psi_0 \neq$ constant.

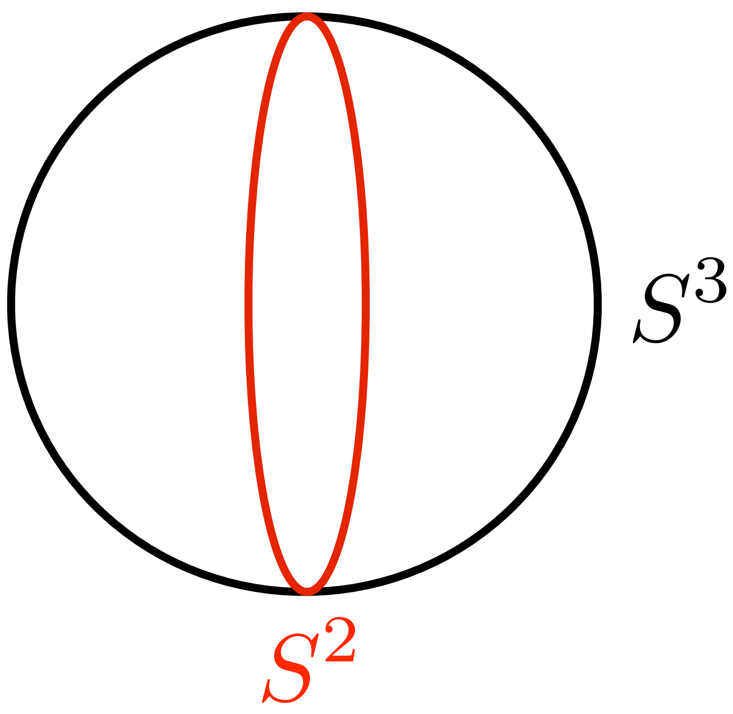
So the nearly-massless graviton has **non-universal couplings** to other fields.

3. Holographic defect CFTs

Gravity in **AdS$_{D+1}$** is dual to **CFT$_D$**

an **AdS$_D$** brane is dual to a **conformal domain wall**

Karch, Randall '01
DeWolfe, Freedman, Ooguri '01
CB, de Boer, Dijkgraaf, Ooguri '01



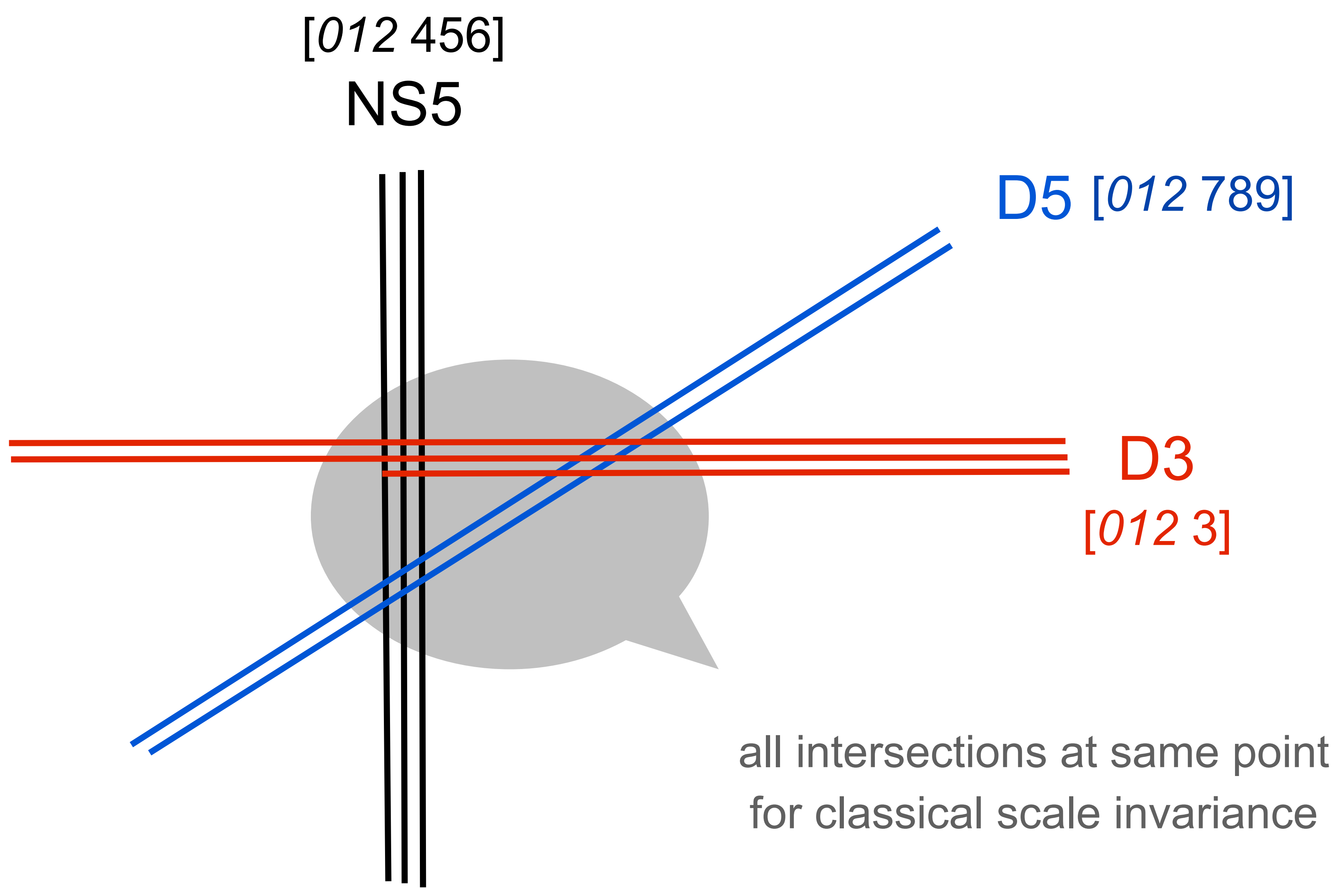
Boundary in global coordinates

(almost) **massless** 4D graviton is dual to (almost) **conserved** 3D $T_{\mu\nu}$

The most symmetric case:

1/2 superconformal **domain walls** of N=4 4D **super Yang-Mills**

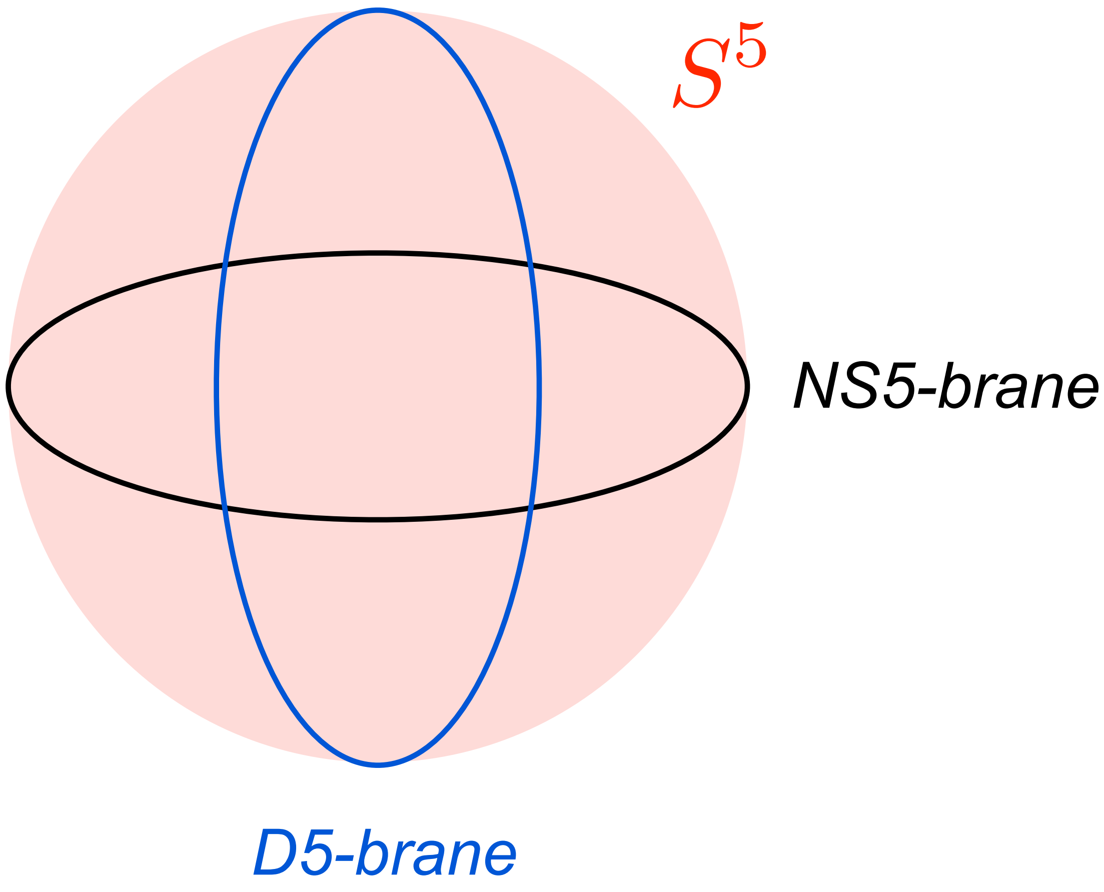
Brane engineering (weak coupling):



all intersections at same point
for classical scale invariance

Strongly-coupled SYM better described as gravity in **AdS_5 x S^5**

Probe 5-branes have worldvolumes **AdS_4 x S^2** with radius L



... but to localize gravity, need back-reacting branes.

4. The UCLA solutions

The exact solutions of IIB supergravity have been discovered

in D'Hoker, Estes and Gutperle '07

The N=4 SYM has symmetry $SU(2,2|4) \supset S0(2,4) \times SO(6)$

The wall breaks this to $OSp(2,2|4) \supset S0(2,3) \times SO(4)$

The solutions are thus $AdS_4 \times S^2 \times S^2$ fibrations over a surface $\sum$

There are also form-fields $F_5, \; H_3, \; F_3$ consistent with these isometries,

and sourced by the corresponding branes.

DEG show that the **general local solution** depends on

two harmonic functions on Σ : h_1, h_2

metric : $$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{\mathrm{S}_1^2} + f_2^2 ds^2_{\mathrm{S}_2^2} + 4\rho^2 dz d\bar{z} \ ,$$

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} \ , \quad f_1^8 = 16\, h_1^8 \frac{N_2 W^2}{N_1^3} \ , \quad f_2^8 = 16\, h_2^8 \frac{N_1 W^2}{N_2^3}$$

$$\rho^8 = \frac{N_1 N_2 W^2}{h_1^4 h_2^4}$$

dilaton : $$e^{4\phi} = \frac{N_2}{N_1}$$

where
$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial\bar{\partial}(h_1 h_2) \ ,$$
$$N_1 = 2h_1 h_2 |\partial h_1|^2 - h_1^2 W \ , \qquad N_2 = 2h_1 h_2 |\partial h_2|^2 - h_2^2 W \ .$$

For the p-form backgrounds we need also the dual harmonic functions:

$$h_1 = -i(\mathcal{A}_1 - \bar{\mathcal{A}}_1) \qquad \rightarrow \qquad h_1^D = \mathcal{A}_1 + \bar{\mathcal{A}}_1$$

$$h_2 = \mathcal{A}_2 + \bar{\mathcal{A}}_2 \qquad \rightarrow \qquad h_2^D = i(\mathcal{A}_2 - \bar{\mathcal{A}}_2)$$

and the volume forms of the unit-radius symmetric spaces:

ω^{0123}	AdS_4
ω^{45}	S_1^2
ω^{67}	S_2^2

3-forms : $$H_{(3)} + iF_{(3)} = \omega^{45} \wedge db_1 + i\,\omega^{67} \wedge db_2$$

where:

$$b_1 = 2ih_1 \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_1} + 2h_2^D$$

$$b_2 = 2ih_2 \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_2} - 2h_1^D$$

5-form : $$F_{(5)} = -4\, f_4^4\, \omega^{0123} \wedge \mathcal{F} + 4\, f_1^2 f_2^2\, \omega^{45} \wedge \omega^{67} \wedge (*_2 \mathcal{F}) \;,$$

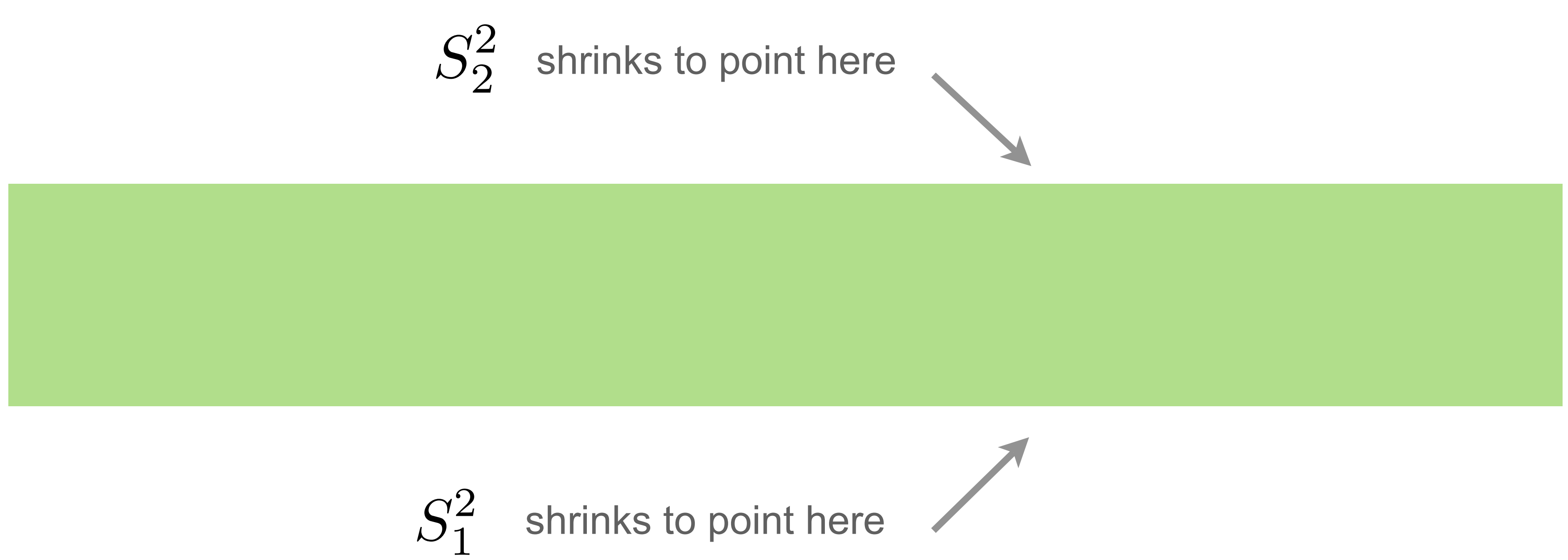
where

$$f_4^4 \mathcal{F} = dj_1 \qquad \text{with} \qquad j_1 = 3\mathcal{C} + 3\bar{\mathcal{C}} - 3\mathcal{D} + i\frac{h_1 h_2}{W}\,(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)$$

$$\partial \mathcal{C} = \mathcal{A}_1 \partial \mathcal{A}_2 - \mathcal{A}_2 \partial \mathcal{A}_1 \qquad\qquad \mathcal{D} = \bar{\mathcal{A}}_1 \mathcal{A}_2 + \mathcal{A}_1 \bar{\mathcal{A}}_2$$

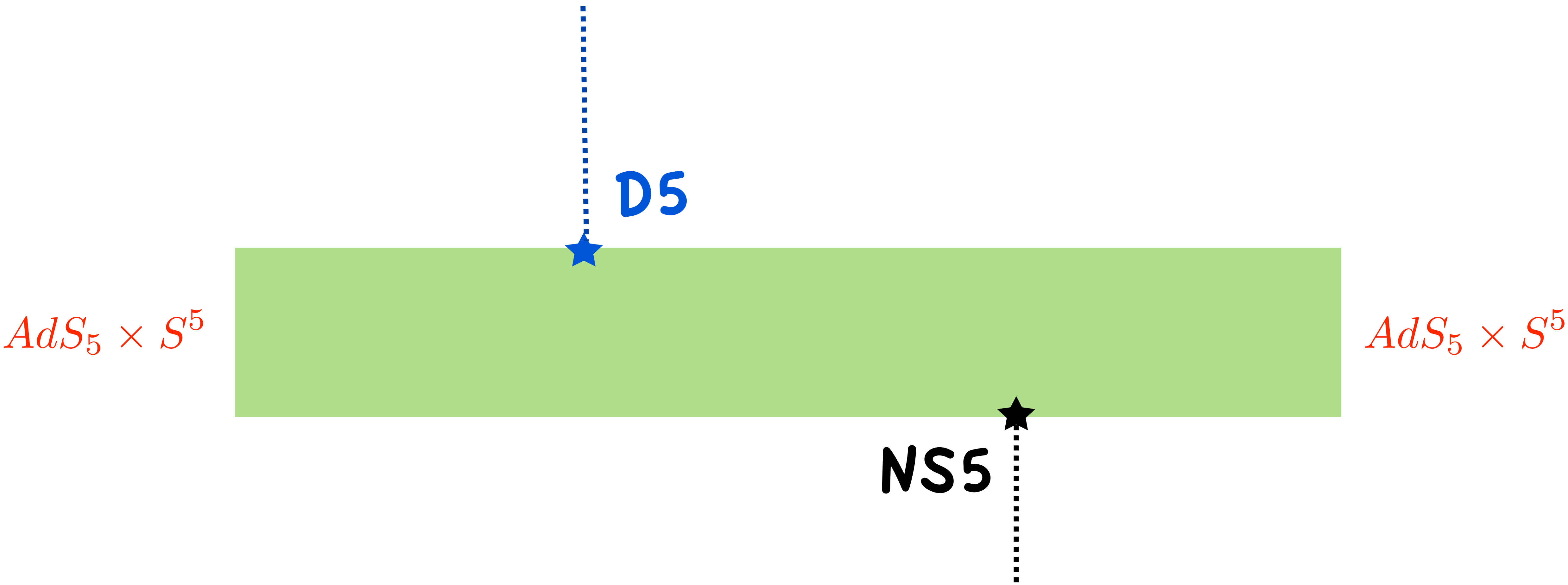
The above expressions provide the **general form** of the **local** solution.

The **absence of singularities** greatly constrains, however, the choice of harmonic functions. Taking Σ = **infinite strip**, requires Neumann (Dirichlet) conditions on the upper (lower) boundary for h_1 and the other way around for h_2 , so that the strip boundaries are are **interior points** of the 10D geometry.



Singularities on the boundary correspond to **asymptotic regions**.

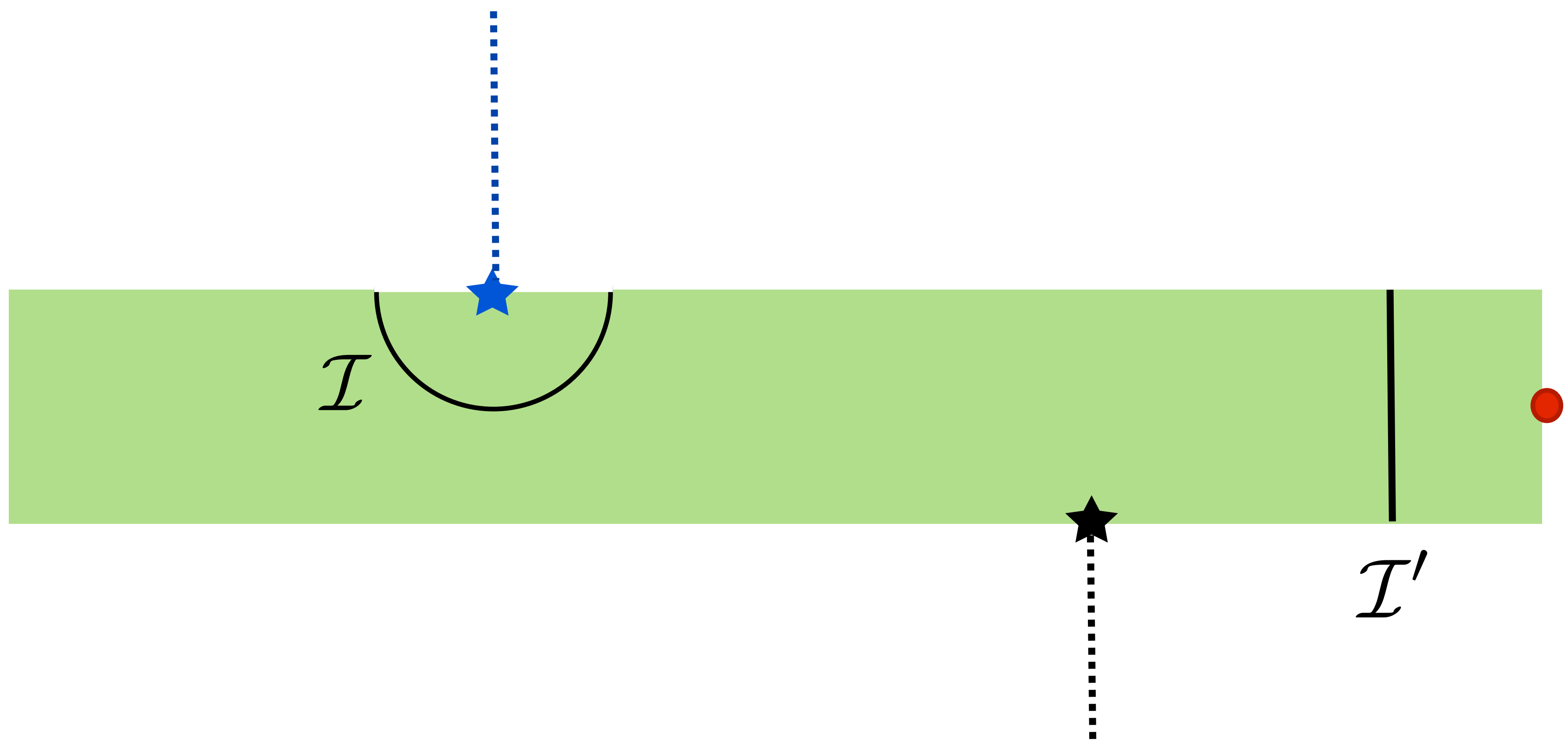
A solution with all necessary ingredients is:



$$h_1 = \left[-i\alpha_1 \sinh(z-\beta_1) - \gamma_1 \ln\left(\tanh\left(\frac{i\pi}{4} - \frac{z-\delta_1}{2}\right)\right)\right] + \text{c.c.}\ ,$$

$$h_2 = \left[\alpha_2 \cosh(z-\beta_2) - \gamma_2 \ln\left(\tanh\left(\frac{z-\delta_2}{2}\right)\right)\right] + \text{c.c.}\ .$$

Non-contractible cycles supporting brane charges:

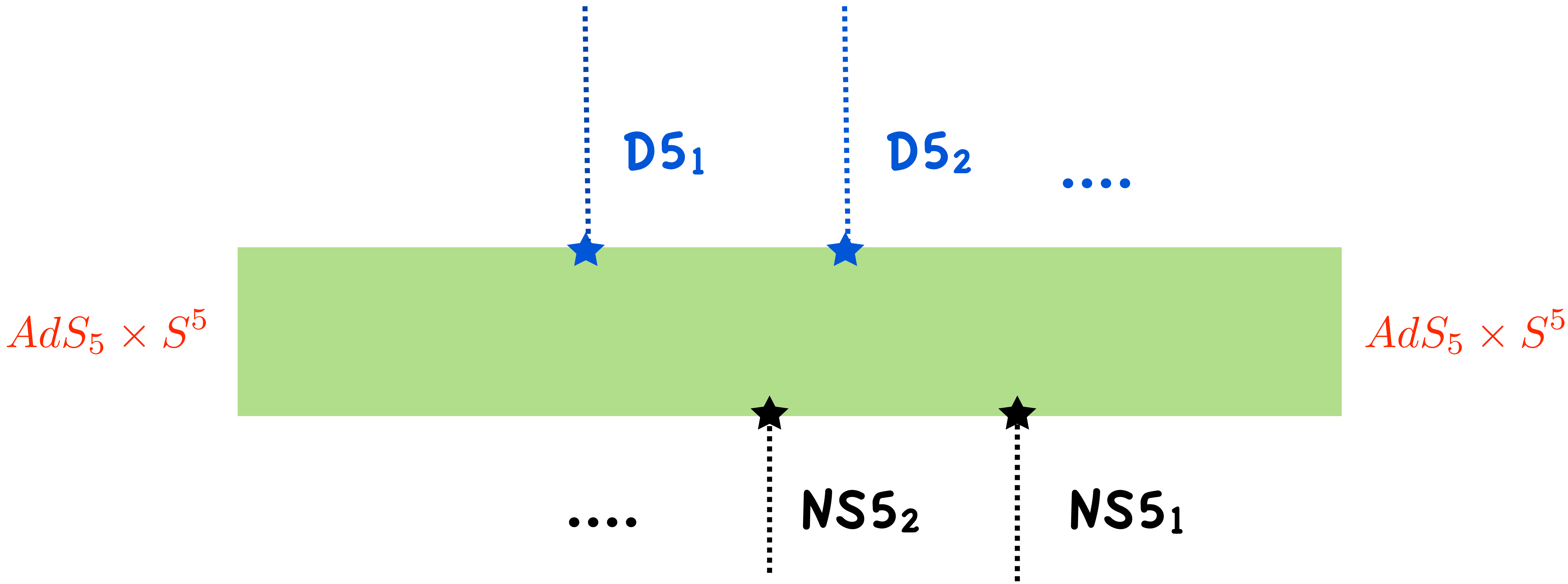


$$C_3 = \mathcal{I} \times S_2^2 \;\sim\; S^3$$ D5-brane

$$C_5 = \mathcal{I} \times S_1^2 \times S_2^2 \;\sim\; S^3 \times S^2$$ D3-brane in D5-brane

$$C_5' = \mathcal{I}' \times S_1^2 \times S_2^2 \;\sim\; S^5$$ D3-brane

More generally, there can be many different stacks of 5-branes:



$$h_1 = \left[-i\alpha \sinh(z-\beta) - \sum_{a=1}^{q} \gamma_a \ln \left(\tanh \left(\frac{i\pi}{4} - \frac{z-\delta_a}{2} \right) \right) \right] + c.c.$$

$$h_2 = \left[\hat{\alpha} \cosh(z-\hat{\beta}) - \sum_{b=1}^{\hat{q}} \hat{\gamma}_b \ln \left(\tanh \left(\frac{z-\hat{\delta}_b}{2} \right) \right) \right] + c.c.$$

5. Parameters & Page charges

Parameter count :

real-axis origin		D3-charge conservation
$\alpha, \beta, \hat{\alpha}, \hat{\beta}$	⟷	$\phi^{(\pm\infty)}, \, Q_{D3}^{(\pm\infty)}$
$\gamma_a \, , \delta_a$		$Q_{D5}^{(a)} \, , \, Q_{D3}^{(a)}$
$\hat{\gamma}_b \, , \hat{\delta}_b$		$Q_{NS5}^{(b)} \, , \, \hat{Q}_{D3}^{(b)}$

... minus one

In matching the parameters, one faces a subtlety:

$$dF_5 = H_3 \wedge F_3$$

implies there is **no local, gauge-invariant** definition of D3-charge

The quantized **Page charge** is either non-local or gauge-variant. ... Marolf '00

In our case:

$$Q_{D3}^{\mathrm{inv}(a)} = \int_{\mathcal{C}_a} F_5 - B_2 \wedge F_3 + \int_{\mathcal{C}_a} F_3 \wedge B_2 \Big|_{z=\infty}$$

$$= 2^8 \pi^3 \left(\hat{\alpha}\, \gamma_a \sinh(\delta_a - \hat{\beta}) - 2\, \gamma_a \sum_{b=1}^{\hat{q}} \hat{\gamma}_b \arctan(e^{\hat{\delta}_b - \delta_a}) \right)$$

$$\hat{Q}_{D3}^{\mathrm{inv}(b)} = \int_{\hat{\mathcal{C}}_b} F_5 + C_2 \wedge H_3 - \int_{\hat{\mathcal{C}}_b} H_3 \wedge C_2 \Big|_{z=-\infty}$$

$$= 2^8 \pi^3 \left(\alpha\, \hat{\gamma}_b \sinh(\hat{\delta}_b - \beta) + 2\, \hat{\gamma}_b \sum_{a=1}^{q} \gamma_a \arctan(e^{\hat{\delta}_b - \delta_a}) \right) \quad .$$

The 5-brane charges are unambiguous:

$$Q_{D5}^{(a)} = 16\pi^2\,\gamma_a \qquad , \qquad Q_{NS5}^{(b)} = 16\pi^2\,\hat{\gamma}_b\ .$$

The remaining parameters control the **asymptotic AdS$_5$ x S^5 regions**:

$$\phi^{(\pm\infty)},\, Q_{D3}^{(\pm\infty)}$$

We don't need their explicit expression here, but note that

$$Q_{D3}^{(\pm\infty)} \propto L^4_{(\pm\infty)}$$

vanish as $\alpha, \hat{\alpha} \longrightarrow 0$.

To “mimic” the Karch-Randall model, we must **fine-tune** parameters

so as to **flatten the brane** relative to the “bulk” AdS_5 .

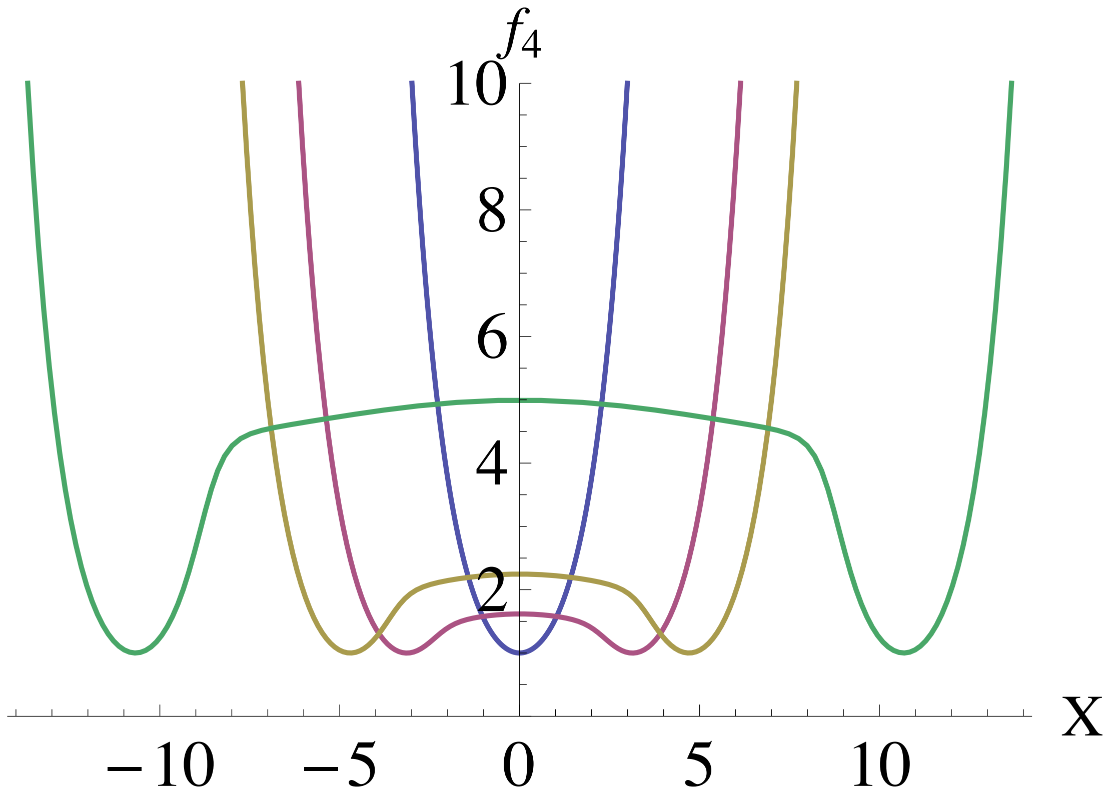
This required **both NS5 and D5 charges**, to stabilize the dilaton, and

attractor mechanism

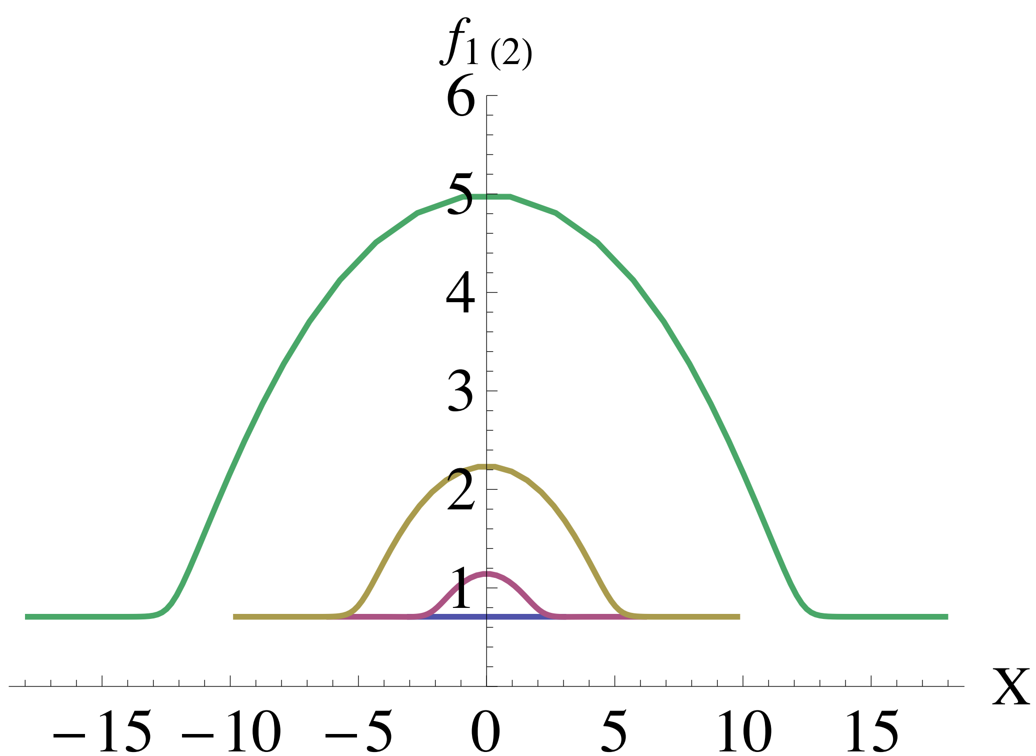
$$\gamma_a, \hat{\gamma}_b \gg \alpha, \hat{\alpha} \qquad \Longrightarrow \qquad Q_{D5}^{(a)}, Q_{NS5}^{(b)} \gg Q_{D3}^{\pm\infty}$$

“more in domain wall than in bulk”

But: in this limit the brane world-volume **decompactifies.**

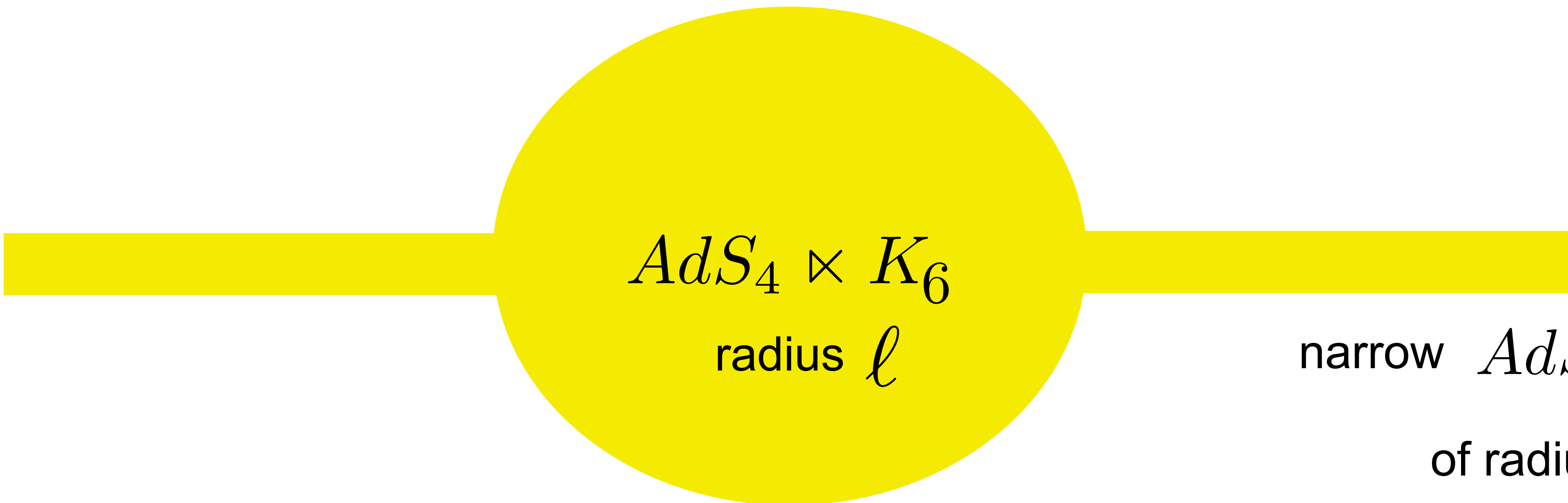


warp factor



sphere radii

The 10d geometry looks like this:



narrow $AdS_5 \times S^5$ throat
of radius L

graviton mass $\sim \frac{L}{\ell} \ll 1$

(numerical, to appear)

The limit geometry $\alpha, \hat{\alpha} \to 0$ is smooth; the **throats cap off**

and $S_1^2 \times S_2^2 \ltimes \Sigma \sim K_6$ ‘compactifies’

ABEG ; Aharony, Berdichevsky, Berkooz, Shamir ‘11

6. New AdS4 - SCFT3 dualities

The "decompactification problem" may (or may not) be solved with fewer supersymmetries [and R-symmetries].

But these geometries are interesting in their own right:

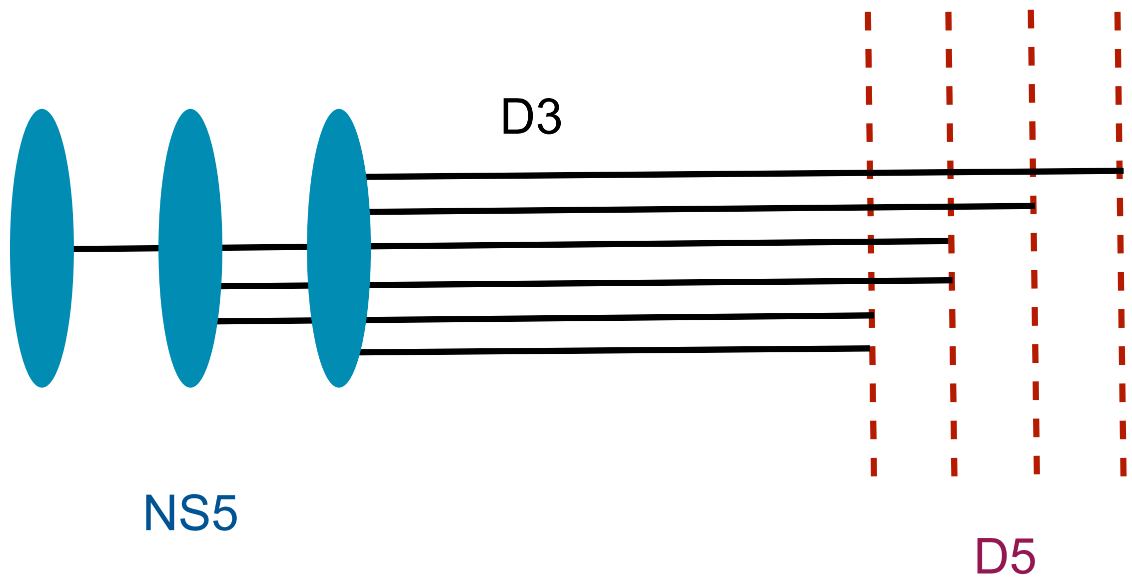
* learn about (strongly-coupled) 3D CFTs
* understand gravitational throats from gauge theory

The candidate dual SCFTs have been discussed in a series of papers by **Gaiotto, Witten '08 .** They are conjectured strongly-coupled fixed points of 3D **quiver gauge theories**.

One way to introduce these theories, is starting with **N D3-branes** suspended between **NS5-branes** on left, and **D5-branes** on right:

This is described by **two partitions of N** , $\rho\,,\hat{\rho}$,

for example:



$$N = 6\ ;\ \rho = (2,2,1,1)\ ;\ \hat{\rho} = (3,2,1)$$

The supersymmetric configurations are in 1-to-1 correspondence with solutions of **Nahm's equations:**

$$\frac{dX^a}{dt} = i\epsilon_{abc}[X^b, X^c]$$

on the **interval,** with boundary conditions that are **simple poles,**

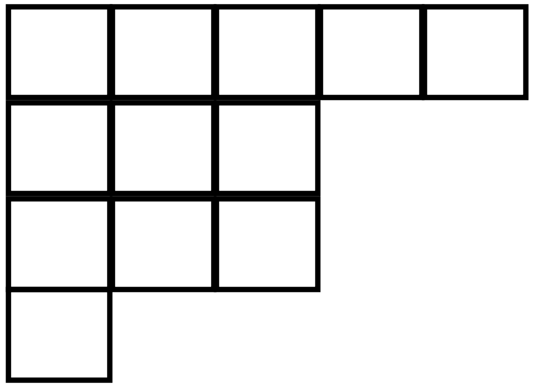
$$X^a \sim \frac{J^a}{t}$$

← N-dimensional generators of SU(2)

The choice of J^a at each end determines the two partitions of N,

conveniently described by Young tableaux,

e.g. ρ : 12 = 5 + 3 + 3 + 1



Non-trivial solutions to Nahm's problem exist iff

$$\rho^T > \hat{\rho}$$

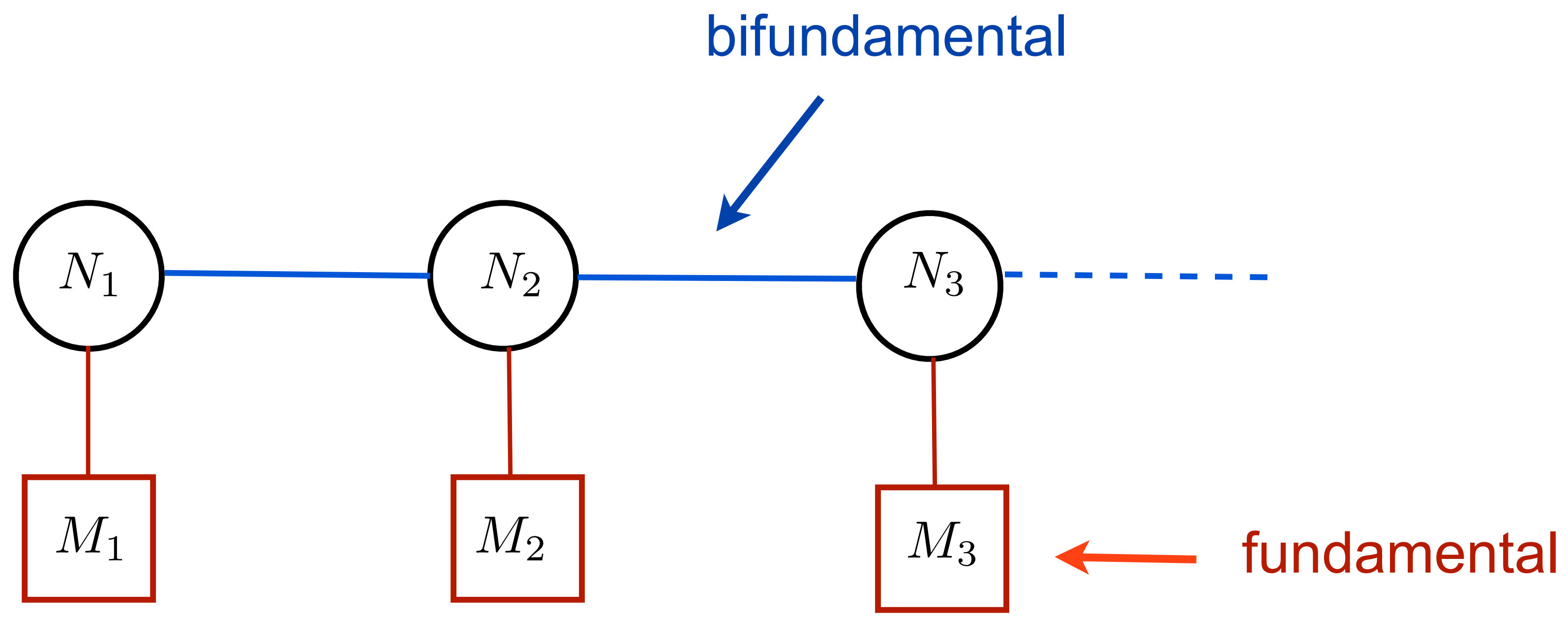
Kronheimer ; Nakajima

Gaiotto, Witten conjecture that **non-trivial SCFTs** , $T_\rho^{\hat{\rho}}(SU(N))$ exist in each of these cases.

Computing these partitions from the brane charges, we show that these inequalities are identically obeyed by the supergravity solutions !

Our backgrounds have furthermore **gauge symmetries**, realized on the 5-branes. These are dual to **global symmetries** of the SCFTs, some of which are explicitly realized in the microscopic theories.

The underlying gauge theories are described by **linear quivers**



gauge symmetry $U(N_1) \times U(N_2) \times U(N3) \times \cdots$

manifest global symmetry $U(M_1) \times U(M_2) \;\; \cdots$

& from “mirror”

Another interesting limit ($\hat{\rho} \simeq \rho^T$) correspond to **severing**

one (or more) **link**, by taking $N_i \longrightarrow 0$

This corresponds to **factorizing the 5-brane singularities** on the strip.

The ensuing theories are **2-graviton theories with weak mixing.**

this is a string-theory “wormhole”

7. Conclusions & outlook

We have embedded the Karch-Randall model in string theory

The graviton obtains a mass, but the brane is decompactified

New AdS_4/CFT_3 dualities

"Narrow bridges" (or throats) are weak links of quiver gauge theories