The Perturbative Ultraviolet Structure of N=4 Supergravity

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Outline

- Statement of the problem: UV divergences in supergravity
  - BCJ color-kinematics duality
  - Double-copy construction of gravity integrands
- Calculation of UV divergences in N=4 SG
  - N=4 SG, n=4, L=3
- Main result: N=4 SG, n=4, L=4
- Interpretation of main result
  - U(1) duality anomaly found by N. Marcus

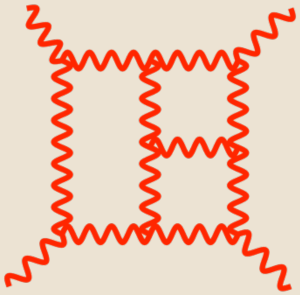
$$\mathcal{A}_n^{\text{loop}} = i^L g^{n-2+2L} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{D_j}$$

$$\mathcal{M}_n^{\text{loop}} = i^{L+1} \left(\frac{\kappa}{2}\right)^{n-2+2L} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{D_j}$$

ULTRAVIOLET DIVERGENCES AND THE DOUBLE COPY METHOD

UV Divergences in Supergravity

- Naively, two derivative coupling in gravity makes the theory badly ultraviolet divergent



$$\text{gravity:} \quad \int \prod \frac{d^D p_i}{(2\pi)^D} \frac{(\kappa p_i^\mu p_j^\nu)\cdots}{\text{propagators}}$$

$$\text{gauge theory:} \quad \int \prod \frac{d^D p_i}{(2\pi)^D} \frac{(g p_i^\mu)\cdots}{\text{propagators}}$$

- Non-renormalizable by power counting

- But: extra symmetry enforces extra cancellations
  - To what extent can observed cancellations be explained by known symmetries?

UV Divergences in Supergravity

- Naturally, the theory with the most symmetry is the best bet for ultraviolet finiteness
  - *N* = 8 supergravity

Cremmer, Julia (1978)

helicity	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+1$	$+\frac{3}{2}$	$+2$
state count	1	8	28	56	70	56	28	8	1

- I will mostly discuss half-maximal supergravity
  - *N* = 4 supergravity

Das (1977);
Cremmer, Scherk, Ferrara (1978)

helicity	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+1$	$+\frac{3}{2}$	$+2$
state count	1	4	6	4	2	4	6	4	1

Expectations about Divergences

- 1970's-1980's: Supersymmetry delays UV divergences until three loops in all 4D pure supergravity theories
  - Expected counterterm is R^4
- In N=8, SUSY and duality symmetry rule out couterterms until 7 loops
  - Expected counterterm is D^8R^4
- 7-loop counterterm has an analog in N = 4 supergravity at three loops
  - But the divergence is not present

Grisaru; Tomboulis; Deser, Kay, Stelle; Ferrara, Zumino; Green, Schwarz, Brink; Howe, Stelle; Marcus, Sagnotti; etc.

Bern, Dixon, Dunbar; Perelstein, Rozowsky (1998);
Howe and Stelle (2003, 2009);
Grisaru and Siegel (1982);
Howe, Stelle and Bossard (2009);
Vanhove; Bjornsson, Green (2010);
Kiermaier, Elvang, Freedman (2010);
Ramond, Kallosh (2010); Beisert et al (2010);
Kallosh; Howe and Lindström (1981);
Green, Russo, Vanhove (2006)
Bern, Carrasco, Dixon, Johansson, Roiban (2010)
Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger (2010)

Duality Symmetries

- Analogs of $E_{7(7)}$ for lower supersymmetry

$\mathcal{N}$=8: $E_{7(7)}$	$E_{7(7)}/SU(8)$
$\mathcal{N}$=6: $SO^*(12)$	$SO^*(12)/U(6)$
$\mathcal{N}$=5: $SU(5,1)$	$SU(5,1)/U(5)$
$\mathcal{N}$=4: $SU(4) \times SU(1,1)$	$SU(1,1)/U(1)$

- Can help UV divergences in these theories
  - **Still have candidate counterterms at $L = \mathcal{N} - 1$ ($1/\mathcal{N}$ BPS)**

Bossard, Howe, Stelle, Vanhove (2010)

- Nice analysis for $\mathcal{N} = 8$ counterterms

Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger (2010)

Recent Field Theory Calculations

- ***N*=8 Supergravity**
  - **Four points, *L*=2,3,4**
  - **Five points, *L*=1,2,3**
  - **Superfinite: Critical Dimension $D = 4 + 6/L$ ($L>1$)**
    - UV finite theory if critical dimension holds for all L
    - But trouble is predicted starting at $L = 5$: $D = 26/5$ or $D = 24/5$?

Bern, Carrasco, Dixon, Johansson, Kosower, Roiban (2007)
Bern, Carrasco, Dixon, Johansson, Roiban (2009)
Carrasco, Johansson (2011)

- ***N*=4 Supergravity**
  - **Four points, $L = 3$, $D = 4$**
    - Unexpected cancellation of R^4 counterterm
    - Counterterm appears valid under all known symmetries, but ¼ BPS
  - **Four points, $L = 2$, $D = 5$**
    - Valid non-BPS counterterm R^4 does not appear
  - **Four points, $L = 4$, $D = 4$**
    - Valid non-BPS counterterm D^2R^4

Bern, Davies, Dennen, Huang (2012)

Bern, Davies, Dennen, Smirnov^2 (2013)

Recent Field Theory Calculations

- How are the calculations done?

1. Find a representation of SYM that satisfies color-kinematics duality (hard)
2. Construct the integrand for a gravity amplitude using the double copy method (easy)
3. Extract the ultraviolet divergences from the integrals (straightforward, but a practical challenge)

Color-Kinematics Duality

- Color-kinematics duality provides a construction of gravity amplitudes from knowledge of Yang-Mills amplitudes

Bern, Carrasco, Johansson (2008)

- In general, Yang-Mills amplitudes can be written as a sum over trivalent graphs

$$\mathcal{A}_n = g^{n-2} \sum_i \frac{n_i c_i}{D_i}$$

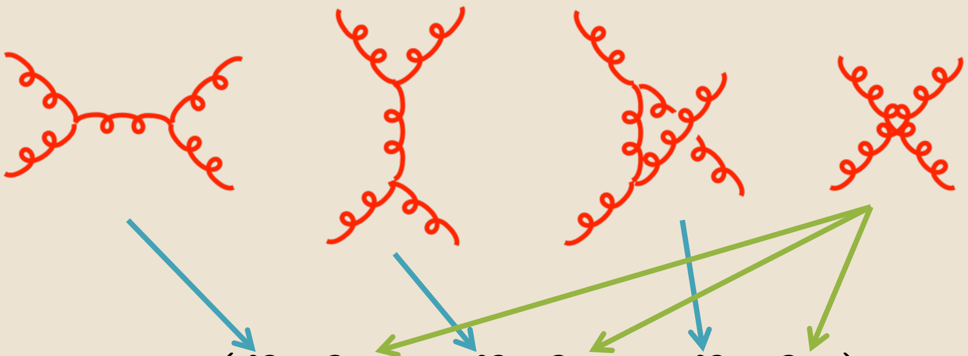
  - **Color factors** $c_i \sim f^{abc} f^{cde}$
  - **Kinematic factors** $n_i \sim (\epsilon_1 \cdot k_2)\, (\epsilon_2 \cdot k_3)\, (\epsilon_3 \cdot \epsilon_4) + \ldots$

- Duality rearranges the amplitude so color and kinematics satisfy the same identities (Jacobi)

$$c_i + c_j + c_k = 0 \leftrightarrow n_i + n_j + n_k = 0$$

Example: Four Gluons

- Four Feynman diagrams
- Color factors based on a Lie algebra



$$c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$$
$$c_t = f^{a_1 a_4 b} f^{b a_2 a_3}$$
$$c_u = f^{a_1 a_3 b} f^{b a_4 a_2}$$

$$\mathcal{A}_4^{\text{tree}} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

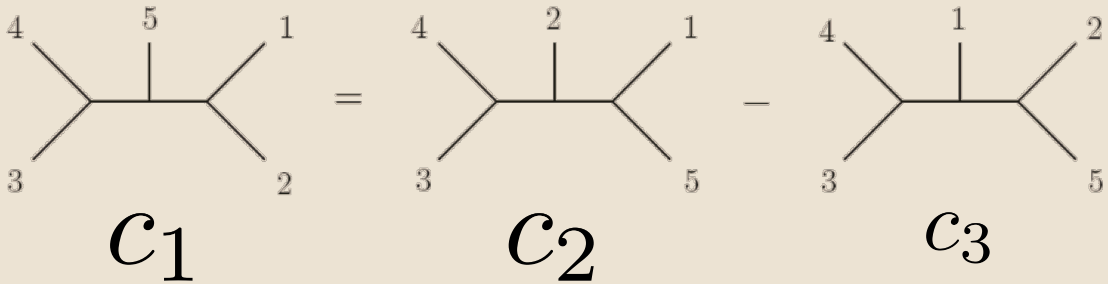
$$n = \epsilon_1 \cdot k_2 \, \epsilon_2 \cdot \epsilon_3 \, \epsilon_4 \cdot k_1 + \ldots$$

- Color factors satisfy Jacobi identity: $c_s + c_t + c_u = 0$
- Numerator factors satisfy similar identity: $n_s + n_t + n_u = 0$
  - Color and kinematics satisfy the same identity!

Five gluons and more

- At higher multiplicity, rearrangement is nontrivial
  - But still possible

$$\mathcal{A}_5^{\text{tree}} = g^3 \sum_{j=1}^{15} \frac{n_j c_j}{D_j}$$



$$c_1 - c_2 + c_3 = 0 \leftrightarrow n_1 - n_2 + n_3 = 0$$

- Claim: We can always find a rearrangement so color and kinematics satisfy the same Jacobi constraint equations.

Recent Field Theory Calculations

- **How are the calculations done?**

1. **Find a representation of SYM that satisfies color-kinematics duality (hard)**
2. **Construct the integrand for a gravity amplitude using the double copy method (easy)**
3. **Extract the ultraviolet divergences from the integrals (straightforward, but a practical challenge)**

Gravity from Double Copy

- Once numerators are in color-dual form, "square" to construct a gravity amplitude

Bern, Carrasco, Johansson (2008)

$$\mathcal{A}_n = g^{n-2} \sum_i \frac{n_i c_i}{D_i} \longrightarrow \mathcal{M}_n = i \left(\frac{\kappa}{2}\right)^{n-2} \sum_i \frac{n_i \tilde{n}_i}{D_i}$$

  - **Gravity numerators are a double copy of gauge theory ones!**

- Proved using BCFW on-shell recursion

Bern, Dennen, Huang, Kiermaier (2010)

- The two copies of gauge theory don't have to be the same theory.

Gravity from Double Copy

- The two copies of gauge theory don't have to be the same theory.
  - Spectrum controlled by tensor product of Yang-Mills theories

N = 8 sugra: (N = 4 SYM) x (N = 4 SYM)
N = 6 sugra: (N = 4 SYM) x (N = 2 SYM)
N = 4 sugra: (N = 4 SYM) x (N = 0 SYM)
N = 0 sugra: (N = 0 SYM) x (N = 0 SYM)

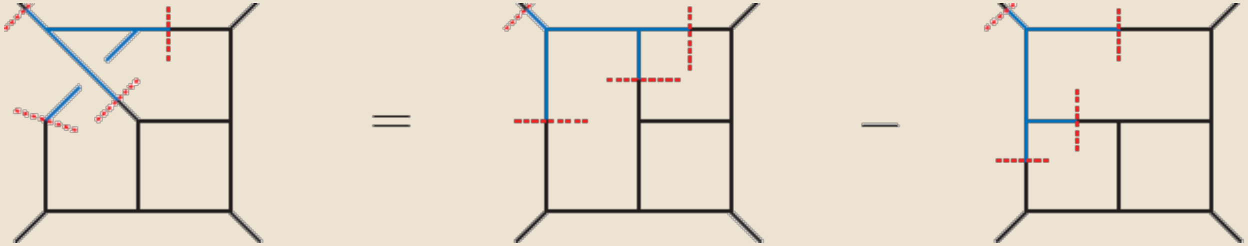
- Recent papers show more-sophisticated lower-SUSY theories

Damgaard, Huang, Sondergaard, Zhang (2012)
Carrasco, Chiodaroli, Gunaydin, Roiban (2013)
Borsten, Duff, Hughes, Nagy (2013)

- Relatively compact expressions for gravity amplitudes

Loop Level

- What we really want is *multiloop* gravity amplitudes
- Color-kinematics duality at loop level
  - Consistent loop labeling between three diagrams
  - Non-trivial to find duality-satisfying sets of numerators



$$c_i = c_j - c_k$$
$$n_i = n_j - n_k$$

- Double copy gives gravity

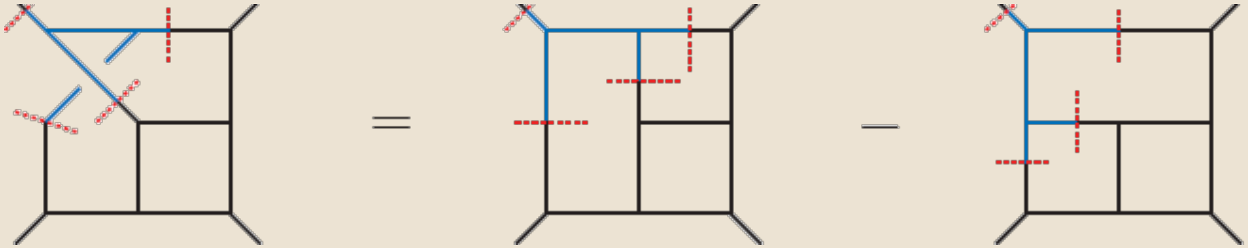
Bern, Carrasco, Johansson (2010)

$$\mathcal{A}_n^{\text{loop}} = i^L g^{n-2+2L} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{D_j}$$

Just replace c with n

$$\mathcal{M}_n^{\text{loop}} = i^{L+1} \left(\frac{\kappa}{2}\right)^{n-2+2L} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{D_j}$$

Gravity Integrands are Trivial



$$c_i = c_j - c_k$$
$$n_i = n_j - n_k$$

✧ **If you have a set of duality satisfying numerators, to get:**

Gauge theory ⟶ gravity theory

simply take

Color factor ⟶ kinematic numerator

$$\mathcal{A}_n^{\text{loop}} = i^L g^{n-2+2L} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{D_j}$$

$$\mathcal{M}_n^{\text{loop}} = i^{L+1} \left(\frac{\kappa}{2}\right)^{n-2+2L} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{D_j}$$

Known Color-Dual Numerators

N = 4 SYM	1 Loop	2 Loops	3 Loops	4 Loops	*L* Loops
4 point	trivial	trivial	ansatz	ansatz	
5 point	construction	ansatz	ansatz		
6 point	construction				
7 point	construction				
n point	construction				

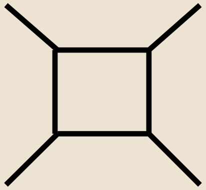
Bern, Carrasco, Johansson (2010)
Carrasco, Johansson (2011)
Bern, Carrasco, Dixon, Johansson, Roiban (2012)
Yuan (2012)
Bjerrum-Bohr, Dennen, Monteiro, O'Connell (2013)

Pure YM	1 Loop	2 Loops	*L* Loops
4 point	ansatz	(All-plus)	
n point	(All-plus and single minus)		

Boels, Isermann, Monteiro, O'Connell (2013)
Bern, Davies, Dennen, Huang, Nohle (2013)

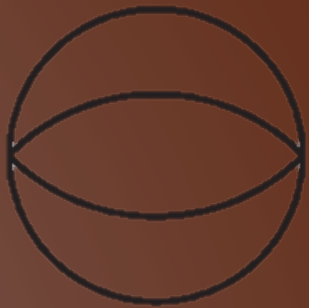
Numerators by Ansatz

- **Strategy:**
  - **Write down an ansatz for a master numerator**
  - **All possible terms**
    - Subject to power counting assumptions
    - Symmetries of the graph



$$n_{\text{box}} = \alpha_1 s_{12}^2 (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) + \alpha_2 s_{12} s_{23} (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) + \ldots$$

- **Take unitarity cuts of the ansatz and match against the known amplitude**
  - **Gives a set of constraints**
- **Very powerful, but relies on having a good ansatz**
  - **Not always possible to write down *all* possible terms**



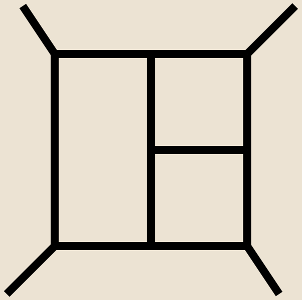
$$\mathcal{S}\left[\int \prod_{i=1}^{L} dp_i I\right] \equiv \mathrm{Div}\left[\int \prod_{i=1}^{L} dp_i I\right] - \sum_{l=1}^{L-1} \sum_{\substack{l-\mathrm{loop} \\ \mathrm{subloops}}} \mathrm{Div}\left[\int \prod_{j=l+1}^{L} dp_j \mathcal{S}\left[\int \prod_{i=1}^{l} dp'_i I\right]\right]$$

EXTRACTING ULTRAVIOLET DIVERGENCES IN N=4 SUPERGRAVITY

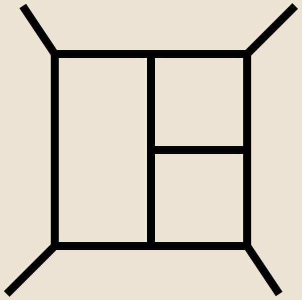
Three Loop Construction

Bern, Davies, Dennen, Huang (2012)

- *N* = 4 SYM copy
  - Use BCJ representation
- Pure YM copy
  - Use Feynman diagrams in Feynman gauge
  - Only one copy needs to satisfy the duality
- Double copy gives *N* = 4 SG
  - Power counting suggests linear divergence
  - Valid counterterm under all known symmetries



$$\sim k^3 \ell (st A_4^{\text{tree}})$$



$$\sim (\epsilon_i \cdot \ell)^4 \ell^4$$

$$\sum_j \int \prod_{l=1}^{L} \frac{d^D \ell_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{P_j}$$

$$\int (d^D \ell)^3 \frac{k^7 \ell^9}{\ell^{20}} \sim \frac{k^8}{\epsilon} \sim \frac{1}{\epsilon} R^4$$

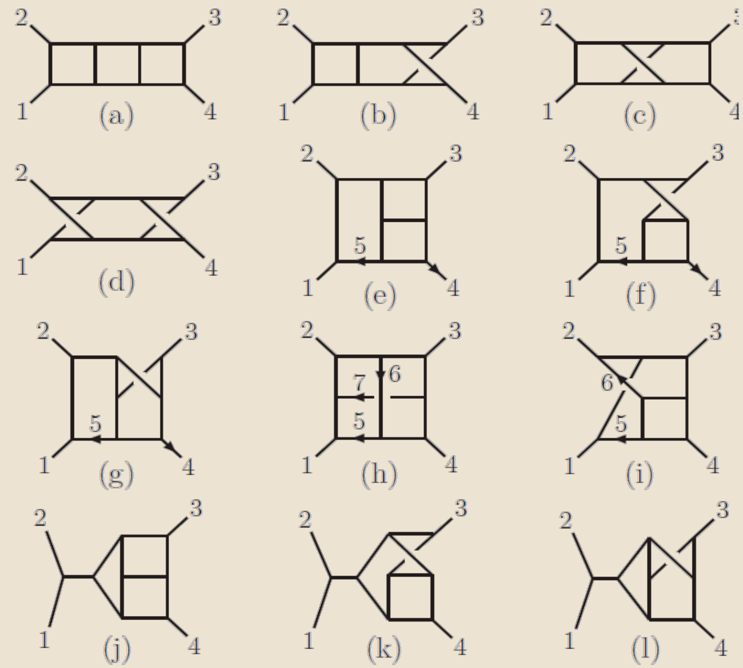
$\mathcal{N} = 4$ SYM Copy

- **Numerators satisfy BCJ duality**

  Bern, Carrasco, Johansson (2010)

- **Factor of stA_4^{tree} pulls out of every graph**

- **Graphs with triangle subdiagrams have vanishing numerators**



Integral $I^{(x)}$	$\mathcal{N} = 4$ Super-Yang-Mills ($\sqrt{\mathcal{N}} = 8$ supergravity) numerator
(a)–(d)	s^2
(e)–(g)	$\big(s(-\tau_{35} + \tau_{45} + t) - t(\tau_{25} + \tau_{45}) + u(\tau_{25} + \tau_{35}) - s^2\big)/3$
(h)	$\big(s(2\tau_{15} - \tau_{16} + 2\tau_{26} - \tau_{27} + 2\tau_{35} + \tau_{36} + \tau_{37} - u)$ $+t(\tau_{16} + \tau_{26} - \tau_{37} + 2\tau_{36} - 2\tau_{15} - 2\tau_{27} - 2\tau_{35} - 3\tau_{17}) + s^2\big)/3$
(i)	$\big(s(-\tau_{25} - \tau_{26} - \tau_{35} + \tau_{36} + \tau_{45} + 2t)$ $+t(\tau_{26} + \tau_{35} + 2\tau_{36} + 2\tau_{45} + 3\tau_{46}) + u\,\tau_{25} + s^2\big)/3$
(j)-(l)	$s(t-u)/3$

$$\tau_{ij} = 2k_i \cdot l_j$$

Pure YM Copy

- Pros and cons of Feynman diagrams
  - 😃 Straightforward to write down
  - 😃 Analysis is relatively easy to pipeline
  - 😃 *D*-dimensional – simple to introduce extra scalars
  - 😡 Lots of diagrams
  - 😡 Time and memory constraints
- Many of the N=4 SYM BCJ numerators vanish
  - If one numerator vanishes, the other is irrelevant

$$\sum_j \int \prod_{l=1}^{L} \frac{d^D \ell_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \cancelto{0}{\tilde{n}_j}}{P_j}$$

  - Power counting for divergences – can throw away most terms very quickly

Ultraviolet Analysis

✧ **To extract ultraviolet divergences from integrals:**

1. **Series expand the integrand and select the logarithmic terms**
2. **Reduce all the tensors in the integrand**
3. **Regulate infrared divergences (uniform mass)**
4. **Subtract subdivergences**
5. **Evaluate vacuum integrals**

1. Series Expansion

- Counterterms are polynomial in external kinematics
  - Count up the degree of the polynomial using dimension operator $\Delta = \sum k^\mu \frac{\partial}{\partial k^\mu} + 2m^2 \frac{\partial}{\partial m^2}$

$$\Delta \int \frac{d^{6-2\epsilon}\ell}{\ell^2(\ell+k)^2} = 2\int \frac{d^{6-2\epsilon}\ell}{\ell^2(\ell+k)^2}$$

- Derivatives reduce the dimension of the integral by at least 1.
  - Apply again to reduce further… all the way down to logarithmic.
  - Now can drop dependence on external momenta.

$$\int \frac{d^{6-2\epsilon}\ell}{\ell^2(\ell+k)^2} = \int d^{6-2\epsilon}\ell \left\{ \frac{4(k\cdot\ell)^2}{[\ell^2]^4} - \frac{k^2}{[\ell^2]^3} \right\} + O(\epsilon^0)$$

1. Series Expansion

$$\int \frac{d^{6-2\epsilon}\ell}{\ell^2(\ell+k)^2} = \int d^{6-2\epsilon}\ell \left\{ \frac{4(k\cdot\ell)^2}{[\ell^2]^4} - \frac{k^2}{[\ell^2]^3} \right\} + O(\epsilon^0)$$

- Counterterms are polynomial in external kinematics
- Series expansion of the propagators

$$\frac{1}{(\ell+k)^2} = \frac{1}{\ell^2}\left\{ 1 + \frac{2\ell\cdot k - k^2}{\ell^2} + \frac{4(\ell\cdot k)^2 - 4\ell\cdot k\, k^2 + [k^2]^2}{[\ell^2]^2} + \ldots \right\}$$

- Terms less than logarithmic have no divergence
- Terms more than logarithmic vanish as the IR regulator is set to zero
- Only log-divergent terms remain

2. Tensor Reduction

$$\int \frac{d^{6-2\epsilon}\ell}{\ell^2(\ell+k)^2} = \int d^{6-2\epsilon}\ell \left\{ \frac{4(k\cdot\ell)^2}{[\ell^2]^4} - \frac{k^2}{[\ell^2]^3} \right\} + O(\epsilon^0)$$

- **The tensor integral knows nothing about external vectors**
  - **Must be proportional to metric tensor** $\int d^{6-2\epsilon}\ell \frac{\ell^\mu \ell^\nu}{[\ell^2]^4} = A\,\eta^{\mu\nu}$
- **Contract both sides with metric to get** $\int d^{6-2\epsilon}\ell \frac{\ell^2}{[\ell^2]^4} = A(6-2\epsilon)$

$$\int \frac{d^{6-2\epsilon}\ell}{\ell^2(\ell+k)^2} = \int d^{6-2\epsilon}\ell \frac{1}{[\ell^2]^3} \left\{ \frac{4k^2}{(6-2\epsilon)} - k^2 \right\} + O(\epsilon^0)$$

- **Generalizes to arbitrary rank – Need rank 8 for 3 and 4 loops**

3. Infrared Regulator

- Integrals have infrared divergences (in 4 dimensions)
- One strategy: Use dimensional regulator for both IR and UV
  - Subdivergences will cancel automatically,
  - But, integrals will generally start at $\epsilon_{\rm IR}^{-L}\epsilon_{\rm UV}^{-L}$
  - Very difficult to do analytically
- Another strategy: Uniform mass regulator for IR
  - Integrals will start at $\epsilon_{\rm UV}^{-L}$ -- much easier!
  - Regulator dependence only enters through subleading terms

$$\int \frac{d^{6-2\epsilon}\ell}{[\ell^2]^3} = \int \frac{d^{6-2\epsilon}\ell}{[\ell^2 - m^2]^3} + O(\epsilon^0)$$

- Now we have a sensible integral!

Marcus, Sagnotti (1985)
Vladimirov (1980)

4. Subdivergences

- What about higher loops?
- At three loops, the integrals have logarithmic subdivergences
  - Integrals are $O(\epsilon^{-3})$
  - Mass regulator can enter subleading terms!
- Recursively remove all contributions from divergent subintegrals
  - Much like adding counterterm diagrams integral by integral
  - IR regulator dependence drops out

Marcus, Sagnotti (1985)
Vladimirov (1980)

$$\mathcal{S}\left[\int\prod_{i=1}^{L}dp_i I\right] \equiv \mathrm{Div}\left[\int\prod_{i=1}^{L}dp_i I\right] - \sum_{l=1}^{L-1}\sum_{\substack{l-\text{loop}\\ \text{subloops}}} \mathrm{Div}\left[\int\prod_{j=l+1}^{L}dp_j\,\mathcal{S}\left[\int\prod_{i=1}^{l}dp_i' I\right]\right]$$

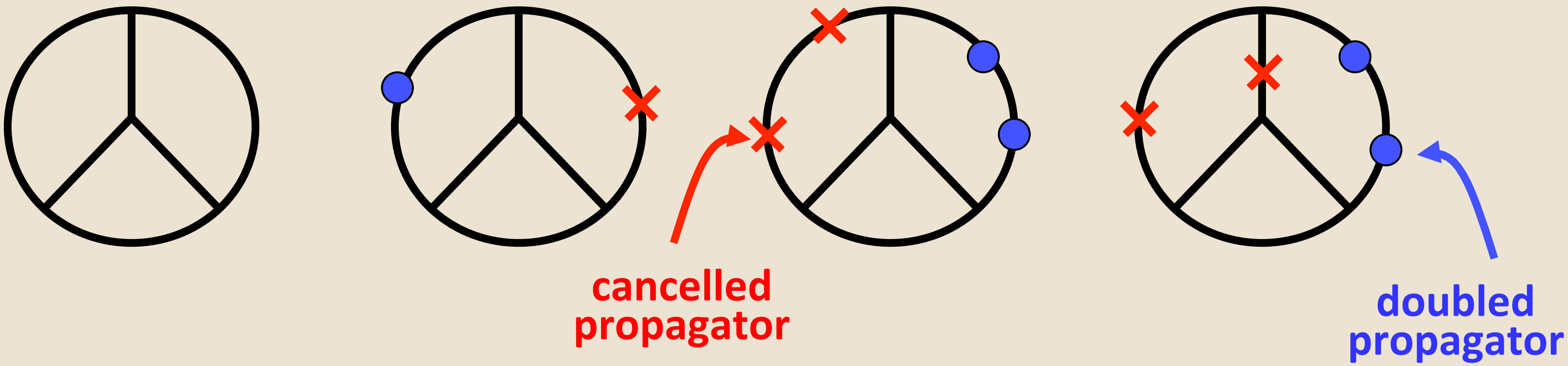
Regulator dependent

Regulator independent

Reparametrize subintegral

5. Vacuum Integrals

- Get about 600 vacuum integrals containing UV information



- Evaluation:
  - MB: Mellin Barnes integration
  - FIESTA: Sector decomposition
  - FIRE: Integral reduction using integration by parts identities
- Rich literature on single-scale vacuum integrals

Czakon

A.V. Smirnov & Tentyukov

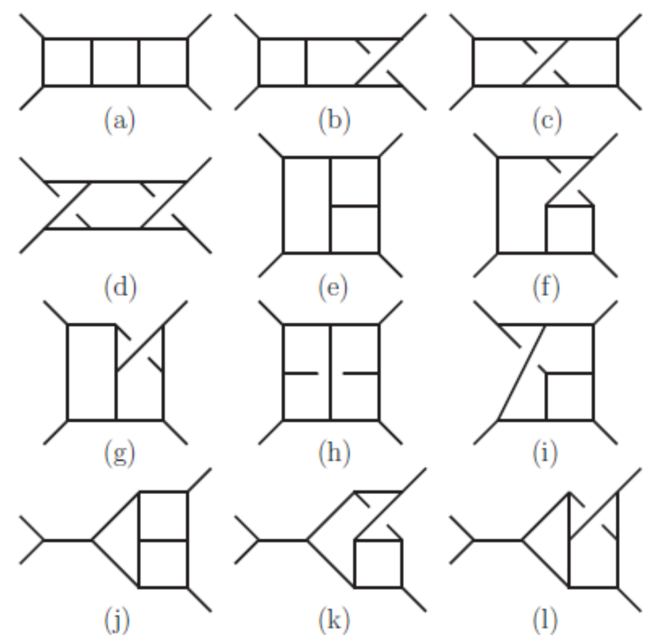
A.V. Smirnov

Davydychev, Kalmykov (2003)
Chetyrkin, Misiak, Munz (1997)
Czakon (2004)

Three Loop Result

- ✓ **Series expand the integrand and select the logarithmic terms**
- ✓ **Reduce all the tensors in the integrand**
- ✓ **Regulate infrared divergences**
- ✓ **Subtract subdivergences**
- ✓ **Evaluate vacuum integrals**

Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$



- ✓ **The sum of all 12 graphs is finite!**

Bern, Davies, Dennen, Huang (2012)

Perspectives on the 3-loop Result

- **If R^4 counterterm is allowed by supersymmetry, why is it not present?**
  - **Heterotic string computation**
    - Solid

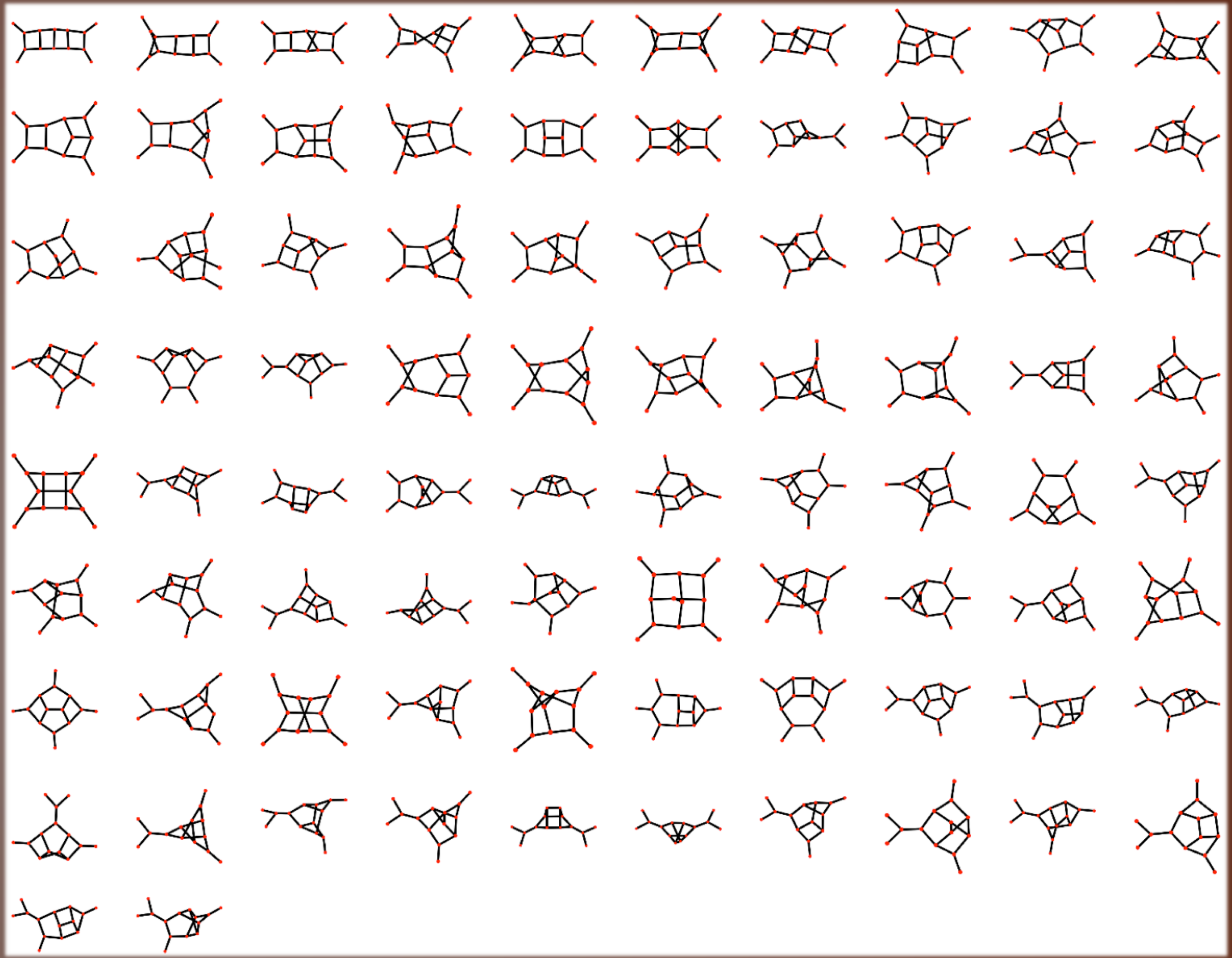
    Tourkine, Vanhove (2012)
  - **Violates Noether-Gaillard-Zumino current conservation**
    - Controversial

    Kallosh (2012)
  - **Hidden superconformal symmetry**
    - Lacks consideration of U(1) anomaly

    Kallosh, Ferrara, Van Proeyen (2012)
  - **Existence of off-shell superspace formalism**
    - Ruled out by three-loop matter amplitudes

    Bossard, Howe, Stelle (2012)

- **Different perspectives lead to different expectations for a four-loop divergence.**



FOUR-LOOP UV DIVERGENCE IN N=4 SUPERGRAVITY

Four Loop Setup

- Allowed counterterm D^2R^4, non-BPS
- Same approach as three loops.

Bern, Davies, Dennen, A. V. Smirnov, V. A. Smirnov (2013)

- $N = 4$ SYM numerators: 82 nonvanishing (comp. 12)

Bern, Carrasco, Dixon, Johansson, Roiban (2012)

- Pure YM numerators: ~30000 Feynman diagrams (comp. ~1000)
- Integrals are generally quadratically divergent
  - Requires a deeper series expansion → proliferation of terms

Four Loop Calculation

- ✓ Series expand the integrand and select the logarithmic terms
- ✓ Reduce all the tensors in the integrand
- ✓ Regulate infrared divergences
- ✓ Subtract subdivergences
- ✓ Evaluate vacuum integrals   Czakon (2004)

✧ **Result is consistent**

- ✓ Overall cancellation of ϵ^{-4}, ϵ^{-3} and ϵ^{-2}
- ✓ Cancellation of transcendental constants related to the mass regulator
- ✓ Gauge invariant

Four Loop Result

$$\mathcal{M}^{\text{4-loop}}_{\mathcal{N}=4}\Big|_{\text{div}} = \frac{1}{(4\pi)^8}\left(\frac{\kappa}{2}\right)^{10}\frac{1-264\zeta_3}{288\epsilon}\, st A^{\text{tree}}_{\mathcal{N}=4}\left(\mathcal{O}^{--++} + 3\mathcal{O}^{-+++} + 60\mathcal{O}^{++++}\right)$$

$$\mathcal{O}^{--++} = -4s^2 t\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$\mathcal{O}^{-+++} = -12s^2t^2\frac{[24]^2}{[12]\langle 23\rangle\langle 34\rangle[41]}$$

$$\mathcal{O}^{++++} = 3stu\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

- **Double copy makes SU(4) R-symmetry manifest**
- **Three distinct counterterms**
- **--++ is 4-graviton sector**
- **The latter two configurations would vanish if duality symmetry were not anomalous**
  - **E.g. in N>4 SG**
- **All three independent configurations have a similar divergence!**
  - **How much can we really read into this? There is very little information in the transcendental coefficient.**

Matter Multiplets in the Loops

$$\mathcal{M}^{\text{4-loop}}_{\mathcal{N}=4}\Big|_{\text{div}} = \frac{1}{(4\pi)^8}\left(\frac{\kappa}{2}\right)^{10} \boxed{\frac{1-264\zeta_3}{288\epsilon}} st A^{\text{tree}}_{\mathcal{N}=4}\left(\mathcal{O}^{--++} + 3\mathcal{O}^{-+++} + 60\mathcal{O}^{++++}\right)$$

↓

$$\boxed{\frac{(n_V+2)}{4608}\left(\frac{6(n_V+2)n_V}{\epsilon^2} + \frac{(n_V+2)(3n_V+4) - 96(22-n_V)\zeta_3}{\epsilon}\right)}$$

- **Couple to matter multiplets to get more info**
- **Requires honest subtraction of subdivergences, since matter amplitudes diverge already at one loop**

Fischler (1979)

- **Kinematic factor is the same as pure SUGRA**
- **Transcendental constants factorize out**

The Structure of the Result

$$\mathcal{M}^{\text{4-loop}}_{\mathcal{N}=4}\Big|_{\text{div}} = \frac{1}{(4\pi)^8}\left(\frac{\kappa}{2}\right)^{10} \boxed{\frac{1-264\zeta_3}{288\epsilon}}\, st A^{\text{tree}}_{\mathcal{N}=4}\left(\mathcal{O}^{--++} + 3\mathcal{O}^{-+++} + 60\mathcal{O}^{++++}\right)$$

↓

$$\boxed{\frac{(n_{\text{V}}+2)}{4608}\left(\frac{6(n_{\text{V}}+2)n_{\text{V}}}{\epsilon^2} + \frac{(n_{\text{V}}+2)(3n_{\text{V}}+4) - 96(22-n_{\text{V}})\zeta_3}{\epsilon}\right)}$$

- **All three independent configurations still have a similar divergence**
  - **Peculiar because the nonanomalous sector should naively have a very different analytic structure. Not related by any supersymmetry Ward identities.**
- **Factorization of transcendental constants is less trivial than it looks**
  - **ζ_4 and ζ_5 cancel away unexpectedly**
- **n_V dependence is apparently consistent with U(1) anomaly**

U(1) Anomaly

Marcus (1985)

Carrasco, Kallosh, Tseytlin, Roiban (2013)

- There is an anomaly in a U(1) subgroup of the SU(4) x SU(1,1) duality symmetry
  - Scalar degrees of freedom parameterize SU(1,1)/U(1)
  - Can gauge the U(1) to linearize the action of SU(1,1)
    - scalars become complex doublet under global SU(1,1)
    - Pick up a phase under local U(1)

$$\Phi^\alpha \Phi_\alpha = 1 \qquad\qquad \Phi'_\alpha = e^{-i\gamma(x)} U_\alpha^\beta \Phi_\beta$$

- Anomalous means different gauge choices for the U(1) give different theories at the quantum level
  - Theories differ by a local, finite term in the effective action

U(1) Anomaly

Carrasco, Kallosh, Tseytlin, Roiban (2013)

- **Double copy perspective:**
  - **All-plus and single-minus YM amplitudes**
    - Vanish at tree level
    - Finite at one-loop level – proportional to (n_S+2)
  - **Feed through double copy to build anomalous amplitudes in N=4 SG**
- **Key feature: One-loop SG anomalous amplitudes are proportional to (n_V+2)**

Anomalies in unitarity cuts

- As pointed out by Carrasco, Kallosh, Tseytlin & Roiban, the anomalous sectors are poorly behaved, and contribute to a four-loop UV divergence (unless somehow cancelled, as they are at three loops)
- Anomalous sector feeds poor UV behavior into MHV sector

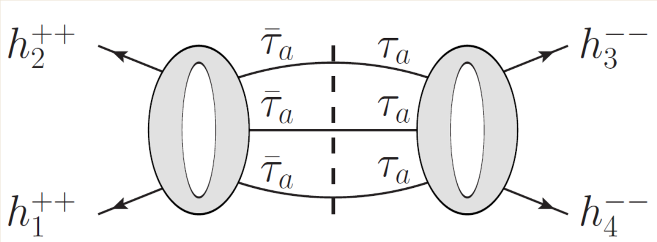


Figure from arXiv:1303.6219
Carrasco, Kallosh, Tseytlin, Roiban

- Each anomaly insertion gives a factor of (n_V+2)
  - This cut contributes $(n_V+2)^2$ times a two-loop integral
  - To get ζ_3 requires a three-loop integral, which leaves only enough room for one anomaly insertion.

Connection Between Sectors?

$$\mathcal{M}^{\text{4-loop}}_{\mathcal{N}=4}\Big|_{\text{div}} = \frac{1}{(4\pi)^8}\left(\frac{\kappa}{2}\right)^{10} \boxed{\frac{1-264\zeta_3}{288\epsilon}} st A^{\text{tree}}_{\mathcal{N}=4}\left(\mathcal{O}^{--++} + 3\mathcal{O}^{-+++} + 60\mathcal{O}^{++++}\right)$$

$$\downarrow$$

$$\boxed{\frac{(n_{\text{V}}+2)}{4608}\left(\frac{6(n_{\text{V}}+2)n_{\text{V}}}{\epsilon^2} + \frac{(n_{\text{V}}+2)(3n_{\text{V}}+4) - 96(22-n_{\text{V}})\zeta_3}{\epsilon}\right)}$$

- **Result looks consistent with being entirely due to the anomaly**
  - **$(n_V+2)^2$ for rational numbers**
  - **$(n_V+2)\ \zeta_3$ consistent with a single anomaly insertion**
- **Bottom line: This divergence looks specific to *N* = 4 SG, and likely due to the anomaly.**

Conclusions

- UV analysis of gravity integrals
  - 3 loop, $\mathcal{N} = 4$ SG is ultraviolet finite.
  - 4 loop diverges, but related to the anomaly.
- Why are there apparently no four-loop divergences unrelated to the U(1) anomaly?
- Is it possible to understand the connection between the anomalous divergences and the 4-graviton divergence through standard symmetries, or is something new needed?
- SU(4) invariant formulation of N=4 SG corresponds to the U(1) gauge choice

$$\mathrm{Im}\Phi_1 = \mathrm{Im}\Phi_2$$

  - Does anything interesting happen in other gauges?

Thank You!