Testing gravity with INPOP planetary ephemerides

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General introduction

- Planetary ephemerides: what for ?
- INPOP: what's new ?
- MESSENGER analysis
- Testing GR with INPOP

Planetary ephemerides

Theory of planetary (and usually Moon) motions

What for ?

- celestial mechanics and reference frames
- tests of fundamental physics
- planetology: physics of asteroids, Moon
- solar physics

- preparation of space missions
- paleoclimatology and geological time scales

- other topics: preparation of stellar occultations, public outreach

3 generations of planetary ephemerides

	Gaillot 1913		DE200 1983		INPOP10a 2011	
	angle	distance Earth-	angle	distance Earth-	angle	distance Earth-
	"	km	"	km	"	km
Mercury	1	450	0.050	5	0.050	0.002
Venus	0.5	100	0.050	2	0.001	0.004
Mars	0.5	150	0.050	0.050	0.001	0.002
Jupiter	0.5	1400	0.1	10	0.010	2
Saturn	0.5	3000	0.1	600	0.010	0.015
Uranus	1	12700	0.2	2540	0.100	1270
Neptune	1	22000	0.2	4400	0.100	2200
Pluto	1	24000	0.2	4800	0.100	2400

The planetary ephemerides today

3 Teams

DE	JPL	DE405 (Standish et al. 1998)	NASA
		DE421 (Folkner et al. 2008)	s/c dedicated
		DE430 (Folkner et al. 2013)	
EMP	IAA	EMP20.. (Pitjeva 2009, 2013)	close to DE
			Limited distribution
INPOP	IMC/OCA	INPOP06,08 (Fienga et al. 2008, 2009)	Science, Innovative
		INPOP10a (Fienga et al. 2011)	IAU TT-TDB, $GM_\odot$
			1Myr solution (La04)
		INPOP10e (Fienga et al. 2013)	ESA Gaia release
		INPOP13a (Verma et al. 2014)	Messenger

The planetary ephemerides today

DE,EMP, INPOP: what they have in common ...

- Numerical integration of the (Einstein-Imfeld-Hoffmann, c^{-4} PPN approximation) equations of motion.

$$\ddot{x}_{Planet} = \sum_{A \neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{GR}(\beta, \gamma, c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_2^{\odot}}$$

- Adams-Cowell in extended precision
- 8 planets + Pluto + Moon + asteroids (point-mass, ring), GR, $J_2^{\odot}$, Earth rotation (Euler angles)
- Moon: orbit and librations
- Simultaneous numerical integration TT-TDB, TCG-TCB
- Fit to observations in ICRF

Rely mainly on space navigation

Specific INPOP developments for testing gravity

Simulation of a Pioneer anomaly type of acceleration

$$\ddot{x}_{Planet} = \ddot{x}_{Newton} + \ddot{x}_{GR}(\beta, \gamma, c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_2^{\odot}} + \ddot{x}_{constant}$$

Supplementary advance of perihelia $\dot{\varpi}$ and nodes $\dot{\Omega}$

At each step of integration t_i,

$$\varpi(t_i) = \varpi(t_0) + \dot{\varpi}(t_i - t_0)$$

$$\Omega(t_i) = \Omega(t_0) + \dot{\Omega}(t_i - t_0)$$

$$\ddot{x}_{Planet} = R(\varpi(t_i), \Omega(t_i))\, \ddot{x}_{Planet}$$

Specific INPOP developments for testing gravity

Equivalence Principle @ astronomical scale

$$m^I \ddot{x} = F(m^G, x_i, \dot{x}_i, m_i^G ...)$$

For each planet j,

$$\ddot{x}_j = \frac{m_j^G}{m_j^I} F(x_i, \dot{x}_i, m_i^G, ...) = (1+\eta) F(x_i, \dot{x}_i, m_i^G, ...)$$

implemented but still preliminary

Specific INPOP developments for testing gravity

With $\mu_\odot = GM_\odot$, $\mu_j = GM_j$ for planet j

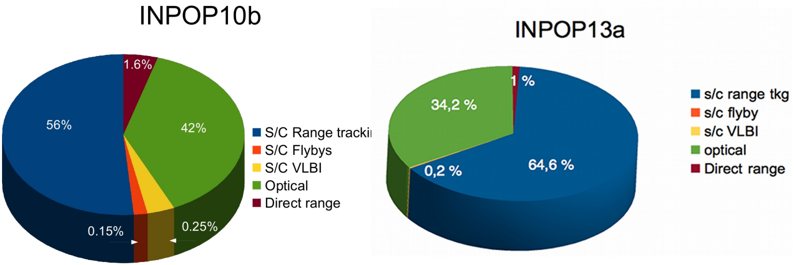
$\frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{G}}{G}$ with $\frac{\dot{\mu_\odot}}{\mu_\odot} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{\mu_j}}{\mu_j} = \frac{\dot{G}}{G}$

$$
\begin{aligned}
M_\odot(\mathrm{t}_i) &= M_\odot(t_0) + (t_i - t_0) \times \dot{M}_\odot \\
\mathrm{G}(\mathrm{t}_i) &= G(t_0) + (t_i - t_0) \times \dot{G}
\end{aligned}
$$

$$
\begin{aligned}
\mu_\odot(t_i) &= G(t_i) \times M_\odot(t_i) \\
\mu_j(t_i) &= G(t_i) \times M_j
\end{aligned}
$$

- by fixing $\dot{M}_\odot$ or $\dot{G} \rightarrow \frac{\dot{\mu}}{\mu}$
- $\forall t_i, M_\odot(t_i)$ and $G(t_i) \rightarrow \ddot{\mathrm{x}}_{Planet}, \ddot{\mathrm{x}}_{Ast}, \ddot{\mathrm{x}}_{Moon}$
- What values of $\frac{\dot{\mu}}{\mu}$ (and then $\frac{\dot{M}_\odot}{M_\odot}$ or $\frac{\dot{G}}{G}$) are acceptable / data accuracy ?

INPOP s/c navigation dependency

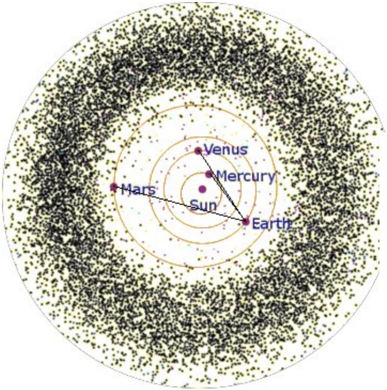


		α	δ	ρ
S/C VLBI	V, Ma, J, S	1/10 mas	1/10 mas	
S/C Flybys	Me, J, S, U, N	0.1/1 mas	0.1/1 mas	1/30 m
S/C Range tracking	Me, V, Ma			2/30 m
Direct range	Me,V			1 km
Optical	J, S, U, N, P	300 mas	300 mas	
LLR	Moon			1cm

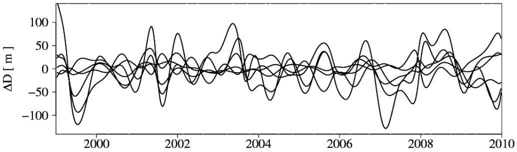
INPOP Evolution

INPOP08	4Dplanetary ephemerides: TT-TDB	TT-TDB 1st release
(Fienga et al. 2009)	New method for fit (a priori sigma)	www.imcce.fr/inpop
	Fitted to planetary data and LLR	30 GM_{ast}, 3ρ
		AU, $J_2^{\odot}$, EMRAT
INPOP10a	289 asteroids, no mean density, ring	Long-term La2010
(Fienga et al. 2011)	Direct fit with constraints	145 GM_{ast}, GM_{ring}
	Improvement of outer planet orbits	$GM_{\odot}$, $J_2^{\odot}$, EMRAT,
	Fixed AU, β, γ, $\dot{\varpi}$, $\dot{\Omega}$	Tests of GR
INPOP10e	Direct fit with constraints + a priori sigma	GAIA last release
(Fienga et al. 2013)	Solar corona studies and corrections	152 GM_{ast}, GM_{ring}
(Verma et al. 2013)	Improvement of Mars extrapolation	$GM_{\odot}$, $J_2^{\odot}$, EMRAT
	Use of raw MGS tracking data (GINS)	
INPOP13a	MESSENGER independant	Tests of GR
(Verma et al 2014)	orbit determination	62 GM_{ast}, GM_{ring}
	β, γ, $(\dot{G}/G)$	$GM_{\odot}$, $J_2^{\odot}$, EMRAT

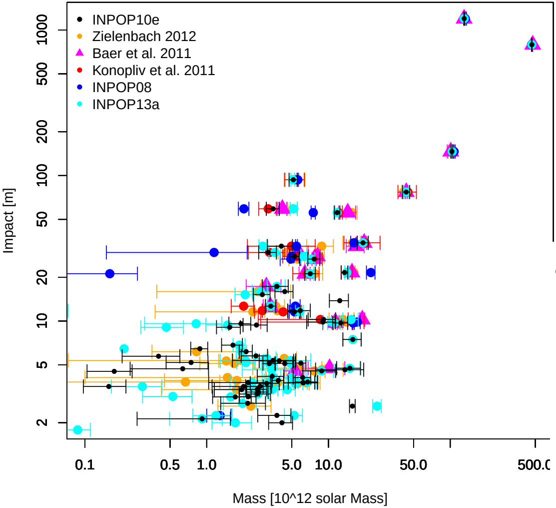
INPOP and the asteroids



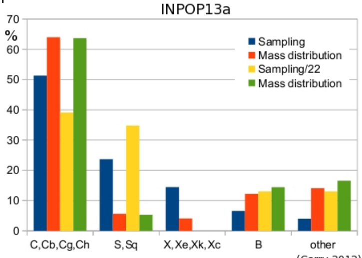
- How to model all these perturbations ... with unknown masses?
- Observed impact: mainly Earth-Mars distances
- Projected accelerations of asteroids over the Earth-Mars distances



- How to distangle ?
- How to identify ?
- LS with constraints + A priori σ



- Uncertainty is directly related with the impact on Mars-Earth orbits
- 20 Biggest perturbers (I >10m) have consistent masses with $\sigma \lesssim 25\%$ *
- $\rightarrow$ Constraints for Solar System formation



(Carry 2012)

INPOP13a

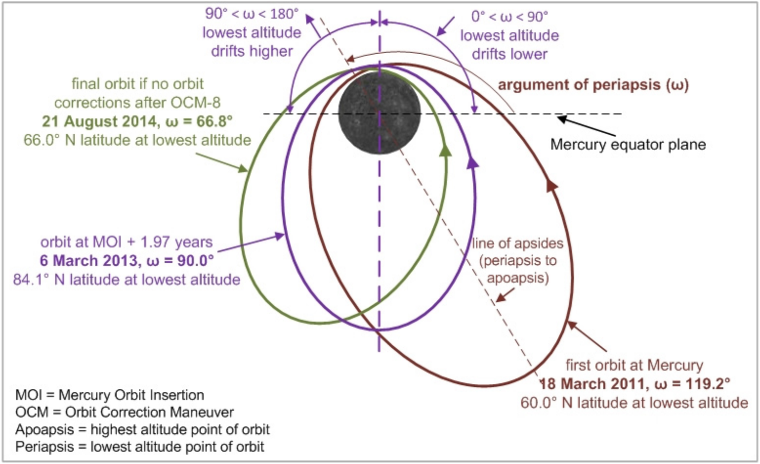
MESSENGER

- 1.5 yr of Doppler + range data (level 2) @ PDS
- Original orbit analysis with GINS/CNES software
- with hypothesis on Macro-model, manouvers

Results

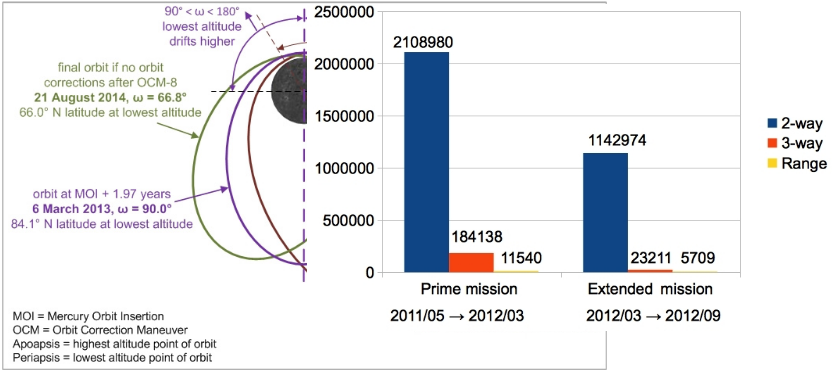
- accurate orbit determination / (Smith et al. 2013)
- Full fit of all planets: INPOP13a
- New constraints over β, γ, $J_2^{\odot}$
- Verma et al. 2014
- $\frac{\dot{G}}{G}$

MESSENGER: NASA mission with 2 periods

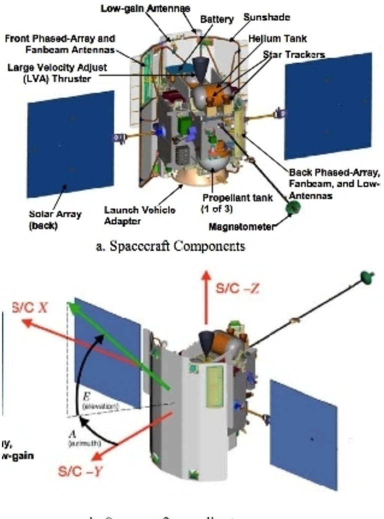


MESSENGER mission: 2 periods

[2011/05:2012/03] + [2012/03:2012/09]



MESSENGER orbit determination with GINS/CNES



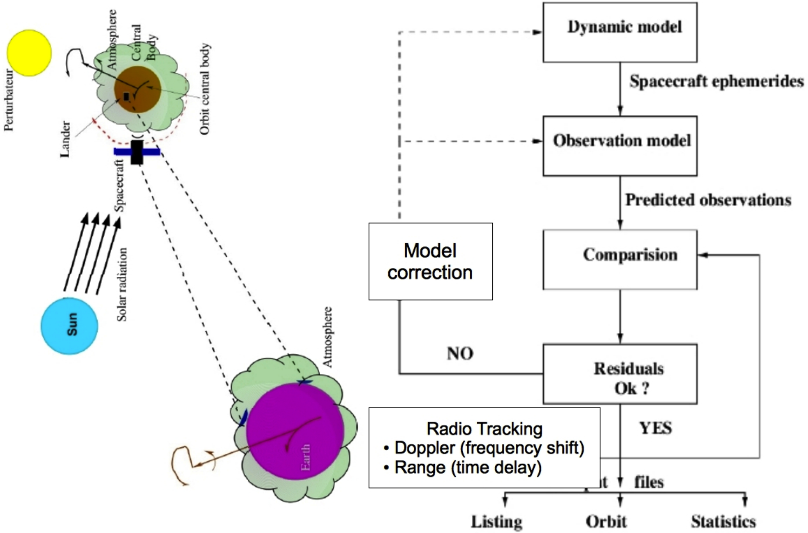
a. Spacecraft Components

b. Spacecraft coordinate axes

Main characteristics:

1. GINS original multi-arc analysis
2. Rotation (Margot 2009) + gravity (Smith et al.,2012)
3. Macro-model: Box-and-wings model (Vaughan et al. 2006)
4. Manouvers: optimization of the data arc length < period of manouvers
5. 3+4 → 1-day data arc for the fit of each arc of orbit

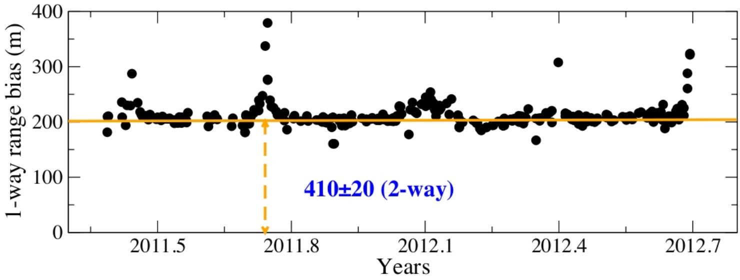
S/C orbit determination (OD)



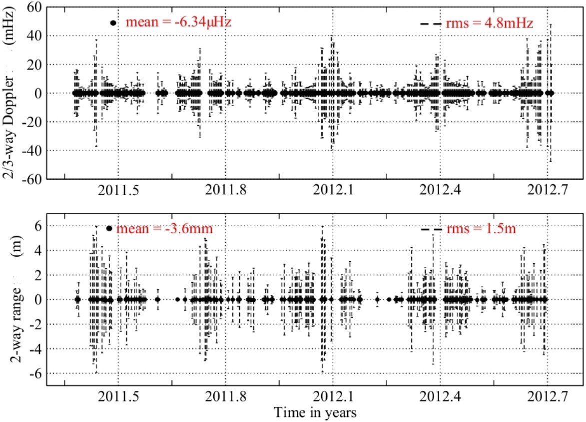
MESSENGER OD validation I

Group Delay

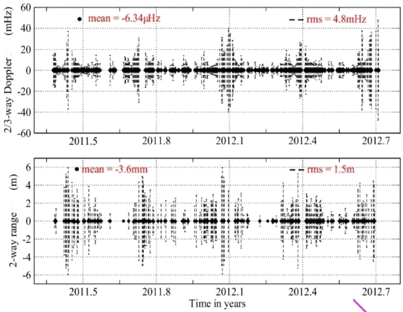
- Offset in range measurement due to on-board transponder
- 410$\pm$20 m
- Srinivasan et al. 2007: 407-415 m



MESSENGER OD validation II



Author	Doppler @ 10s	Range
Verma et al. 2014	-0.00063±4.8 mHz	-0.003±1.5 m
Genova et al. 2013	-0.00088±3.6 mHz	-0.06±1.87 m
Smith et al. 2012	0.4±2.0 mm/s	-

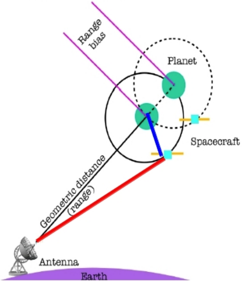


2-way Doppler

Differences between estimated **velocities** of s/c orbiting the planet and the observed Doppler shift

2-way range

Differences between estimated **distances of s/c** **orbiting the planet** and the Earth observed time delay

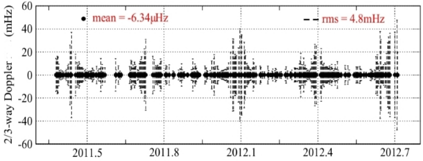


Orbit of the s/c about the planet very well known

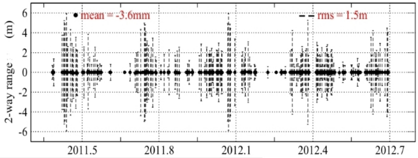
Constraints on **distances between the planet and the Earth**

MESSENGER Range Bias for INPOP

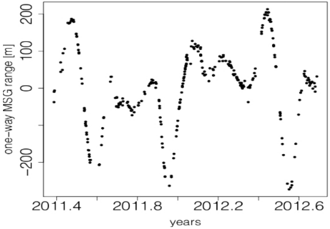
MESSENGER Range Bias for INPOP

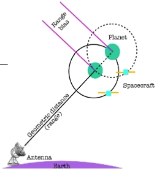


Differences between estimated **velocities** of s/c orbiting the planet and the observed Doppler shift



Differences between estimated **distances of s/c <u>orbiting the planet</u>** and the Earth observed time delay





Constraints on **distances between the planet and the Earth**

INPOP13a: Important improvement of the Mercury orbit

- same structure as INPOP10e (Fienga et al. 2013)
- Messenger range biais deduced from GINS OD
  $\rightarrow$ 314 data points from 2011.4 to 2012.6

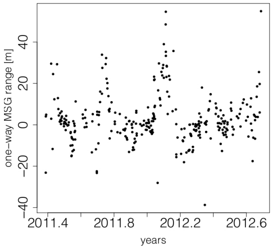
INPOP13a: Important improvement of the Mercury orbit

- same structure as INPOP10e (Fienga et al. 2013)
- Messenger range biais deduced from GINS OD
  $\rightarrow$ 314 data points from 2011.4 to 2012.6
- Refit over full data sets (INPOP10e + MSG)
  $\rightarrow$ IC, $GM_{\odot}$, 62 GM_{ast}, $J_2^{\odot}$

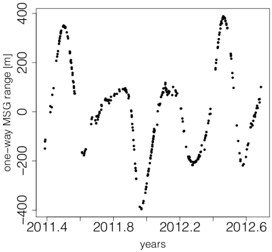
	INPOP13a $\pm 1\sigma$	INPOP10e $\pm 1\sigma$	DE423 $\pm 1\sigma$
$J_2^{\odot} \times 10^{-7}$	(2.40 ± 0.20)	(1.80 ± 0.25)	1.80 (2.0 ± 0.20) [P13] (2.1 ± 0.70) [DE430]
$GM_{\odot}$ - 132712440000 [$km^3. s^{-2}$]	(48.063 ± 0.4)	(50.16 ± 1.3)	40.944
GM(Ceres) [10^{12} x $M_{\odot}$]	468.430 ± 1.18	467.267 ± 1.85	473.485 ± 1.33
GM(Pallas)	103.843 ± 0.98	102.65 ± 1.60	103.374 ± 6.92
GM(Bamberga)	5.087 ± 0.19	4.769 ± 0.43	5.422 ± 1.00
GM(Metis)	3.637 ± 0.40	4.202 ± 0.67	4.524 ± 0.67

INPOP13a improvement of the Mercury orbit

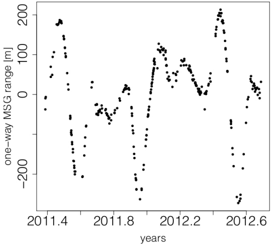
INPOP13a MSG range



INPOP10e MSG range



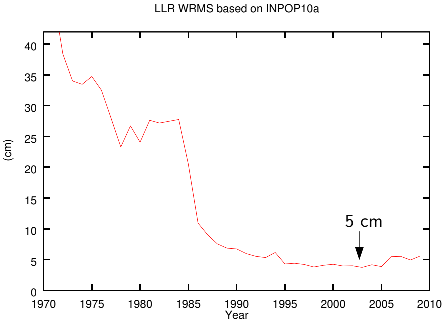
DE423 MSG range

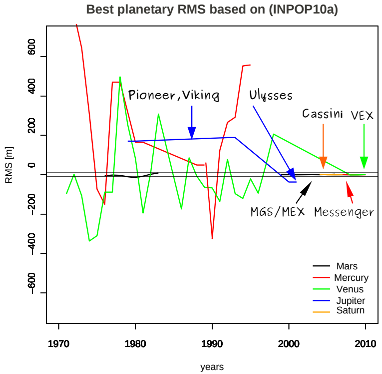


Mercury			$\Delta\chi^2$	INPOP13a	INPOP10e
Direct range [m]	462	1971-1998	3%	-108 $\pm$ 866	-45 $\pm$ 872
Mariner range [m]	2	1974-1975	1%	124 $\pm$ 56	-52 $\pm$ 113
MSG flyby ra [mas]	3	2008-2009	1%	0.8 $\pm$ 1.3	0.7 $\pm$ 1.5
MSG flyby de [mas]	3	2008-2009	1%	2.4 $\pm$ 2.4	2.4 $\pm$ 2.5
MSG flyby range [m]	3	2008-2009	1%	-1.9 $\pm$ 7.7	-5.0 $\pm$ 5.8
MSG range [m]	314	2011.3-2012.7	94%	-0.4 $\pm$ 8.4	6.2$\pm$ 205

The Solar system and the tests of gravity

With such accuracy, the solar system is still the ideal lab for testing gravity





and the modified gravity comes ...!

For example,

Theories	Phenomenology	Object
Standard Model	violation of EP	Moon-LLR
MOND	$d\dot{\varpi}_{supp}$, $d\dot{\Omega}_{supp}$	planets
Scalar field theories	$\dot{G}/G$	Moon-LLR, planets
	variation of β, γ	planets
Dark Energy	$\dot{G}/G$	Moon-LLR, planets
AWE/chameleons	variation of β, γ	planets
Dark Matter	linear drift of AU	planets
	a_{supp}	Moon-LLR,planets
	$d\dot{\varpi}_{supp}$, $d\dot{\Omega}_{supp}$	planets
ISL	$d\dot{\varpi}_{supp}$	planets,Moon-LLR
f(r)	a_{supp}	planets

Limits of solar system gravity tests with spacecraft tracking

- Accuracy $\approx$ 1 cm over 1 to 5 years
- deflection of light $\rightarrow \gamma$

- navigation unknowns (AMDs, solar panel, accelerations)
- planet unkowns (potential, rotation...)
- solar plasma
- correlation with planet ephemerides ?
- .. or a dedicated mission

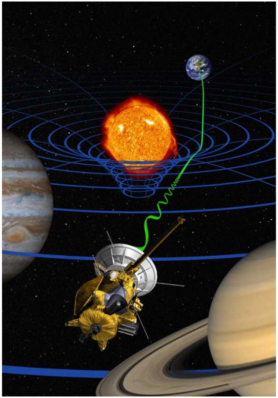
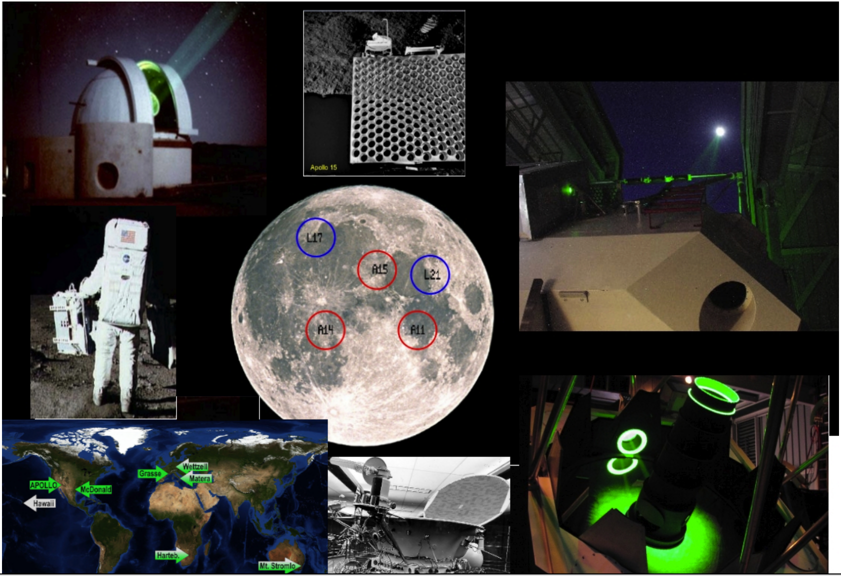


Figure: (Bertotti et al. 2003)
$(\gamma - 1) \times 10^4 = (0.21 \pm 0.23)$

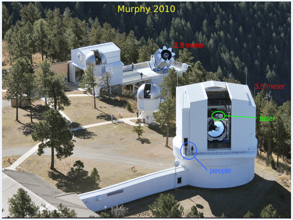
Gravity tests with the Moon



Gravity tests with the Moon

- Accuracy $\approx$ 10 to 1 cm over 40 years
- EP, preferred-frame tests, frame dragging effects, ISL, $\dot{G}/G$

- APOLLO $\rightarrow$ 1 mm accuracy



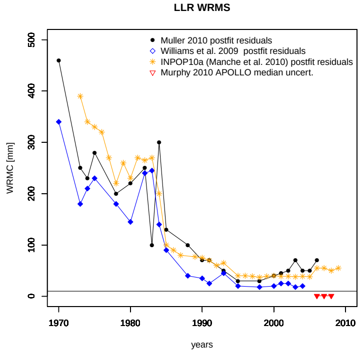
(Merkowitz et al. 2009)

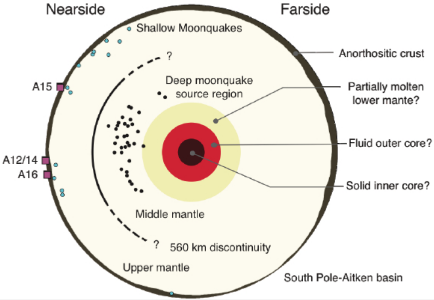
Science	Timescale	Current (cm)	1 mm	0.1 mm
Weak Equivalence Principle	Few years	$\lvert\Delta a/a\rvert<1.3\times10^{-13}$	10^{-14}	10^{-15}
Strong Equivalence Principle	Few years	$\lvert\eta\rvert<4.4\times10^{-4}$	3×10^{-5}	3×10^{-6}
Time variation of G	~10 years	9×10^{-13} yr^{-1}	5×10^{-14}	5×10^{-15}
Inverse Square Law	~10 years	$\lvert\alpha\rvert<3\times10^{-11}$	10^{-12}	10^{-13}
PPN β	Few years	$\lvert\beta-1\rvert<1.1\times10^{-4}$	10^{-5}	10^{-6}

Limits of gravity tests with LLR

- how to convert 1 cm to 1 mm ?
- New reflectors
- New Southern Hemisphere station (SHELLI)

- Lunar interior
- Earth rotation
- Planets, asteroids
- = cm-level accuracy





INPOP and gravity tests

In Planetary and Lunar ephemerides (like INPOP), GR plays a role in

$$\Delta t_{SHAP} = (1+\gamma) GM_{\odot}(t) ln \frac{l_0 + l_1 + t}{l_0 + l_1 - t}$$

$$\Delta \dot{\varpi}_{PLA} = \frac{2\pi(2\gamma - \beta + 2) GM_{\odot}(t)}{a(1-e^2)c^2} + \frac{3\pi J_2 R_{\odot}^2}{a^2(1-e^2)c^2} + \Delta \dot{\varpi}_{AST}$$

$$\Delta \dot{\varpi}_{Moon} = \frac{2\pi(2\gamma - \beta + 2) GM_{\odot}(t)}{a(1-e^2)c^2} + \Delta \dot{\varpi}_{GEO} + \Delta \dot{\varpi}_{SEL} + \Delta \dot{\varpi}_{S,PLA}$$

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GR tests are then limited by

- Contributions by $J_2^{\odot}$, Asteroids, $2\gamma - \beta + 2$
- Lunar and Earth physics

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GR tests are then limited by

- Contributions by $J_2^{\odot}$, Asteroids, $2\gamma - \beta + 2$
- Lunar and Earth physics

BUT

- Decorrelation with all the planets
- Benefit of PE global fit versus single space mission

$2\gamma - \beta + 2$ and the solar J_2

the biggest constraints are given by

- INPOP08: Mars data
- INPOP10a: Mercury flybys (2 NP in 1972-1973 + 3 NP in 2008-2009)
- INPOP13a: Mercury full tracking

Planets	INPOP angle	accuracy distance	GR effect in longitude, Φ	S/N	over period
Mercure	0.050"	1km	0.43 "/yr	300	35 years
Venus	0.001"	4m	0.086 "/yr	172	2 years
				344	4 years
Mars	0.001"	2m	0.013 "/yr	390	30 years

$2\gamma - \beta + 2$ and the solar J_2

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- INPOP13a: Mercury full tracking

Planets	INPOP angle	accuracy distance	GR effect in longitude, Φ	S/N	over period
Mercure	0.050"	1km	0.43 "/yr	300	35 years
	0.5 mas	10 m		860	1 yr
Venus	0.001"	4m	0.086 "/yr	172	2 years
				344	4 years
Mars	0.001"	2m	0.013 "/yr	390	30 years

INPOP and tests of GR: the method

"Real" uncertainty/LS estimations + "my theory proposes this violation of GR. Is it compatible with INPOP ?"

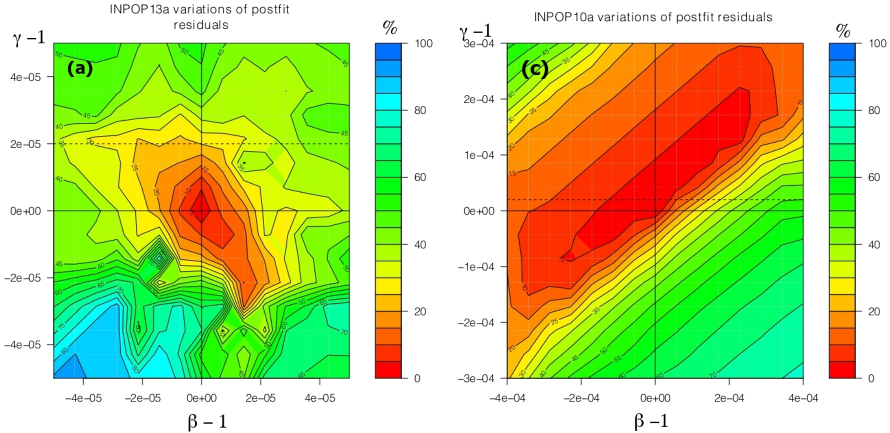
Grid of sensitivity for GRP determinations

(Fienga et al. 2009, 2011), (Verma et al. 2014)

- GRP: PPN β,γ, $\dot{\varpi}$, $\dot{\Omega}$, a_{supp}, $\dot{G}/G$
- Construction of different INPOP for different values of GRP
- For each value of GRP , all parameters (IC planets, GM_{Ast}, $GM_{\odot}$) of INPOP are fitted.
- Iteration = all correlations are taken into account
- Tests of consistency with s/c orbits (Verma 2013)
- Postfit residuals /INPOP $\rightarrow$ GRP intervals with Δ residuals $< 5\%$

What values of GRP are acceptable at the level of data accuracy ?

INPOP13a and tests of GR: PPN β and γ



Decorrelation + improvement of a factor 10

PPN β and γ detectable intervals for $J_2^{\odot} = 2.40 \pm 0.20$

	$(\beta-1)\times(\gamma-1)\times 10^5$	Limit [%]	$(\beta-1)\times(\gamma-1)\times 10^5$
INPOP10a	$(\beta$-1$) = (-6.2 \pm 8.1)$ $(\gamma$-1$) = (4.5 \pm 7.5)$		
K11	$(\beta$-1$) = (4\pm 24)$ $(\gamma$-1$) = (18 \pm 26)$	25*	$(\beta$-1$) = (0.2 \pm 2.5)$ $(\gamma$-1$) = (-0.3 \pm 2.5)$
M08-LLR-SEP W09-LLR-SEP	$(\beta$-1$) = (15 \pm 18)$ $(\beta$-1$) = (12 \pm 11)$	10	$(\beta$-1$) = (-0.15 \pm 0.70)$ $(\gamma$-1$) = (0.0 \pm 1.1)$
B03-CASS	$(\gamma$-1$) = (2.1 \pm 2.3)$	5	$(\beta$-1$) = (0.02 \pm 0.12)$ $(\gamma$-1$) = (0.0 \pm 0.18)$
L11-VLB	$(\gamma$-1$) = (-8 \pm 12)$		
P13	$(\beta$-1$) = (-2\pm 3)$ $(\gamma$-1$) = (4 \pm 6)$	Least squares 3-σ	$(\beta$-1$) = (1.34 \pm 0.13)$ $(\gamma$-1$) = (4.53 \pm 1.62)$

(Verma et al. 2014)

$\dot{\mu}/\mu$ with $\mu = GM_\odot$

Method

- Implementation with $\frac{\dot{\mu}}{\mu} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot}$ and

$$
\begin{aligned}
M_\odot(\mathrm{t}) &= M_\odot(t_0) + (t - t_0) \times \dot{M}_\odot \\
\mathrm{G(t)} &= G(t_0) + (t - t_0) \times \dot{G} \\
\mu(t) &= G(t) \times M_\odot(t)
\end{aligned}
$$

- by fixing $\dot{M}_\odot$ or $\dot{G} \rightarrow \frac{\dot{\mu}}{\mu}$
- At each step, t_i, of the numerical integration of the Eq.of motions of planets, asteroids $\rightarrow M_\odot(\mathrm{t}_i)$ and $\mathrm{G}(\mathrm{t}_i)$ are injected.
- Same method as PPN $\beta,\gamma \rightarrow$ grid of $\frac{\dot{\mu}}{\mu}$ + construction of full PE
- What values of $\frac{\dot{\mu}}{\mu}$ are acceptable / data accuracy ?

$\dot{\mu}/\mu$ with $\mu = GM_\odot$

with PPN $\beta,\gamma = 1$, $J_2^\odot = 2.40 \pm 0.20$

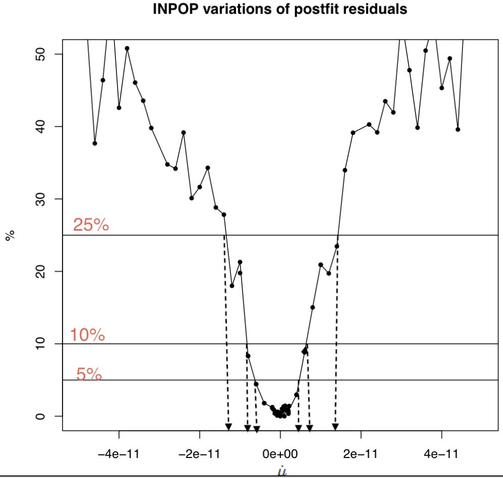
Method	$\dot{G}/G$ $\times 10^{13}$ yr^{-1}
LLR-M05	(6 ± 8)
Binary pulsar	(40 ± 50)
Helioseismology	(0 ± 16)
Big Bang nucleo.	(0 ± 4)
Planck +WP+BAO	(-1.42 ± 2.48)
EMP (P12)	$(0.166 \pm 0.724)^{*}$
DE (K11)	$(1.0 \pm 1.6)^{**}$
5%	$(0.62 \pm 0.86)^{*}$ $(0.85 \pm 0.55)^{**}$
10%	$(0.595 \pm 1.035)^{*}$ $(0.825 \pm 0.725)^{**}$
25 %	$(0.72 \pm 1.71)^{*}$ $(0.95 \pm 1.40)^{**}$

* $\dot{M_\odot}/M_\odot = (-0.67 \pm 0.31) \times 10^{13}$ yr^{-1}

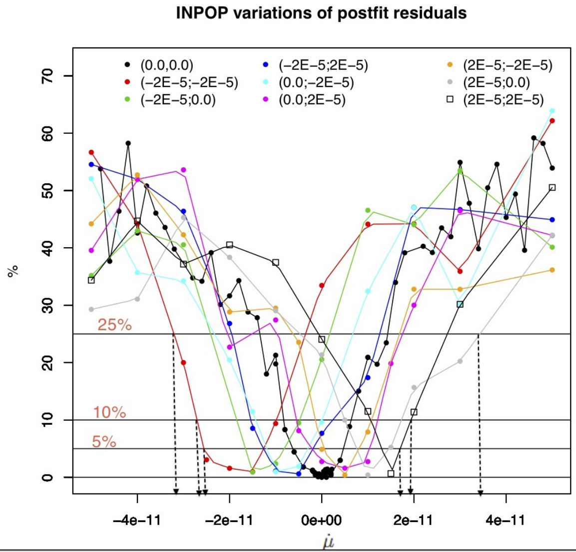
** $\dot{M_\odot}/M_\odot = -0.9 \times 10^{13}$ yr^{-1}

DE (K11) with $J_2^\odot$ fixed

EMP (P12) with $J_2^\odot$, β and γ fixed



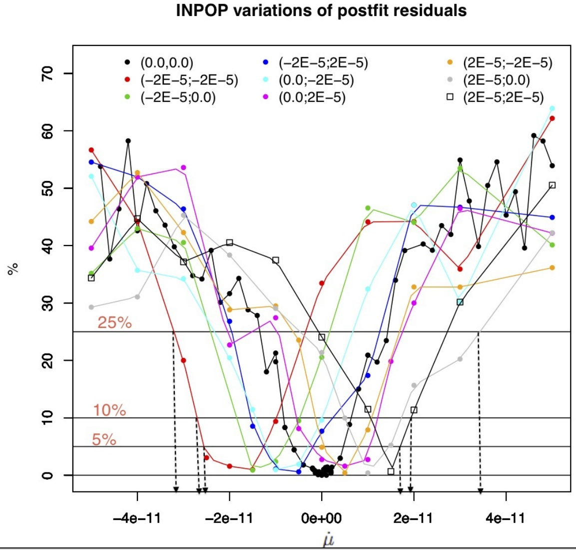
Preliminary results about $\dot{\mu}/\mu$ with $\mu = GM_\odot$



with PPN $\beta,\gamma \neq 1$

Shift of the minimum of residual variation with $\dot{\mu}/\mu$ AND β,γ

Preliminary results about $\dot{\mu}/\mu$ with $\mu = GM_\odot$

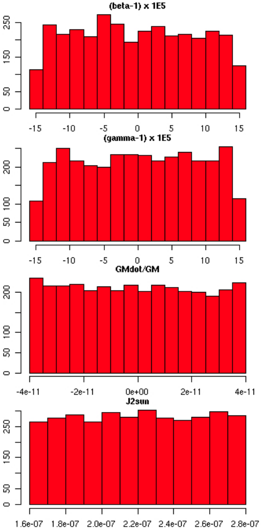


with PPN $\beta,\gamma \neq 1$

Shift of the minimum of residual variation with $\dot{\mu}/\mu$ AND β,γ

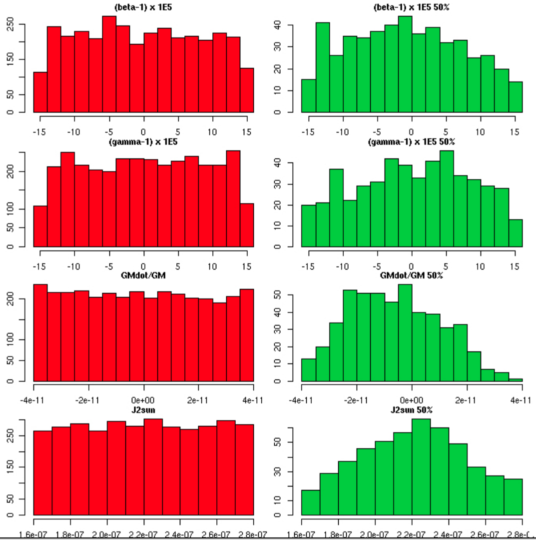
→ 3D grid of $\dot{\mu}/\mu$ and $J_2^\odot$ + random β, γ

Direct Monte Carlo of $\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ



- 4000 INPOP runs with random selection of ($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ)
- 1 run = 4 iterations (1hr/iteration @ 16 itanium processors)
- Selection of INPOP($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ) inducing differences to INPOP13a residuals $<$ 50 %

Direct Monte Carlo of $\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ



- Only 15 % INPOP($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ) < 50 %
- 1.4% for INPOP() < 25 %
- No clear gaussian distribution especially for β and γ

$\rightarrow$ Optimisation of the MC by a genetic algorithm

$< J_2^{\odot} >$	$(2.21 \pm 0.29) \times 10^{-7}$ *W-test = 0.984*
$< \beta - 1 >$	$(-0.8 \pm 8.2) \times 10^{-5}$? *0.969*
$< \gamma - 1 >$	$(0.2 \pm 8.2) \times 10^{-5}$? *0.968*
$< \dot{G}/G >$ $\times 10^{13}$ yr^{-1}	(0.04 $\pm$ 2.46)* (0.27 $\pm$ 1.66)** *0.987*

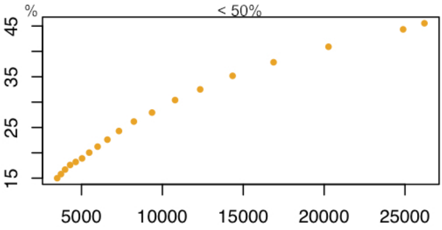
Simple Genetic Algorithm with mutation (SGAM)

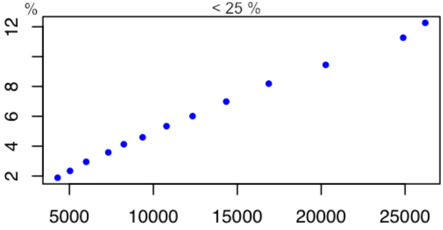
- 1 individual = INPOP ($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ)
- 1 chromosome = a set of ($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ)
- fitness of each individual = differences to INPOP13a residuals < 50 % or 25 %
- 2 crossovers + 1/10 mutation (= new random value each over 10)

set i	$[(\dot{\mu}/\mu)_i,\ (J_2^{\odot})_i,\ \beta_i,\ \gamma_i]$
set j	$[(\dot{\mu}/\mu)_j,\ (J_2^{\odot})_j,\ \beta_j,\ \gamma_j]$
1 crossover	$[(\dot{\mu}/\mu)_i,\ (J_2^{\odot})_i, \beta_j,\ \gamma_j]$ $[(\dot{\mu}/\mu)_j,\ (J_2^{\odot})_j, \beta_i,\ \gamma_i]$
2 crossovers	$[(\dot{\mu}/\mu)_i,\ (J_2^{\odot})_j, \beta_i,\ \gamma_j]$ $[(\dot{\mu}/\mu)_j,\ (J_2^{\odot})_i, \beta_j,\ \gamma_i]$

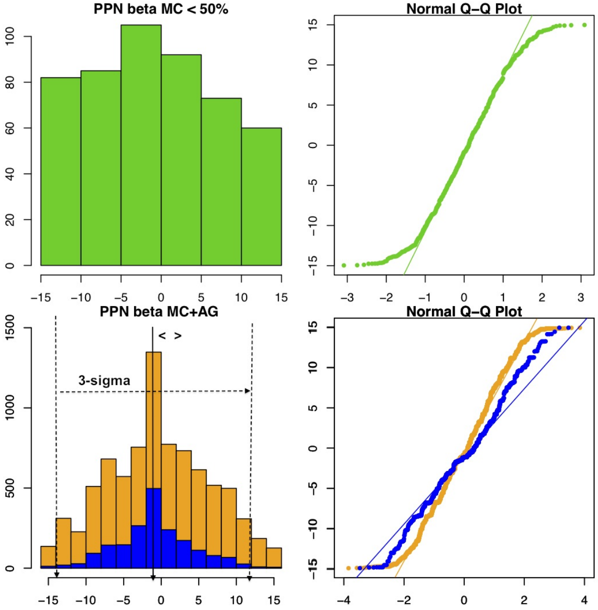
26 200 runs with SGAM

- @ PSL mesocentre : NEC 1472 kernels on 92 nodes
- 2 nodes allocated for INPOP
- 12 runs (= 12 x 4 iterations) @ 1hr / node
- 4000 MC simulation = population 0 @ SGAM



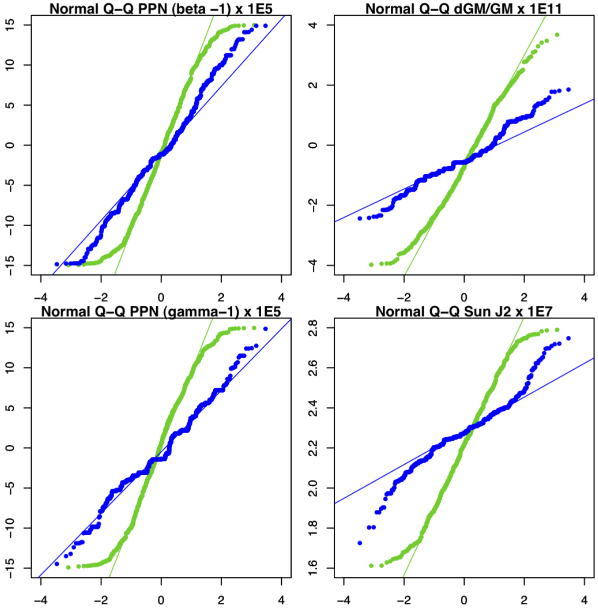


After 26 200 runs, 45% runs with INPOP < 50%
and 11% INPOP < 25 %

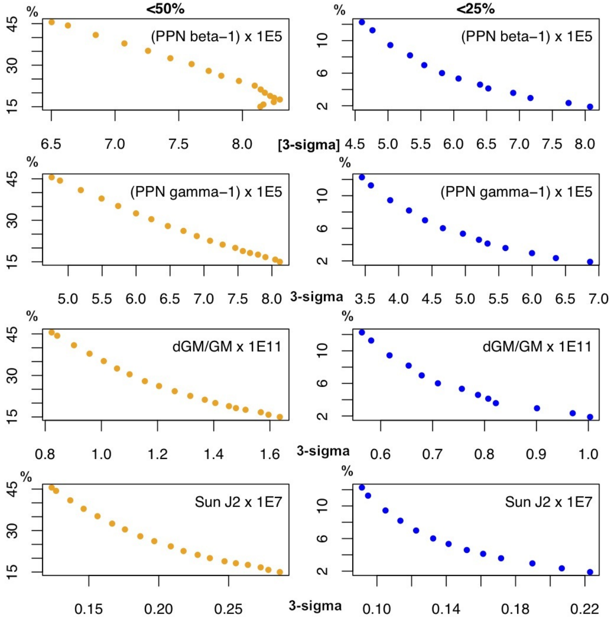


Improvement in gaussianity after 26 200 runs:

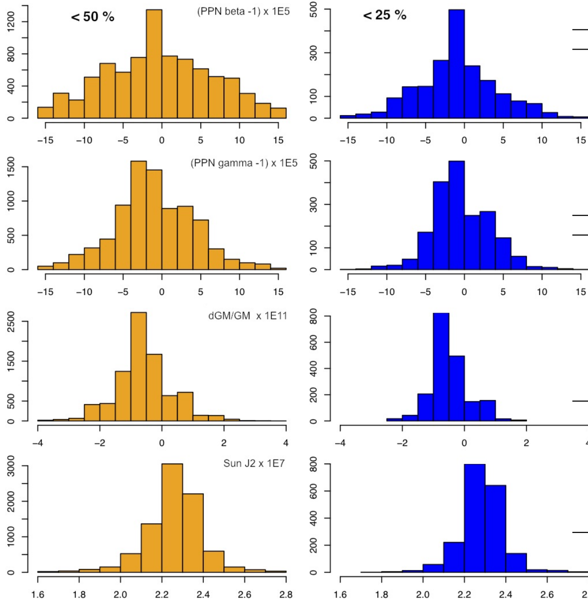
→ Proper definition of mean and 3-sigma



Improvement in gaussianity after 26 200 runs



Reduction of the 3-sigma intervals for $\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ with the increase of $<50\%$ or $<25\%$ populations



After 26 200 runs,

INPOP $< 50\%$	
$< J_2^{\odot} >$ $\times\ 10^7$	(2.26 $\pm$ 0.12)
$< \beta - 1 >$ $\times\ 10^{-5}$	(-0.43 $\pm$ 6.50)
$< \gamma - 1 >$ $\times\ 10^{-5}$	(-1.1 ± 4.76)
$< \dot{G}/G >$ $\times\ 10^{13}$ yr^{-1}	(0.11 $\pm$ 1.65)* (0.34 $\pm$ 0.84)**
INPOP $< 25\%$	
$< J_2^{\odot} >$ $\times\ 10^7$	(2.280 $\pm$ 0.09)
$< \beta - 1 >$ $\times\ 10^{-5}$	(-0.94 $\pm$ 4.76)
$< \gamma - 1 >$ $\times\ 10^{-5}$	(-0.59 ± 3.58)
$< \dot{G}/G >$ $\times\ 10^{13}$ yr^{-1}	(0.19 $\pm$ 1.38)* (0.42 $\pm$ 0.58)**
LLR-M05	(6 $\pm$ 8)
Binary pulsar	(40 $\pm$ 50)
Helioseismology	(0 $\pm$ 16)
Big Bang nucleo.	(0 $\pm$ 4)
Planck +WP+BAO	(-1.42$\pm$ 2.48)
EMP (P13)	(0.166 $\pm$ 0.724)*
DE (K11)	(1.0 $\pm$ 1.6)**

Bi-modal distribution for γ ?

PPN β, γ, $\dot{\mu}/\mu$, $J_2^{\odot}$

Method	PPN $\beta - 1$ $\times 10^{-5}$	PPN $\gamma - 1$ $\times 10^{-5}$	$\dot{G}/G$ $\times 10^{13}$ yr^{-1}	$J_2^{\odot}$ $\times 10^7$
2-D Grid	0.2 ± 2.5	-0.3 ± 2.5	0.0	2.4 ± 0.20
MC	-0.8 ± 8.2	0.2 ± 8.2	0.04 ± 2.46	2.21 ± 0.29
MC + SGAM	-0.9 ± 4.8	-0.6 ± 3.6	0.19 ± 1.38	2.28 ± 0.09
B03-Cass	0.0	2.1 ± 2.3	0.0	NC
L11-VLB	0.0	-8 ± 12	0.0	fixed
W09-LLR	12 ± 11	fixed	0.0	fixed
M05-LLR	15 ± 18	fixed	6 ± 8	fixed
K11-DE	4 ± 24	18 ± 26	1.0 ± 1.6	fixed to 1.8
F13-DE	0.0	0.0	0.0	2.1 ± 0.70
P13-EMP	-2 ± 3	4 ± 6		2.0 ± 0.2
			0.166 ± 0.724	
Planck +WP+BAO	0.0	0.0	-1.42± 2.48	

$\dot{G}/G \approx 10^{-13}$ yr^{-1} $\beta - 1 \approx 5 \times 10^{-5}$ $\gamma - 1 \approx 4 \times 10^{-5}$

EP $\eta = 2 \times 10^{-4}$

Other tests: Pioneer anomaly

Context

- unexplained acceleration of about 8×10^{-10} $m.s^{-2}$
- detected on Pioneer 10 and 11 after the Saturn (?), Uranus orbits
- First detected in 1988 and investigated since 2004

Investigations

- Thermal models
- Alternative physics on s/c dynamics
- Alternative physics on planet dynamics ?

Other tests: Pioneer anomaly

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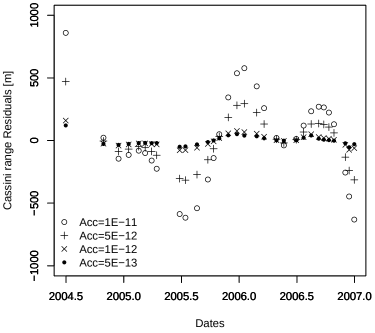
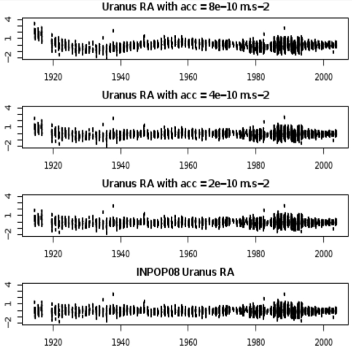
Investigations

- Thermal models
- Alternative physics on s/c dynamics
- Alternative physics on planet dynamics ? No

Other tests: Pioneer anomaly

Test of a constant sun-oriented acceleration of about 8×10^{-10} m.s^{-2} on EIH equations with Cassini range tracking but also Neptune and Uranus optical observations





(Fienga et al. 2009): $< 5 \times 10^{-13}$ m.s^{-2}

$< 2 \times 10^{-10}$ m.s^{-2}

(Folkner 2009): $< 10^{-14}$ m.s^{-2}

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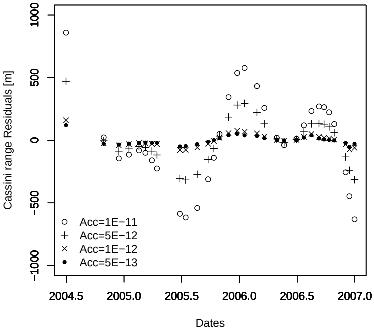
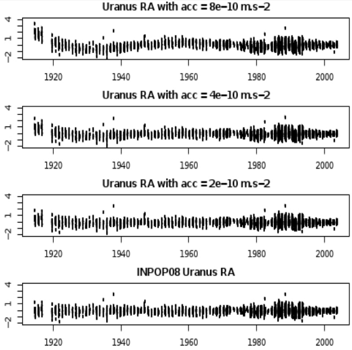




(Fienga et al. 2009): $< 5 \times 10^{-13}$ m.s^{-2}

$< 2 \times 10^{-10}$ m.s^{-2}

(Folkner 2009): $< 10^{-14}$ m.s^{-2}

No unexplained 10^{-10} m.s^{-2} on planet orbits

Other tests: Anomalous precession in nodes and perihelia?

Same procedure as for PPN β and γ

- For each value of $\dot{\varpi}_k$, $\dot{\Omega}_k$, all parameters (IC planets, GM_{Ast}, $GM_{\odot}$ of INPOP are fitted.
- postfit residuals /INPOP $\rightarrow \dot{\varpi}_k$ or $\dot{\Omega}_k$ intervals with Δ residuals $< 5\%$
- INPOP08: Only planet IC refitted
- INPOP10a: ALL the parameters are refitted: IC, $GM_{\odot}$, GM_{ast}
- New Observations in INPOP10a: Cassini VLB, Jupiter flybys, Mercury flybys

Anomalous precession in perihelia ?

$\dot{\varpi}_{\rm sup}$ mas.cy^{-1}	INPOP08	INPOP10a	P09	P10
Mercury	-10 $\pm$ 30	1.2 $\pm$ 1.6	-3.6 $\pm$ 5	-4 $\pm$ 5
Venus	-4 $\pm$ 6	0.2 $\pm$ 1.5	-0.4 $\pm$ 0.5	
EMB	0.0 $\pm$ 0.2	-0.2 $\pm$ 0.9	-0.2 $\pm$ 0.4	
Mars	0.4 $\pm$ 0.6	-0.04 $\pm$ 0.15	0.1 $\pm$ 0.5	
Jupiter	142 $\pm$ 156	-41 $\pm$ 42		
Saturn	-10 $\pm$ 8	0.15$\pm$ 0.65	-6 $\pm$ 2	-10 $\pm$ 15

Anomalous precession in nodes ?

$\dot{\Omega}_{sup}$ mas.cy^{-1}	INPOP08	INPOP10a
Mercury		1.4 $\pm$ 1.8
Venus	200 $\pm$ 100	0.2 $\pm$ 1.5
EMB	0.0 $\pm$ 10.0	0.0 $\pm$ 0.9
Mars	0.0 $\pm$ 2	-0.05 $\pm$ 0.13
Jupiter	-200 $\pm$ 100	-40 $\pm$ 42
Saturn	-200 $\pm$ 100	-0.1 $\pm$ 0.4

Improvements INPOP10a / INPOP08 due to:

- new observations
- Fit

No supplementary advances in perihelia and nodes

Constraints on MOND (Blanchet et Novak 2011)

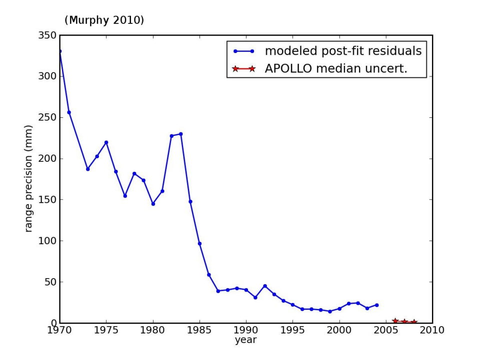
Discussions

Tests		Accuracy	
EP η	LLR	4×10^{-4}	
	planet	2×10^{-4}	Metric theories
PPN γ	Spacecraft	2×10^{-5}	
	planet	4×10^{-5}	
	LLR	10^{-3}	Metric theories
PPN β	LLR	10^{-4}	Metric theories
	planet	5×10^{-5}	
$\dot{G}/G$ [yr^{-1}]	planet	10^{-13}	
$\dot{\varpi}_{\mathrm{sup}}, \dot{\Omega}_{\mathrm{sup}}$	LLR	10	
[mas.cy^{-1}]	planet	$40 \rightarrow 0.1$	MOND
a_{supp}	LLR	10^{-16}	Dark Matter density
	planet	10^{-14}	Pioneer anomaly

Tests	
EP η	LLR
PPN γ	Spacecraft
	planet
	LLR
PPN β	LLR
	planet
$\dot{\varpi}_{\mathrm{sup}}, \dot{\Omega}_{\mathrm{sup}}$	LLR
mas.cy^{-1}	planet

LLR

- 1 cm limitation in the dynamics
- 1 mm accuracy for observations
- Efforts to compare and improve the Moon dynamics



Discussions

LLR

- 1 cm limitation in the dynamics
- 1 mm accuracy for observations
- Efforts to compare and improve the Moon dynamics

Planetary Ephemerides

- Jupiter to be improved
- Equivalence Principal for all the planets and the Moon
- MC + SGAM for EP
- β,γ decorrelation linked to spacecraft orbits
- Efforts to estimate/limit correlations with spacecraft orbits
- New tests to implement

The end