Extracting black-hole rotational energy: the generalized Penrose process

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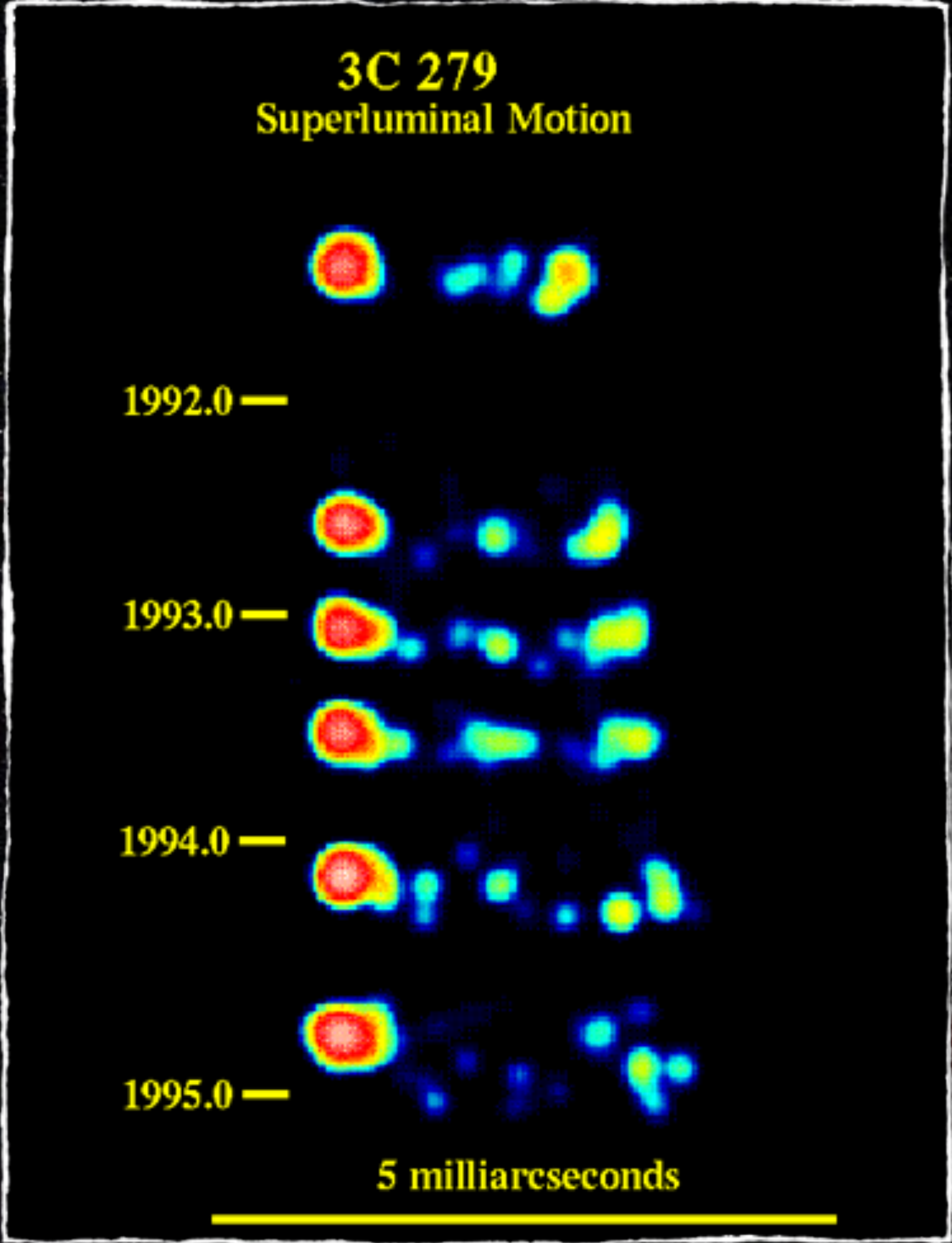
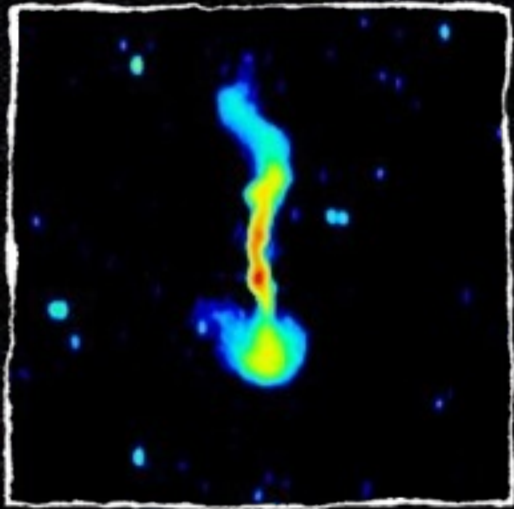
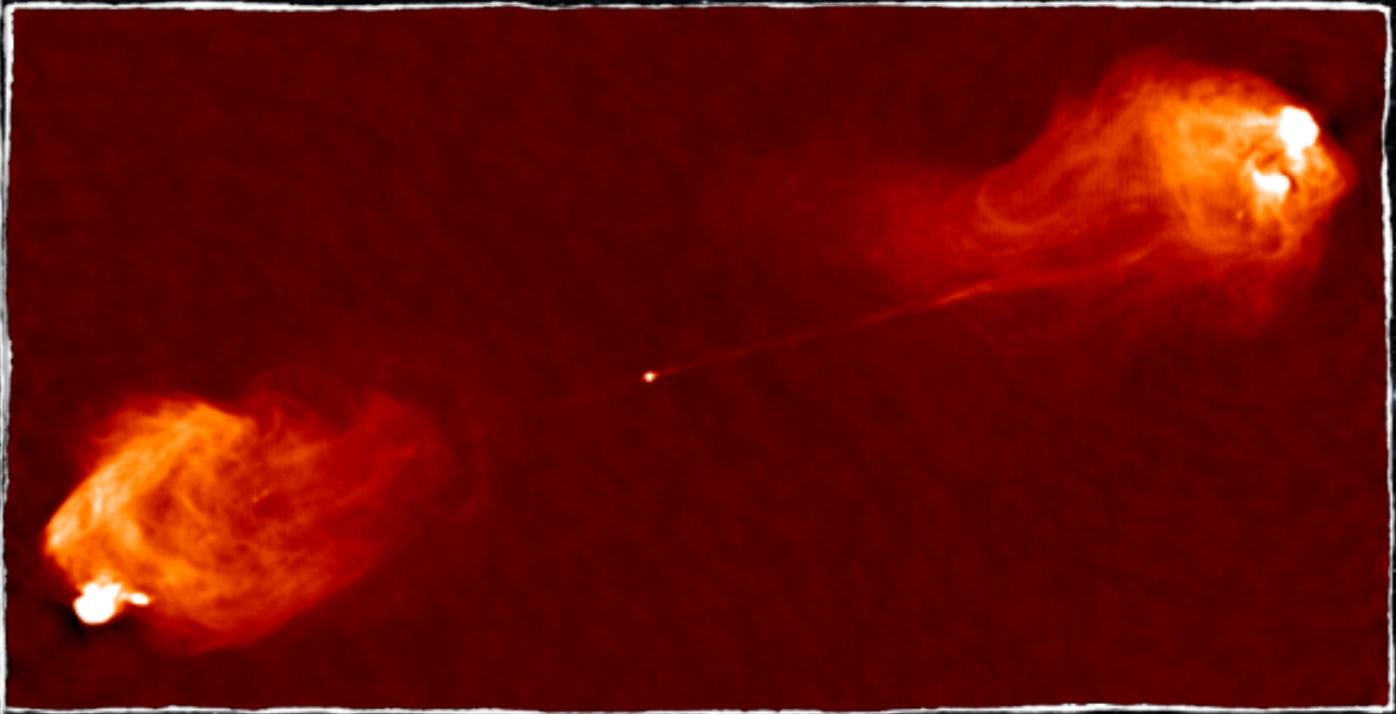
Based on *Lasota, Gourgoulhon, Abramowicz, Tchekhovskoy & Narayan* ; Phys. Rev. D 89, 024041 (2014)

IHES, 6th of February 2014

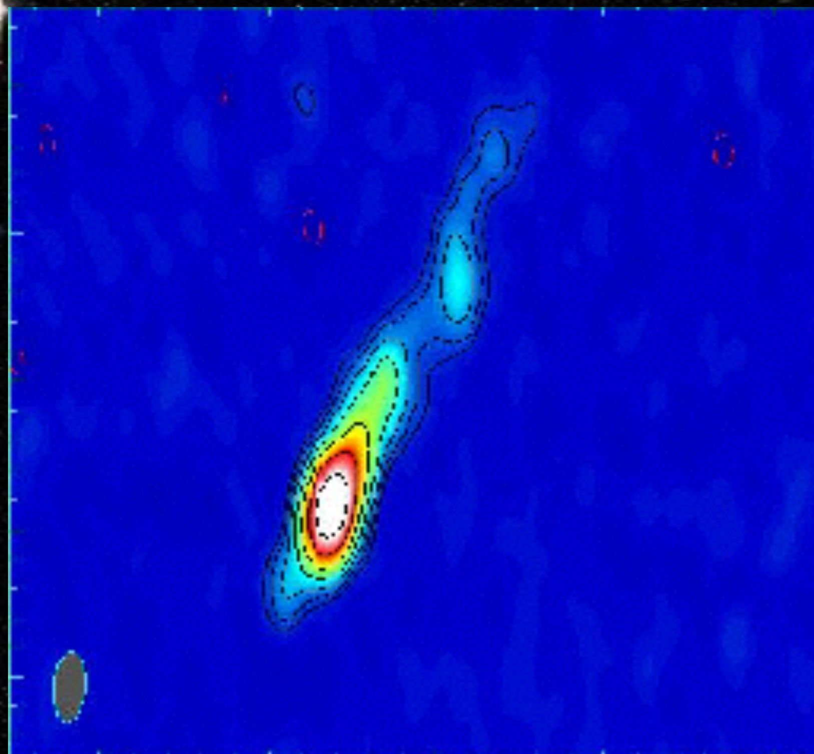
Relativistic jets in Active Galactic Nuclei

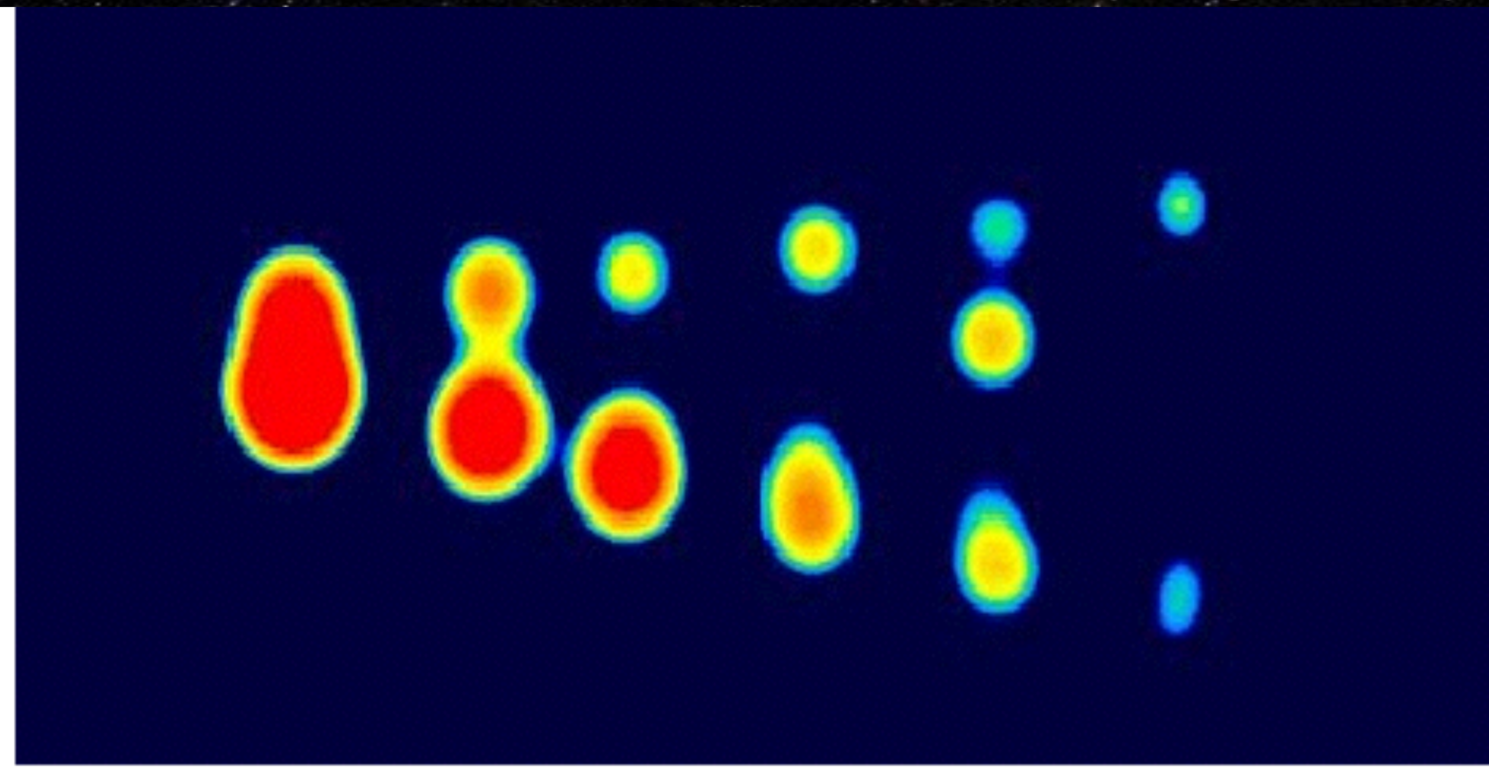


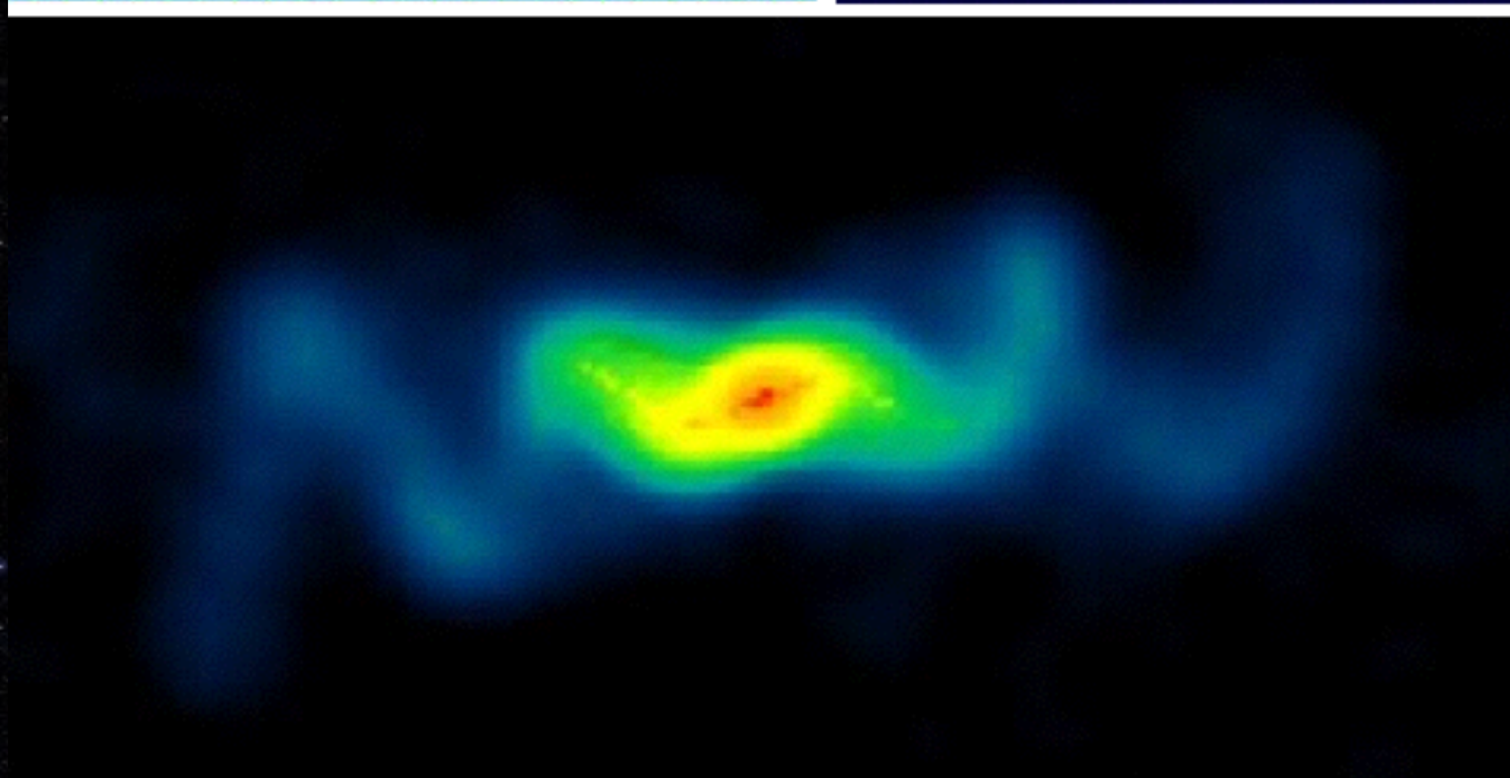


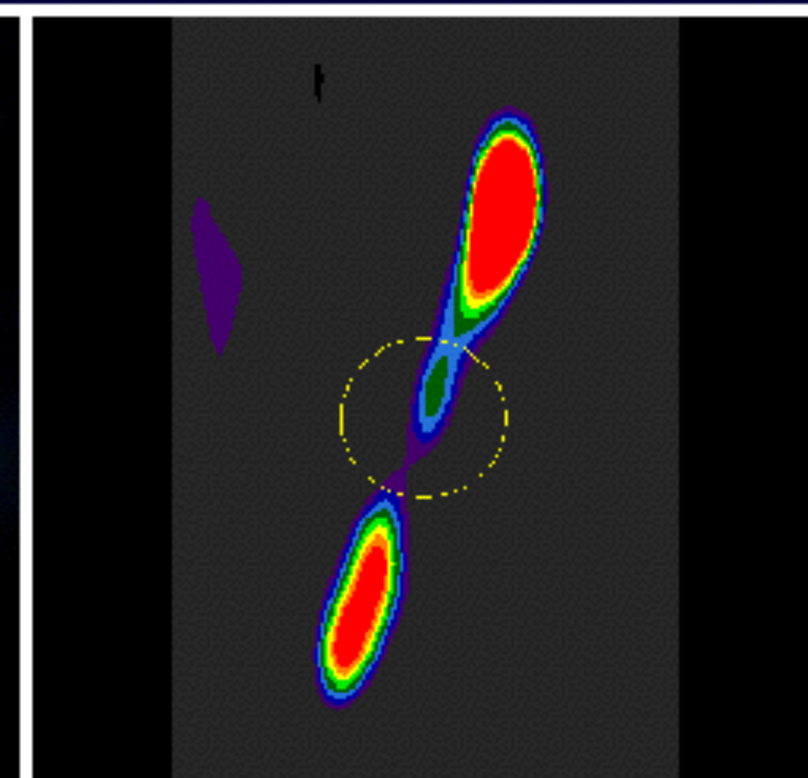


Relativistic jets in compact binaries (microquasars)

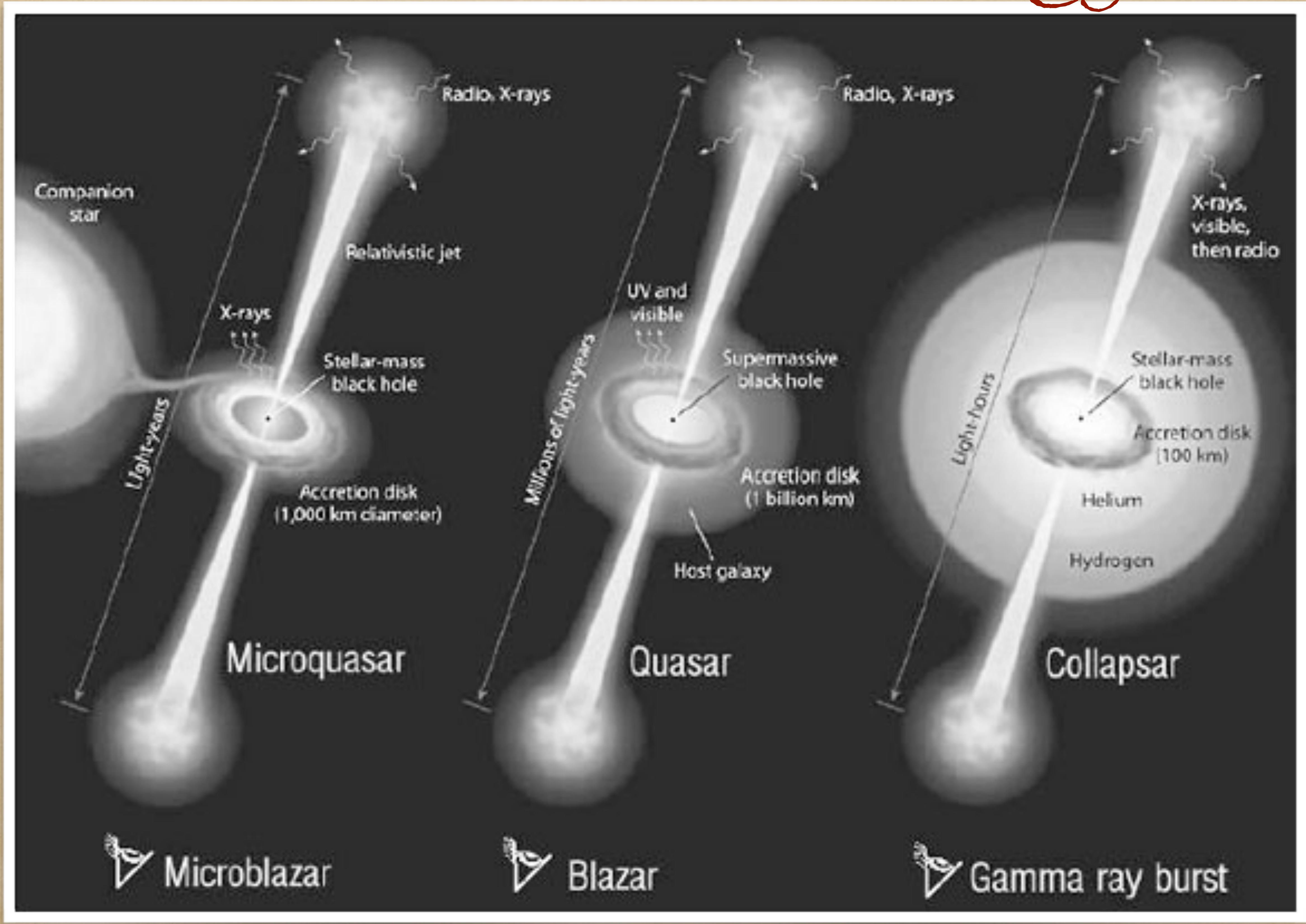




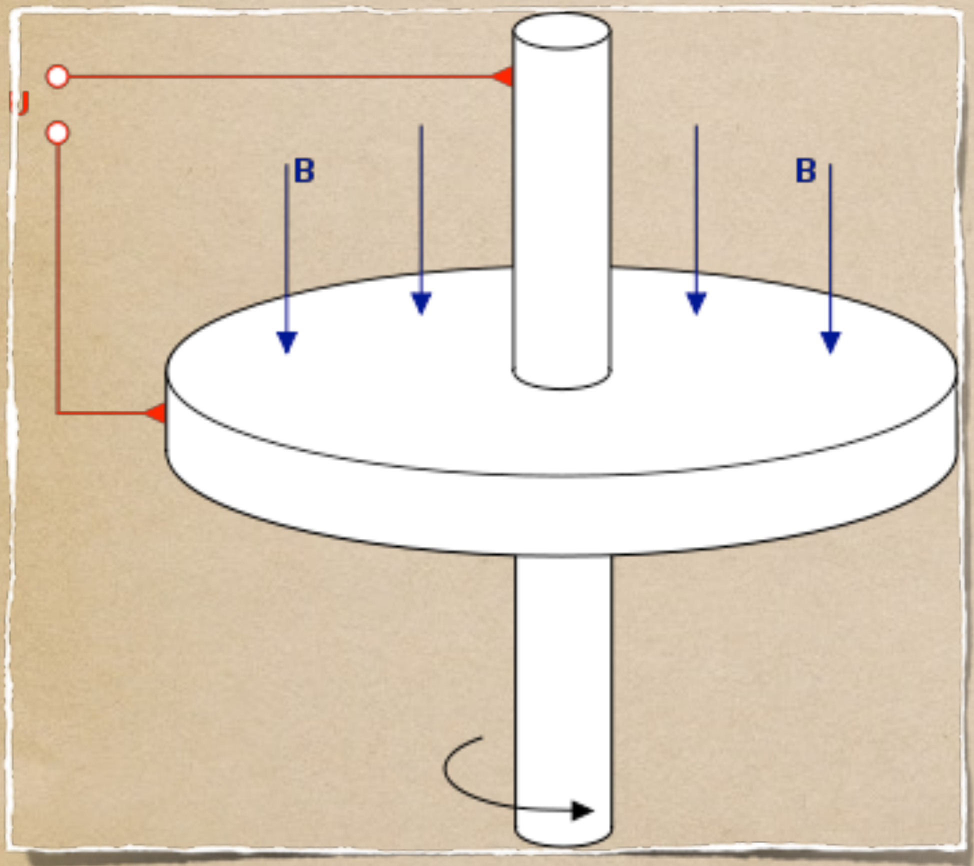


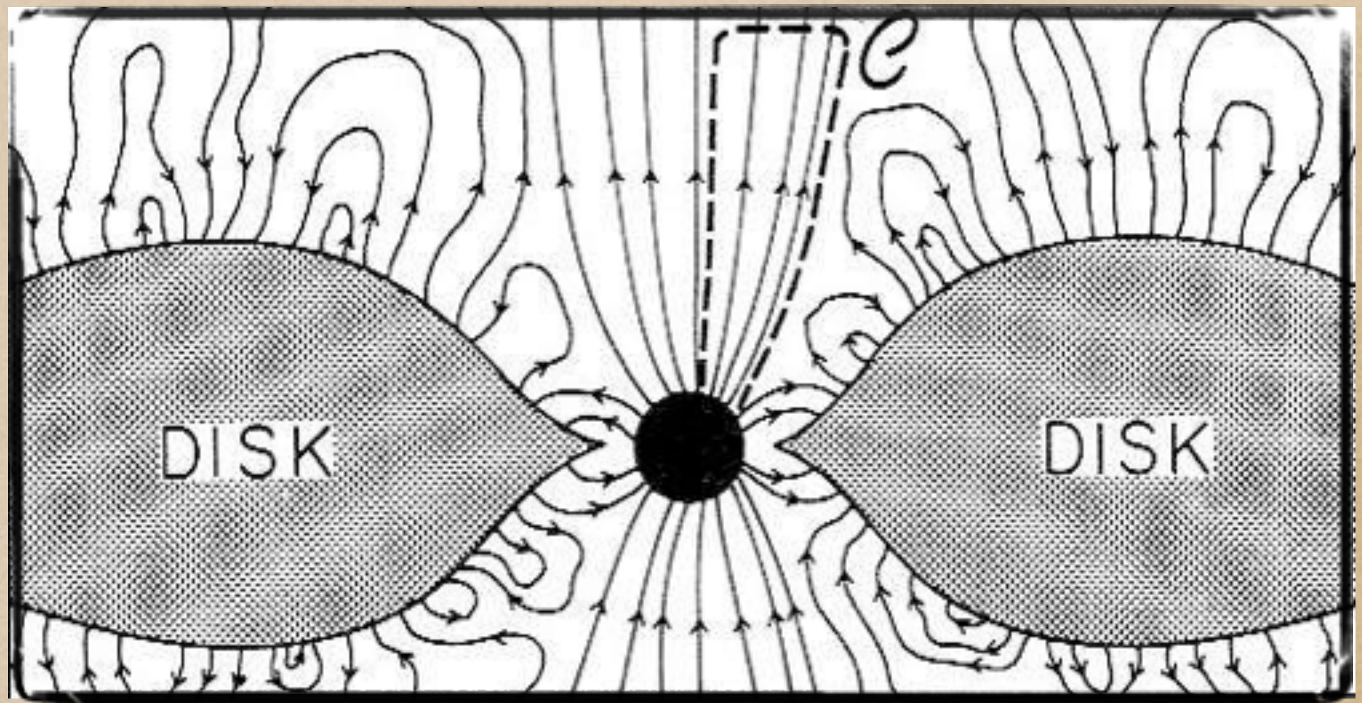


Common source of energy ?



Tapping black-hole rotational energy by unipolar induction

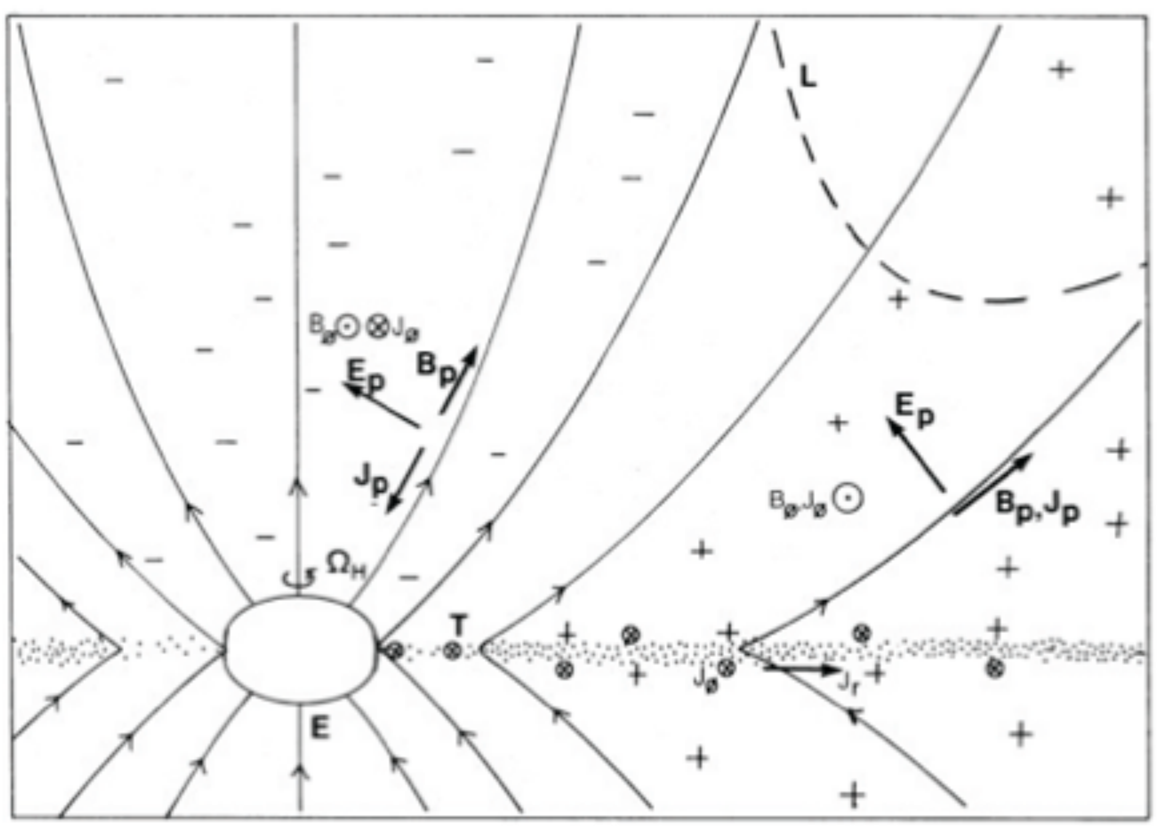


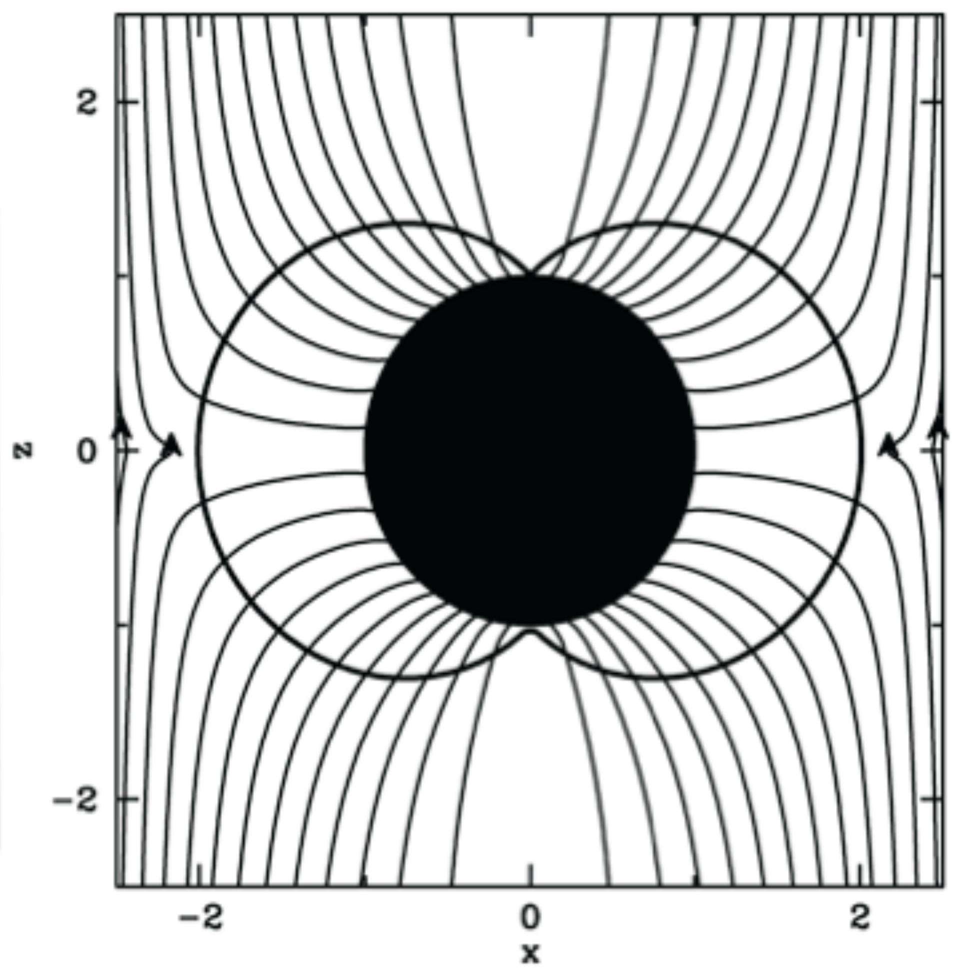


Ruffini & Wilson 1975, Damour 1978, Blandford & Znajek 1977

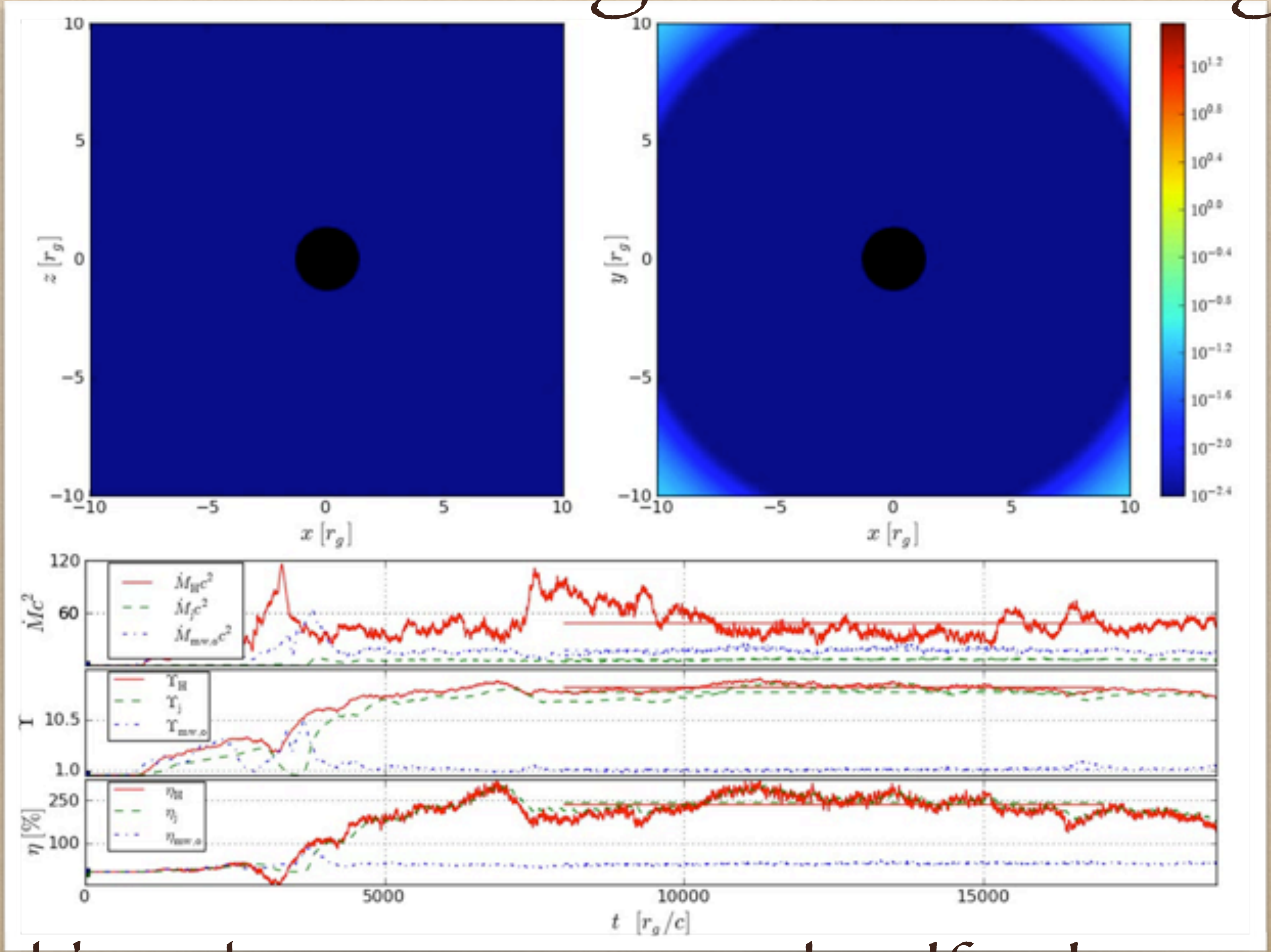
Controversy:

- is the BH surface an analogue of a Faraday disc (causality)
- is the Blandford-Znajek mechanism efficient (rotation of black-hole or disc) ?



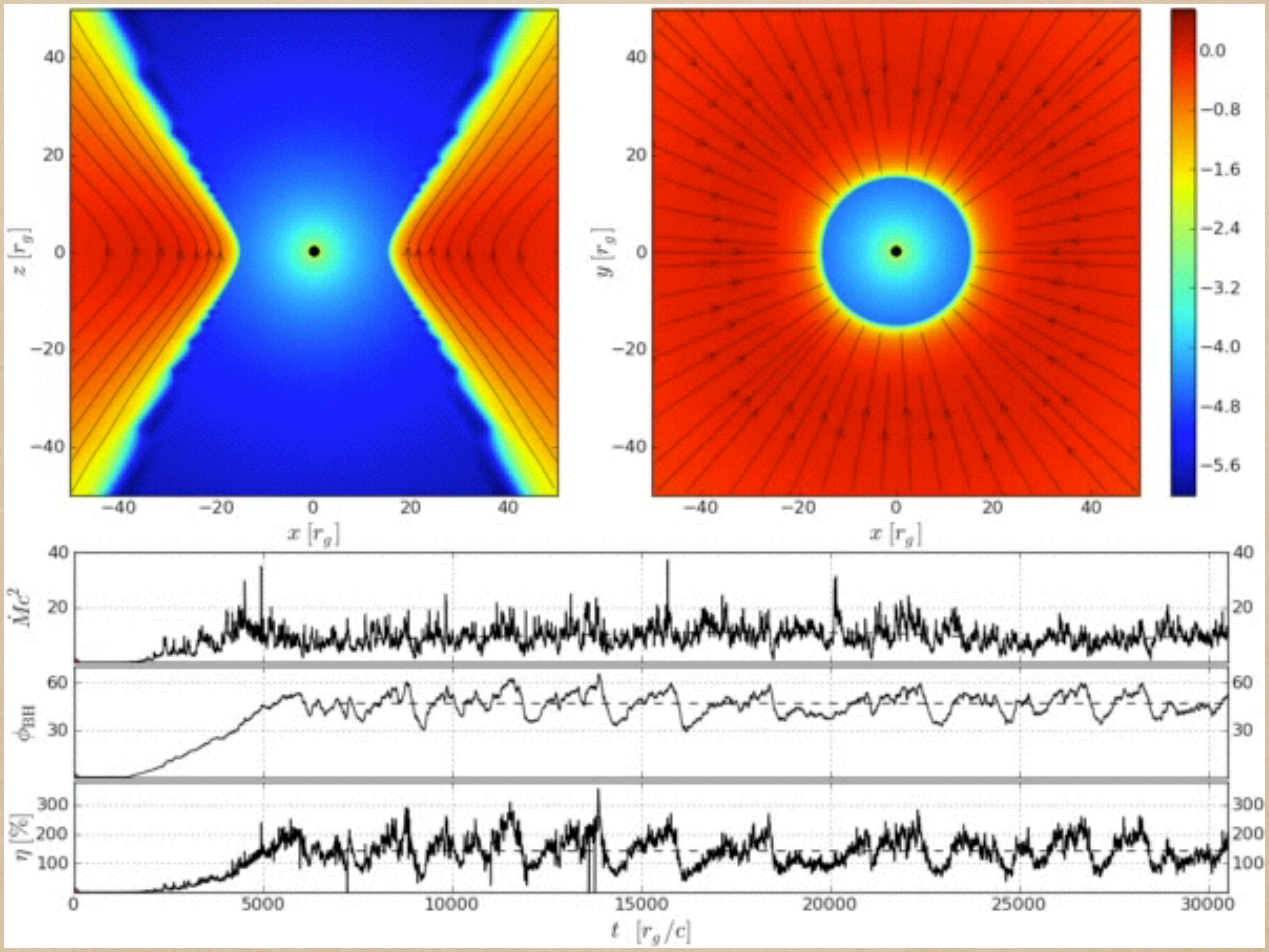


Recent (2011-2013) GRMHD simulations clearly showed BH rotational energy extraction in a particular (MAD) magnetic field configuration

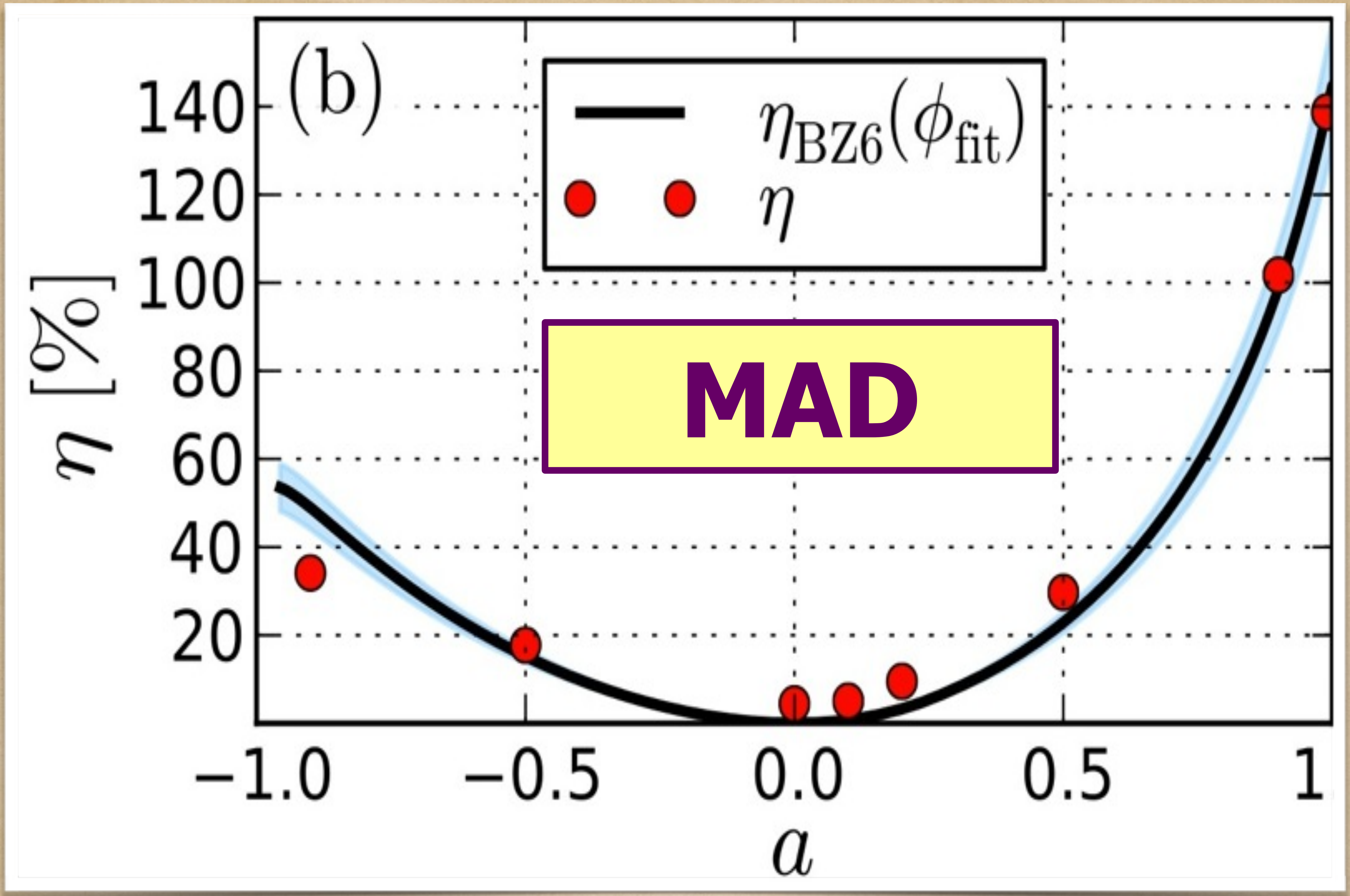


Tchekhovskoy, McKinney, Blandford 2012

MAD simulation

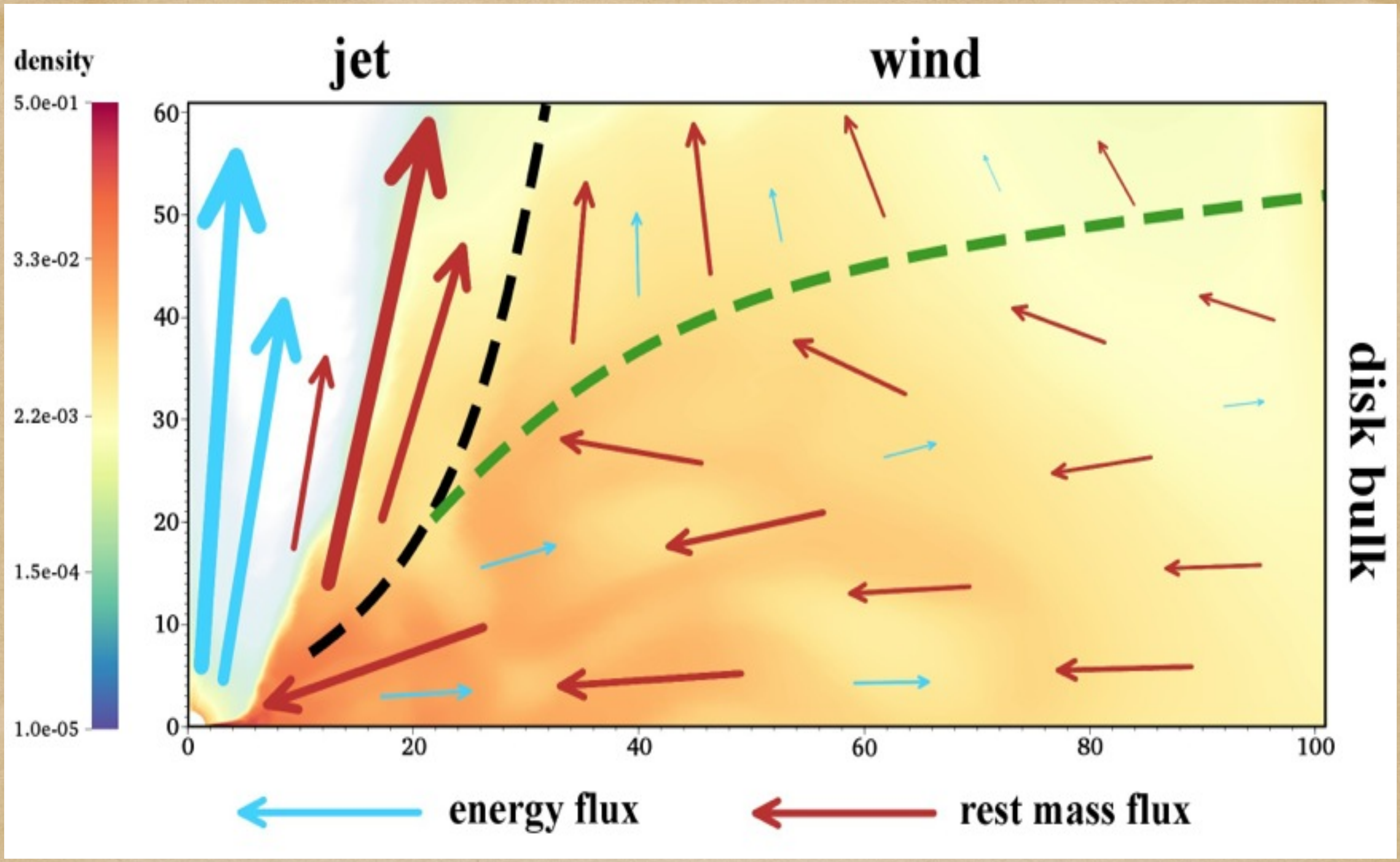


Tchekhovskoy, McKinney, Narayan 2011



BH Jet in MAD state has a large efficiency:

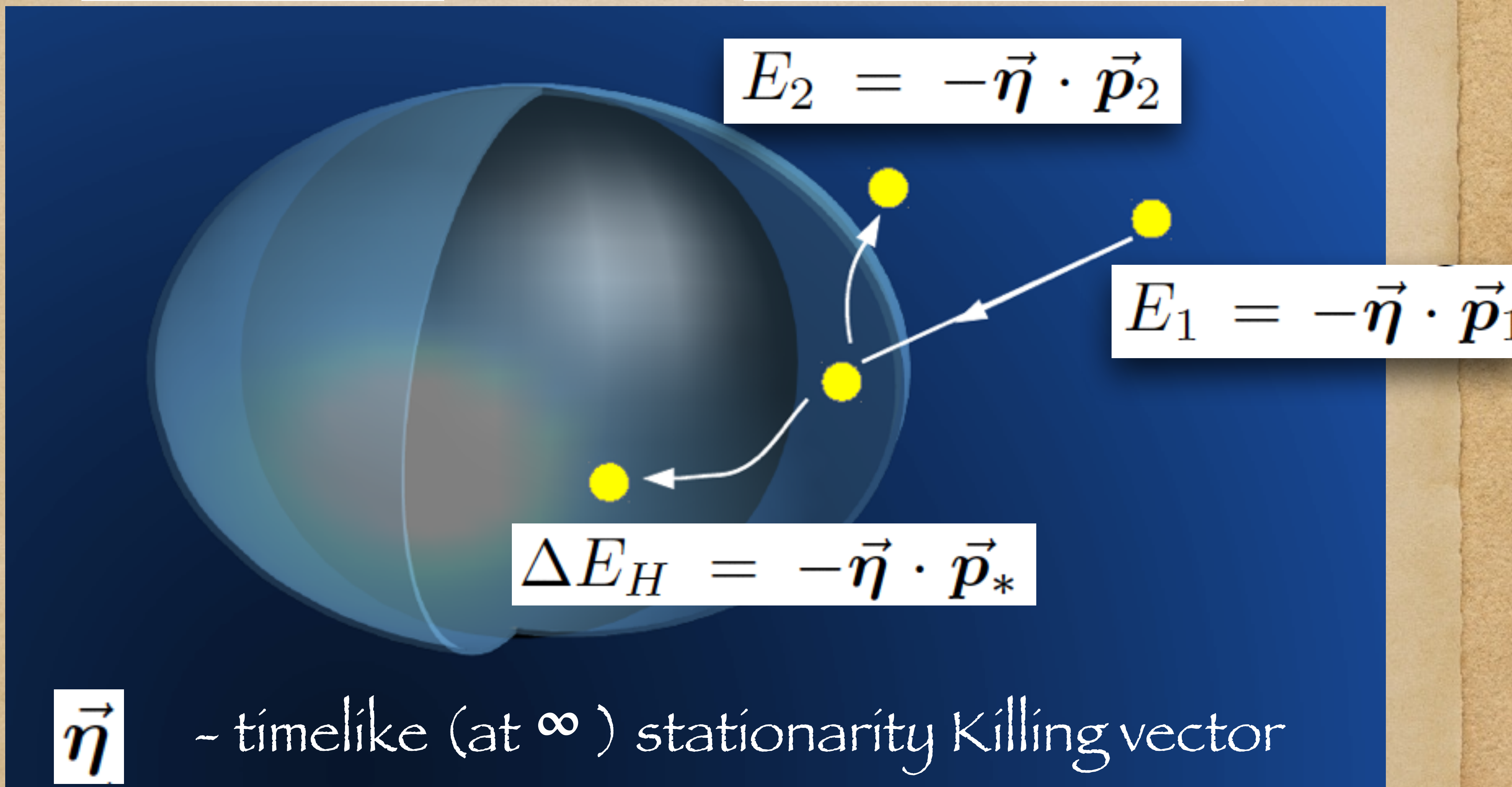
$$\eta = P_{jet}/\dot{M}c^2 > 100\%$$



Sądowski et al. (2013)

Penrose process

$$\vec{p}_1 = \vec{p}_2 + \vec{p}_* \longrightarrow E_1 = E_2 + \Delta E_H$$



For $\Delta E_H < 0$ $E_2 > E_1$

$\vec{\eta}$ - timelike (at ∞) stationarity Killing vector

$\vec{\xi}$ - spacelike axisymmetry Killing vector

$$\vec{u} = q\left(\vec{\eta} + \omega\vec{\xi}\right)$$ -ZAMO, $$\omega = \frac{\vec{\eta}\cdot\vec{\xi}}{\vec{\xi}\cdot\vec{\xi}}$$

Energy measured by ZAMOs always non-negative:

$$-\left(\vec{\eta} + \omega\vec{\xi}\right)\vec{p}_* = (\Delta E_H - \omega_H \Delta J_H) \geq 0$$

Hence for $\Delta E_H < 0$ $\omega_H \Delta J_H \leq \Delta E_H$.

Since $\omega_H \geq 0$

$\omega_H \neq 0$ and $\Delta J_H < 0.$

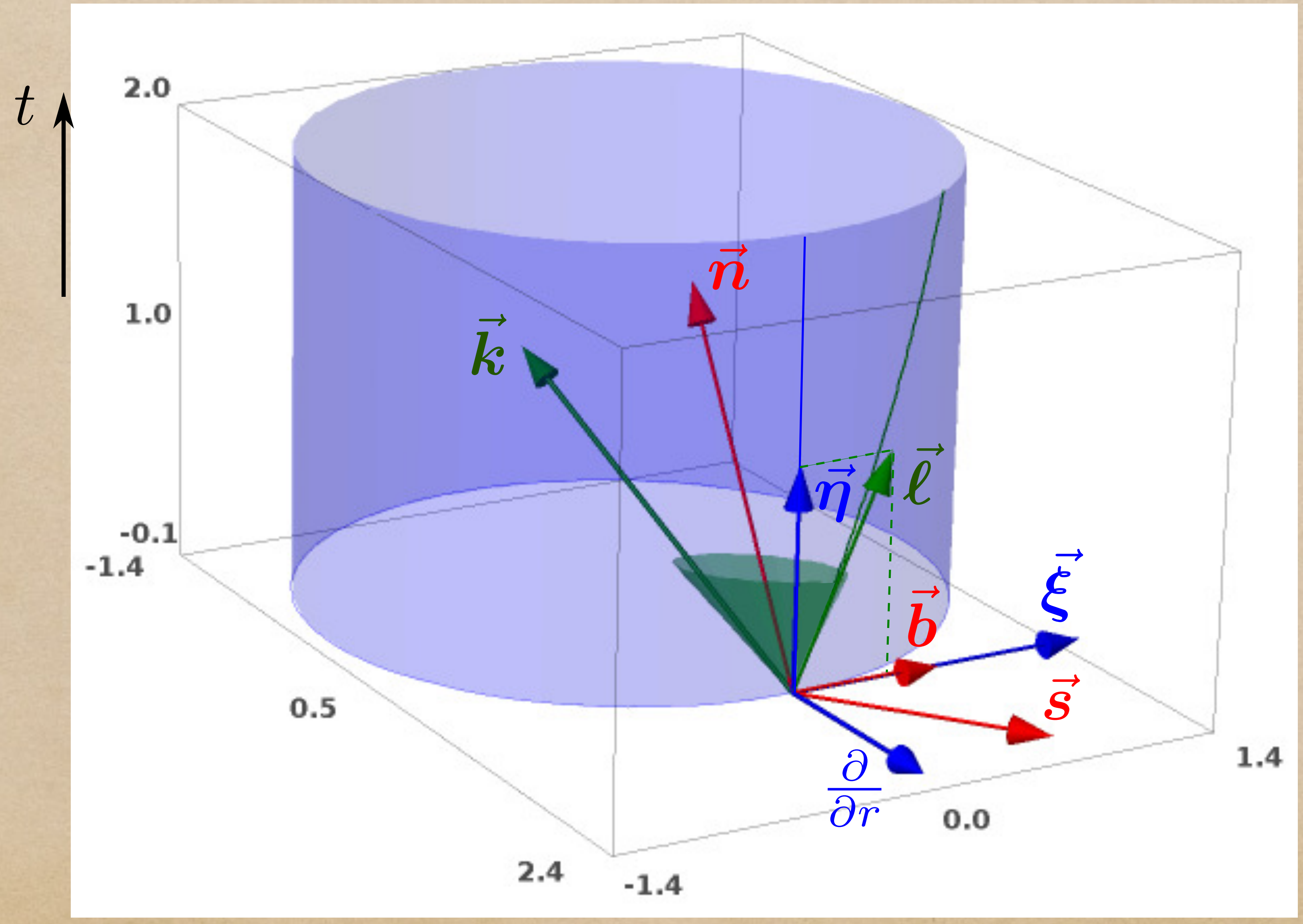
Horizon

$$\vec{\ell} = \vec{\eta} + \omega_H \vec{\xi},$$

$$\vec{\ell} \cdot \vec{\ell} = 0$$

$$\omega_H = a/[2mr_H],$$

$$r_H = m + \sqrt{m^2 - a^2}$$



T - energy moment tensor

$$T_{\mu\nu}\ell^{\mu}\ell^{\nu}|_{\mathcal{H}} \geq 0.$$

- null energy condition

- **Energy conservation**

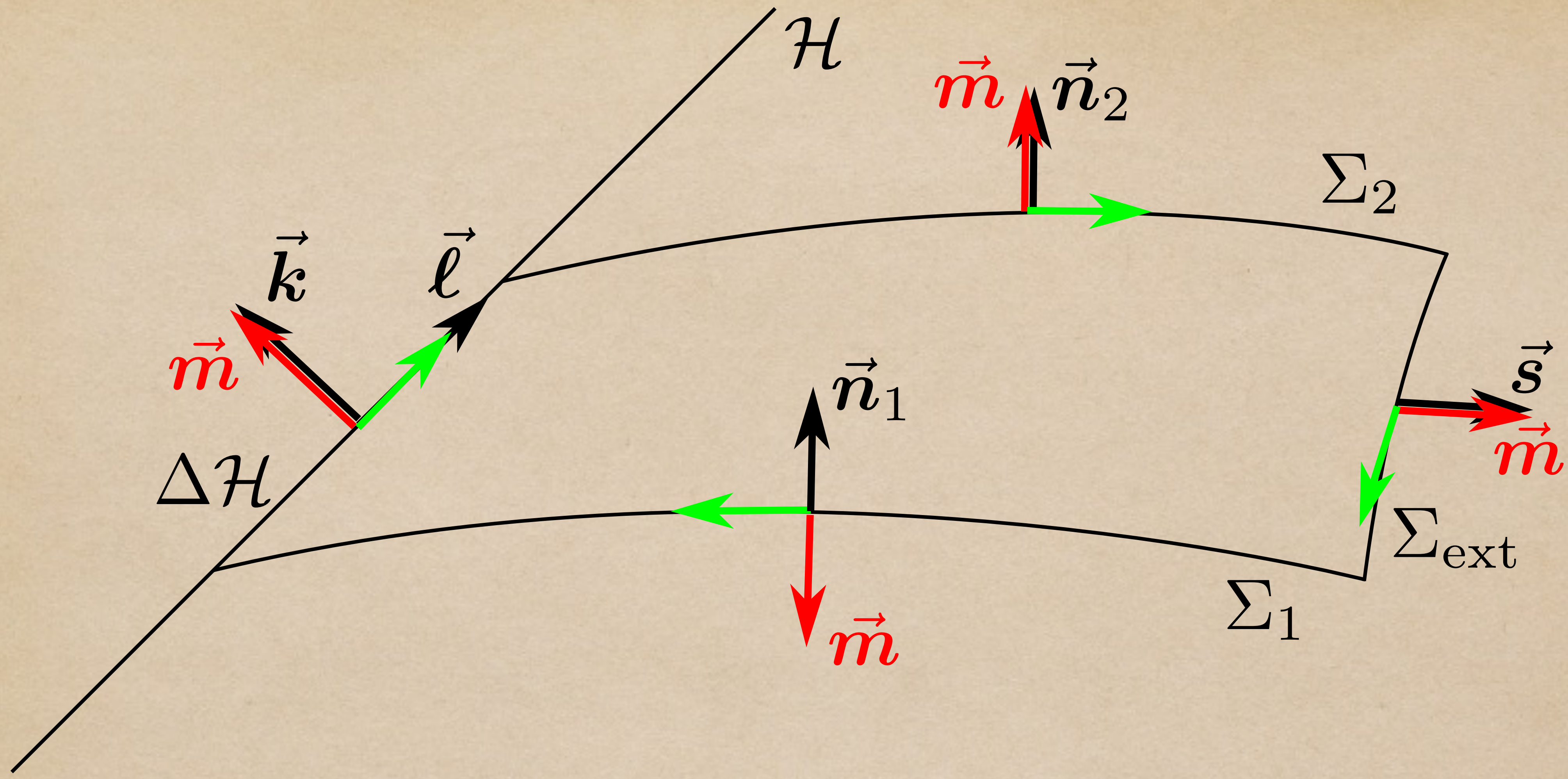
$$P^{\alpha} = -T^{\alpha}_{\ \mu}\eta^{\mu}$$

- Noether current (« energy momentum density vector »)

$$\nabla_{\mu}P^{\mu} = 0$$

by Stoke's theorem

$$\oint_{\mathcal{V}} \epsilon(\vec{\boldsymbol{P}}) = 0,$$



$$\int_{\Sigma_1\downarrow} \epsilon(\vec{P}) + \int_{\underleftarrow{\Delta\mathcal{H}}} \epsilon(\vec{P}) + \int_{\Sigma_2\uparrow} \epsilon(\vec{P}) + \int_{\underrightarrow{\Sigma_{\rm ext}}} \epsilon(\vec{P}) = 0$$

$$E_1 := \int_{\Sigma_1 \uparrow} \epsilon(\vec{\boldsymbol{P}}) = -\int_{\Sigma_1} P_\mu n_1^\mu \, \mathrm{d}V$$

$$E_2 := \int_{\Sigma_2 \uparrow} \epsilon(\vec{\boldsymbol{P}}) = -\int_{\Sigma_2} P_\mu n_2^\mu \, \mathrm{d}V$$

$$\Delta E_{\mathrm{ext}} := \int_{\underrightarrow{\Sigma_{\mathrm{ext}}}} \epsilon(\vec{\boldsymbol{P}}) = \int_{\Sigma_{\mathrm{ext}}} P_\mu s^\mu \, \mathrm{d}V$$

$$\Delta E_H := \int_{\underleftarrow{\Delta\mathcal{H}}} \epsilon(\vec{\boldsymbol{P}}) = -\int_{\Delta\mathcal{H}} P_\mu \ell^\mu \, \mathrm{d}V$$

$$E_2 + \Delta E_{\mathrm{ext}} - E_1 = -\Delta E_H$$

****

$M^\alpha = T^\alpha_\mu \xi^\mu$

- angular-momentum density vector

$$J_2 + J_{\mathrm{ext}} - J_1 = -\Delta J_H$$

Energy « gain »: $\Delta E := E_2 + \Delta E_{\rm ext} - E_1$

can be positive, if and only if $\Delta E_H < 0$

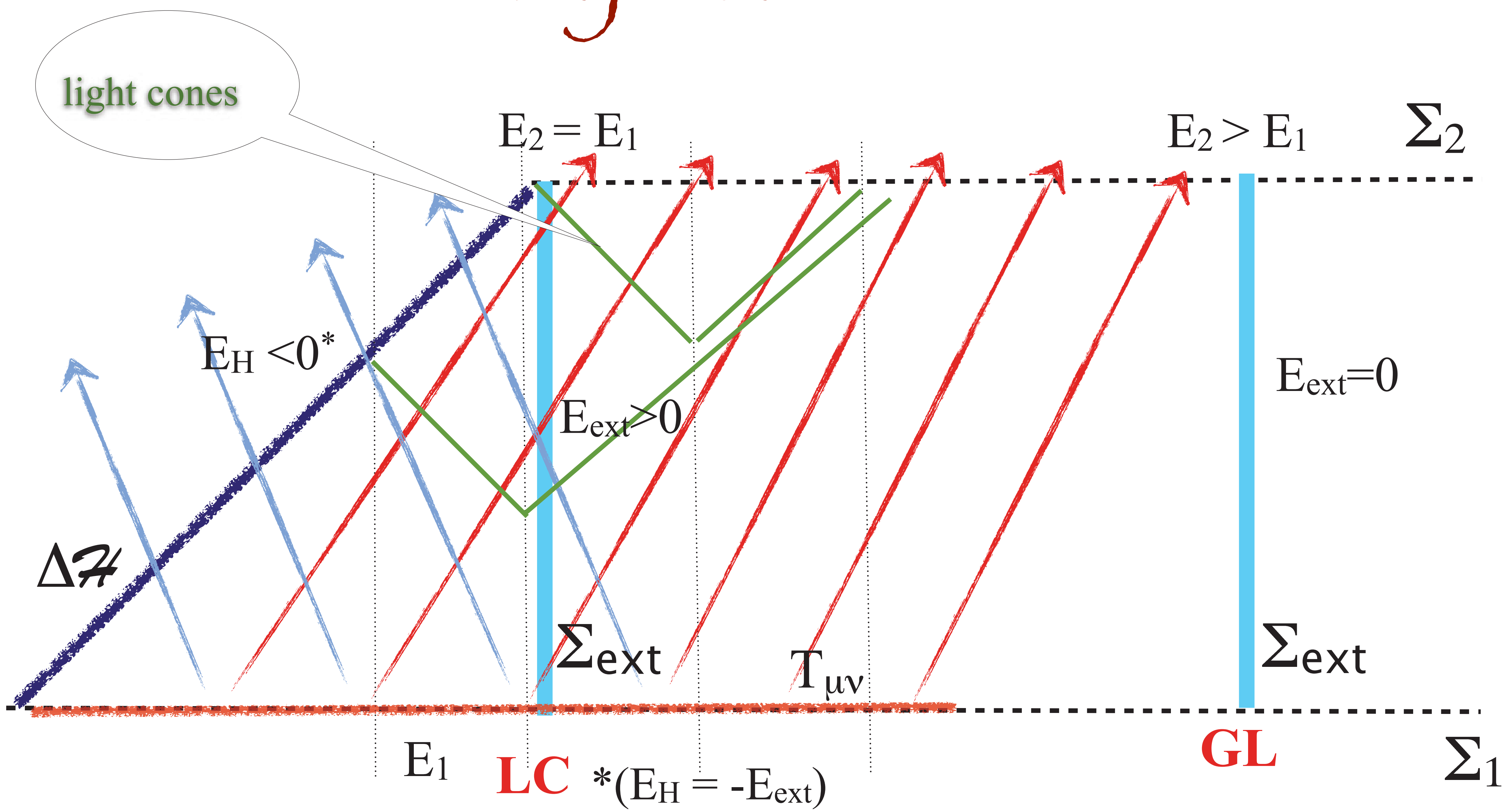
We refer to any such process as a Penrose process.

$$T_{\mu\nu}\ell^\mu \ell^\nu = T_{\mu\nu}(\eta^\nu + \omega_H \xi^\nu)\ell^\mu = -P_\mu \ell^\mu + \omega_H M_\mu \ell^\mu$$

$$-\int_{\Delta\mathcal{H}} P_\mu \ell^\mu \, \mathrm{dV} + \omega_H \int_{\Delta\mathcal{H}} M_\mu \ell^\mu \, \mathrm{dV} \geq 0 \qquad \omega_H \Delta J_H \leq \Delta E_H \qquad \Delta J_H < 0$$

For a matter distribution or a nongravitational field obeying the null energy condition, a necessary and sufficient condition for energy extraction from a rotating black hole is that it absorbs negative energy ΔE_H and negative angular momentum ΔJ_H.

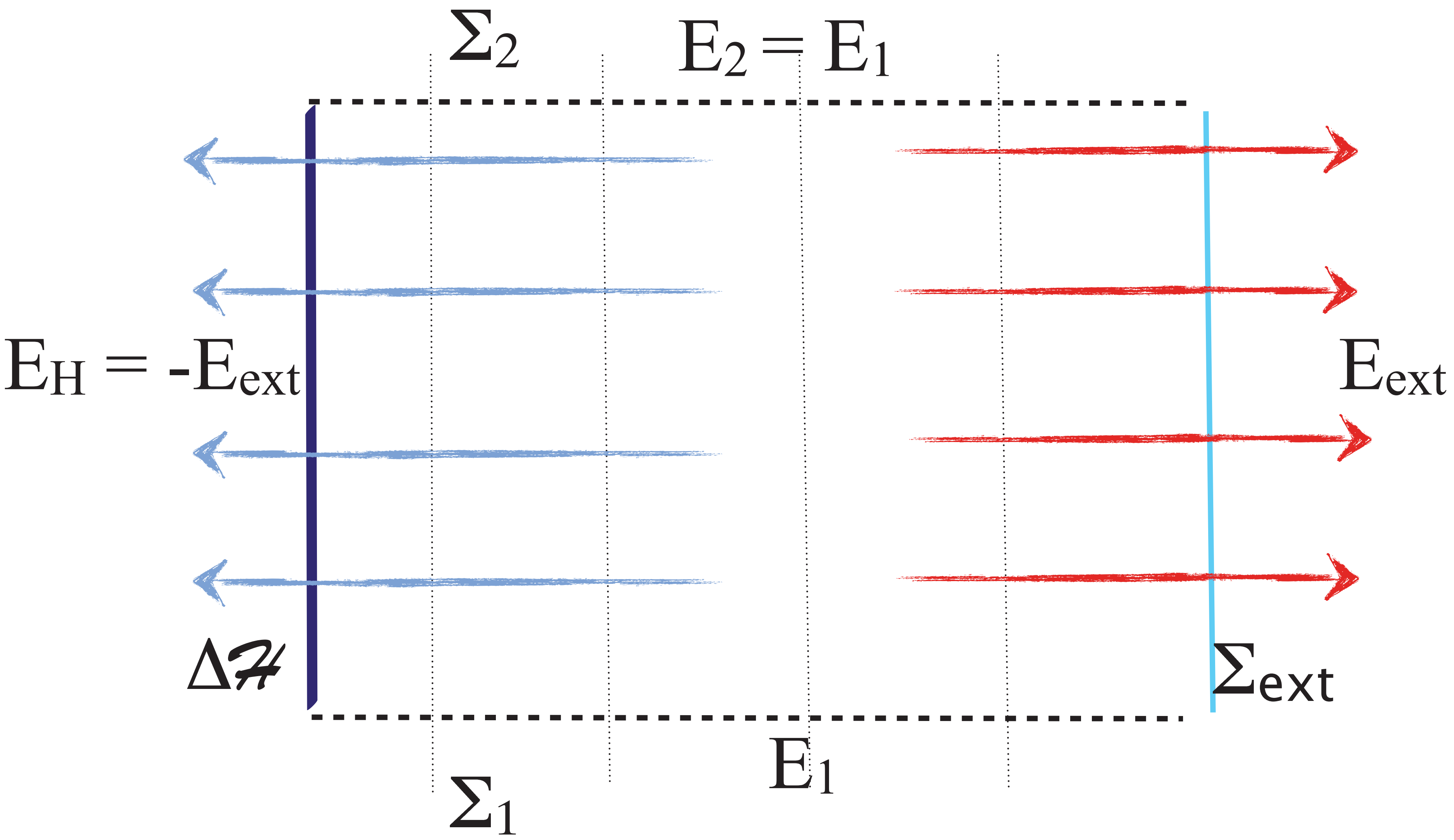
Physical view



LC: Since energy is conserved, from $E_2 = E_1$ and $E_{ext} > 0$ it follows that $E_H = -E_{ext}$

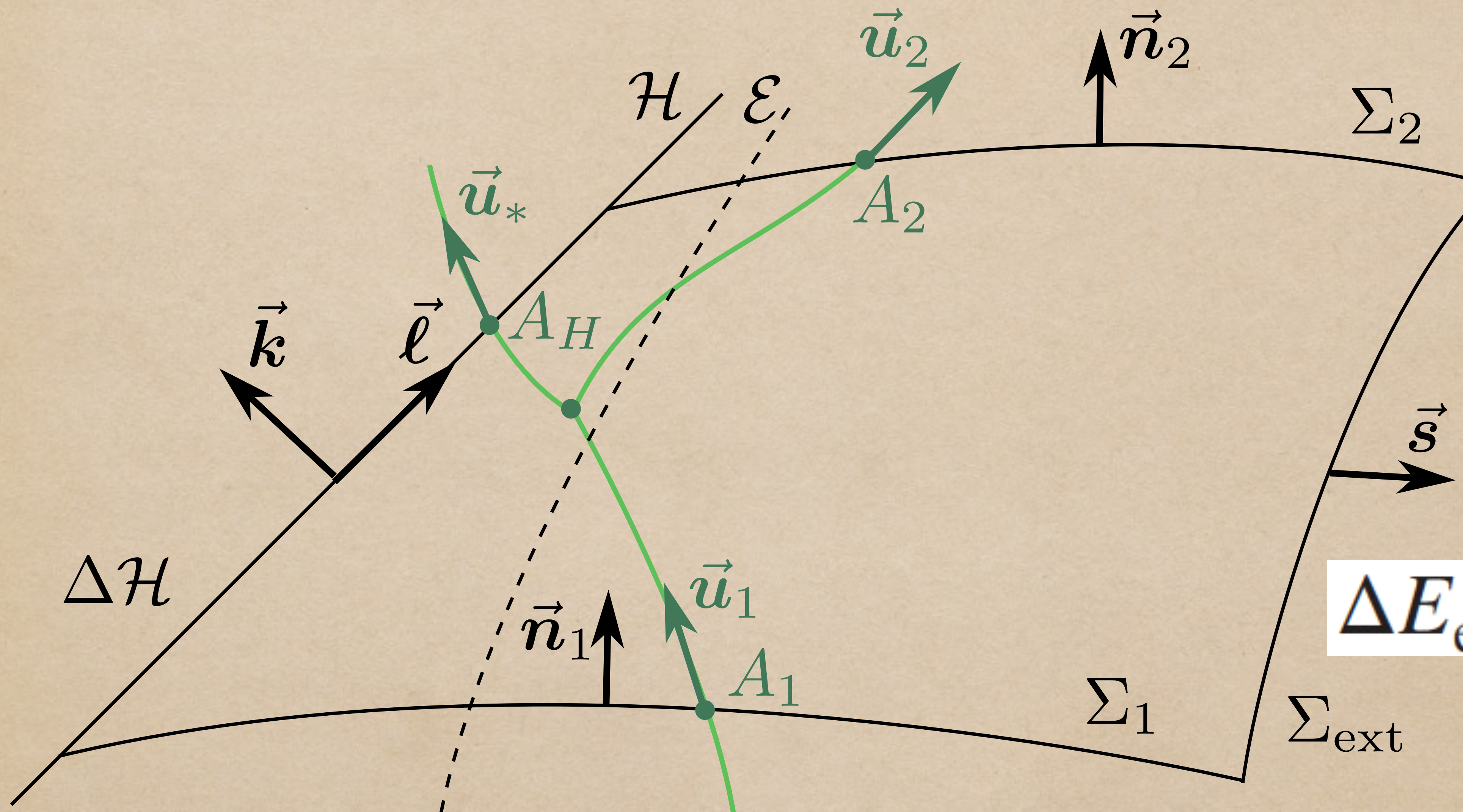
GL: Since energy is conserved, from $E_2 > E_1$ and $E_{ext} = 0$ it follows that $E_H < 0$

Numerical view



Since energy is conserved, from $E_2 = E_1$ and $E_{ext} > 0$ it follows that $E_H = -E_{ext}$

Mechanical Penrose process



$$\Delta E_{\rm ext} = 0$$

$$T_{\alpha\beta}(M) = \mathfrak{m} \int_{-\infty}^{+\infty} \delta_{A(\tau)}(M)\; g_\alpha{}^\mu(M, A(\tau)) u_\mu(\tau) \times g_\beta{}^\nu(M, A(\tau)) u_\nu(\tau)\, \mathrm{d}\tau$$

$$\delta_A(M) = \frac{1}{\sqrt{-g}}\,\delta(x^0 - z^0)\,\delta(x^1 - z^1)\,\delta(x^2 - z^2)\,\delta(x^3 - z^3),$$

$$P_\alpha(M) = \mathfrak{m}\int_{-\infty}^{+\infty} \delta_{A(\tau)}(M)\,\boxed{[-\,g_\sigma{}^\nu(M, A(\tau))u_\nu(\tau)\eta^\sigma(M)]}\,\times g_\alpha{}^\mu(M, A(\tau))u_\mu(\tau)\,\mathrm{d}\tau.$$

$$E_2 = -\mathfrak{m}_2\,\eta_\mu u_2^\mu \qquad \Delta E_H = -\mathfrak{m}_*\,(\eta_\mu u_*^\mu)\big|_{A_H} = -\mathfrak{m}_*\,\eta_\mu u_*^\mu$$

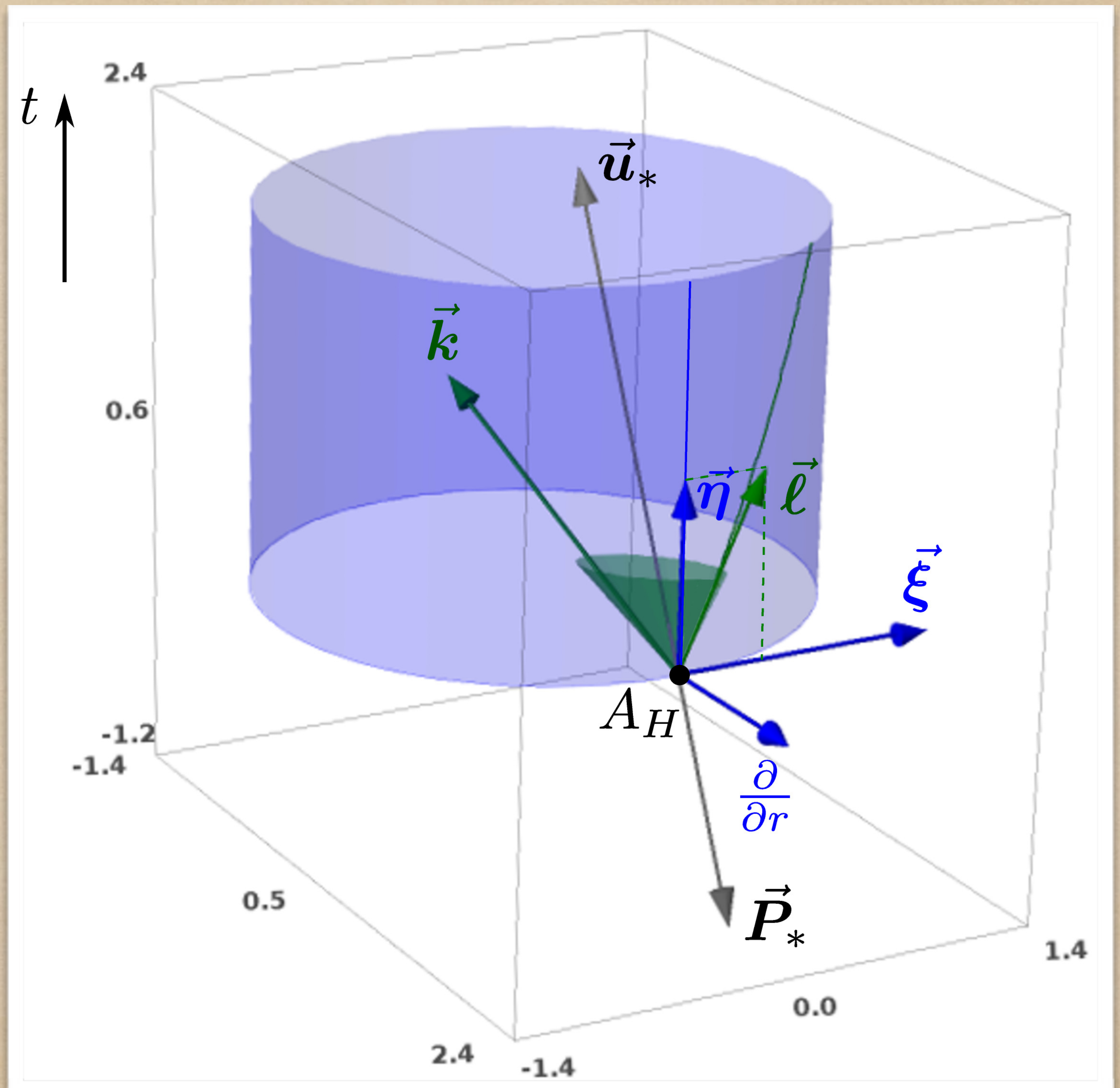
$$\Delta E = E_2 - E_1$$

$\Delta E_H < 0,$ if and only if $\eta_\mu u_*^\mu > 0$

(possible only in the ergosphere)

$\vec{P}_*$ is collinear to $\vec{u}_*$ so it is timelike and past-directed

because [ ] is negative.



t
2.4
0.6
-1.2
-1.4
0.5
2.4
-1.4
0.0
1.4
$\vec{u}_*$
$\vec{k}$
$\vec{\eta}$
$\vec{\ell}$
$\vec{\xi}$
A_H
$\frac{\partial}{\partial r}$
$\vec{P}_*$

General electromagnetic field

$$T_{\alpha\beta} = \frac{1}{\mu_0}\left(F_{\mu\alpha}F^{\mu}{}_{\beta} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ g_{\alpha\beta}\right)$$

Therefore the integrand in $\Delta E_H = -\int_{\Delta\mathcal{H}} P_\mu \ell^\mu \, \mathrm{dV}$ is:

$$\boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \frac{1}{\mu_0}\left(F_{\mu\rho}\eta^\rho F^{\mu}{}_{\sigma}\ell^\sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ \vec{\boldsymbol{\eta}}\cdot\vec{\boldsymbol{\ell}}\right)$$

since $\vec{\boldsymbol{\eta}}\cdot\vec{\boldsymbol{\ell}} = 0$

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = F_{\mu\rho}\eta^\rho F^{\mu}{}_{\sigma}\ell^\sigma$$

- pseudoelectric field 1-form on $\mathscr{H}$

$$\boldsymbol{E} := \boldsymbol{F}(., \vec{\boldsymbol{\ell}})$$

Hence $$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \boldsymbol{F}(\vec{\boldsymbol{E}}, \vec{\boldsymbol{\eta}})$$

or $$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}} - \omega_H \boldsymbol{F}(\vec{\boldsymbol{E}}, \vec{\boldsymbol{\xi}})$$

therefore $$\Delta E_H < 0,$$

if $$\omega_H \boldsymbol{F}(\vec{\boldsymbol{E}}, \vec{\boldsymbol{\xi}}) > \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}} \text{ in some part of } \Delta\mathcal{H}.$$

This is the most general condition on <u>any electromagnetic field</u> configuration allowing black-hole energy extraction through a Penrose process

(Since $\vec{\boldsymbol{E}}$ is tangent to $\mathscr{H}$ $\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}} \geq 0$)

- <u>Stationary and axisymmetric electromagnetic field</u>

$$\mathcal{L}_{\vec{\eta}} \boldsymbol{F} = 0 \qquad \text{and} \qquad \mathcal{L}_{\vec{\xi}} \boldsymbol{F} = 0.$$

therefore

$$\boldsymbol{F}(.,\vec{\eta}) = \mathbf{d}\Phi$$
$$\boldsymbol{F}(.,\vec{\xi}) = \mathbf{d}\Psi$$
$${}^{\star}\boldsymbol{F}(\vec{\eta},\vec{\xi}) = I,$$

Φ, Ψ and I are gauge-invariant. Introducing a 1-form A such that F=dA one can choose A so that

$$\Phi = \langle \boldsymbol{A}, \vec{\eta} \rangle = A_t \qquad \Psi = \langle \boldsymbol{A}, \vec{\xi} \rangle = A_\varphi.$$

$$\mathcal{L}_{\vec{\eta}} \Phi = \mathcal{L}_{\vec{\xi}} \Phi = 0 \qquad \mathcal{L}_{\vec{\eta}} \Psi = \mathcal{L}_{\vec{\xi}} \Psi = 0$$

and $\boldsymbol{E} = \mathbf{d}(\Phi + \omega_H \Psi)$ is a pure gradient.

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{\nabla}} \Phi \qquad \mu_0 \boldsymbol{T}(\vec{\boldsymbol{\xi}}, \vec{\boldsymbol{\ell}}) = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{\nabla}} \Psi$$

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \vec{\boldsymbol{\nabla}} \Phi \cdot \vec{\boldsymbol{\nabla}} (\Phi + \omega_H \Psi)$$

Force free case (Blandford-Znajek)

$$\boldsymbol{F}(\vec{\boldsymbol{j}}, .) = 0$$

$\vec{\boldsymbol{j}}$ - electric 4-current. From stationarity

$$\vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{\nabla}} \Phi = 0 \quad \text{and} \quad \vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{\nabla}} \Psi = 0$$ so

there exists a function $\omega(\Psi)$ such that

$$\mathbf{d}\Phi = -\omega(\Psi) \mathbf{d}\Psi$$

One gets

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \omega(\Psi)\,(\omega(\Psi) - \omega_H)\; \vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi.$$

$$\vec{\boldsymbol{\ell}} \cdot \vec{\boldsymbol{\nabla}}\Psi = \vec{\boldsymbol{\eta}} \cdot \vec{\boldsymbol{\nabla}}\Psi + \omega_H \vec{\boldsymbol{\xi}} \cdot \vec{\boldsymbol{\nabla}}\Psi = \underbrace{\mathcal{L}_{\vec{\boldsymbol{\eta}}}\Psi}_{0} + \omega_H \underbrace{\mathcal{L}_{\vec{\boldsymbol{\xi}}}\Psi}_{0} = 0$$

therefore on $\mathscr{H}$

$$\vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi \geq 0$$ and

$$\boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) < 0 \iff \begin{cases} 0 < \omega(\Psi) < \omega_H \\ \vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi \neq 0 \end{cases}$$

(Blandford & Znajek 1977)

Blandford-Znajek = Penrose

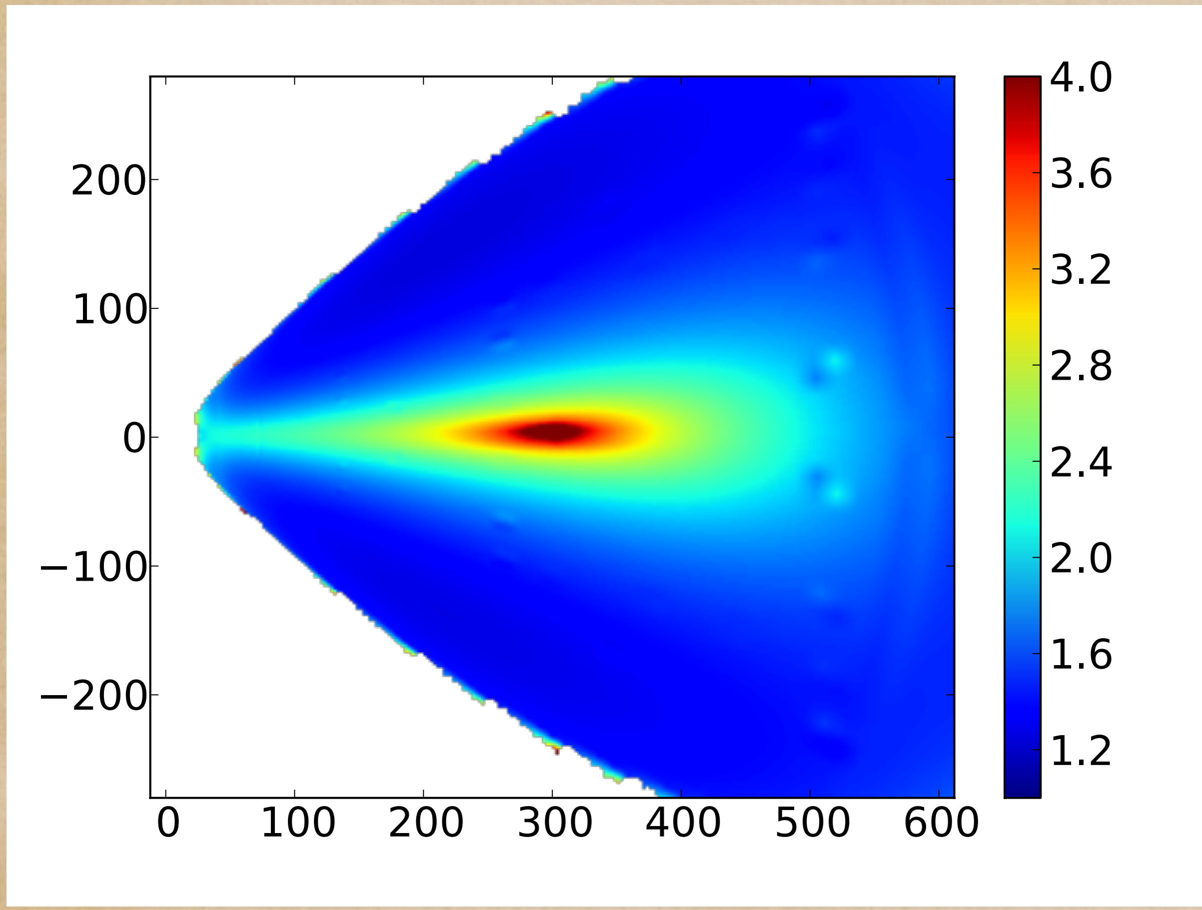
For a stationary and axisymmetric force-free electromagnetic field, a necessary condition for the Penrose process to occur is

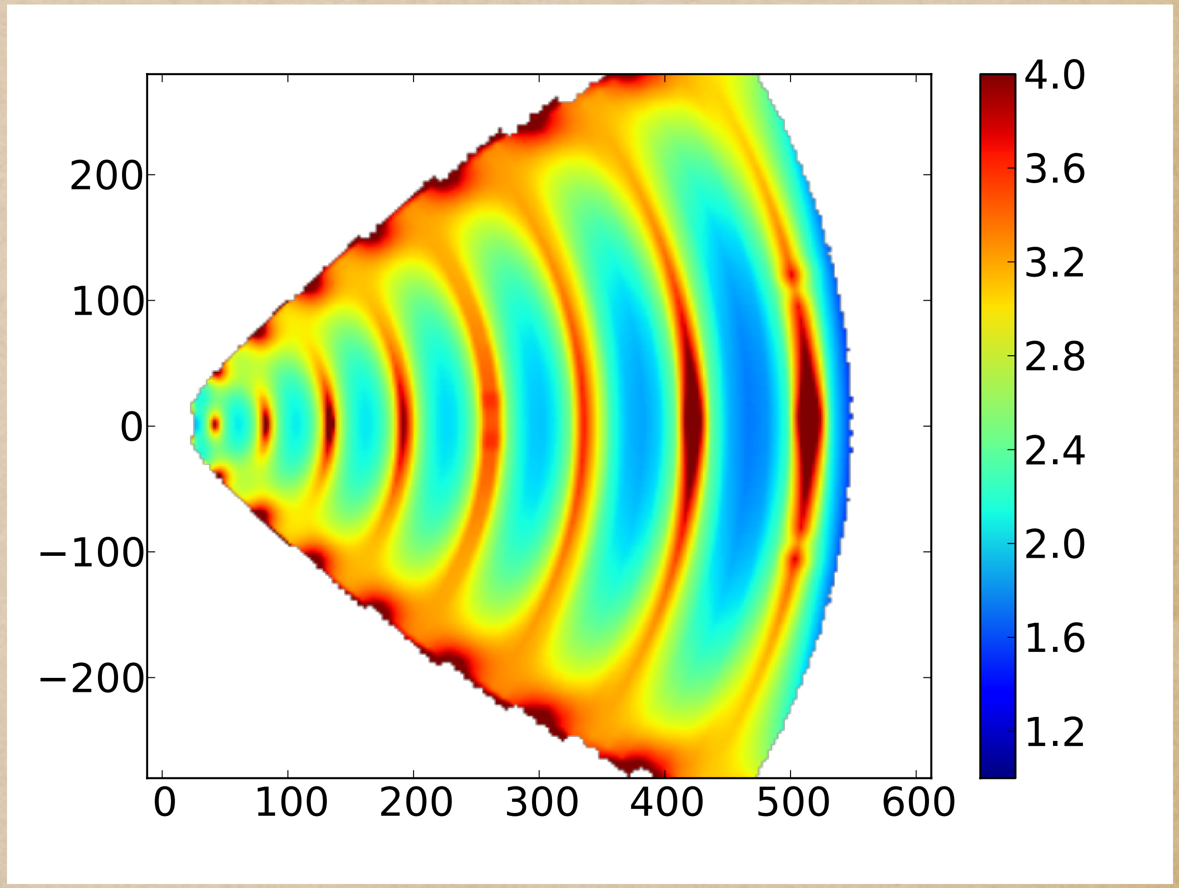
$$0 < \omega(\Psi) < \omega_H \quad \text{in some part of } \Delta\mathcal{H}.$$

General Relativistic MagnetoHydroDynamics (GRMHD)

$$\boldsymbol{F}\left(\vec{\boldsymbol{u}},.\right)=0$$

GRMHD *HARM* (Gammie, McKinney, Tóth 2003)





MAD
(Magnetically Arrested discs)

SANE
(Standard And Normal Evolution)

(McKinney, Tchekhovskoy, Narayan, Blandford)

Blandford-Znajek efficiency

$$\eta_{\rm BZ} = \frac{[P_{\rm BZ}]_t}{\left[\dot{M}\right]_t c^2} = \frac{\kappa}{4\pi c}\left[\phi_{BH}^2\right]_t \left(\frac{\omega_H r_g}{c}\right)^2 f(\omega_H)$$

$\dot{M}$ is the accretion rate $[\ldots]_t$ - time average, $r_g = Gm/c^2$

$\varphi_{BH}^2 = \Phi_{BH}^2/\dot{M} r_g^2 c$, -normalized magnetic flux

$f(\omega_H) \approx 0.77$ for $a_* = 1$

Magnetic flux can be accumulated only if the disc is not thin, h/r ~ 1. Here discs are slim, h/r ~ 0.3.

Energy-momentum tensor

$$T_{\mu\nu} = T_{\mu\nu}^{(\mathrm{MA})} + T_{\mu\nu}^{(\mathrm{EM})}$$

Flux densities:

$$\dot{e}_{\mathrm{MA}} := T^{(\mathrm{MA})r}{}_{t} \qquad \dot{j}_{\mathrm{MA}} := -T^{(\mathrm{MA})r}{}_{\varphi} \quad \text{etc.}$$

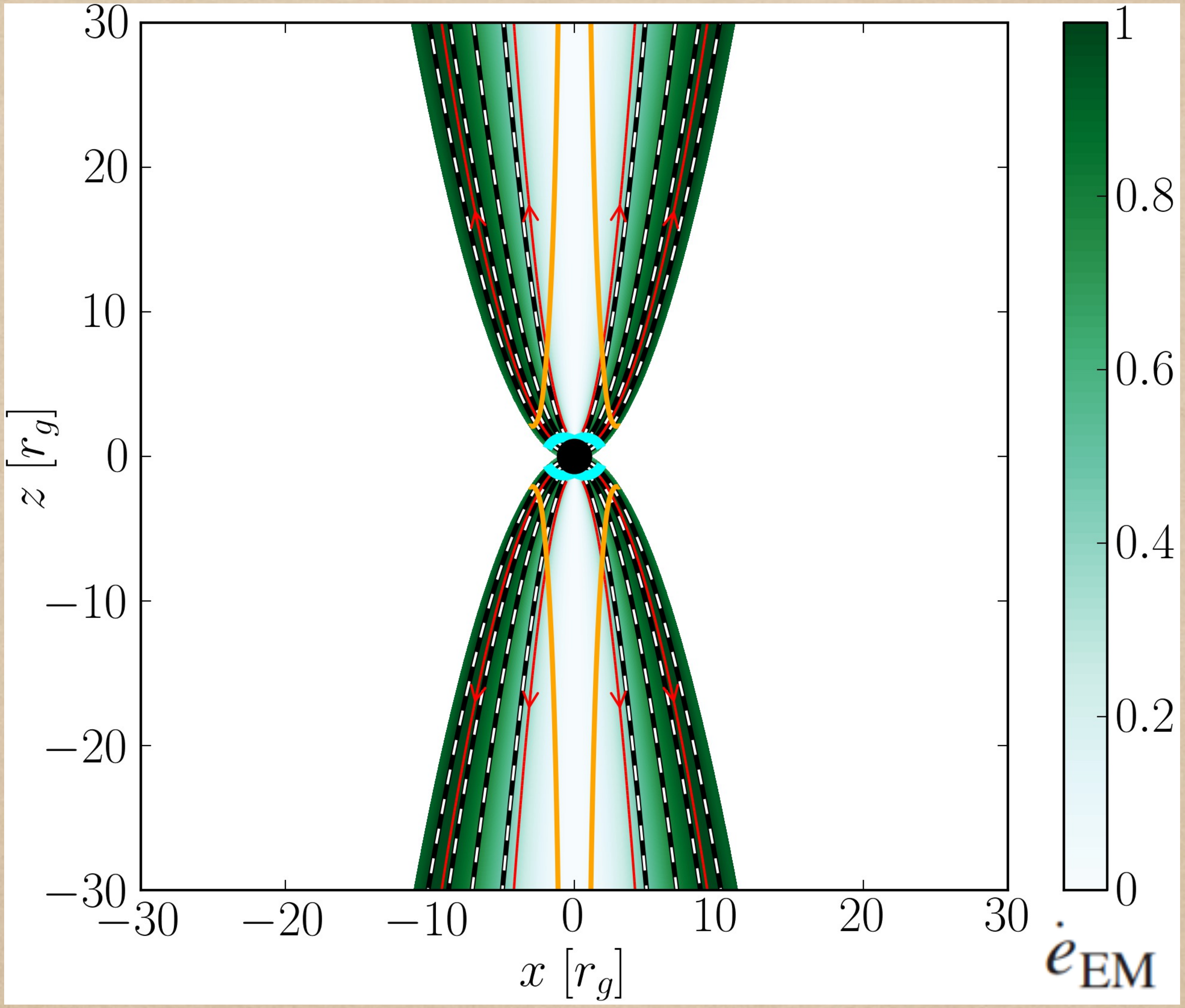
At horizon

$$\Delta E_H = \int_{\Delta\mathcal{H}} \dot{e}_H (r_H^2 + a^2 \cos^2\theta) \sin\theta \mathrm{d}t \mathrm{d}\theta \mathrm{d}\varphi$$

$$\dot{e}_H := -P^r|_{\mathcal{H}} = T^r{}_t|_{\mathcal{H}}$$

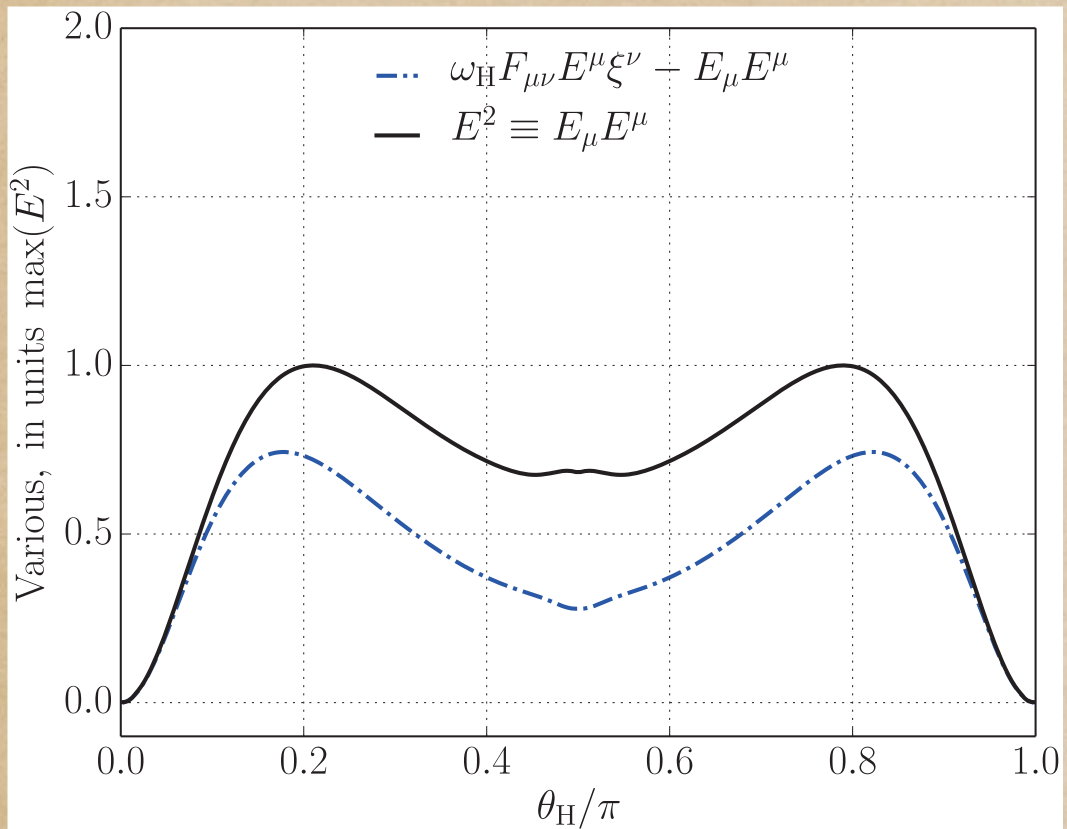
$$\dot{j}_H := -M^r|_{\mathcal{H}} = -T^r{}_{\varphi}|_{\mathcal{H}}$$

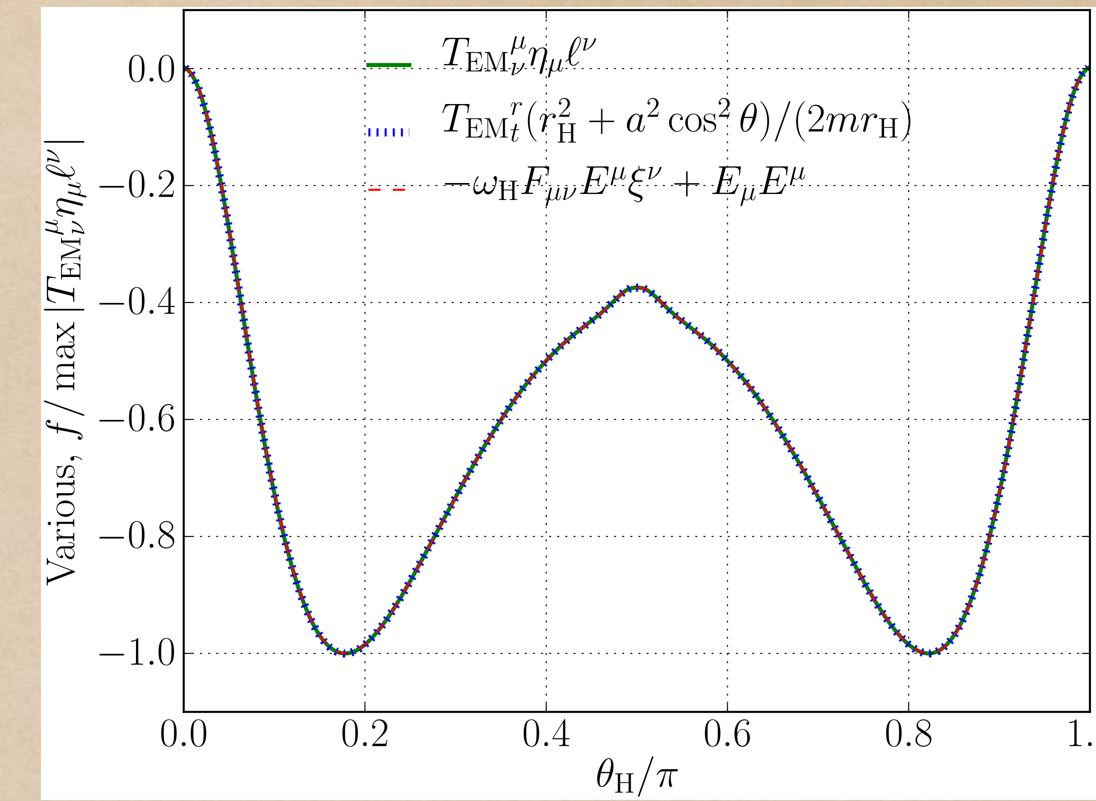
Force-free

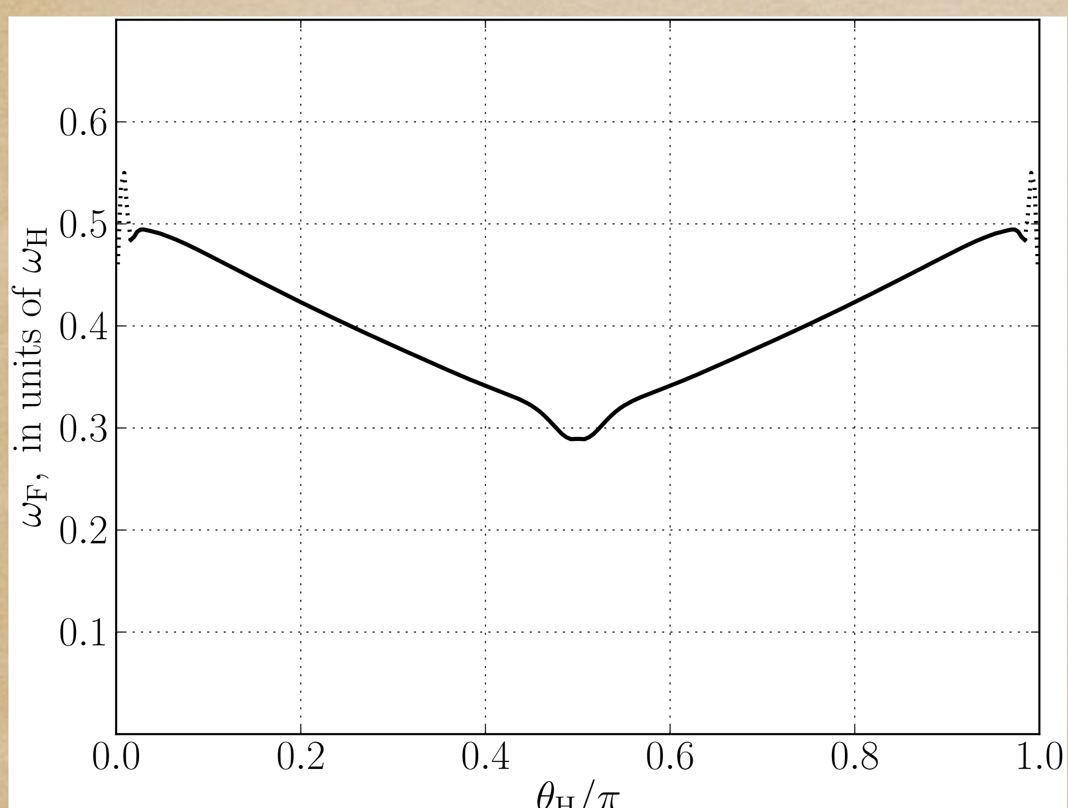


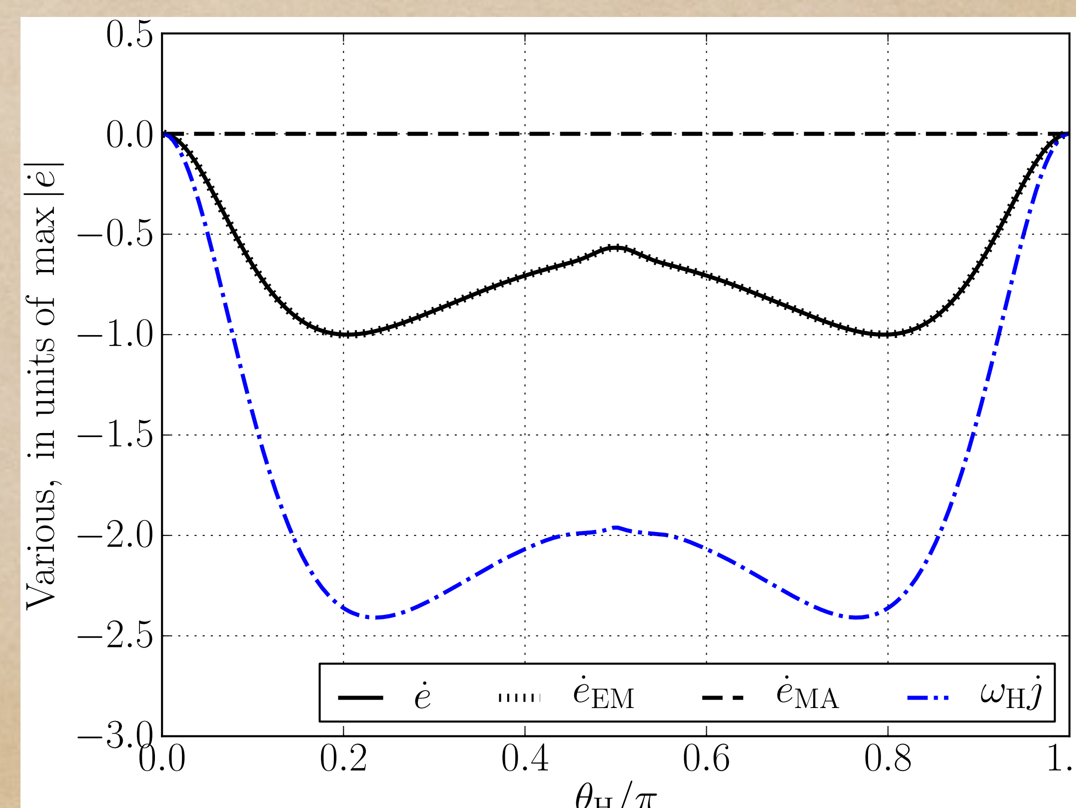
30
20
10
0
−10
−20
−30
$z\ [r_g]$
−30
−20
−10
0
10
20
30
$x\ [r_g]$
1
0.8
0.6
0.4
0.2
0
$\dot{e}_{\mathrm{EM}}$

Force-free at horizon

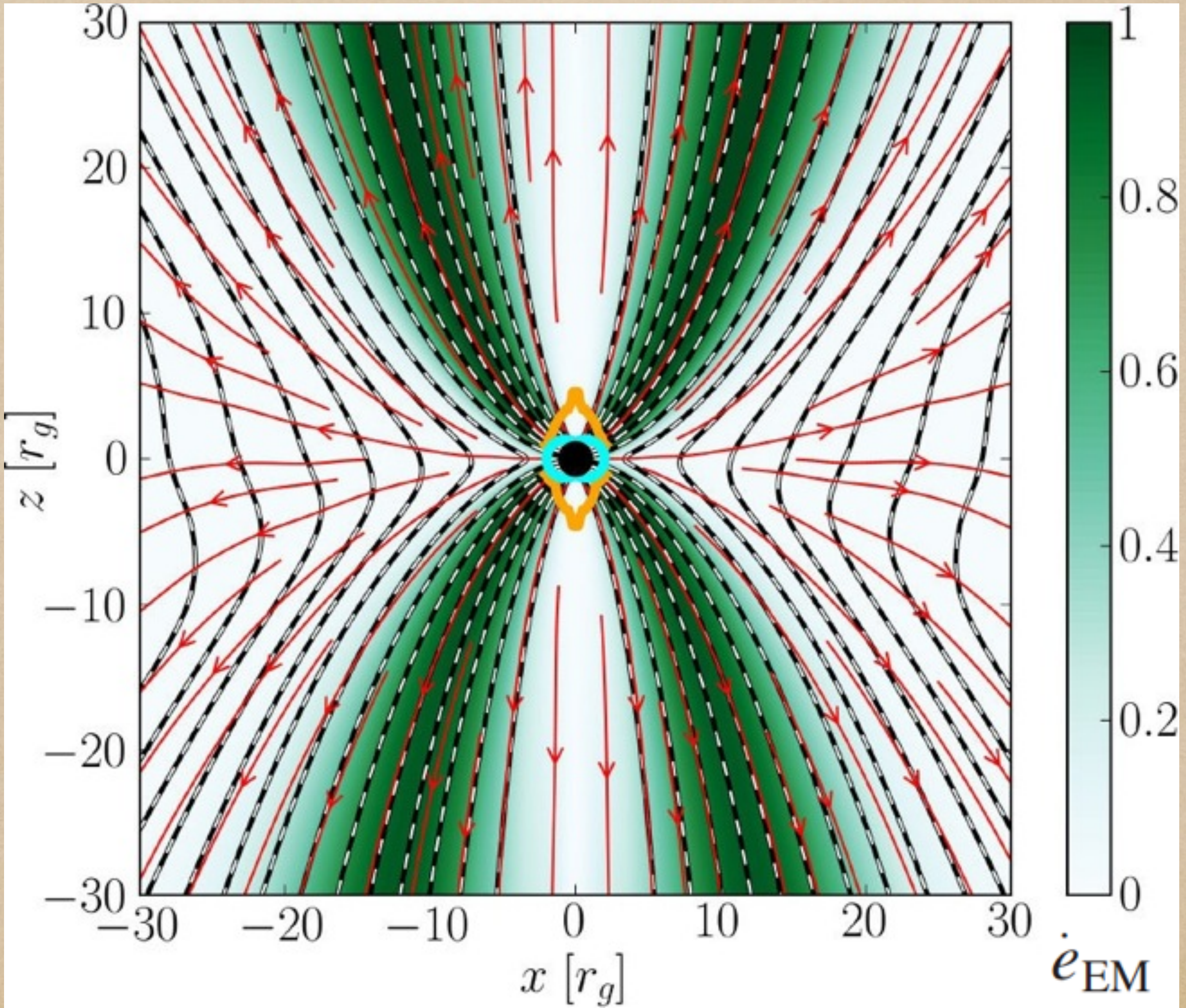






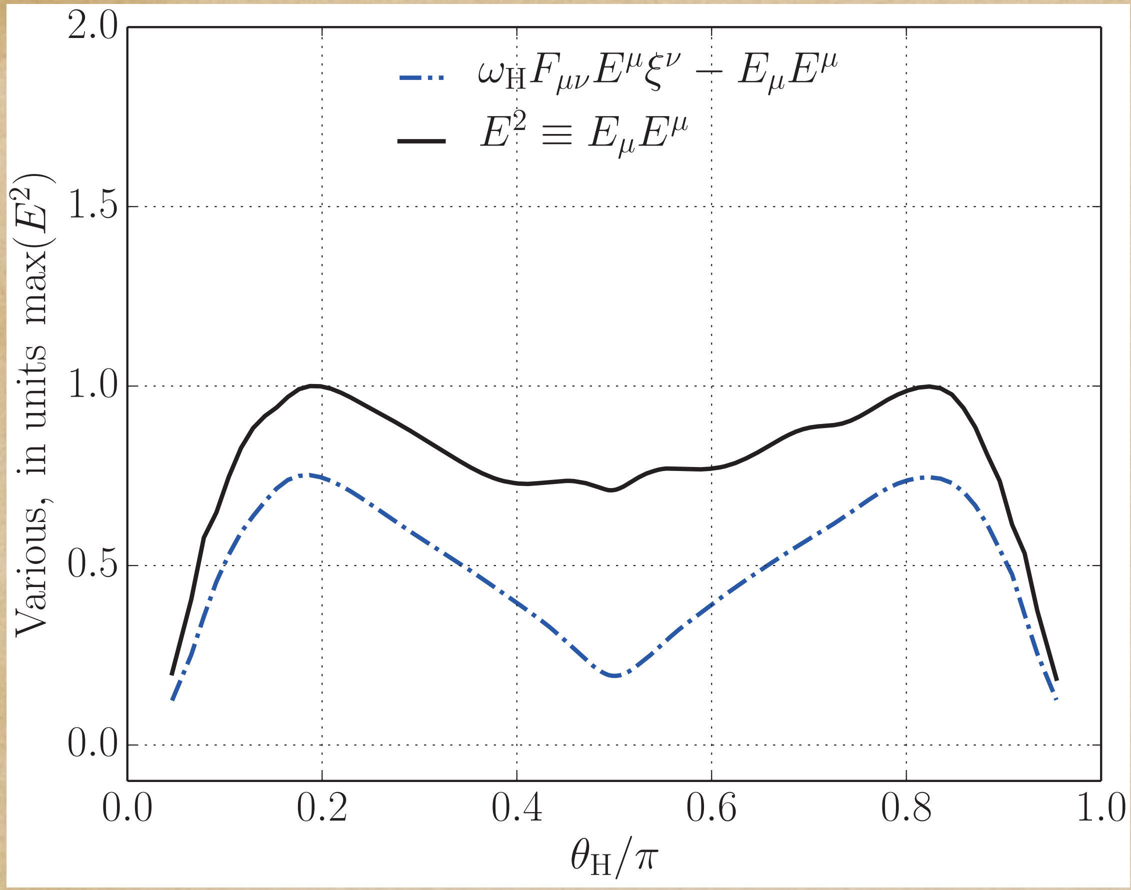


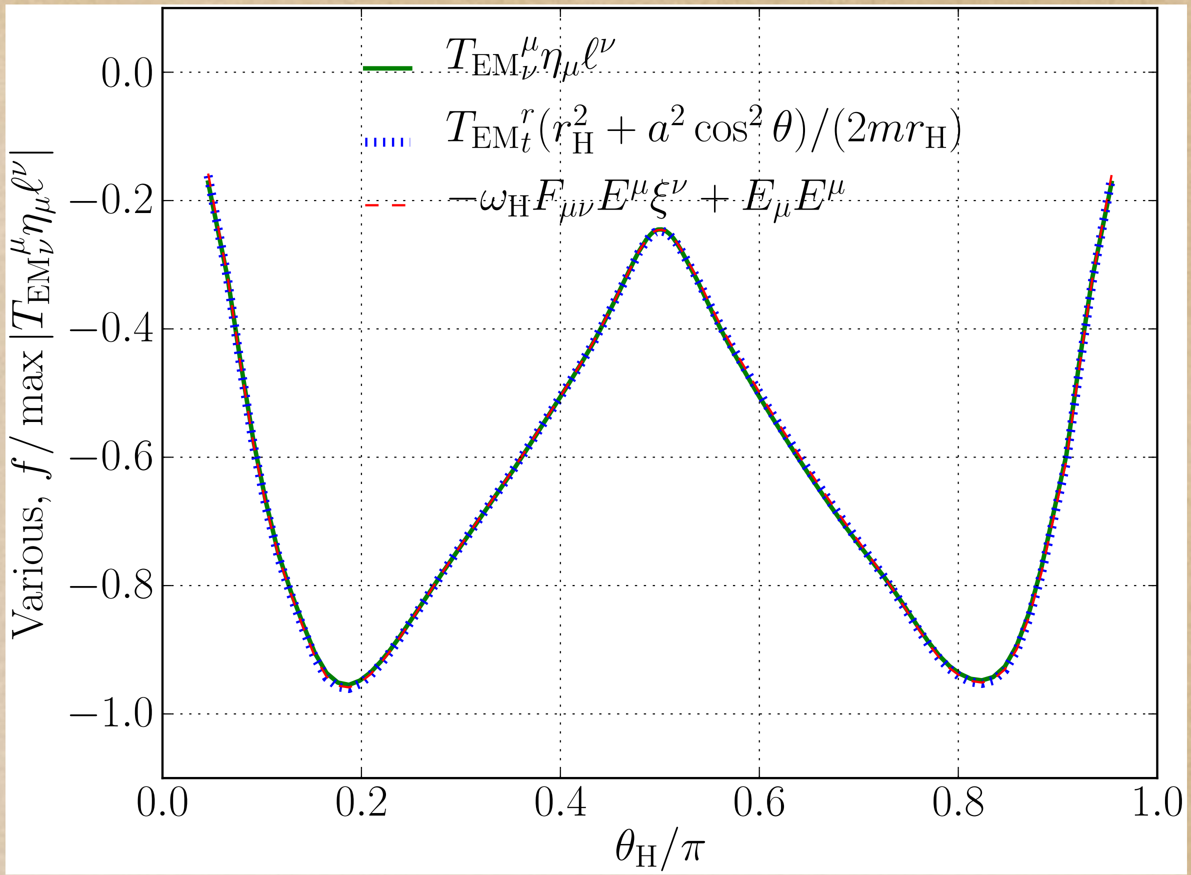
MAD

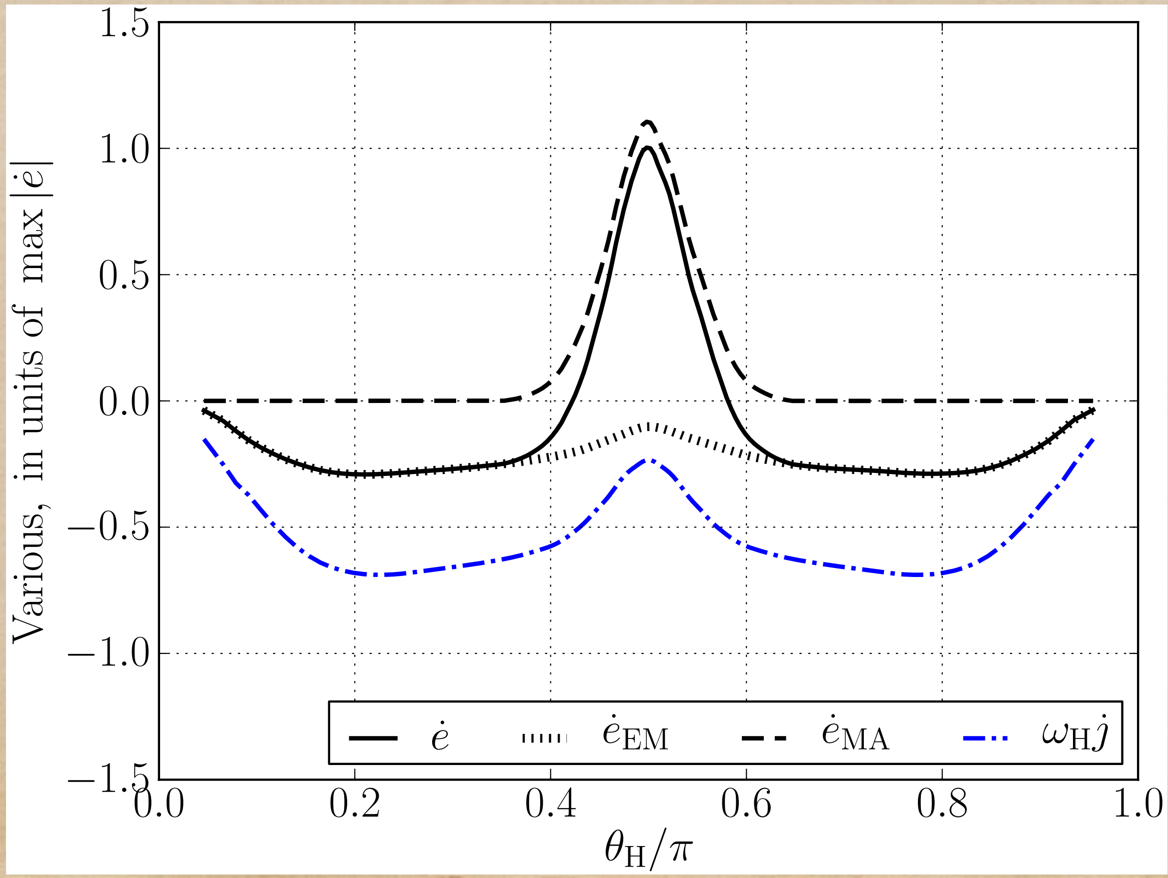


30
20
10
0
−10
−20
−30
$z\ [r_g]$
−30
−20
−10
0
10
20
30
$x\ [r_g]$
1
0.8
0.6
0.4
0.2
0
$\dot{e}_{\rm EM}$

MAD at horizon







Noether current in GRMHD

MHD: $u_\mu F^{\mu\nu} = 0$

Magnetic field vector $b^\mu := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$

$b_\mu u^\mu = 0,$

Hence the energy-momentum tensor

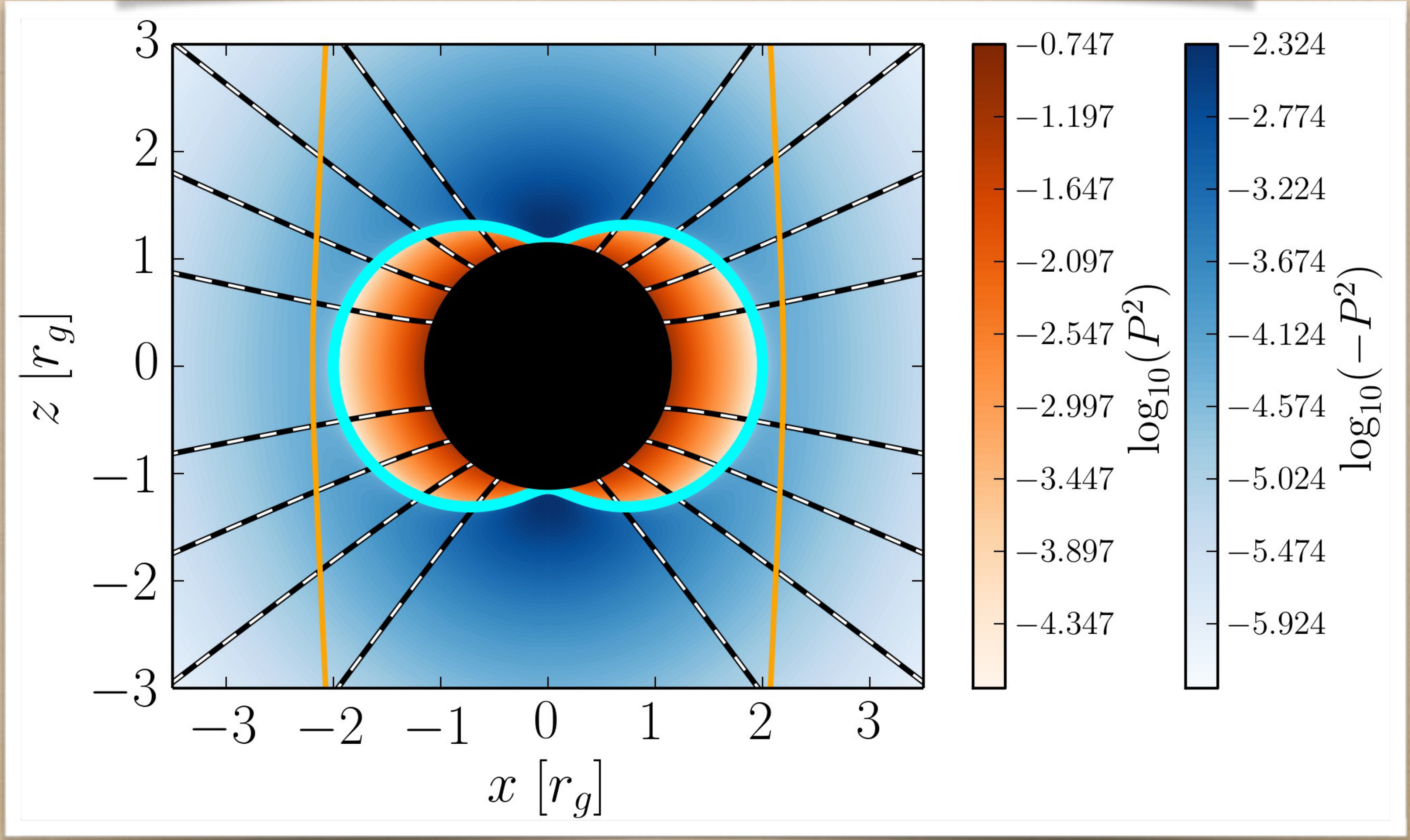
$$T^{(\mathrm{EM})}_{\mu\nu} = b^2 u_\mu u_\nu + \frac{1}{2} b^2 g_{\mu\nu} - b_\mu b_\nu$$

Noether current $P^{(\mathrm{EM})}_\mu = T^{(\mathrm{EM})}_{\mu\nu}\eta^\nu$

$P^\mu_{(\mathrm{EM})} P^{(\mathrm{EM})}_\mu = P^2_{(\mathrm{EM})} = \frac{1}{4} b^4 g_{tt}$ >0 in the ergosphere

Noether current: force-free

$$P^2_{(\mathrm{EM})} > 0 \quad \text{inside ergosphere,}$$
$$P^2_{(\mathrm{EM})} < 0 \quad \text{outside ergosphere}$$

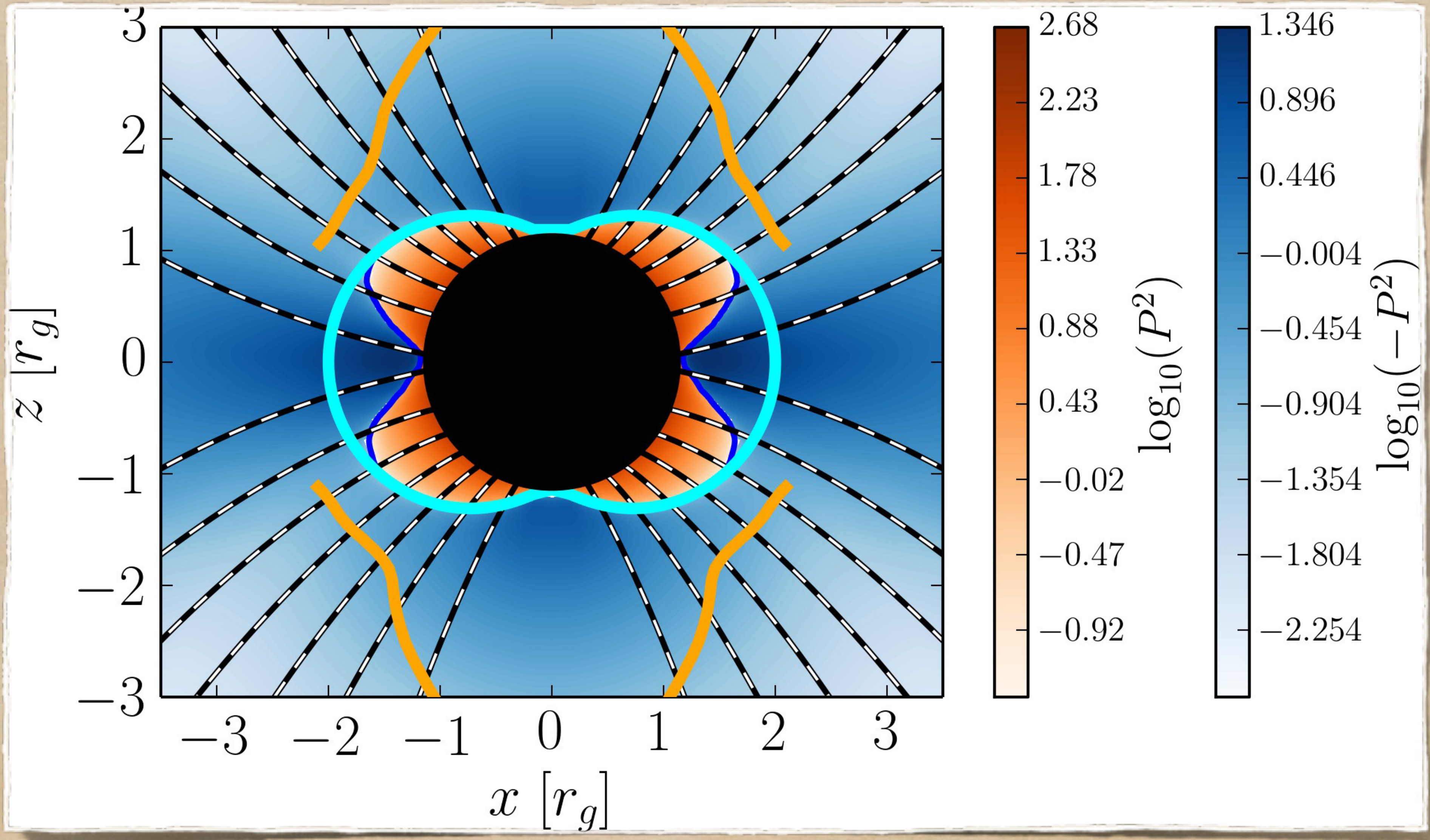


Noether current: MAD

$$T_{\mu\nu} = T_{\mu\nu}^{(\mathrm{MA})} + T_{\mu\nu}^{(\mathrm{EM})}$$

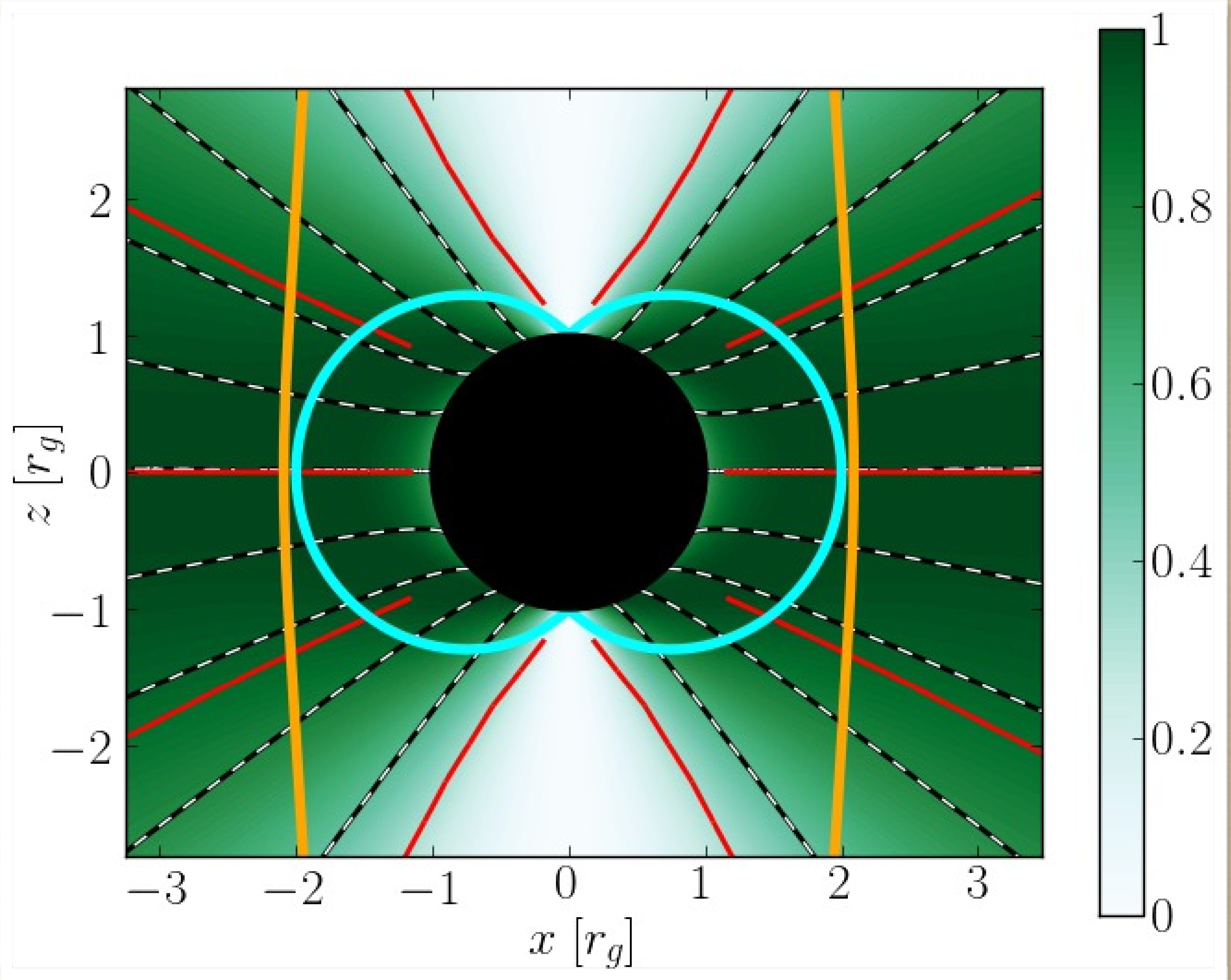
$$P^2 = \left(\frac{1}{2}b^2 + p\right)^2 g_{tt} - A,$$

$$A = 2(\Gamma-1)ub_t^2 + u_t^2(\rho+u+p+b^2)[(2-\Gamma)u+\rho],$$



Conclusions

- The Blandford-Znajek mechanism is rigorously a Penrose process.
- GRMHD simulations of Magnetically Arrested Discs correctly (from the point of view of general relativity) describe extraction of black-hole rotational energy through a Penrose process.



z [r_g]
x [r_g]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
1
0.8
0.6
0.4
0.2
0