Extracting black-hole rotational energy: the generalized Penrose process

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Relativistic jets in Active Galactic Nuclei

3C 279
Superluminal Motion

1992.0
1993.0
1994.0
1995.0
5 milliarcseconds
Relativistic jets in compact binaries (microquasars)
Common source of energy?

Microquasar

Quasar

Collapsar

Microblazar

Blazar

Gamma ray burst
Tapping black-hole rotational energy by unipolar induction

Controversy:
• is the BH surface an analogue of a Faraday disc (causality)
• is the Blandford-Znajek mechanism efficient (rotation of black-hole or disc)?
Recent (2011-2013) GRMHD simulations clearly showed BH rotational energy extraction in a particular (MAD) magnetic field configuration.

Tchekhovskoy, McKinney, Blandford 2012
MAD simulation

Tchekhovskoy, McKinney, Narayan 2011
BH Jet in MAD state has a large efficiency:

\[ \eta = \frac{P_{\text{jet}}}{\dot{M}c^2} > 100\% \]
Sądowski et al. (2013)
Penrose process

\[ \vec{p}_1 = \vec{p}_2 + \vec{p}_* \]

\[ E_1 = E_2 + \Delta E_H \]

\[ E_2 = -\vec{\eta} \cdot \vec{p}_2 \]

\[ E_1 = -\vec{\eta} \cdot \vec{p}_1 \]

\[ \Delta E_H = -\vec{\eta} \cdot \vec{p}_* \]

\( \vec{\eta} \) - timelike (at \( \infty \)) stationarity Killing vector

For \( \Delta E_H < 0 \) \( E_2 > E_1 \)
- timelike (at $\infty$) stationarity Killing vector
- spacelike axisymmetry Killing vector

$\vec{u} = q \left( \eta + \omega \xi \right)$

-ZAMO, $\omega = \frac{\eta \cdot \xi}{\xi \cdot \xi}$

Energy measured by ZAMOs always non-negative:

$$- \left( \eta + \omega \xi \right) \vec{p}_* = (\Delta E_H - \omega_H \Delta J_H) \geq 0$$

Hence for $\Delta E_H < 0$

Since $\omega_H \geq 0$

$\omega_H \neq 0$ and $\Delta J_H < 0$. 
\[ \vec{\ell} = \vec{\eta} + \omega_H \vec{\xi}, \]
\[ \vec{\ell} \cdot \vec{\ell} = 0 \]
\[ \omega_H = a/[2m r_H], \]
\[ r_H = m + \sqrt{m^2 - a^2} \]
$T$ - energy moment tensor

$T_{\mu \nu} \ell^\mu \ell^\nu |_{\mathcal{H}} \geq 0.$ - null energy condition

- **Energy conservation**
  - Noether current ("energy momentum density vector")

$P^\alpha = -T^\alpha_{\mu} \eta^\mu$

$\nabla_\mu P^\mu = 0$

by Stoke's theorem

$\int_\gamma \epsilon(\vec{P}) = 0.$
\[
\int_{\Sigma_1 \downarrow} \varepsilon(\vec{P}) + \int_{\Delta \mathcal{H}} \varepsilon(\vec{P}) + \int_{\Sigma_2 \uparrow} \varepsilon(\vec{P}) + \int_{\Sigma_{\text{ext}}} \varepsilon(\vec{P}) = 0
\]
\[ E_1 := \int_{\Sigma_1} \epsilon(\vec{P}) = -\int_{\Sigma_1} P_{\mu} n_1^{\mu} \, dV \]
\[ E_2 := \int_{\Sigma_2} \epsilon(\vec{P}) = -\int_{\Sigma_2} P_{\mu} n_2^{\mu} \, dV \]
\[ \Delta E_{\text{ext}} := \int_{\Sigma_{\text{ext}}} \epsilon(\vec{P}) = \int_{\Sigma_{\text{ext}}} P_{\mu} s^{\mu} \, dV \]
\[ \Delta E_H := \int_{\Delta \mathcal{H}} \epsilon(\vec{P}) = -\int_{\Delta \mathcal{H}} P_{\mu} \ell^{\mu} \, dV \]

\[ E_2 + \Delta E_{\text{ext}} - E_1 = -\Delta E_H \]

\[ M^\alpha = T^\alpha_{\mu} \xi^{\mu} \]

\[ J_2 + J_{\text{ext}} - J_1 = -\Delta J_H \]

- angular-momentum density vector
For a matter distribution or a nongravitational field obeying the null energy condition, a necessary and sufficient condition for energy extraction from a rotating black hole is that it absorbs negative energy $\Delta E_H$ and negative angular momentum $\Delta J_H$.

Energy «gain»:

$$\Delta E := E_2 + \Delta E_{\text{ext}} - E_1$$

$\Delta E_H < 0$

We refer to any such process as a Penrose process.

$$T_{\mu \nu} \ell^\mu \ell^\nu = T_{\mu \nu} (\eta^\nu + \omega_H \xi^\nu) \ell^\mu = -P_{\mu} \ell^{\mu} + \omega_H M_{\mu} \ell^{\mu}$$

$$- \int_{\Delta \mathcal{H}} P_{\mu} \ell^{\mu} \, dV + \omega_H \int_{\Delta \mathcal{H}} M_{\mu} \ell^{\mu} \, dV \geq 0$$

$$\omega_H \Delta J_H \leq \Delta E_H$$

$$\Delta J_H < 0$$

For a matter distribution or a nongravitational field obeying the null energy condition, a necessary and sufficient condition for energy extraction from a rotating black hole is that it absorbs negative energy $\Delta E_H$ and negative angular momentum $\Delta J_H$. 
**Physical view**

**LC**: Since energy is conserved, from $E_2 = E_1$ and $E_{\text{ext}} > 0$ it follows that $E_H = -E_{\text{ext}}$

**GL**: Since energy is conserved, from $E_2 > E_1$ and $E_{\text{ext}} = 0$ it follows that $E_H < 0$
Since energy is conserved, from $E_2 = E_1$ and $E_{\text{ext}} > 0$ it follows that $E_H = -E_{\text{ext}}$
Mechanical Penrose process

\[ T_{\alpha\beta}(M) = m \int_{-\infty}^{+\infty} \delta_{A(\tau)}(M) \ g_\alpha^\mu(M, A(\tau)) u_\mu(\tau) \times g_\beta^\nu(M, A(\tau)) u_\nu(\tau) \ d\tau \]
\[ \delta_A(M) = \frac{1}{\sqrt{-g}} \delta(x^0 - z^0) \delta(x^1 - z^1) \delta(x^2 - z^2) \delta(x^3 - z^3), \]

\[ P_\alpha(M) = m \int_{-\infty}^{+\infty} \delta_A(\tau)(M) \left[ - g^\nu_\sigma(M, A(\tau)) u_\nu(\tau) \eta^\sigma(M) \right] \times g^\mu_\alpha(M, A(\tau)) u_\mu(\tau) \, d\tau. \]

\[ E_2 = -m_2 \eta_\mu u_2^\mu \]
\[ \Delta E_H = -m_* (\eta_\mu u_*^\mu)|_{A_H} = -m_* \eta_\mu u_*^\mu \]
\[ \Delta E = E_2 - E_1 \]

\[ \Delta E_H < 0, \]

if and only if \( \eta_\mu u_*^\mu > 0 \)

(possible only in the ergosphere)

\( \vec{P}_* \) is collinear to \( \vec{u}_* \)

so it is timelike and past-directed

because \( \eta_\mu u_*^\mu \) is negative.
General electromagnetic field

\[ T_{\alpha\beta} = \frac{1}{\mu_0} \left( F_{\mu\alpha} F^\mu_{\beta} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} g_{\alpha\beta} \right) \]

Therefore the integrand in \[ \Delta E_H = -\int_{\Delta H} P_\mu \ell^\mu \, dV \] is:

\[ T(\vec{\eta}, \vec{\ell}) = \frac{1}{\mu_0} \left( F_{\mu\rho} \eta^\rho F^{\mu}_{\sigma} \ell^\sigma - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \vec{\eta} \cdot \vec{\ell} \right) \]

since \[ \vec{\eta} \cdot \vec{\ell} = 0 \]

\[ \mu_0 T(\vec{\eta}, \vec{\ell}) = F_{\mu\rho} \eta^\rho F^{\mu}_{\sigma} \ell^\sigma \]

• pseudoelectric field 1-form on \( \mathcal{H} \)

\[ E := F(\ldots, \vec{\ell}) \]
Hence \[ \mu_0 T(\eta, \ell) = F(\vec{E}, \eta) \]

or \[ \mu_0 T(\eta, \ell) = \vec{E} \cdot \vec{E} - \omega_H F(\vec{E}, \xi) \]

therefore \[ \Delta E_H < 0, \]

if \[ \omega_H F(\vec{E}, \xi) > \vec{E} \cdot \vec{E} \] in some part of \( \Delta \mathcal{H} \).

This is the most general condition on any electromagnetic field configuration allowing black-hole energy extraction through a Penrose process.

(Since \( \vec{E} \) is tangent to \( \mathcal{H} \) \( \vec{E} \cdot \vec{E} \geq 0 \))
Stationary and axisymmetric electromagnetic field

\[ \mathcal{L}_\eta F = 0 \quad \text{and} \quad \mathcal{L}_\xi F = 0 \]
	herefore

\[
F(., \eta) = d\Phi \\
F(., \xi) = d\Psi \\
*F(\eta, \xi) = \mathcal{I},
\]

\(\Phi, \Psi\) and \(I\) are gauge-invariant. Introducing a 1-form \(A\) such that \(\mathcal{F} = dA\) one can choose \(A\) so that

\[
\Phi = \langle A, \eta \rangle = A_t \\
\Psi = \langle A, \xi \rangle = A_\varphi.
\]

\[ \mathcal{L}_\eta \Phi = \mathcal{L}_\xi \Phi = 0 \]

\[ \mathcal{L}_\eta \Psi = \mathcal{L}_\xi \Psi = 0 \]

and

\[ E = d(\Phi + \omega_H \Psi) \]

is a pure gradient.
Force free case (Blandford-Znajek)

\[ \mu_0 T(\vec{\eta}, \vec{\ell}) = \vec{E} \cdot \vec{\nabla} \Phi \]

\[ \mu_0 T(\vec{\xi}, \vec{\ell}) = \vec{E} \cdot \vec{\nabla} \Psi \]

\[ \mu_0 T(\vec{\eta}, \vec{\ell}) = \vec{\nabla} \Phi \cdot \vec{\nabla} (\Phi + \omega_H \Psi) \]

\[ F(\vec{j}, .) = 0 \]

\[ \vec{j} \cdot \vec{\nabla} \Phi = 0 \quad \text{and} \quad \vec{j} \cdot \vec{\nabla} \Psi = 0 \]

so there exists a function \( \omega(\Psi) \) such that

\[ d\Phi = -\omega(\Psi) d\Psi \]
One gets

\[ \mu_0 T(\vec{\eta}, \vec{\ell}) = \omega(\Psi) (\omega(\Psi) - \omega_H) \vec{\nabla}\Psi \cdot \vec{\nabla}\Psi. \]

\[ \vec{\ell} \cdot \vec{\nabla}\Psi = \vec{\eta} \cdot \vec{\nabla}\Psi + \omega_H \vec{\xi} \cdot \vec{\nabla}\Psi = \mathcal{L}_{\vec{\eta}}\Psi + \omega_H \mathcal{L}_{\vec{\xi}}\Psi = 0 \]

therefore on \( H \)

\[ \vec{\nabla}\Psi \cdot \vec{\nabla}\Psi \geq 0 \quad \text{and} \quad T(\vec{\eta}, \vec{\ell}) < 0 \iff \begin{cases} 0 < \omega(\Psi) < \omega_H \\ \vec{\nabla}\Psi \cdot \vec{\nabla}\Psi \neq 0 \end{cases} \]

(Blandford & Znajek 1977)
For a stationary and axisymmetric force-free electromagnetic field, a necessary condition for the Penrose process to occur is

$0 < \omega(\Psi) < \omega_H$ in some part of $\Delta \mathcal{H}$. 
General Relativistic MagnetoHydroDynamics (GRMHD)

\[ F(\vec{u}, .) = 0 \]

GRMHD HARM (Gammie, McKinney, Tóth 2003)

MAD (Magnetically Arrested discs)

SANE (Standard And Normal Evolution)

(McKinney, Tchekhovskoy, Narayan, Blandford)
Blandford-Znajek efficiency

\[ \eta_{BZ} = \frac{[P_{BZ}]_t}{[\dot{M}]_t c^2} = \frac{\kappa}{4\pi c} \left[ \phi_{BH}^2 \right]_t \left( \frac{\omega_H r_g}{c} \right)^2 f(\omega_H) \]

\( \dot{M} \) is the accretion rate

\[ \phi_{BH}^2 = \Phi_{BH}^2 / \dot{M} r_g^2 c, \]

- normalized magnetic flux

\[ f(\omega_H) \approx 0.77 \text{ for } a_* = 1 \]

Magnetic flux can be accumulated only if the disc is not thin, \( h/r \sim 1 \). Here discs are slim, \( h/r \sim 0.3 \).
Energy-momentum tensor

\[ T_{\mu\nu} = T_{\mu\nu}^{(MA)} + T_{\mu\nu}^{(EM)} \]

Flux densities:

\[ \dot{e}_{MA} := T_{t}^{(MA)r} \]
\[ J_{MA} := -T_{\phi}^{(MA)r} \quad \text{etc.} \]

At horizon

\[ \Delta E_H = \int_{\Delta H} \dot{e}_H (r_H^2 + a^2 \cos^2 \theta) \sin \theta dt dt d\theta d\phi \]

\[ \dot{e}_H := -P_{\nu}^{\nu} \big|_{H} = T_{t}^{r} \big|_{H} \]

\[ J_H := -M_{\nu}^{\nu} \big|_{H} = -T_{\phi}^{r} \big|_{H} \]
Force-free
Force-free at horizon

Various, in units of $\omega H$.

Various, $f / \max |\dot{E}|$.

Various, in units of $\max |\dot{E}|$.

Various, in units of $\max |\dot{\epsilon}|$.
Noether current in GRMHD

**MHD:** \( u_\mu F^{\mu\nu} = 0 \)

Magnetic field vector \( b^\mu := \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta} \)

Hence the energy-momentum tensor

\[
T^{(EM)}_{\mu\nu} = b^2 u_\mu u_\nu + \frac{1}{2} b^2 g_{\mu\nu} - b_\mu b_\nu
\]

Noether current

\[
P^{(EM)}_\mu = T^{(EM)}_{\mu\nu} \eta^\nu
\]

\[
P_{(EM)}^\mu P^{(EM)}_\mu = P^{2(EM)}_{(EM)} = \frac{1}{4} b^4 g_{tt} > 0 \text{ in the ergosphere}
\]
Noether current: force-free

\[ P_{(EM)}^2 > 0 \quad \text{inside ergosphere,} \]
\[ P_{(EM)}^2 < 0 \quad \text{outside ergosphere} \]
Noether current: MAD

\[ T_{\mu\nu} = T^{(MA)}_{\mu\nu} + T^{(EM)}_{\mu\nu} \]

\[ P^2 = \left( \frac{1}{2} b^2 + p \right)^2 g_{tt} - A, \]

\[ A = 2(\Gamma - 1)ub_t^2 + u_t^2(\rho + u + p + b^2)(2 - \Gamma)u + \rho, \]
Conclusions

- The Blandford-Znajek mechanism is rigorously a Penrose process.
- GRMHD simulations of Magnetically Arrested Discs correctly (from the point of view of general relativity) describe extraction of black-hole rotational energy through a Penrose process.