Thermodynamics of a Black Hole with Moon

Alexandre Le Tiec

Laboratoire Univers et Théories
Observatoire de Paris / CNRS

In collaboration with Sam Gralla

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Outline

① Mechanics and thermodynamics of stationary black holes

② Mechanics of a black hole with a corotating moon

③ Surface area, angular velocity and surface gravity

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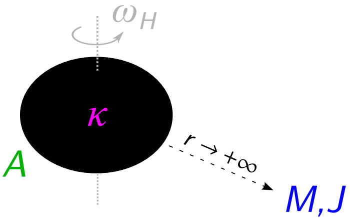
Black hole uniqueness theorem in GR

[Israel (1967); Carter (1971); Hawking (1973); Robinson (1975)]

- The only stationary vacuum black hole solution is the Kerr solution of mass M and angular momentum J

"Black holes have no hair." (J. A. Wheeler)

- Black hole event horizon characterized by:
  - Angular velocity ω_H
  - Surface gravity κ
  - Surface area A

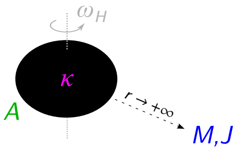


The laws of black hole mechanics

[Hawking (1972); Bardeen *et al.* (1973)]

- Zeroth law of mechanics:

$$\kappa = \text{const.}$$

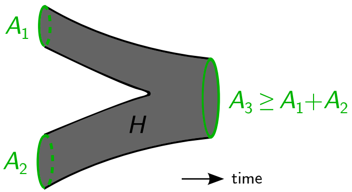


- First law of mechanics:

$$\delta M = \omega_H \, \delta J + \frac{\kappa}{8\pi} \, \delta A$$

- Second law of mechanics:

$$\delta A \geqslant 0$$



Analogy with the laws of thermodynamics

[Bardeen, Carter & Hawking (1973)]

	Black Hole (BH)	Thermo. System
Zeroth law	$\kappa = \text{const.}$	$T = \text{const.}$
First law	$\delta M = \omega_H \, \delta J + \frac{\kappa}{8\pi} \, \delta A$	$\delta E = \delta W + T \, \delta S$
Second law	$\delta A \geqslant 0$	$\delta S \geqslant 0$

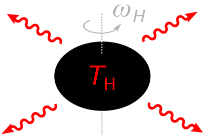
- Black holes should have an entropy $S_{\text{BH}} \propto A$ [Bekenstein (1973)]
- Analogy suggests stationary BHs have temperature $T_{\text{BH}} \propto \kappa$
- However BHs are perfect absorbers, so classically $T_{\text{BH}} = 0$

Thermodynamics of stationary black holes

[Hawking (1975)]

- Quantum fields in a classical curved background spacetime
- *Stationary* black holes radiate particles at the temperature

$$T_{\rm H} = \frac{\hbar}{2\pi}\,\kappa$$



- Thus the entropy of *any* black hole is given by $S_{\rm BH} = A/4\hbar$
- Key results for the search of a quantum theory of gravity: string theory, LQG, emergent gravity, etc.

Going beyond stationarity and axisymmetry

- A fully general theory of radiating black holes is called for
- However, even the *classical notion* of surface gravity for a dynamical black hole is problematic [Nielsen & Yoon (2008)]
- Main difficulty: lack of a horizon Killing field, a Killing field tangent to the null geodesic generators of the event horizon

Objective: explore the mechanics and thermodynamics of a *dynamical* and *interacting* black hole

Main tool: black hole perturbation theory

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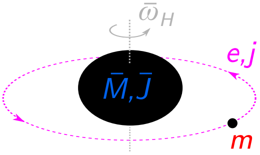
Rotating black hole + orbiting moon

- Kerr black hole of mass $\bar{M}$ and spin $\bar{J}$ perturbed by a moon of mass $m \ll \bar{M}$:

$$g_{ab}(\lambda) = \bar{g}_{ab} + \lambda\, \mathcal{D}g_{ab} + \mathcal{O}(\lambda^2)$$



- Perturbation $\mathcal{D}g_{ab}$ obeys the linearized Einstein equation with point-particle source [Gralla & Wald (2008)]

$$\mathcal{D}G_{ab} = 8\pi\, \mathcal{D}T_{ab} = 8\pi\, m \int_\gamma \mathrm{d}\tau\, \delta_4(x,y)\, u_a u_b$$

- Particle has energy $e = -m\, t^a u_a$ and ang. mom. $j = m\, \phi^a u_a$
- Physical $\mathcal{D}g_{ab}$: retarded solution, no incoming radiation, perturbations $\mathcal{D}M = e$ and $\mathcal{D}J = j$ [Keidl *et al.* (2010)]

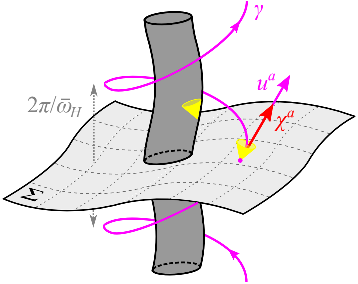
Rotating black hole + corotating moon

- We choose for the geodesic γ the unique equatorial, circular orbit with azimuthal frequency $\bar{\omega}_H$, i.e., the *corotating* orbit
- Gravitational radiation-reaction is $\mathcal{O}(\lambda^2)$ and neglected
  ↳ the spacetime geometry has a helical symmetry
- In adapted coordinates, the helical Killing field reads

$$\chi^a = t^a + \bar{\omega}_H \, \phi^a$$

- Conserved orbital quantity associated with symmetry:

$$z \equiv -\chi^a u_a = m^{-1} \left(e - \bar{\omega}_H \, j \right)$$



Zeroth law for a black hole with moon

- Because of helical symmetry and corotation, the expansion and shear of the *perturbed* future event horizon H vanish
- Rigidity theorems then imply that H is a Killing horizon
  [Hawking (1972); Chruściel (1997); Friedrich *et al.* (1999); etc]
- The horizon-generating Killing field must be of the form

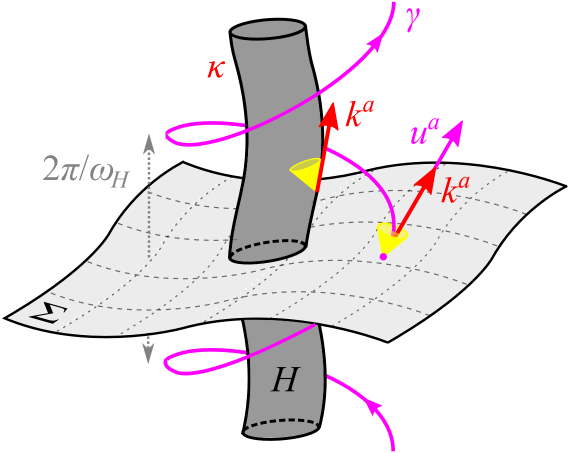
$$k^a(\lambda) = t^a + (\bar{\omega}_H + \lambda\,\mathcal{D}\omega_H)\,\phi^a + \mathcal{O}(\lambda^2)$$

- The surface gravity κ is defined in the usual manner as

$$\kappa^2 = -\frac{1}{2}\,(\nabla^a k^b\,\nabla_a k_b)|_H$$

- Since $\kappa =$ const. over *any* Killing horizon [Bardeen *et al.* (1973)], we have proven a zeroth law for the *perturbed* event horizon

Zeroth law for a black hole with moon



Smarr formula for a black hole with moon

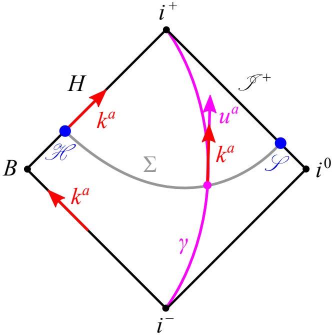
- For *any* spacetime with a Killing field k^a, Stokes' theorem yields the following identity (with $Q_{ab} \equiv -\varepsilon_{abcd} \nabla^c k^d$):

$$\int_{\partial\Sigma} Q_{ab} = 2 \int_{\Sigma} \varepsilon_{abcd} \, R^{de} k_e$$

- Applied to a given BH with moon spacetime, this gives the formula

$$M = 2\omega_H J + \frac{\kappa A}{4\pi} + mz$$

- In the limit $\lambda \to 0$, we recover Smarr's formula [Smarr (1973)]



First law for a black hole with moon

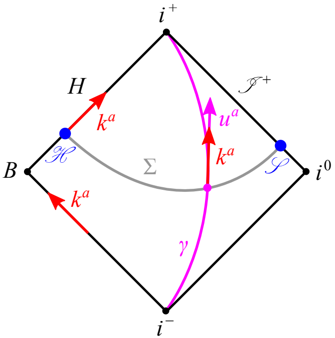
- Adapting [Iyer & Wald (1994)] to non-vacuum perturbations of non-stationary, non-axisymmetric spacetimes we find:

$$\int_{\partial\Sigma} (\delta Q_{ab} - \Theta_{abc} k^c) = 2\,\delta \int_\Sigma \varepsilon_{abcd} G^{de} k_e - \int_\Sigma \varepsilon_{abcd} k^d G^{ef} \delta g_{ef}$$

- Applied to nearby BH with moon spacetimes, this gives the first law

$$\delta M = \omega_H\,\delta J + \frac{\kappa}{8\pi}\,\delta A + z\,\delta m$$

- Features variations of the Bondi mass and angular momentum



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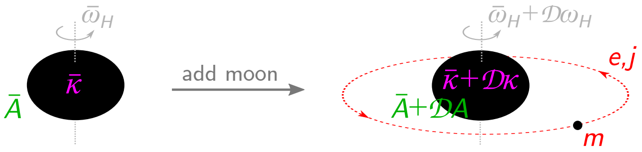
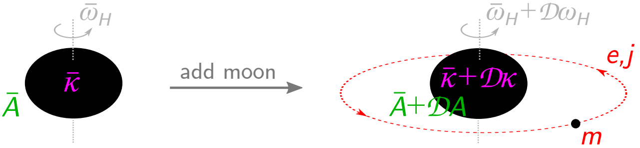
Perturbation in horizon surface area



- Application of the first law to this perturbation gives

$$\mathcal{D}M = \bar{\omega}_H \, \mathcal{D}J + \frac{\bar{\kappa}}{8\pi} \, \mathcal{D}A + m \, z$$

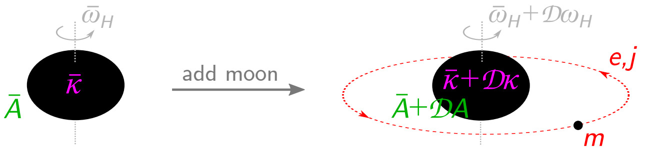
Perturbation in horizon surface area



- Application of the first law to this perturbation gives

$$e = \bar{\omega}_H \, j + \frac{\bar{\kappa}}{8\pi} \mathcal{D}A + (e - \bar{\omega}_H \, j)$$

Perturbation in horizon surface area



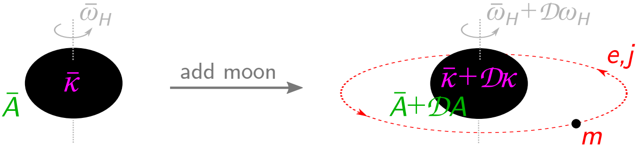
- Application of the first law to this perturbation gives

$$e = \bar{\omega}_H \, j + \frac{\bar{\kappa}}{8\pi} \mathcal{D}A + (e - \bar{\omega}_H \, j)$$

- The perturbation in horizon surface area vanishes:

$$\mathcal{D}A = 0$$

Perturbation in horizon surface area



- Application of the first law to this perturbation gives

$$e = \bar{\omega}_H \, j + \frac{\bar{\kappa}}{8\pi} \mathcal{D}A + (e - \bar{\omega}_H \, j)$$

- The perturbation in horizon surface area vanishes:

$$\mathcal{D}A = 0$$

- The black hole's entropy is unaffected by the moon

Particle Hamiltonian first law

- Geodesic motion of test mass m in Kerr geometry $\bar{g}_{ab}(x; \bar{M}, \bar{J})$ derives from canonical Hamiltonian [Carter (1968)]

$$\mathcal{H}(y, p) = \frac{1}{2}\, \bar{g}^{ab}(y; \bar{M}, \bar{J})\, p_a p_b$$

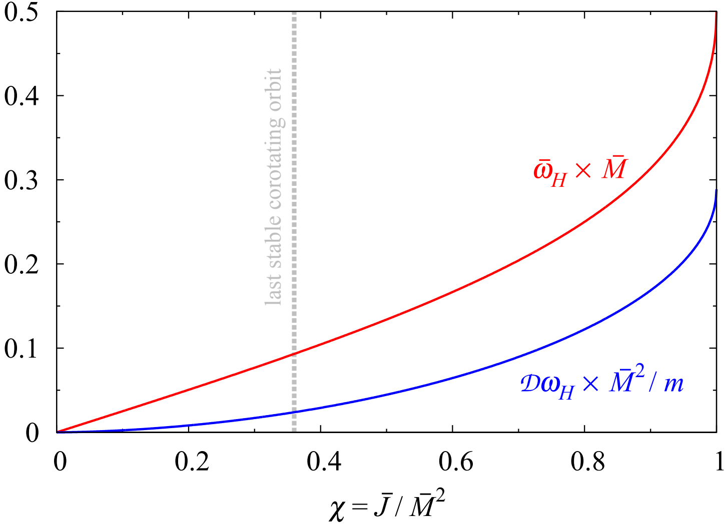
- Varying $\mathcal{H}(y, p)$ yields the Hamiltonian first law [Le Tiec (2013)]

$$\delta e = \bar{\omega}_H\, \delta j + z\, \delta m + \frac{z}{m} \left(\partial_{\bar{M}} \mathcal{H}\, \delta \bar{M} + \partial_{\bar{J}} \mathcal{H}\, \delta \bar{J} \right)$$

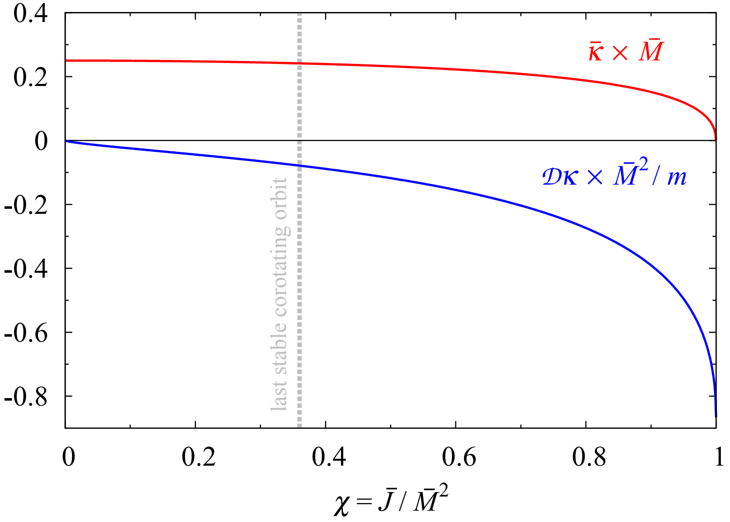
- Combining both first laws, we find for the perturbations in horizon angular velocity and surface gravity

$$\mathcal{D}\omega_H = \frac{z}{m} \left(\bar{\omega}_H\, \partial_{\bar{M}} \mathcal{H} + \partial_{\bar{J}} \mathcal{H} \right) \quad \text{and} \quad \mathcal{D}\kappa = \frac{z}{m}\, \bar{\kappa}\, \partial_{\bar{M}} \mathcal{H}$$

Perturbation in horizon angular velocity



Perturbation in horizon surface gravity



Cooling a black hole with an orbiting moon

- Key properties of semi-classical calculation in Kerr preserved:
  - $k^a = t^a + \omega_H \phi^a$ is infinitesimally related to χ^a
  - k^a is normalized so that $k^a t_a = -1$ at infinity
  - t^a is an *asymptotic* time translation symmetry
- The constant surface gravity κ should thus correspond to the Hawking temperature T_H of the *perturbed* black hole:

$$\mathcal{D} T_H = \frac{\hbar}{2\pi} \mathcal{D}\kappa < 0$$

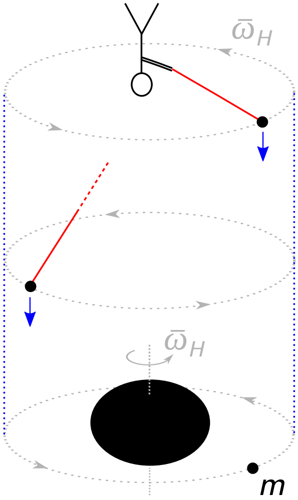
- The moon has a cooling effect on the rotating black hole!
- Both rotationally-induced and tidally-induced deformations of a BH horizon have a cooling effect $\longrightarrow$ generic result?

Physical process version

① Set moon on circular trajectory with angular frequency $\bar{\omega}_H$

② Lower moon adiabatically down "corotating cylinder"

③ Release moon on corotating orbit and retract tensile rod



Summary and prospects

Main results

- We established a zeroth law and a first law of black hole mechanics valid beyond stationarity and axisymmetry
- We studied a realistic, dynamical and interacting black hole whose surface gravity is well-defined
- The moon has a cooling effect on the rotating black hole!

Future work

- Recover our results from a direct analysis of $\mathcal{D}g_{ab}$ near H
- Semi-classical calculation confirming that $T_{\mathrm{H}} = \hbar\,\kappa/2\pi$
- Recover our results from a microscopic point of view

Additional Material

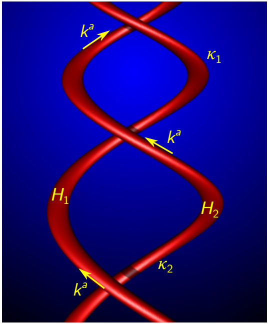
Generalized zeroth law of mechanics

[Friedman, Uryū & Shibata (2002)]

- Black hole spacetimes with *helical* Killing vector field k^a
- On each component H_i of the horizon, the expansion and shear of the null generators vanish
- Generalized rigidity theorem
  ↳ $H = \bigcup_i H_i$ is a Killing horizon
- *Constant* horizon surface gravity

$$\kappa_i^2 = -\frac{1}{2} \left(\nabla^a k^b \, \nabla_a k_b\right)\big|_{H_i}$$

- Lack of asymptotic flatness
  ↳ overall scaling of κ_i is free

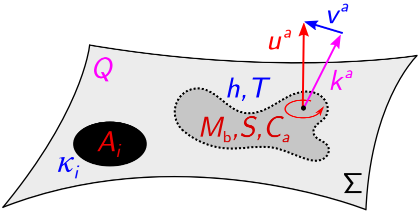


Generalized first law of mechanics

[Friedman, Uryū & Shibata (2002)]

- Spacetimes with black holes + perfect fluid matter sources
- One-parameter family of solutions $\{g_{ab}(\lambda), u^a(\lambda), \rho(\lambda), s(\lambda)\}$
- *Globally* defined Killing field k^a → conseved Noether charge Q

$$\delta Q = \sum_i \frac{\kappa_i}{8\pi}\,\delta A_i + \int_\Sigma \left[\,\bar{h}\,\delta(\mathrm{d}M_\mathrm{b}) + \bar{T}\delta(\mathrm{d}S) + v^a\delta(\mathrm{d}C_a)\right]$$



Issue of asymptotic flatness

[Friedman, Uryū & Shibata (2002)]

- Binaries on circular orbits have a *helical* Killing symmetry k^a
- Helically symmetric spacetimes are *not* asymptotically flat
  [Gibbons & Stewart (1983); Detweiler (1989); Klein (2004)]
- Asymptotic flatness can be recovered if radiation (reaction) can be "turned off" (neglected):
  - Conformal Flatness Condition
  - Post-Newtonian approximation
  - Linear perturbation theory
- For asymptotically flat spacetimes:

$$k^a \to t^a + \Omega\,\phi^a \quad \text{and} \quad \delta Q = \delta M_{\text{ADM}} - \Omega\,\delta J$$