THE EFFECTIVE-ONE-BODY MODELING OF COALESCING COMPACT BINARIES:

INTERFACING NUMERICAL AND ANALYTICAL RESULTS

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Collaboration: T. Damour, S. Bernuzzi (more than 40 papers > 2006)

and many NR (and AR) people: C. Reisswig, D. Pollney, L. Baiotti, B. Giacomazzo, L. Rezzolla, T. Dietrich, B. Bruegmann, M. Hannam, S. Husa, E. Harms, L. Villain, A. Zenginoglu, S. Akcay, D. Bini, B. Iyer, I. Hinder, S. Hopper, F. Guercilena...

OUTLINE

- Coalescing (stellar-mass) relativistic binaries (binary black holes, BBH, and binary neutron star, BNS): most promising sources for ground-based gravitational wave (GW) detectors: strong fields, high velocities. [expected rates 0.4-400 BNS mrgs per yr in aLIGO > 2015]

- Difficult problem: numerical relativity (NR) simulations are needed to accurately tackle the late stages of the inspiral through plunge and merger (and ringdown for BBHs). Computationally expensive effort.

- Analytical modelizations are essential to: (i) interpret, check and predict the (sparse) numerical results; (ii) to cover efficiently and accurately the full parameter space (masses, spins and equation of state) to build waveform templates needed for detection and parameter estimation

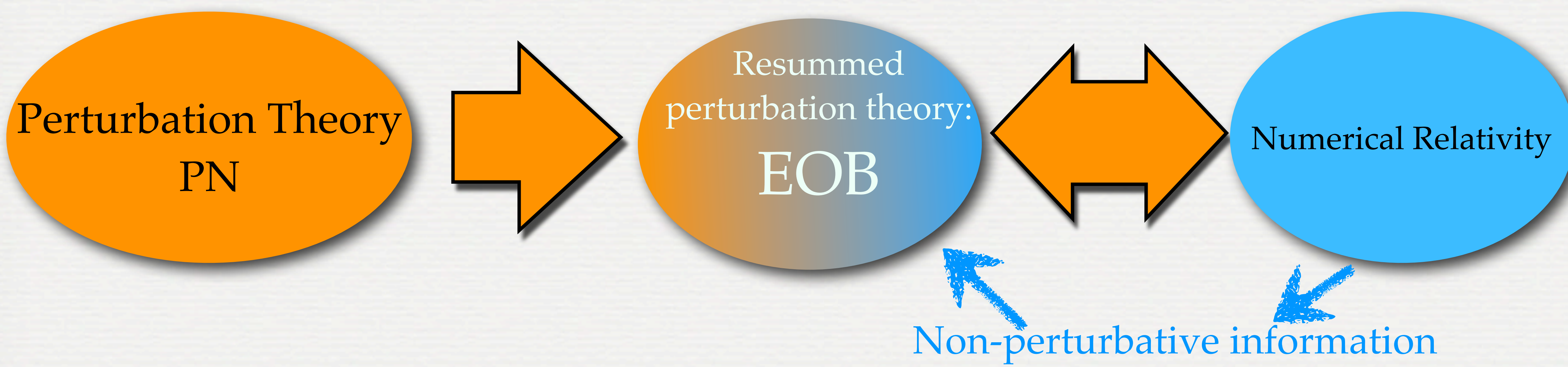
- Standard (post-Newtonian, Taylor-expanded) methods are not suitable to the strong-field and high-velocity regimes: one needs to do better. This "better" is the effective-one-body (EOB), resummed, approach to the two-body GR dynamics.

IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).

- **Practical**: need many thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$

- **Solution**: synergy between analytical & numerical relativity



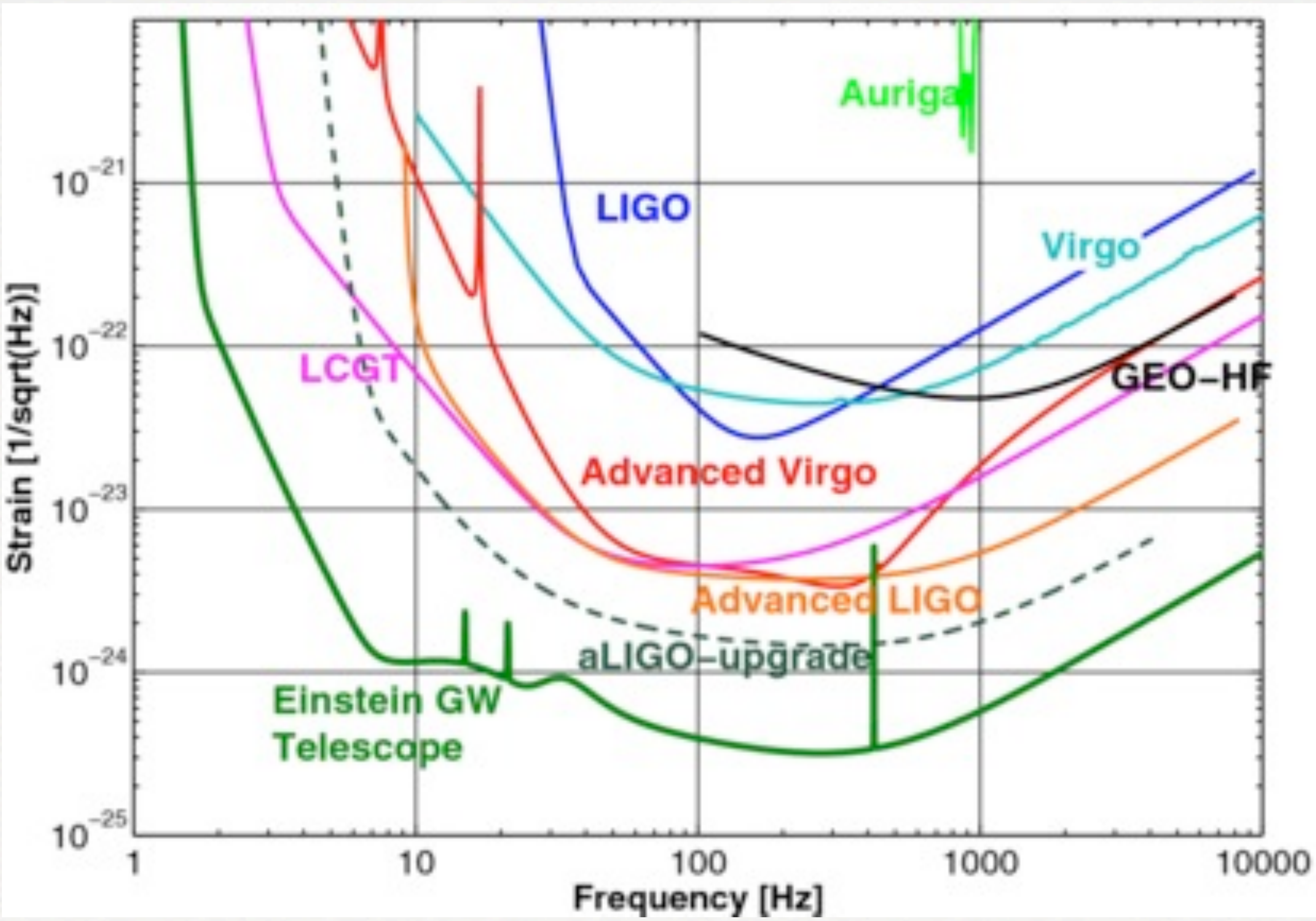
NEED OF TEMPLATES

To extract GW signal from detector's output (lost in broadband noise $S_n(f)$)

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

Detector's output

Template of expected GW signal



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NEWTONIAN PRELIMINARIES

GWS FROM COMPACT BINARIES: BASICS

Newtonian binary systems in circular orbits: Kepler's law

$$GM = \Omega^2 R^3$$

$$M = m_1 + m_2$$

$$\frac{v^2}{c^2} = \frac{GM}{c^2 R} = \left(\frac{GM\Omega}{c^3}\right)^{2/3}$$

Quadrupole formula: GW luminosity (energy flux)

$$x = \left(\frac{v}{c}\right)^2$$

$$P_{\rm gw} = \frac{dE_{\rm gw}}{dt} = \frac{32}{5}\frac{c^5}{G}\nu^2 x^5$$

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{M^2}$$

GWS FROM COMPACT BINARIES: BASICS

$$E^{\rm orbital} = E^{\rm kinetic} + E^{\rm potential} = -\frac{1}{2}\frac{m_1 m_2}{R} = -\frac{1}{2}\mu x$$

Balance argument

$$\frac{dE^{\rm orbital}}{dt} = P_{GW} = \frac{dE_{\rm GW}}{dt}$$

$$\omega_{22}^{\rm GW} = 2\pi f_{22}^{\rm GW} = 2\Omega^{\rm orbital}$$

$$f_{GW}^{22} = \frac{1}{\pi}\left(\frac{5}{256\nu}\right)^{3/8}\left(\frac{1}{t - t_{\rm coalescence}}\right)^{3/8}$$

GWS FROM COMPACT BINARIES: BASICS

Chirp mass: $\mathcal{M} = \nu^{3/5} M$

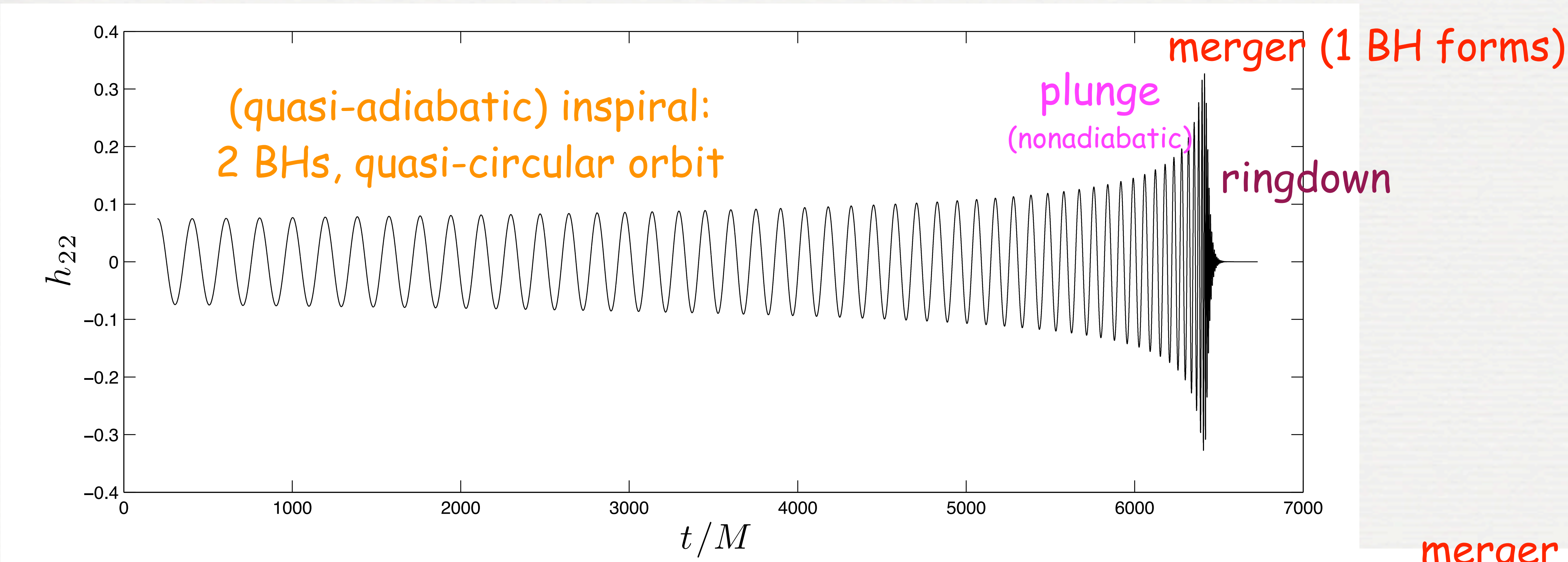
$$f_{GW}^{22} = 134\mathrm{Hz}\left(\frac{1.21 M_\odot}{\mathcal{M}}\right)^{5/8}\left(\frac{1s}{t - t_{\mathrm{coalescence}}}\right)^{3/8}$$

$M = 1.4 M_\odot + 1.4 M_\odot$ “Standard” BNS system

WHAT DO WE NEED? BBHS!

$$h_+ - ih_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\ {}_{-2}Y_{\ell m}(\theta,\phi)$$

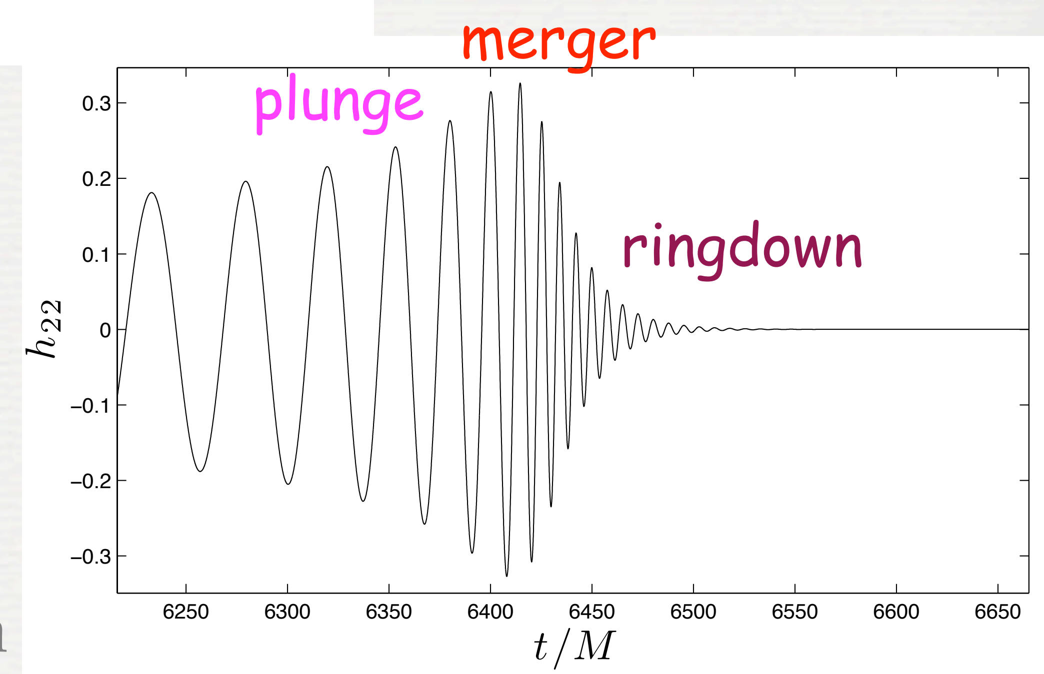
$$h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$$



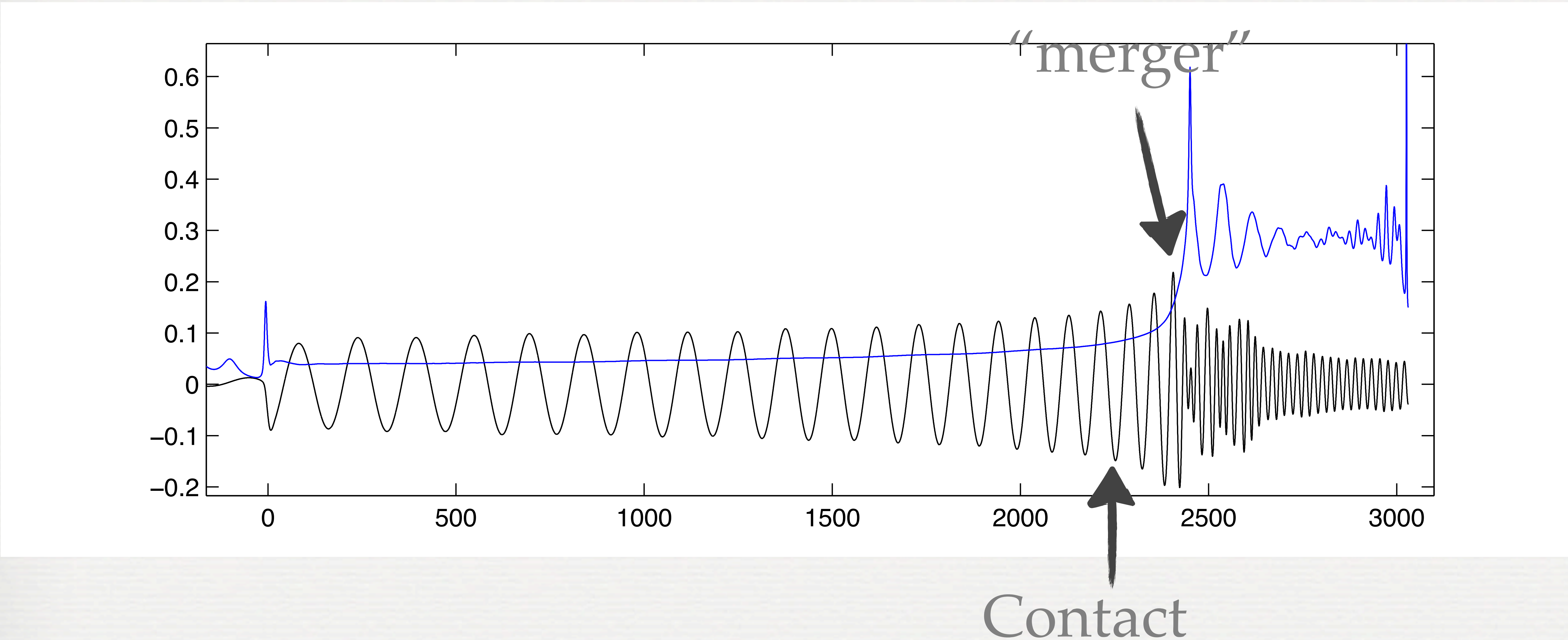
equal-mass BBH, aligned-spins

$$\chi_1 = \chi_2 = +0.98$$

SXS (Simulating eXtreme Spacetimes) public data



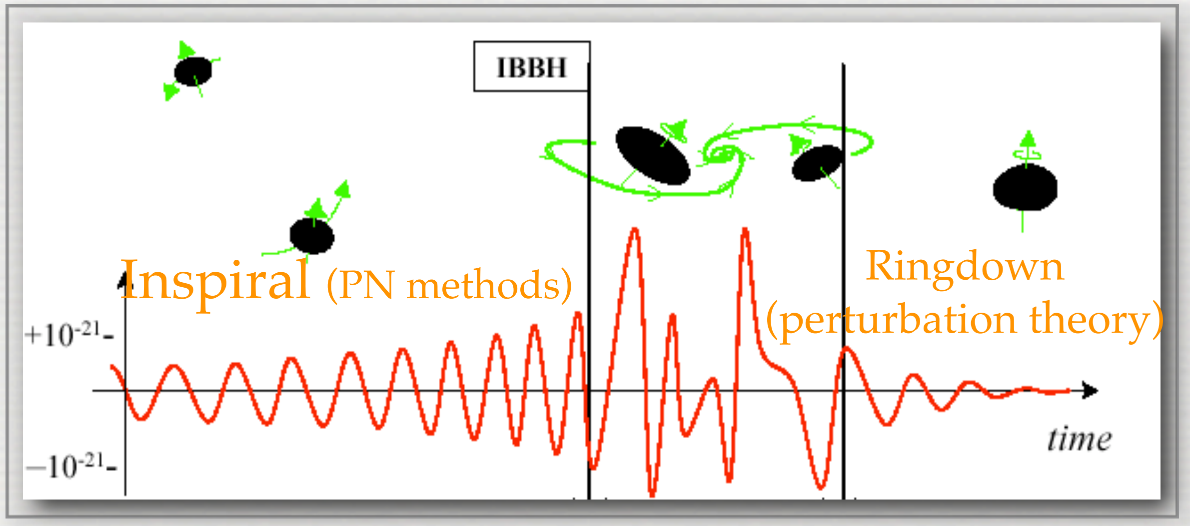
AND BNS



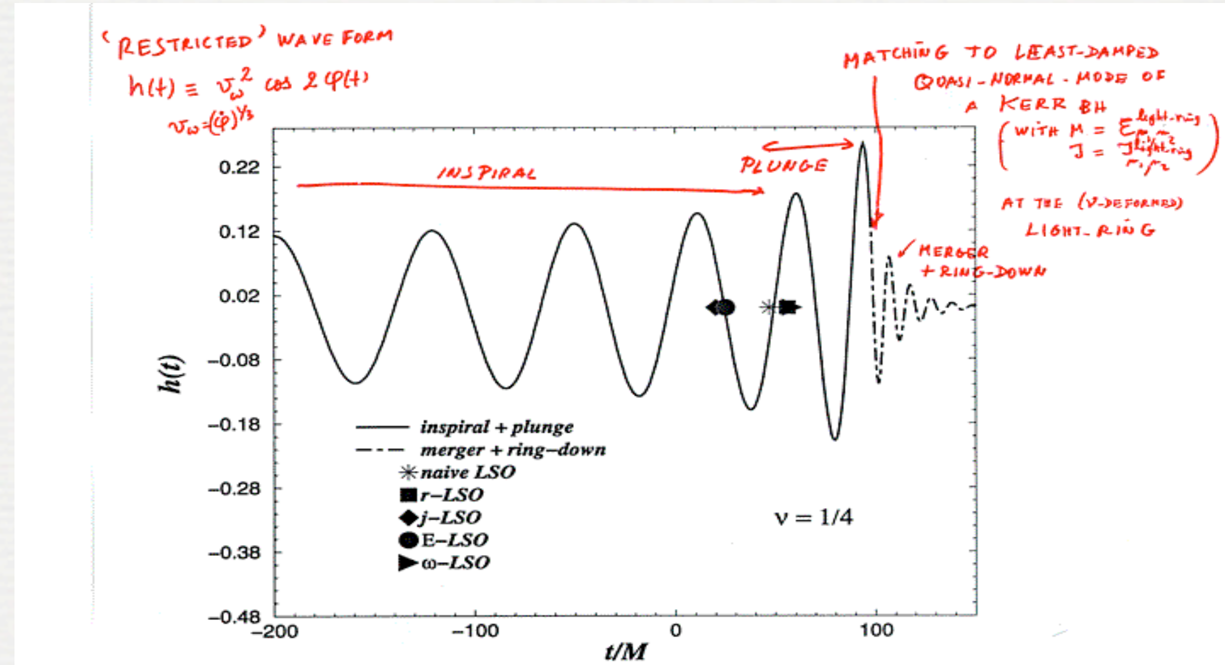
$$q = 1 \qquad M = 2.7M_\odot$$

TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998



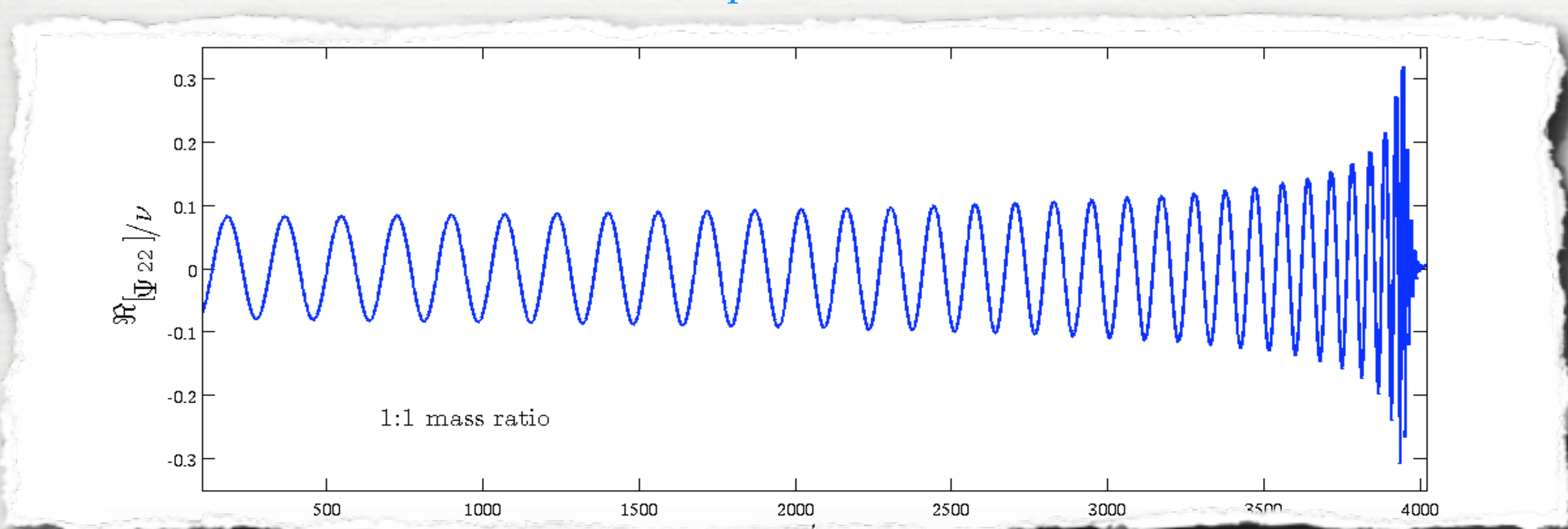
Merger:
Numerical Relativity (only?)



Effective-One-Body (Buonanno & Damour (2000)):
analytical from the beginning to the end

Numerical Relativity: >= 2005 (Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code (with some caveats): M. Scheel et al., 2008



Spectral code
Extrapolation (radius & resolution)

Phase error:
< 0.02 rad (inspiral)
<0.1 rad (ringdown)

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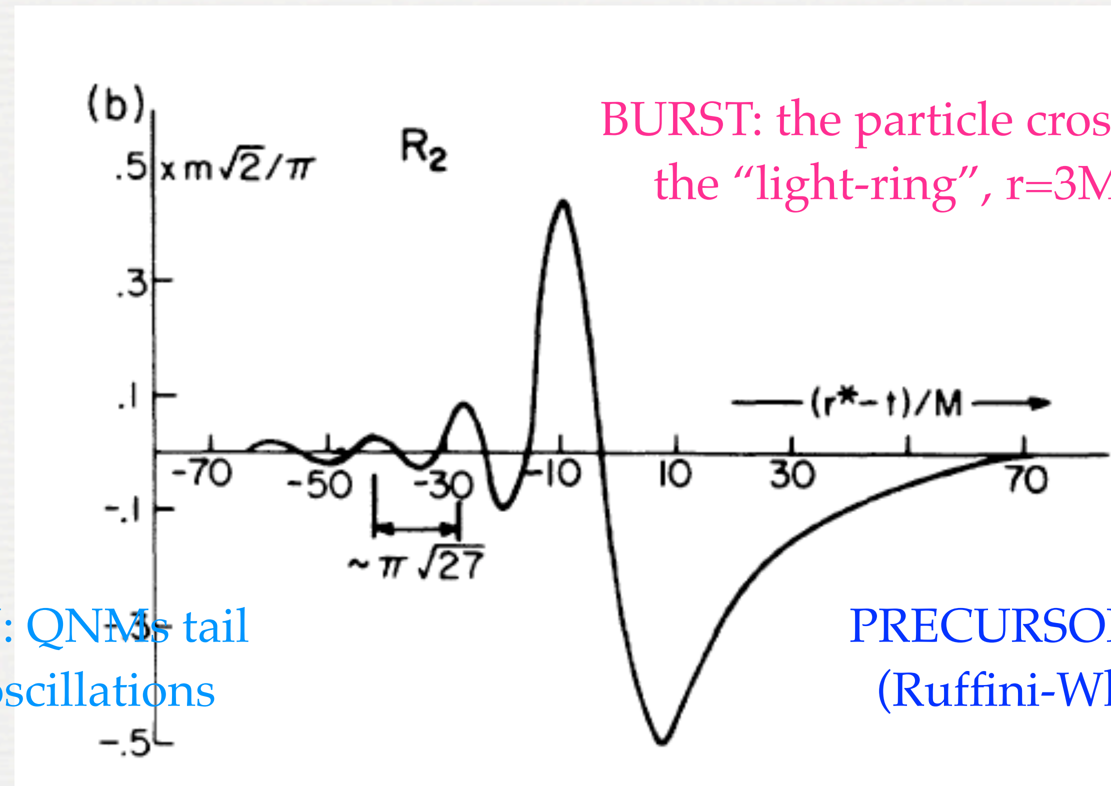
PRECURSOR-BURST-RINGDOWN STRUCTURE :1972

Davis, Ruffini & Tiomno: radial plunge of a test-particle onto a Schwarzschild black hole



BURST: the particle crosses
the "light-ring", r=3M

RINGDOWN: QNMs tail
Spacetime oscillations

PRECURSOR: Quadrupole formula
(Ruffini-Wheeler approximation)

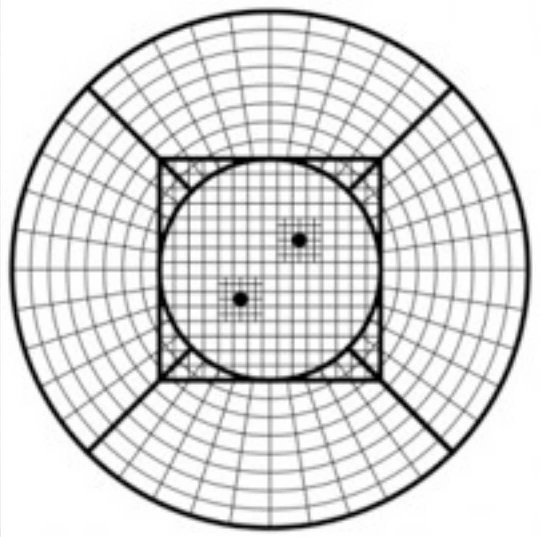
BBH & BNS COALESCENCE: NUMERICAL RELATIVITY

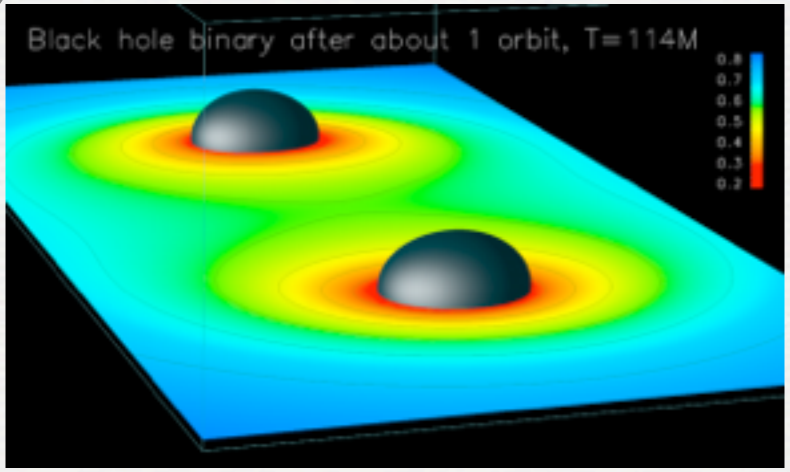
Numerical relativity is complicated & computationally expensive:

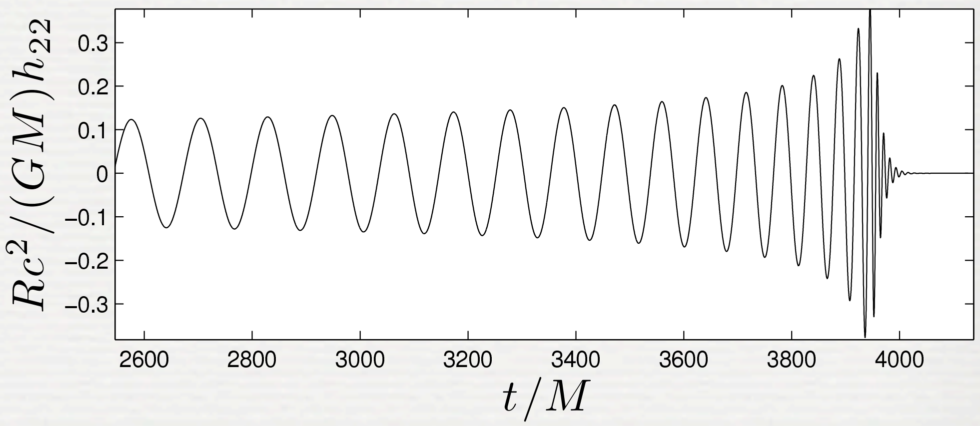
- Formulation of Einstein equations (BSSN, harmonic, Z4c,...)
- Setting up initial data (solution of the constraints)
- Gauge choice
- Numerical approach (finite-differencing (FD, e.g. Llama) vs spectral (SpEC,SXS))
- High-order FD operators
- Treatment of BH singularity (excision vs punctures)
- Wave extraction problem on finite-size grids (Cauchy-Characteristic vs extrapolation)
- Huge computational resources (mass-ratios 1:10; spin)
- Adaptive-mesh-refinement
- Error budget (convergence rates are far from clean...)
- For BNS: further complications due to GR-Hydrodynamics for matter
- Months of running to get one accurate waveform....

Multi-patch grid structure

(Llama FD code, Pollney & Reisswig)







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A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy [PRL 111 (2013) 241104]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4,2] Sergei Ossokine,[1,5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

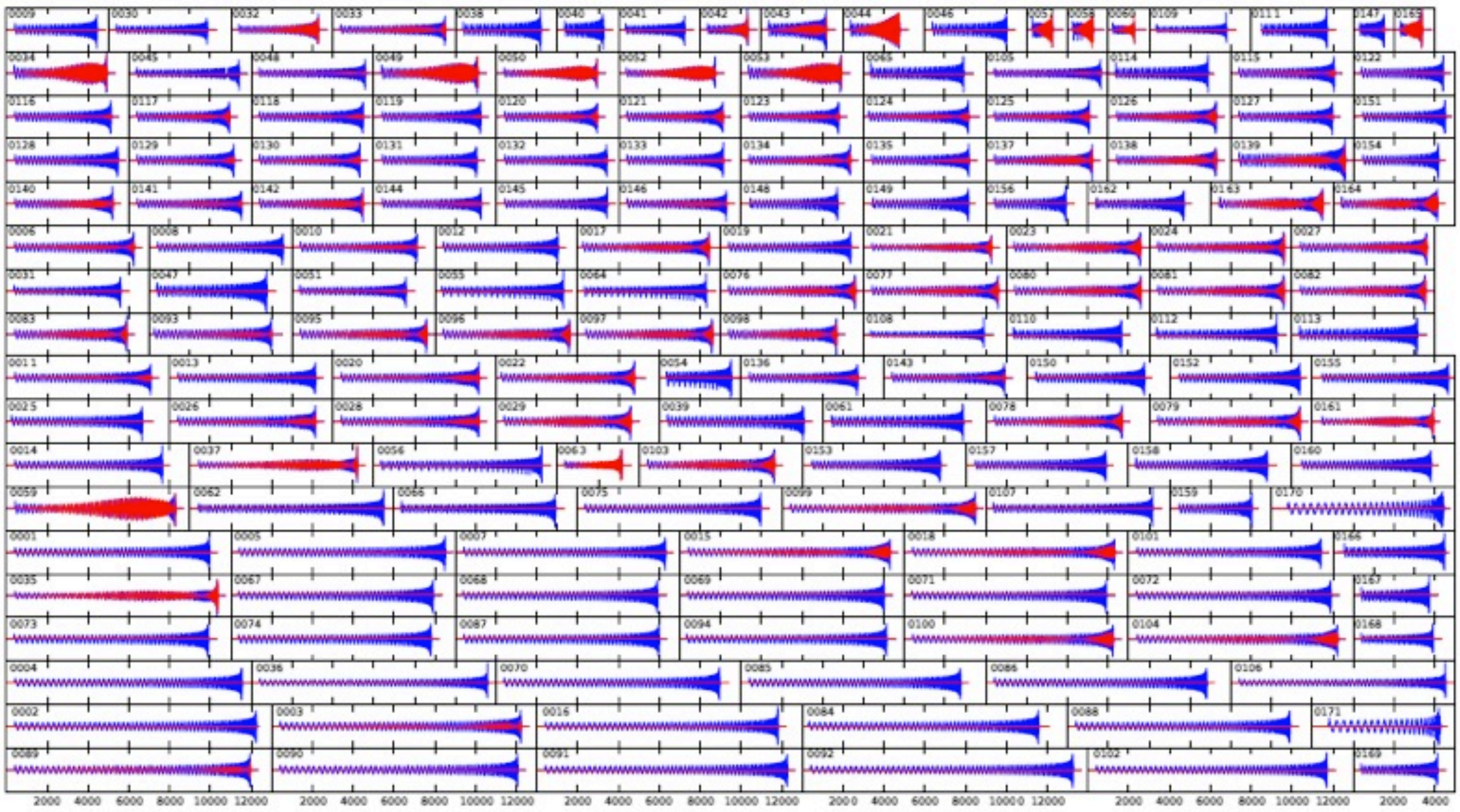

FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

www.black-holes.org

ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian (conservative dynamics)

Radiation reaction (mechanical losses)

Waveform

3PN POST-NEWTONIAN HAMILTONIAN (C.O.M)

$$\widehat{H}^{\rm NR}_{\rm real}(\mathbf{q},\mathbf{p}) = \widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p})\,, \tag{4.27}$$

where

$$\widehat{H}_{\rm N}\,(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{q}, \tag{4.28a}$$

$$\widehat{H}_{\rm 1PN}\,(\mathbf{q},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left[(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q} + \frac{1}{2q^2}, \tag{4.28b}$$

$$\begin{aligned}\widehat{H}_{\rm 2PN}\,(\mathbf{q},\mathbf{p}) &= \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left[\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q}\\ &+ \frac{1}{2}\left[(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3},\end{aligned} \tag{4.28c}$$

$$\begin{aligned}\widehat{H}_{\rm 3PN}\,(\mathbf{q},\mathbf{p}) &= \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4\\ &+ \frac{1}{16}\left[\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\right]\frac{1}{q}\\ &+ \left[\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q^2}\\ &+ \left\{\left[-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right]\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{q^3}\\ &+ \left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2 + \omega_{\rm static}\right)\nu\right]\frac{1}{q^4}.\end{aligned} \tag{4.28d}$$

$$\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$$

From DJS, PRD 62 (2000) 084011

TAYLOR-EXPANDED (CIRCULAR) 3PN WAVEFORM

Blanchet, Iyer&Joguet, 02; Blanchet, Damour, Iyer&Esposito-Farese, 04; Kidder07; Blanchet et al.,08

$$h^{22} = -8\sqrt{\frac{\pi}{5}}\frac{G\nu m}{c^2 R}e^{-2i\phi}x\Big\{1 - x\Big(\frac{107}{42} - \frac{55}{42}\nu\Big) + x^{3/2}\Big[2\pi + 6i\ln\Big(\frac{x}{x_0}\Big)\Big] - x^2\Big(\frac{2173}{1512} + \frac{1069}{216}\nu - \frac{2047}{1512}\nu^2\Big)$$
$$- x^{5/2}\Big[\Big(\frac{107}{21} - \frac{34}{21}\nu\Big)\pi + 24i\nu + \Big(\frac{107i}{7} - \frac{34i}{7}\nu\Big)\ln\Big(\frac{x}{x_0}\Big)\Big]$$
$$+ x^3\Big[\frac{27\,027\,409}{646\,800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln 2 - \frac{428}{105}\ln x$$
$$- 18\Big[\ln\Big(\frac{x}{x_0}\Big)\Big]^2 - \Big(\frac{278\,185}{33\,264} - \frac{41}{96}\pi^2\Big)\nu - \frac{20\,261}{2772}\nu^2 + \frac{114\,635}{99\,792}\nu^3 + \frac{428i}{105}\pi + 12i\pi\ln\Big(\frac{x}{x_0}\Big)\Big] + O(\epsilon^{7/2})\Big\},$$

$$x = (M\Omega)^{2/3} \sim v^2/c^2$$

$$M = m_1 + m_2$$

$$\nu = \frac{m_1 m_2}{M^2}$$

PN-EXPANDED (CIRCULAR) ENERGY FLUX (3.5PN)

$$\frac{dE}{dt} = -\mathcal{L}$$

balance equation

Mechanical loss GW luminosity

$$\mathcal{L} = \frac{32c^5}{5G}\nu^2 x^5 \left\{1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 \right.$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)\right.$$
$$\left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3$$
$$\left. + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\right\}.$$

Newtonian quadrupole formula

$$C = \gamma_E = 0.5772156649...$$

AN IMPROVED ANALYTICAL APPROACH

EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

References:

Buonanno & Damour 1999	(2PN Hamiltonian)
Buonanno & Damour 2000	(Rad. Reac. & full waveform)
Damour,Jaranowski, Schaefer 2000	(3PN Hamiltonian)
Damour 2001	(spin)
Damour&Nagar07, Damour, Iyer, Nagar 08	(factorized waveform)
Damour&Nagar10	(tidal effects)
Nagar11	(NNLO spin-orbit effects)
Barausse&Buonanno11	(NNLO spin-orbit effects)
Balmelli&Jetzer12	(NNLO spin-spin effects)

EOB APPROACH IN A NUTSHELL

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1 m_2/(m_1 + m_2))$ in an effective metric $g_{\mu\nu}^{\rm eff}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g_{\mu\nu}^{\rm eff}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1 m_2/(m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM

PN dynamics
(DD81,D82,DJS01,IF03,BDIF04)

PN rad losses
WW76, BDIWW95, BDEFI 05

PN waveform
BD89, B95&05,ABIQ04,

BH perturbations
RW57, Z70, Z72

Resummed (BD99)

Resummed (DIS98)

Resummed (DN07,DIN08)

QNMs spectrum
$\sigma_N = \alpha_N + i\omega_N$

EOB Hamiltonian
$H_{\rm EOB}$

EOB Rad. Reac. force
$\hat{\mathcal{F}}_\varphi$

Factorized waveform

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\hat{h}_{\ell m}^{(\epsilon)}$$
$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{eff}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

Matching at merger time

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$
$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$
$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$
$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

BNS: tides
(Love numbers)

$$h_{\ell m}^{\rm ringdown}(t) = \sum_N C_N^+ e^{-\sigma_N^+(t-t_m)}$$

EOB waveform

$$h_{\ell m}^{\rm EOB} = \theta(t_m - t)\, h_{\ell m}^{\rm insplunge}(t) + \theta(t - t_m)\, h_{\ell m}^{\rm ringdown}(t)$$

TWO-BODY/EOB "CORRESPONDENCE":

THINK QUANTUM-MECHNICALLY (J.A. WHEELER)

Real 2-body system
(in the c.o.m. frame)
(m_1, m_2)

1:1 map

An effective particle
in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$g_{\mu\nu}^{\rm eff}$$

$$\mathcal{E} = f(E)$$

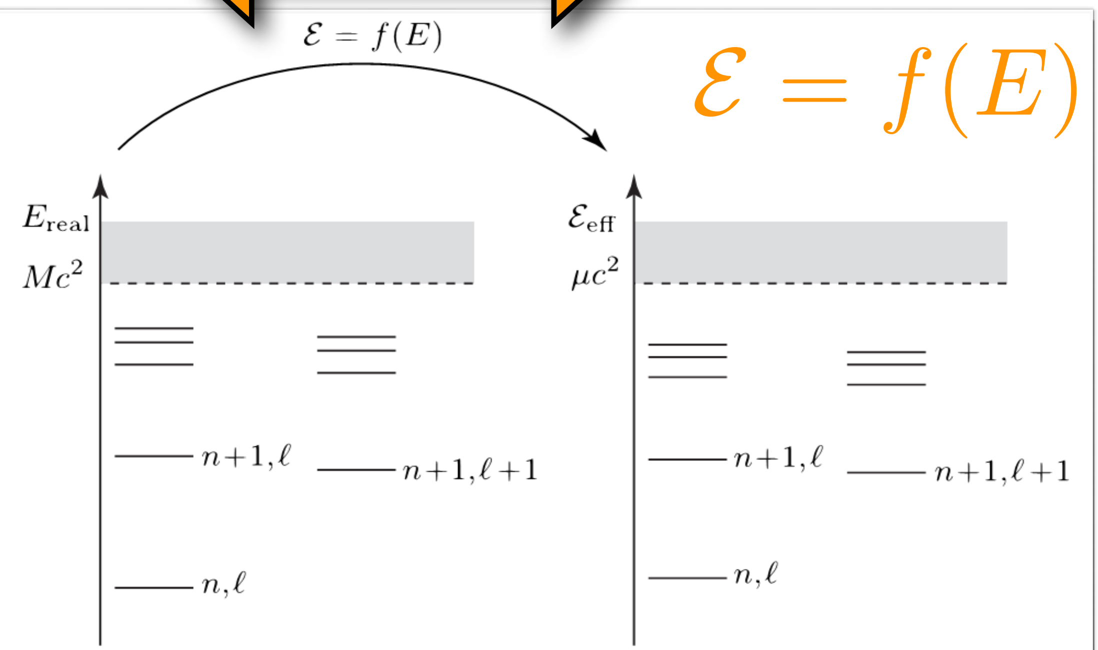

Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

$$\mu^2 + g^{\mu\nu}_{\rm eff} \frac{\partial S_{\rm eff}}{\partial x^\mu} \frac{\partial S_{\rm eff}}{\partial x^\nu} + \mathcal{O}(p^4) = 0$$

Sommerfeld's
"Old Quantum Mechanics"
(action-angle variables &
Delaunay Hamiltonian)

$$J = \ell\hbar = \frac{1}{2\pi}\oint p_\varphi d\varphi$$

$$N = n\hbar = I_r + J$$

$$I_r = \frac{1}{2\pi}\oint p_r dr$$

$$H^{\rm classical}(q,p) \longrightarrow H^{\rm classical}(I_a) \longrightarrow \boxed{E^{\rm quantum}(I_a = n_a h) = f^{-1}[\mathcal{E}^{\rm quantum}_{\rm eff}(I^{\rm eff}_a = n_a h)]}$$

THE EOB ENERGY MAP

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Real 2-body system
(PN-expanded Hamiltonian
in the c.o.m. frame)

$$(m_1, m_2)$$

1:1 map

An effective particle
in some effective metric

$$g_{\mu\nu}^{\text{eff}}$$

Simple energy map:

$$\mathcal{E}_{\text{eff}} = \frac{E_{\text{real}}^2 - m_1^2 - m_2^2}{2M}$$

EOB Hamiltonian:

$$H_{\text{EOB}} = M\sqrt{1 + 2\nu\left(\hat{H}_{\text{eff}} - 1\right)}$$

$$M = m_1 + m_2$$

$$\nu = \frac{\mu}{M}$$

$$\hat{H}_{\text{eff}} = \frac{H_{\text{eff}}}{\mu}$$

EXPLICIT FORM OF THE EOB EFFECTIVE HAMILTONIAN

The effective metric at 3PN:

$$ds^2_{\rm eff} = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

The coefficients are a ν-dependent deformation of the Schwarzschild ones

$$A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

Crucial EOB radial potential

Contribution at 3PN

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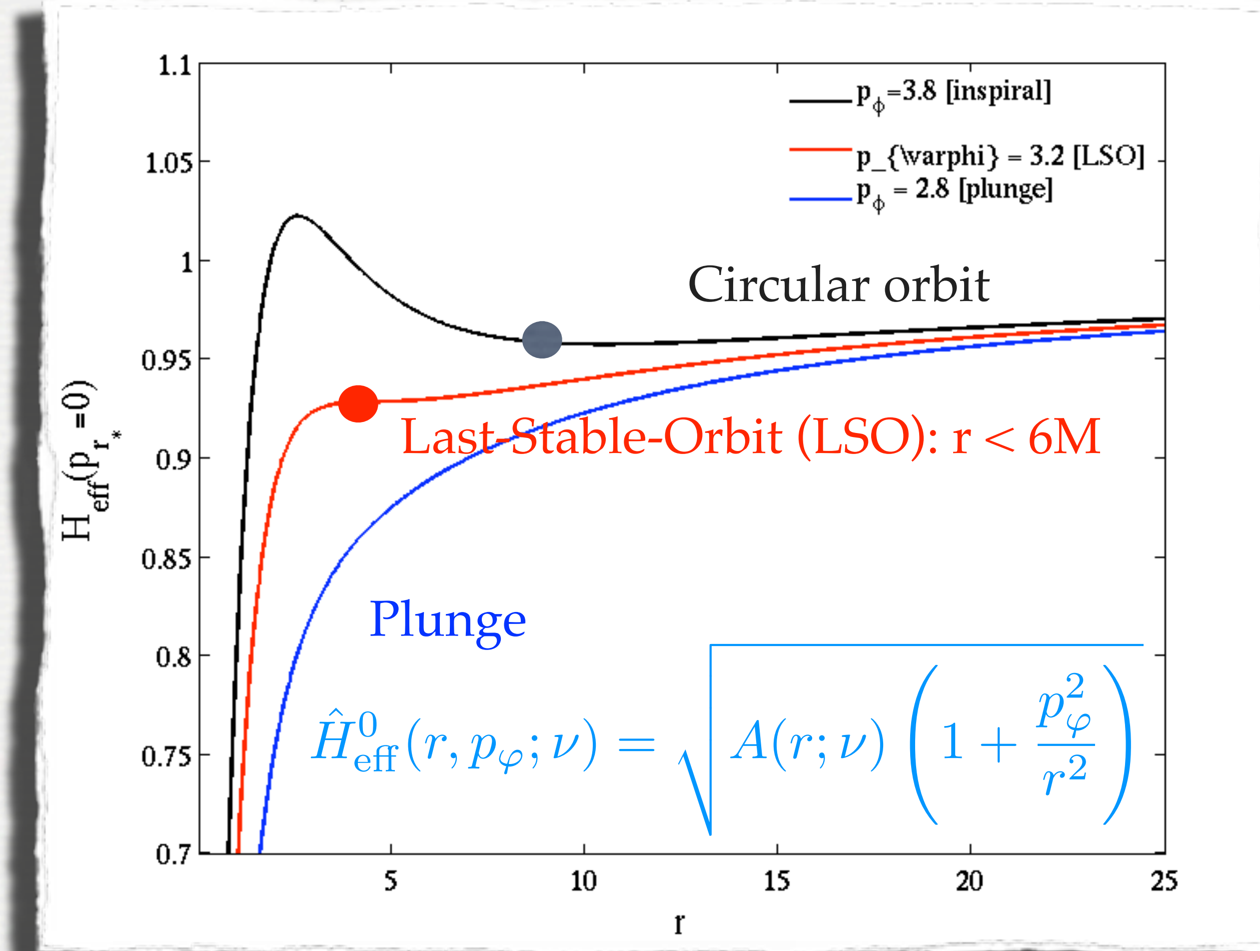
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi$$



- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Resummation multipole by multipole (Damour, Iyer & Nagar 2008, Damour & Nagar, 2009)

Plus horizon contribution [AN&Akcay2012]

WAVEFORM RESUMMATION PROCEDURE

- Resummation of the waveform multipole by multipole [Damour, AN, Iyer 2008]
- Factorized (multipolar) waveform at highest available PN order

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega}\sum_{\ell=2}^{\ell_{\max}}\sum_{m=1}^{\ell}(m\Omega)^2|Rh_{\ell m}^{(\epsilon)}|^2$$

Next-to-quasi-circular correction

$$h_{\ell m} \equiv h_{\ell m}^{(N,\epsilon)}\,\hat{h}_{\ell m}^{(\epsilon)}\,\hat{h}_{\ell m}^{\rm NQC}$$

Newtonian x PN x NQC

PN-correction

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{\rm eff}^{(\epsilon)}\,T_{\ell m}\,e^{i\delta_{\ell m}}\,\rho_{\ell m}^{\ell}$$

Remnant phase and modulus corrections: "improved" PN series

The "Tail factor"

$$T_{\ell m} = \frac{\Gamma(\ell+1-2i\hat{\hat{k}})}{\Gamma(\ell+1)}e^{\pi\hat{\hat{k}}}e^{2i\hat{\hat{k}}\ln(2kr_0)}$$

Resums an infinite number of leading logarithms in tail effects (hereditary contributions)

Effective source:
EOB (effective) energy (even-parity modes)
EOB angular momentum (odd-parity modes)

4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011,Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A^{\rm Taylor}_{\rm 5PN} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$a_5^{\log} = \frac{64}{5}$$
$$a_5^c = a^c_{5_0} + \nu a^c_{5_1}$$
$$a^c_{5_0} = -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma$$
$$a^c_{5_1} = -\frac{221}{6} + \frac{41}{32}\pi^2$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

5PN logarithmic term (analytically known)

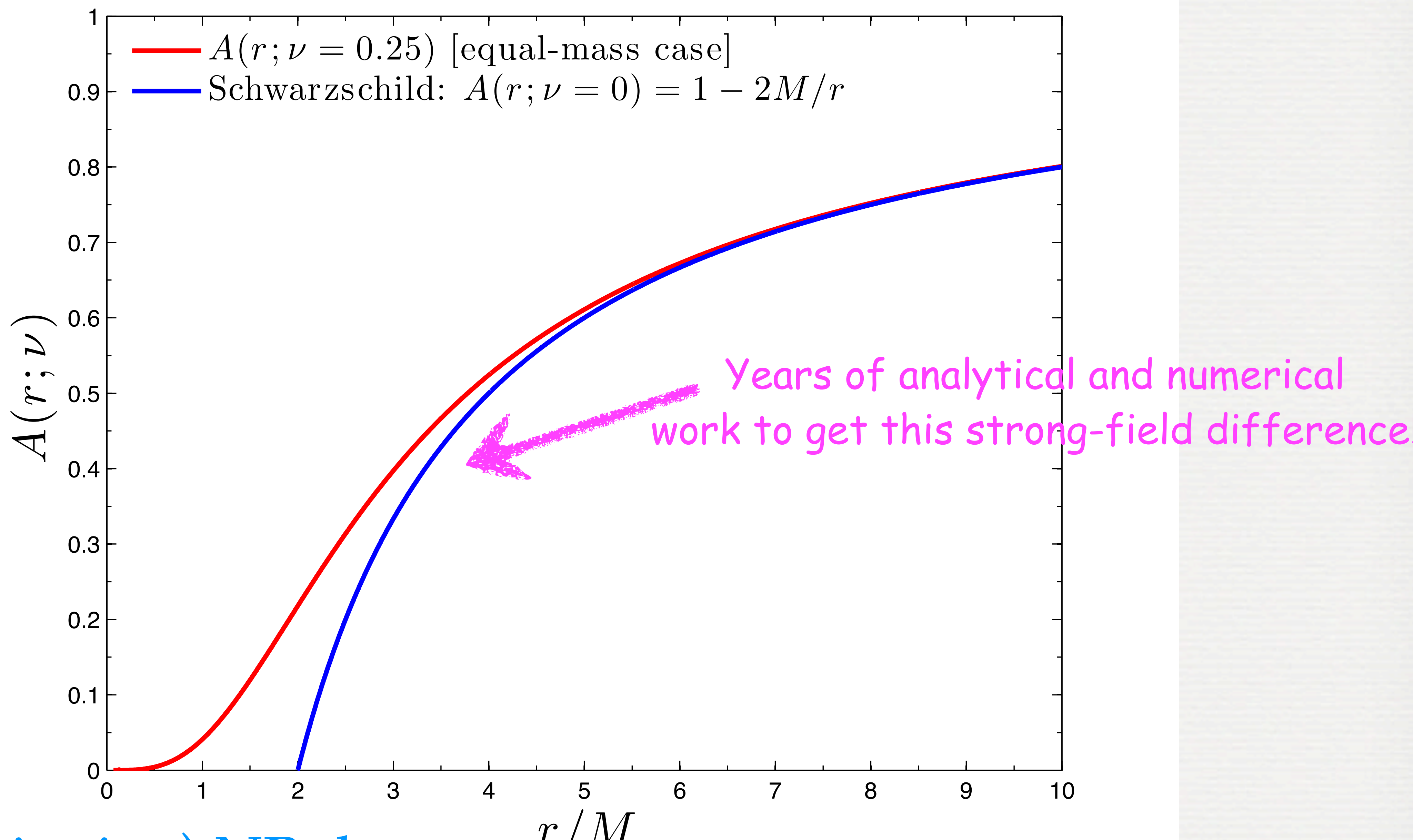
NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu,a_6^c) = P^1_5[A^{\rm Taylor}_{\rm 5PN}(u;\nu,a_6^c)]$$

THE EOB[NR] POTENTIAL

Effect of finite-mass correctios: system is more bound!

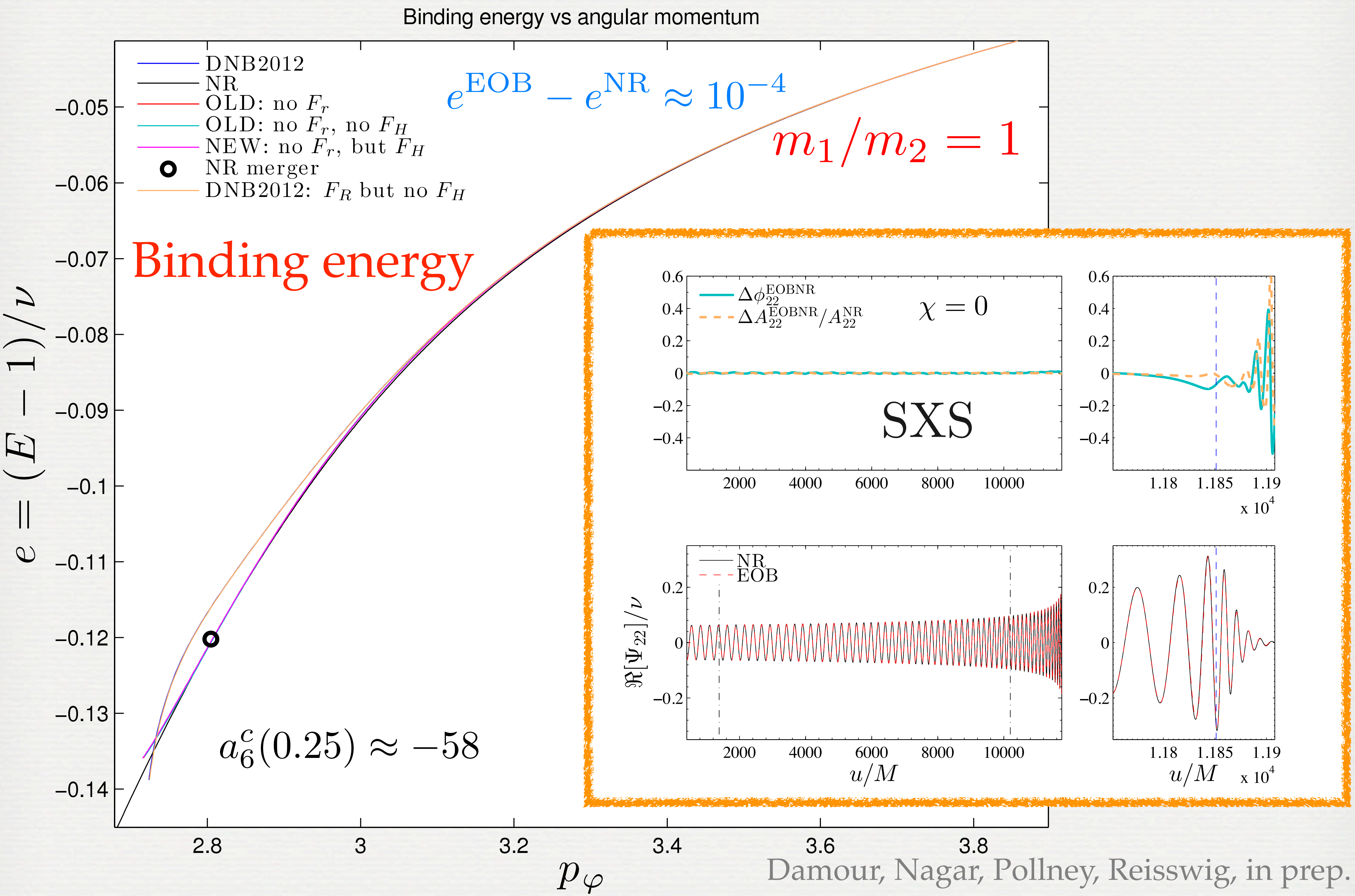


From (6 nonspinning) NR data sets one gets:

$$a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$$

Damour&Nagar 2014

RESULTS: DYNAMICS AND WAVEFORMS (NO SPIN)



Damour, Nagar, Pollney, Reisswig, in prep.

EOB APPROACH TO THE DYNAMICS OF TWO SPINNING BLACK HOLES

Damour01, Buonanno-Chen-Damour06, Damour-Jaranowski-Schafer08, Barausse&Buonanno10,Nagar11,Barausse&Buonanno2011,Taracchini et al. 12, Balmelli&Jetzer2013, Pan et al. 2013

Nonspinning case: EOB description = deformation of test-particle dynamics in a Schwarzschild background

Spinning case: EOB description = deformation of (spinning) test-particle dynamics in a Kerr background

Deformation parameters:

$\nu = \mu/M$ and "effective test spin" $\mathbf{S}^*$

Based on Hamiltonian formulation in the center of mass frame

SO & SS EFFECTS IN EOB HAMILTONIAN

New way of combining available knowledge within some Hamiltonian [Damour&Nagar, PRD 2014]

$$\hat{H}_{\rm eff} = \frac{g_S^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S} + \frac{g_{S^*}^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S}^* + \sqrt{A(1+\gamma^{ij}p_i p_j + Q_4(p))}$$

with the structure

$$g_S^{\rm eff} = 2 + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$g_{S^*}^{\rm eff} = \left(\frac{3}{2} + \text{test mass coupling}\right) + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$A = 1 - \frac{2}{r} + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$\gamma^{ij} = \gamma^{ij}_{\rm Kerr} + \nu(\text{PN corrections}) + \dots$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = M^2(X_1^2\chi_1 + X_2^2\chi_2) \qquad X_i = m_i/M$$

$$\mathbf{S}^* = \frac{m_2}{m1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2 = M^2\nu(\chi_1+\chi_2) \qquad -1 \le \chi_i \le 1$$

Particle: (μ, S_*) Kerr black-hole: (M, S)

$$H_{\rm Kerr} = H^{\rm Kerr}_{\rm orb} + H^{S}_{\rm SO}(\mathbf{S}) + H^{S_*}_{\rm SO}(\mathbf{S}_*)$$

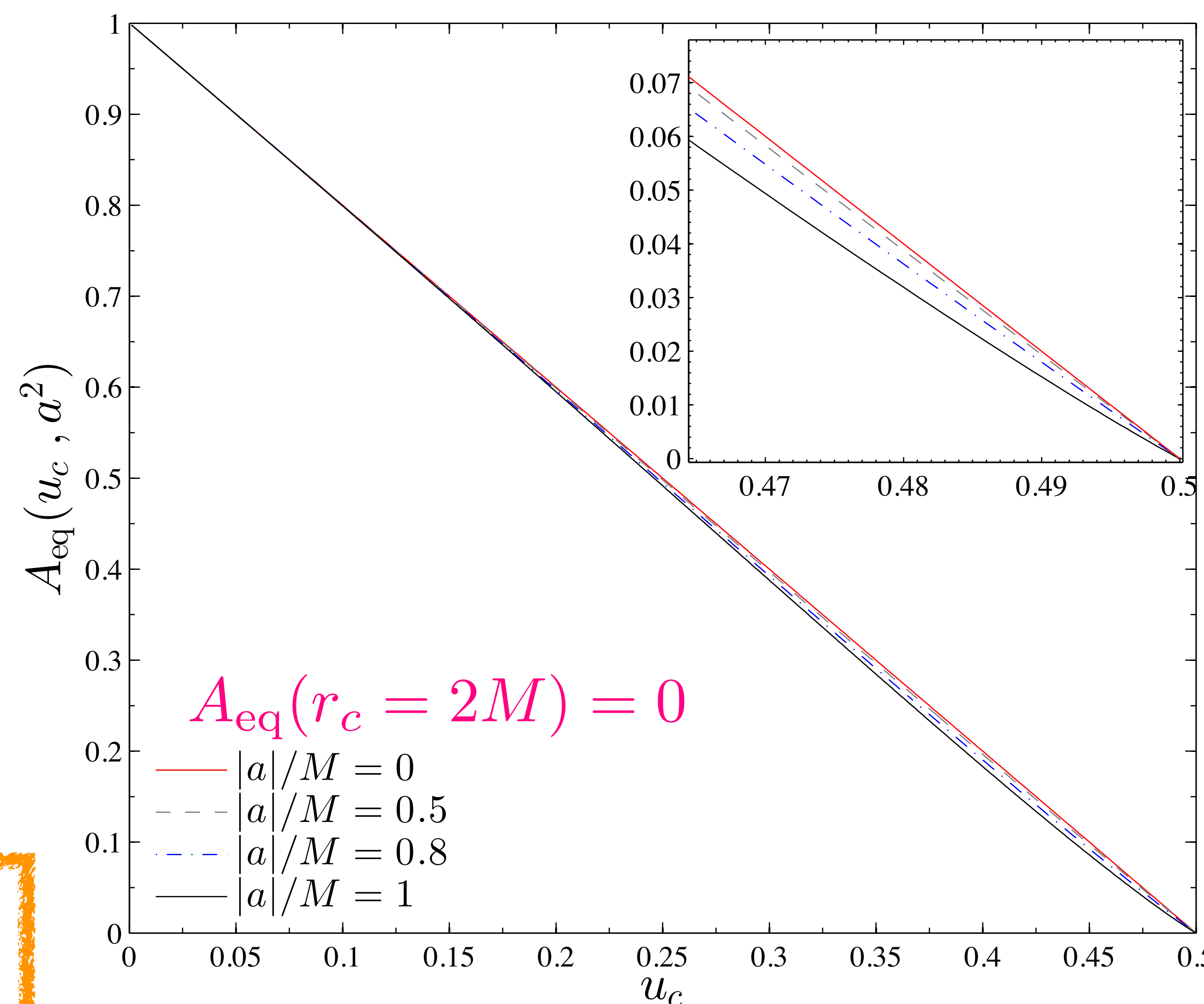
$$H^{\rm Kerr}_{\rm orb,eq}(r,p_r,p_\varphi) = \sqrt{A^{\rm eq}(r)\left(\mu^2 + \frac{p_\varphi^2}{r_c^2} + \frac{p_r^2}{B^{\rm eq}(r)}\right)}.$$

$$A_{\rm eq}(r) \equiv \frac{\Delta(r)}{r_c^2} = \left(1 - \frac{2M}{r_c}\right)\frac{1+\frac{2M}{r_c}}{1+\frac{2M}{r}}$$

centrifugal radius

$$r_c^2 = r^2 + a^2 + \frac{2Ma^2}{r}$$

EOB:Identify a similar centrifugal radius in the comparable mass case and devise a similar deformation of A



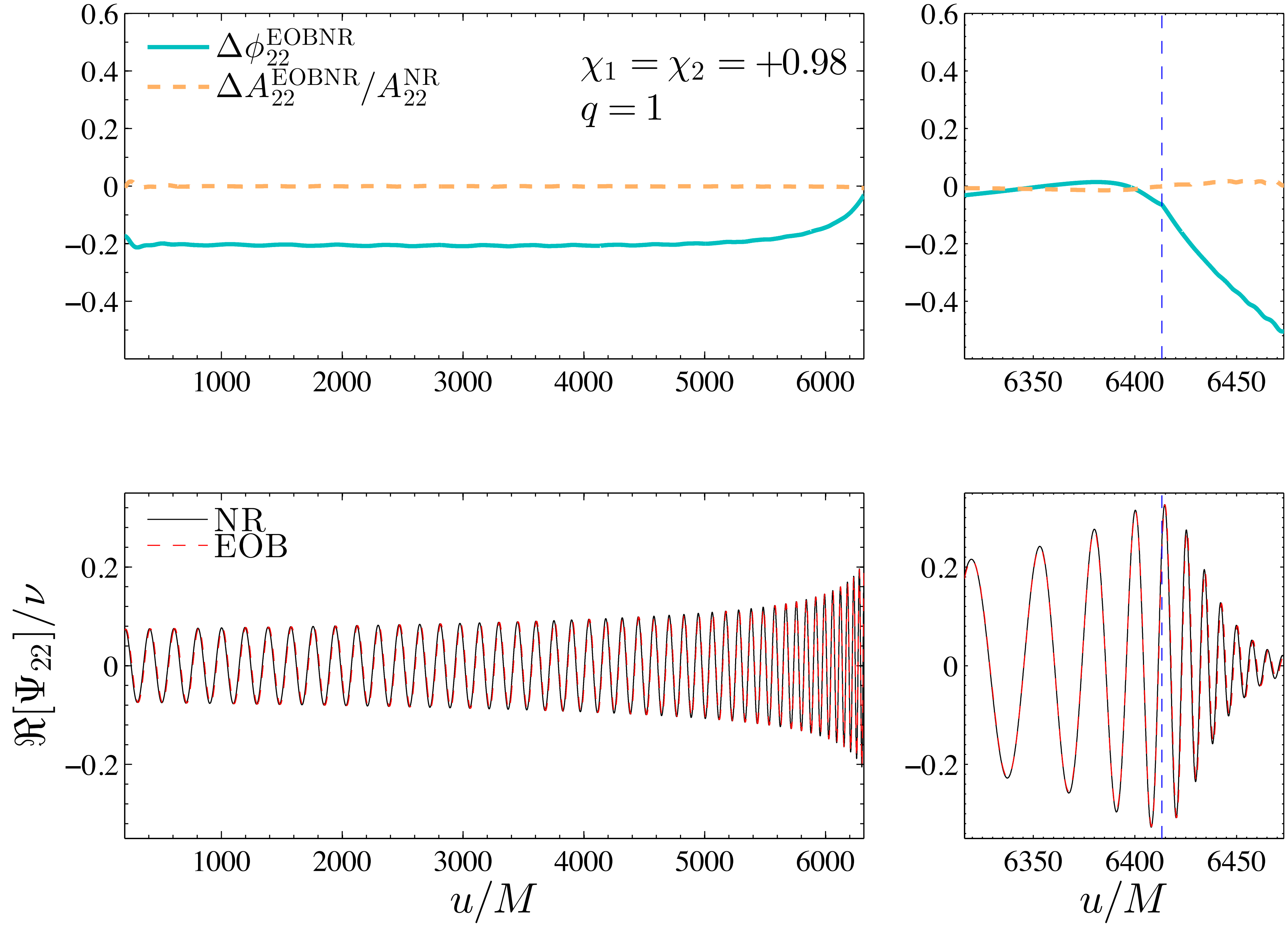
Damour&Nagar, PRD 2014

SPIN-ALIGNED WAVEFORMS

$m_1 = m_2$

$\chi_1 = \chi_2 = +0.98$

NR phase error ≈ 1 *rad*



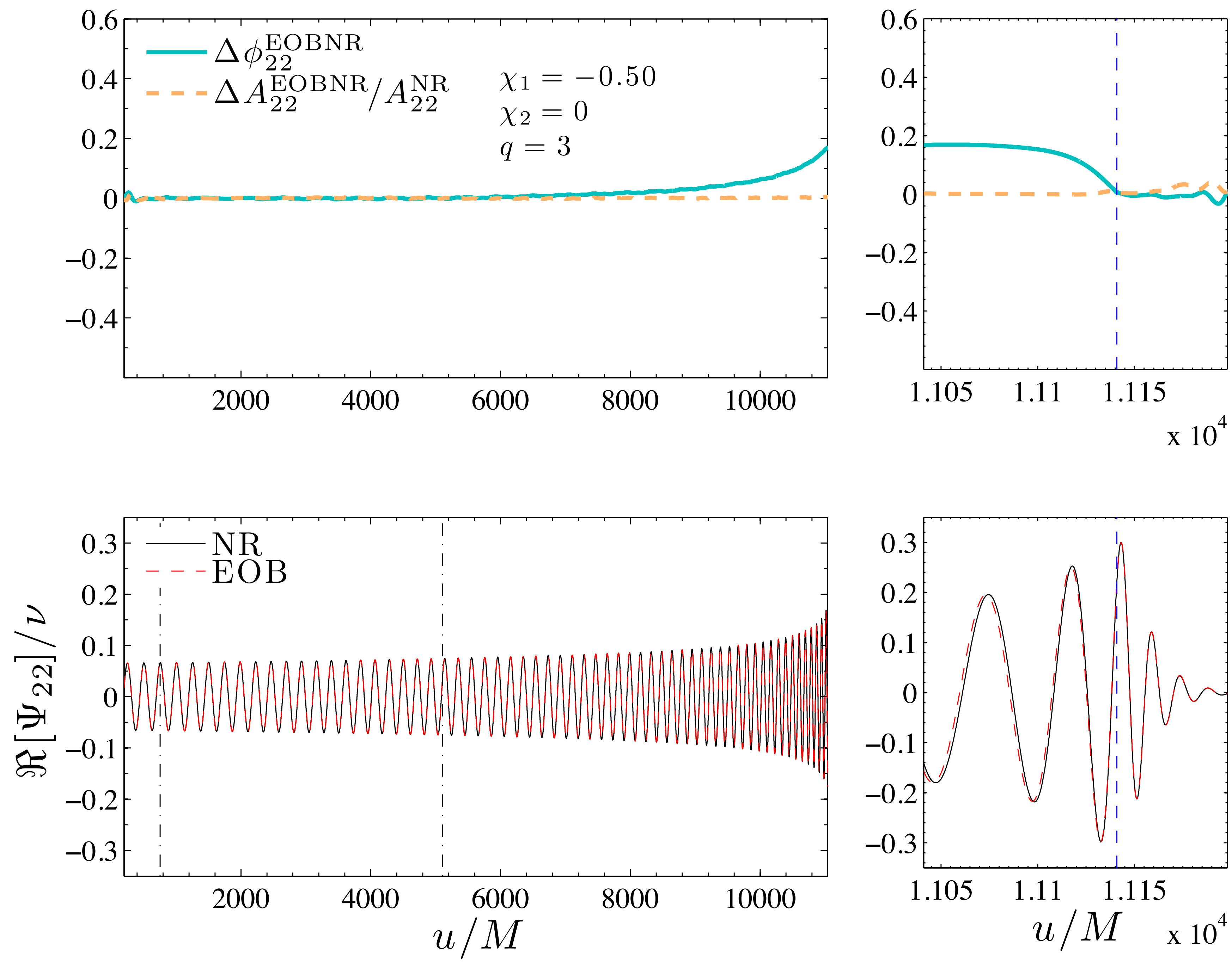
A. Nagar -IHES 2014

Damour&Nagar, in prep.

SPIN-ALIGNED WAVEFORMS

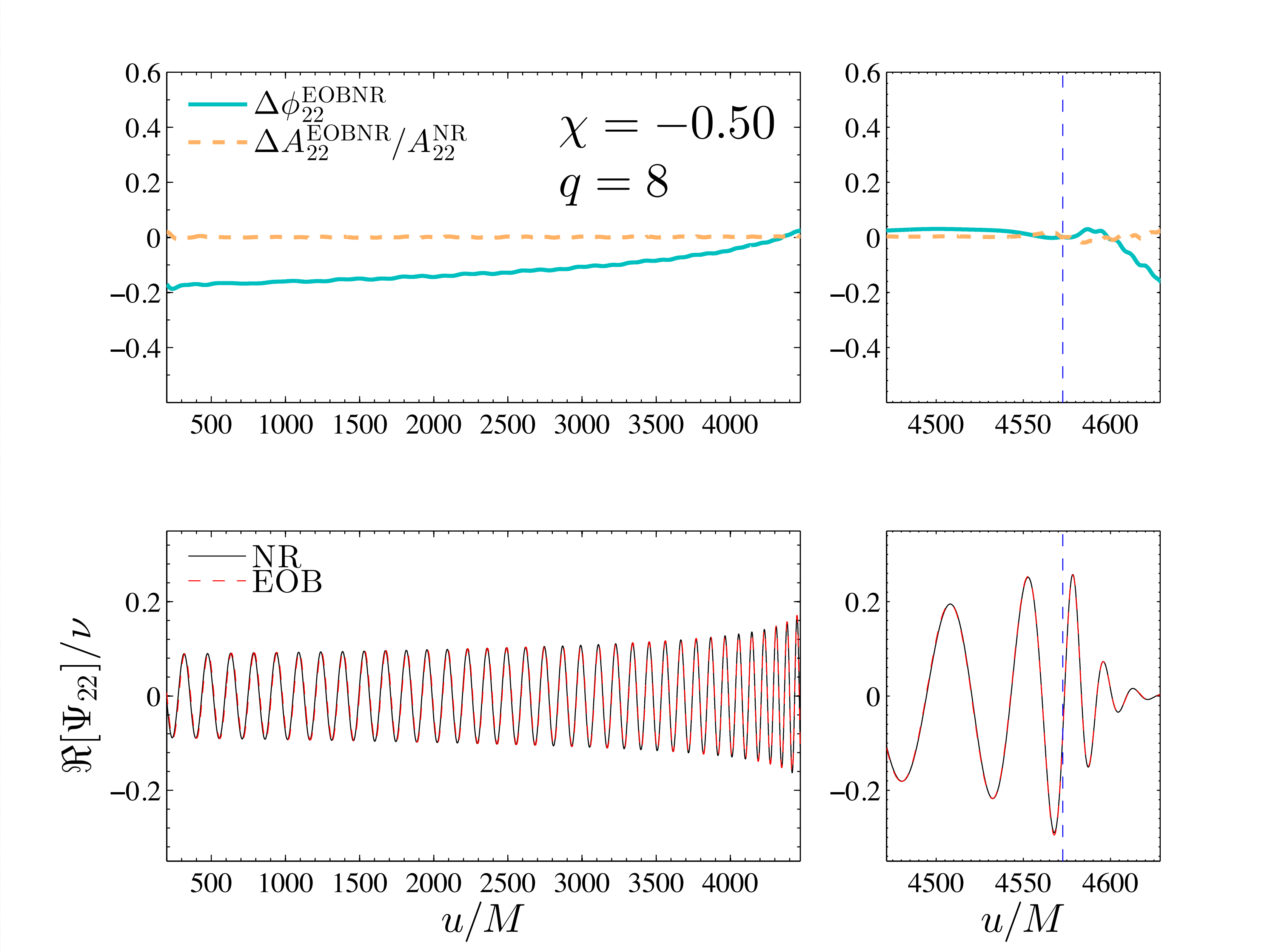
$m_1 = 3m_2$ error $\approx 0.05\,\mathrm{rad}$

$\chi_1 = -0.5; \quad \chi_2 = +0.0$



Damour&Nagar, in prep.

SPIN-ALIGNED WAVEFORMS



error ≈ 0.3 rad

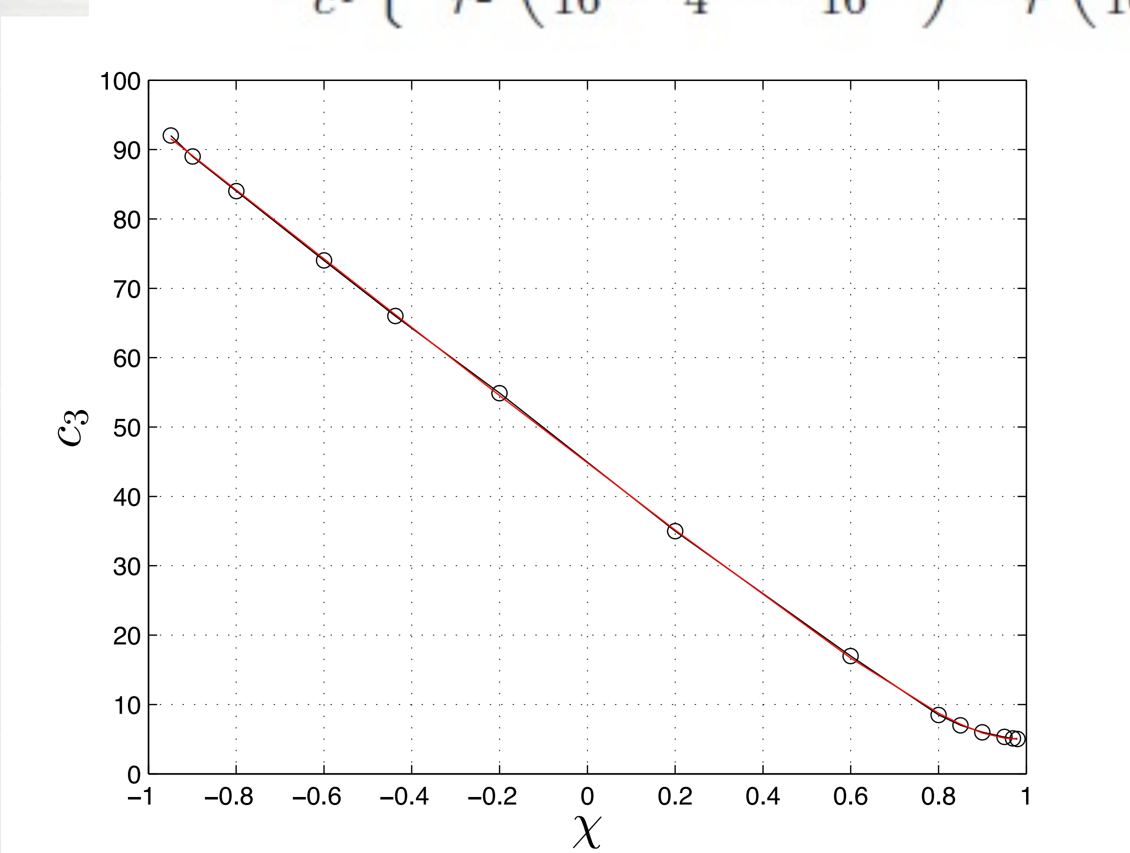
Need to tune a single effective (spin-dependent) 3.5PN spin-orbit coupling parameter using equal-spin, equal-mass BBH data

GYRO-GRAVITOMAGNETIC RATIOS

$$\hat{H}_{\rm SO}^{\rm eff} = G_S \mathbf{L}\cdot\mathbf{S} + G_{S^*}\mathbf{L}\cdot\mathbf{S}^* \qquad G_S = \frac{1}{r^3} g_S^{\rm eff}, \quad G_{S^*} = \frac{1}{r^3} g_{S^*}^{\rm eff}$$

$$g_S^{\rm eff} = 2 + \frac{1}{c^2}\left\{-\frac{1}{r}\frac{5}{8}\nu - \frac{33}{8}(\mathbf{n}\cdot\mathbf{p})^2\right\}$$
$$+\frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{51}{4}\nu + \frac{\nu^2}{8}\right) + \frac{1}{r}\left(-\frac{21}{2}\nu + \frac{23}{8}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2 + \frac{5}{8}\nu(1+7\nu)(\mathbf{n}\cdot\mathbf{p})^4\right\}, \quad + \frac{1}{c^6}\frac{\nu c_3}{r^3}$$

$$g_{S^*}^{\rm eff} = \frac{3}{2} + \frac{1}{c^2}\left\{-\frac{1}{r}\left(\frac{9}{8} + \frac{3}{4}\nu\right) - \left(\frac{9}{4}\nu + \frac{15}{8}\right)(\mathbf{n}\cdot\mathbf{p})^2\right\}$$
$$+\frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{27}{16} + \frac{39}{4}\nu + \frac{3}{16}\nu^2\right) + \frac{1}{r}\left(\frac{69}{16} - \frac{9}{4}\nu + \frac{57}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2 + \left(\frac{35}{16} + \frac{5}{2}\nu + \frac{45}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^4\right\} + \frac{1}{c^6}\frac{\nu c_3}{r^3}$$



$$c_3 = c_0 \frac{1 + n_1\chi}{1 + d_1\chi + d_2\chi^2}$$

BBH TAKE-AWAY

1. New way of blending finite-mass and spin effect in an EOB Hamiltonian based on the structure of the Hamiltonian of a (spinning) particle on a Kerr background

2. Analytical freedom: only two flexibility parameters that are extracted from NR data as simple (separate) functions of symmetric mass ratio and spin magnitude

3. Compatibility (within NR errors) between such EOBNR model and state-of-the art NR data over mass ratio and spin (no precession)

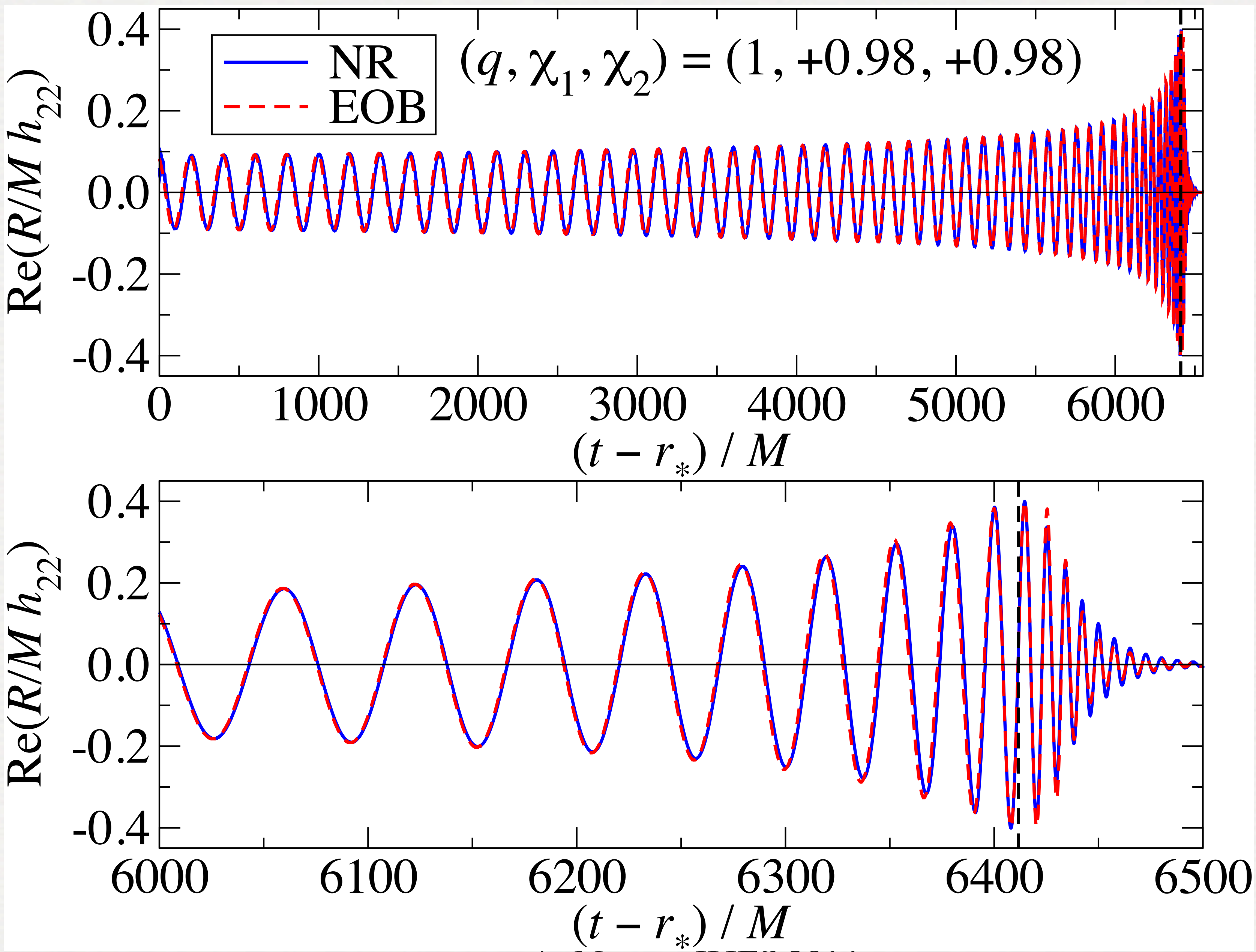
LUCKILY, WE'RE NOT ALONE IN THE EOB[NR]

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

Different choices for the analytic freedom (as well as spin gauge)

From Pan, Buonanno, Taracchini et al., 1307.6232

Taracchini, Buonanno et al., PRD **89**, 061502 (R), 2014



AND PRECESSION

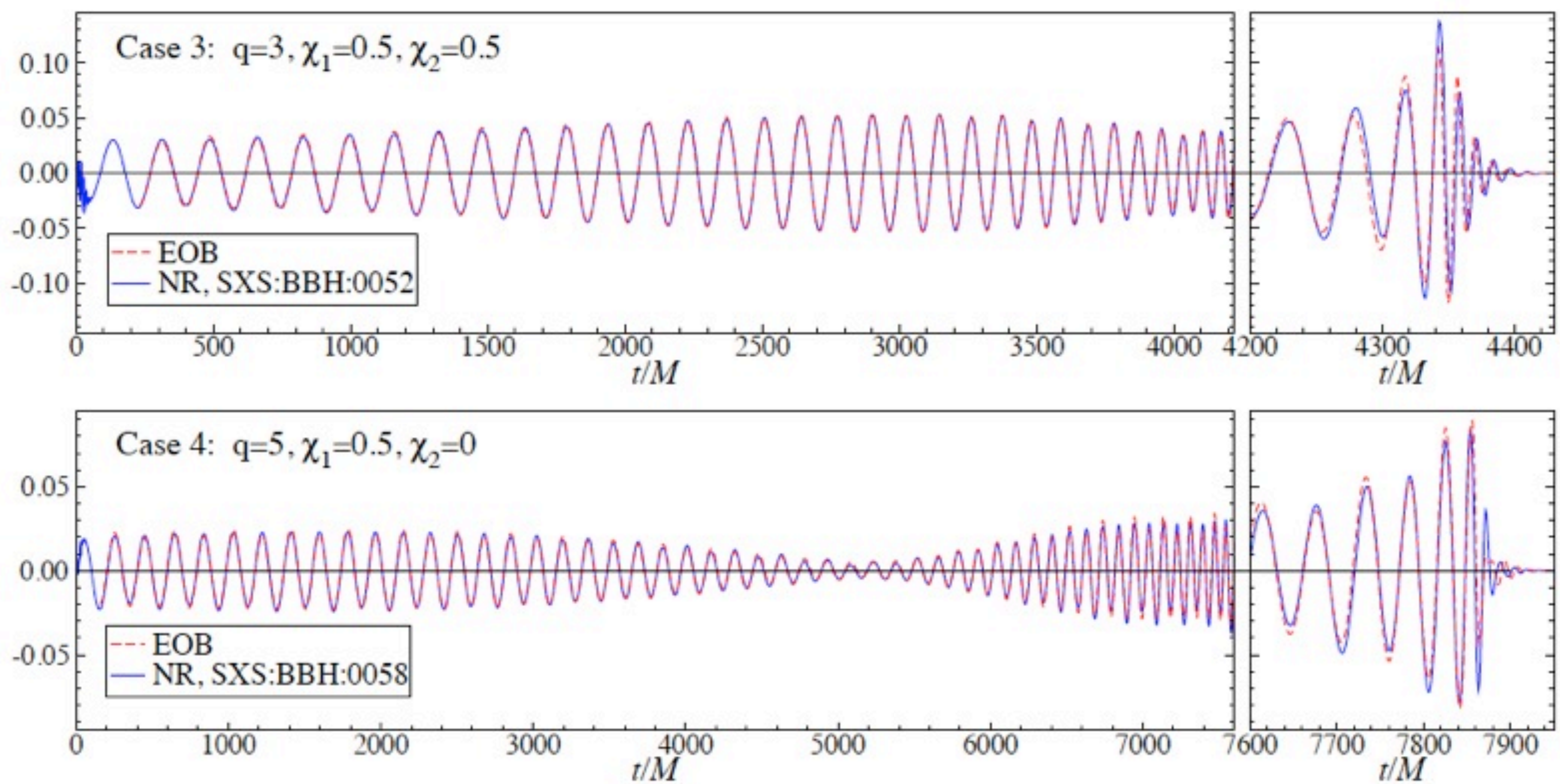
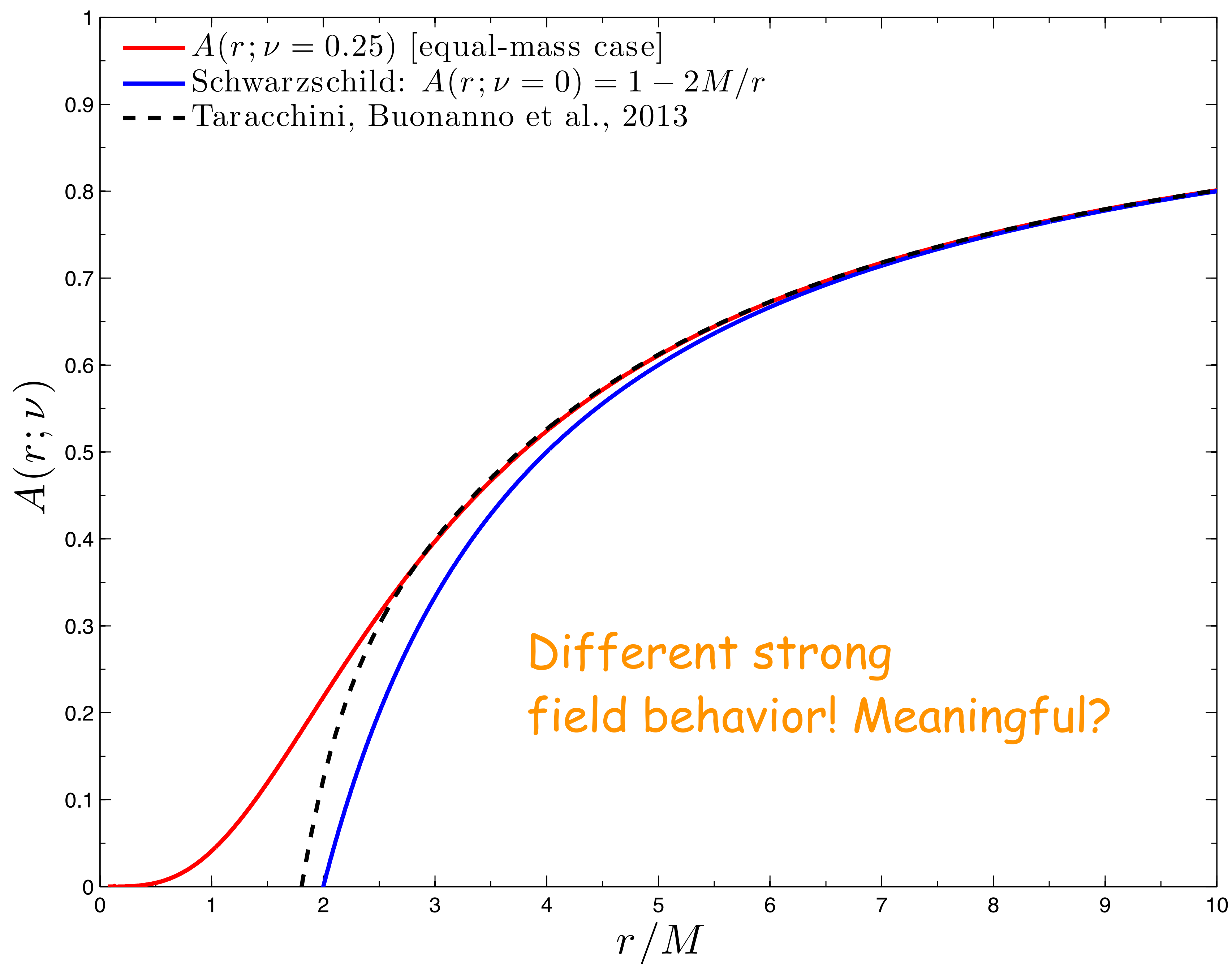


FIG. 9: We show for cases 3 and 4 of Table I the GW polarization h_+, containing contributions from $\ell = 2$ modes, that propagates along a direction $\hat{N}$ specified by spherical coordinates $\theta = \pi/3$ and $\phi = \pi/2$ associated with the inertial source frame $\{e_1^S, e_2^S, e_3^S\}$. The EOB waveforms start at the after-junk-radiation times of $t = 230M$ and $t = 160M$, respectively.

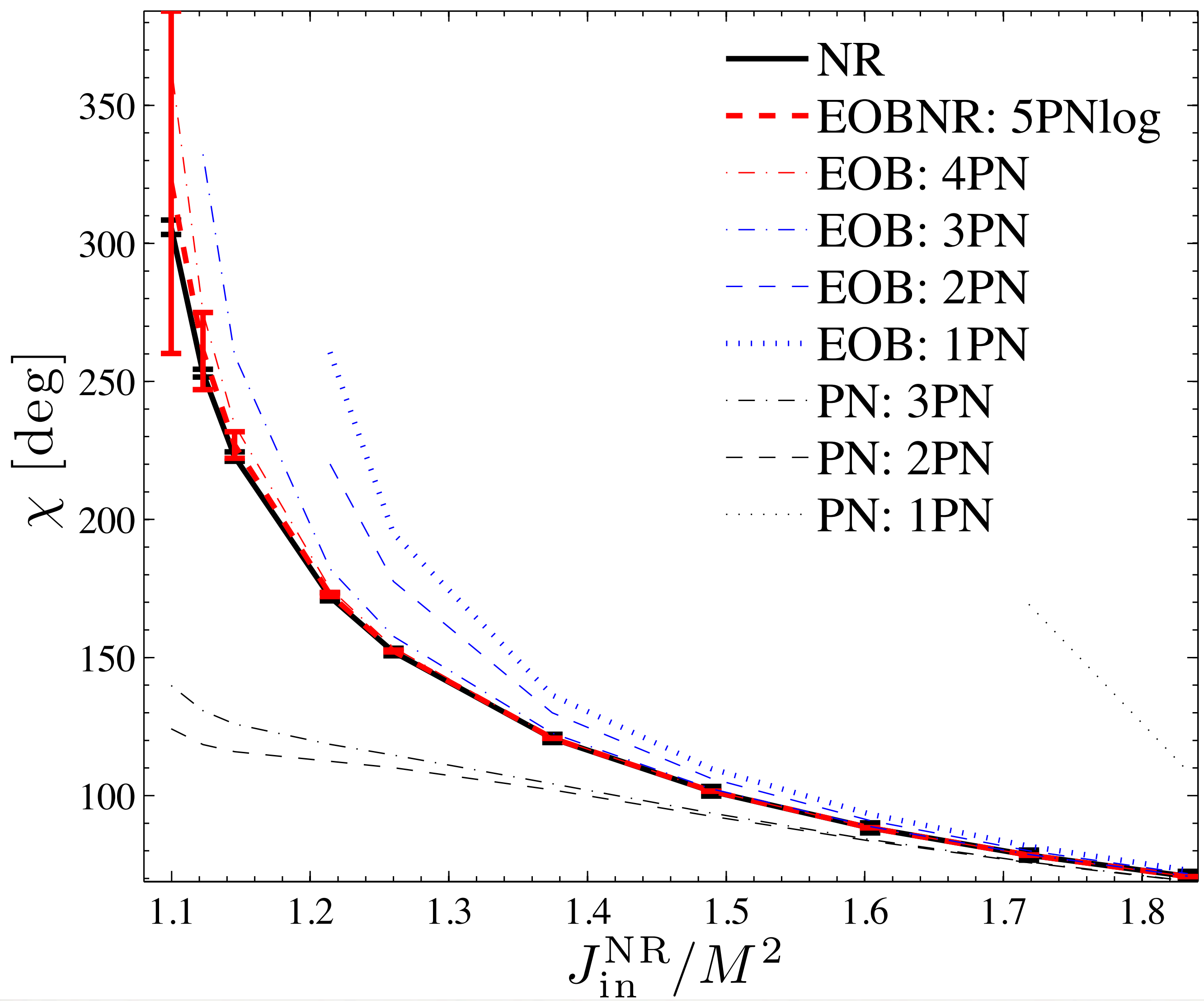
A FUNCTION?

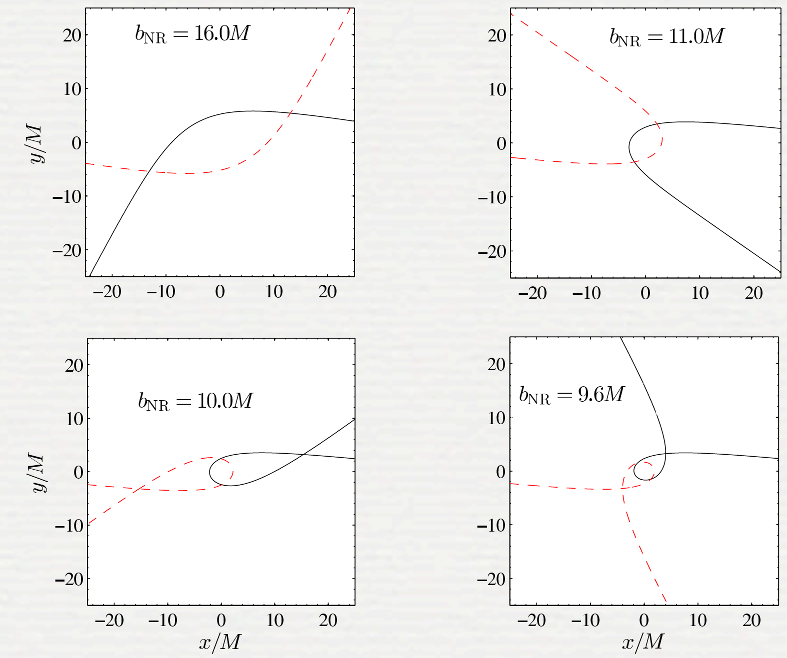
The two models perform similarly; are based on (essentially) the same PN analytical information but have considerable structural differences

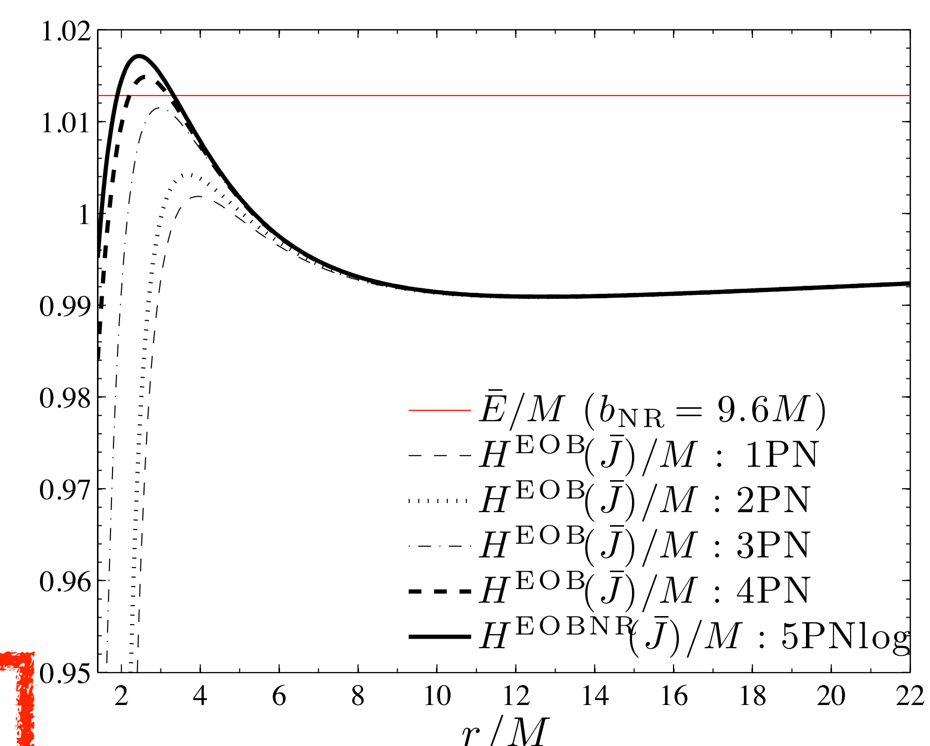


STRONG FIELD: EOB/NR SCATTERING ANGLE

Damour, Guercilena, Hinder, Hopper,
Nagar and Rezzolla, PRD **89**, 081503 (R), 2014







NR uncertainties on scattering angles are still large to firmly distinguish one A function to the other.

Damour&Nagar, wip

venerdì 31 ottobre 2014

BNS:ANALYTICAL NEEDS

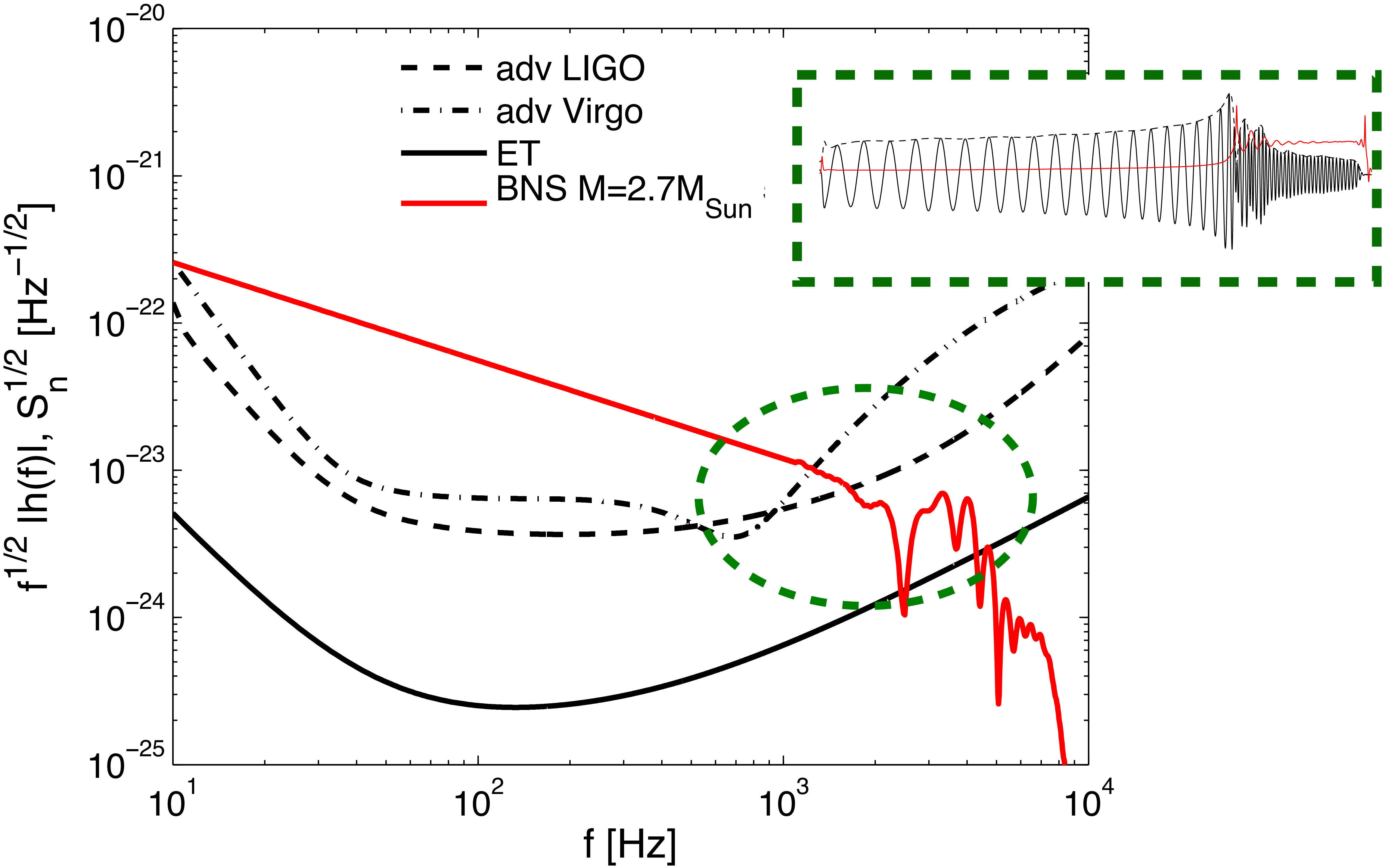
- Study the response of each neutron star to the tidal field of the companion [theory of relativistic Love numbers (i.e. tidal polarizability coefficients) + tidal corrections to dynamics (beyond Newtonian accuracy)]

- Incorporate the corresponding tidal effects within a theoretical framework able to describe the gravitational wave signal emitted by inspiralling compact binaries (possibly up to merger): PN-expanded description vs EOB-resummed description.

- Compare analytical models against NR simulations, possibly calibrating high-order tidal corrections if needed

- Assess the measurability of tidal effects within the signal seen by interferometric detectors

BNS: EOS detection



$$M_{ij} = \mu_2 G_{ij}$$

EOS detection by measuring the tidal polarization coefficient: the Love number

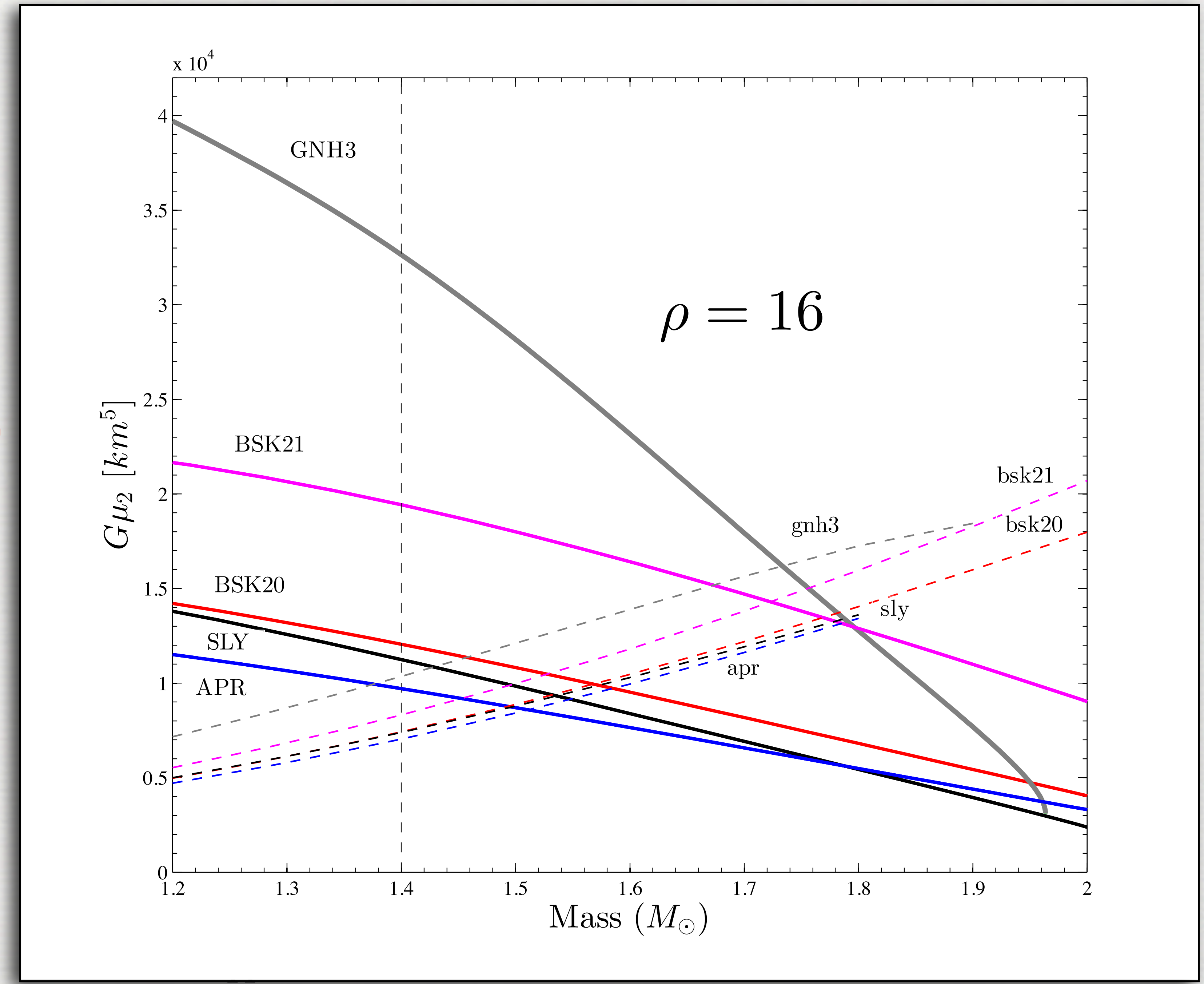
MEASURING LOVE NUMBERS

Common thinking: difficult to get EOS information from BNS (early) inspiral BNS waveforms.
Conclusion: advanced LIGO will be unable to say anything on NS EOS
(Hinderer et al, 2009)

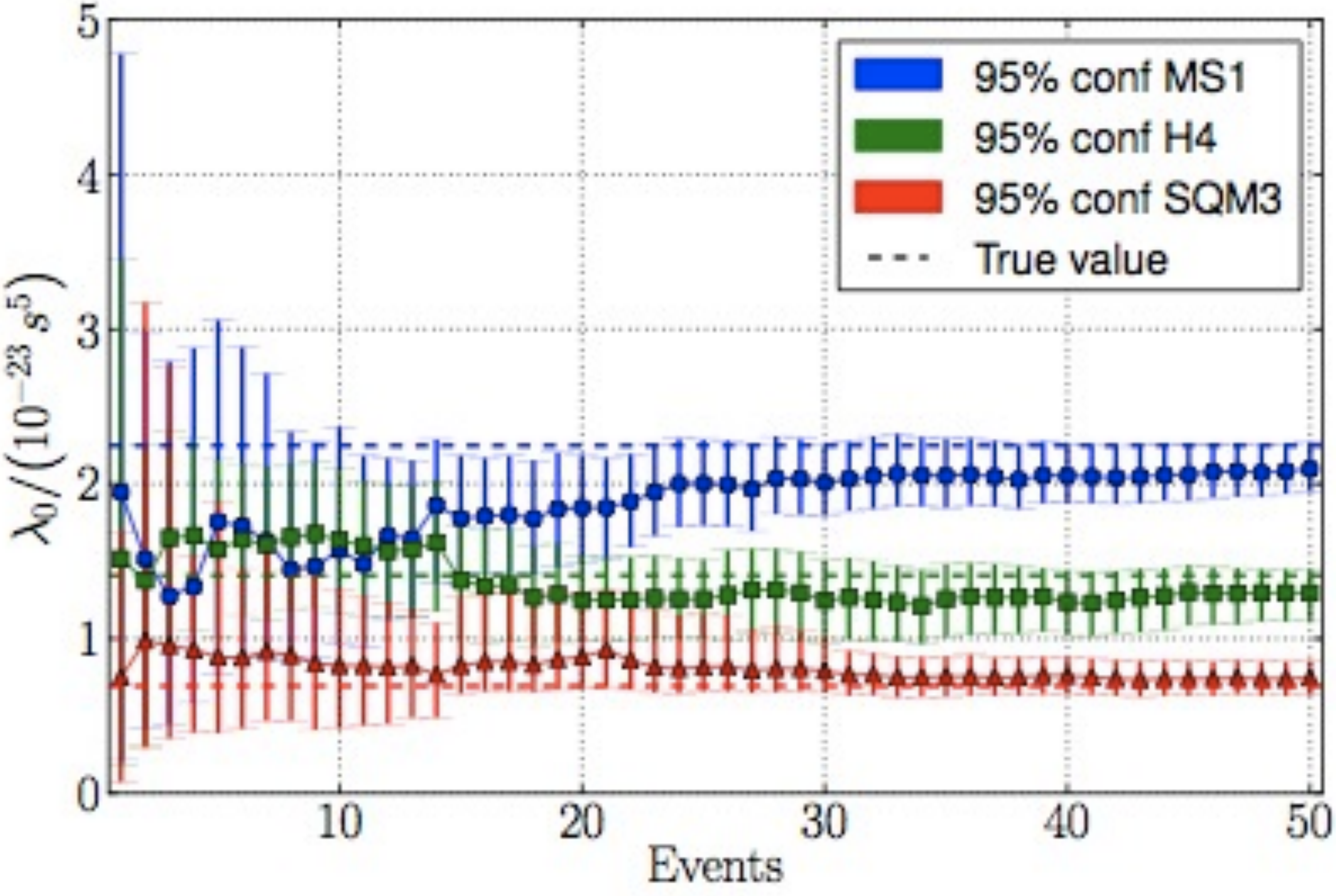
NEW IMPORTANT RESULT (Damour ,Nagar, Villain 2012)

Tidal polarizability parameters *can actually be measured* by adv LIGO with a reasonable SNR=16 (5 events per year)

Use EOB controlled, accurate, description of the phasing up to BNS merger!



[Del Pozzo+ 2013]



1PN tidal F2 to LSO, 2σ, Bayesian analysis/realistic DA

TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: nonminimal worldline couplings

$$\Delta S_{\rm nonminimal} = \sum_A \frac{1}{4}\mu_2^A \int ds_A \left(u^\mu u^\nu R_{\mu\alpha\nu\beta}\right)^2 + \dots$$

Damour&Esposito-Farèse96, Goldberger&Rothstein06, TD&AN09

Relativistic Love number

Modifications of the EOB effective metric...

$$A(r) = A_r^0 + A^{\rm tidal}(r)$$
$$A^{\rm tidal}(r) = -\kappa_2^T u^6 \left(1 + \bar\alpha_1 u + \bar\alpha_2 u^2 + \dots\right) + \dots$$

And tidal modifications of GW waveform & radiation reaction

- Need analytical theory for computing $\mu_2,\ \kappa_2^T,\ \bar\alpha_1 \dots$
- (?)Need accurate NR simulations to "calibrate" the higher-order PN tidal contributions, that may be quite important during the late inspiral

LOVE NUMBERS IN GENERAL RELATIVITY

Relativistic star in an external gravito-electric & gravito-magnetic (multipolar) tidal field

The star acquires induced gravito-electric and gravito-magnetic multipole moments

Linear tidal polarization

Induced multipole moments

$$M_L^{(A)} = \mu_\ell^A G_L^{(A)}$$
$$S_L^{(A)} = \sigma_\ell^A H_L^{(A)}$$

External multipolar field

$$G\mu_\ell = [length]^{2\ell+1}$$
$$G\sigma_\ell = [length]^{2\ell+1}$$

$$2k_\ell \equiv (2\ell-1)!!\frac{G\mu_\ell}{R^{2\ell+1}}$$

$$j_\ell \equiv (2\ell-1)!!\frac{4(\ell+2)}{\ell-1}\frac{G\sigma_\ell}{R^{2\ell+1}}$$

Dimensionless relativistic "second" Love numbers

Actual calculation based on star perturbation theory: Love numbers are obtained as boundary conditions (matching interior to exterior perturbations)

RELATIVISTIC LOVE NUMBERS (POLYTROPIC EOS)

"rest-mass polytrope" (solid lines)

$$p = K\mu^\gamma$$
$$e = \mu + \frac{p}{\gamma - 1}$$

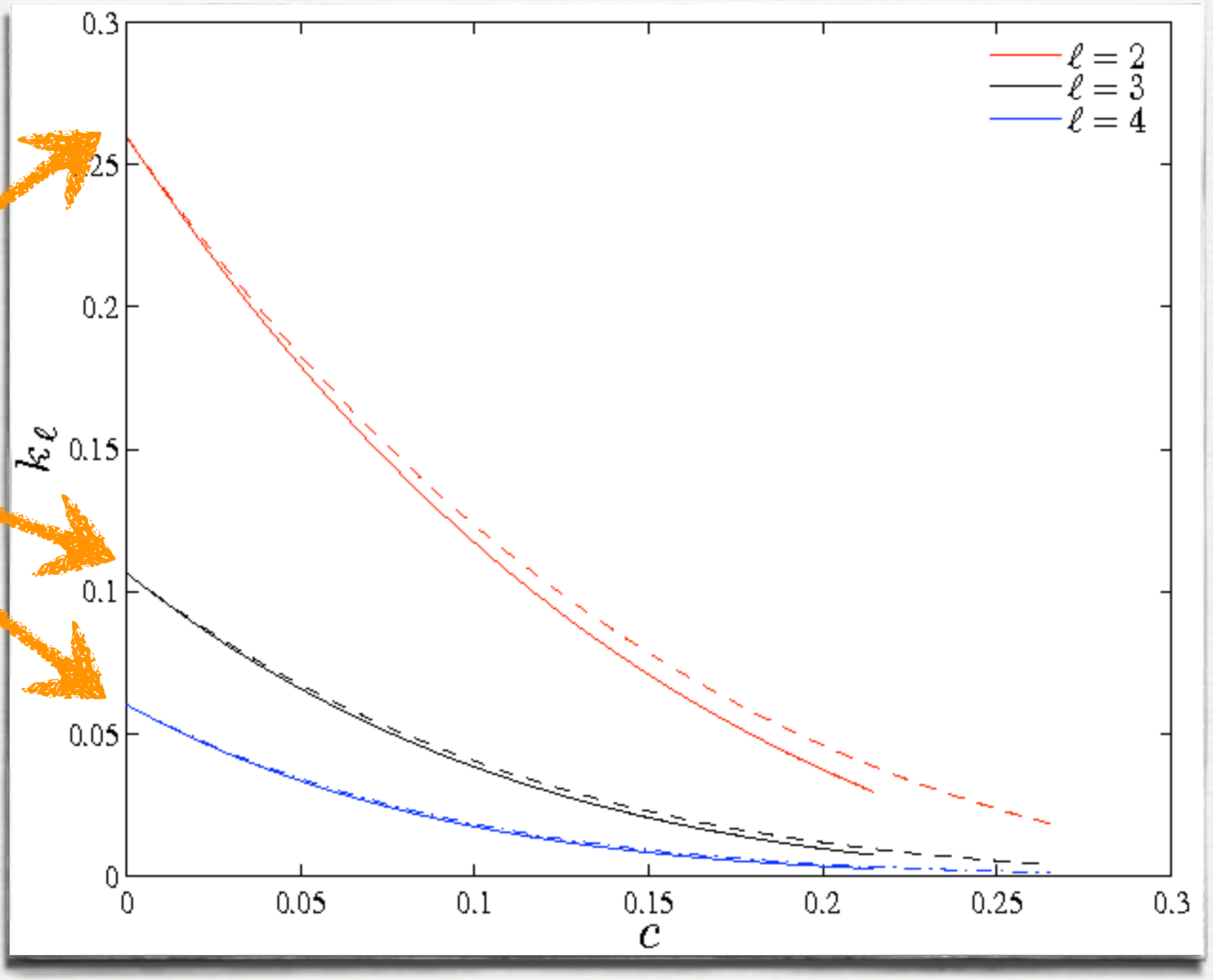
"energy polytrope" (dashed lines)

$$p = K e^\gamma$$

Tidal polarization parameters

$$M_L^{(A)} = \mu_\ell^A G_L^{(A)}$$
$$2k_\ell \equiv (2\ell - 1)!! \frac{G\mu_\ell}{R^{2\ell+1}}$$

Newtonian values



Newtonian values

Relativistic values

A. Nagar -IHES 2014

TIDAL INTERACTION POTENTIAL

Central tidal "coupling constant":

$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$X_{A,B} \equiv M_{A,B}/M$

$M_A = M_B$
$\ell = 2$
$k_2^A = k_2^B$
$\Rightarrow \kappa_2^T = \frac{1}{8}\frac{k_2}{\mathcal{C}^5}$

Function of: masses, compactnesses and relativistic Love numbers

In the dynamics:

$\kappa_2^T \sim 100$

$$A(u) = A^0(u) + A^{\rm tidal}$$

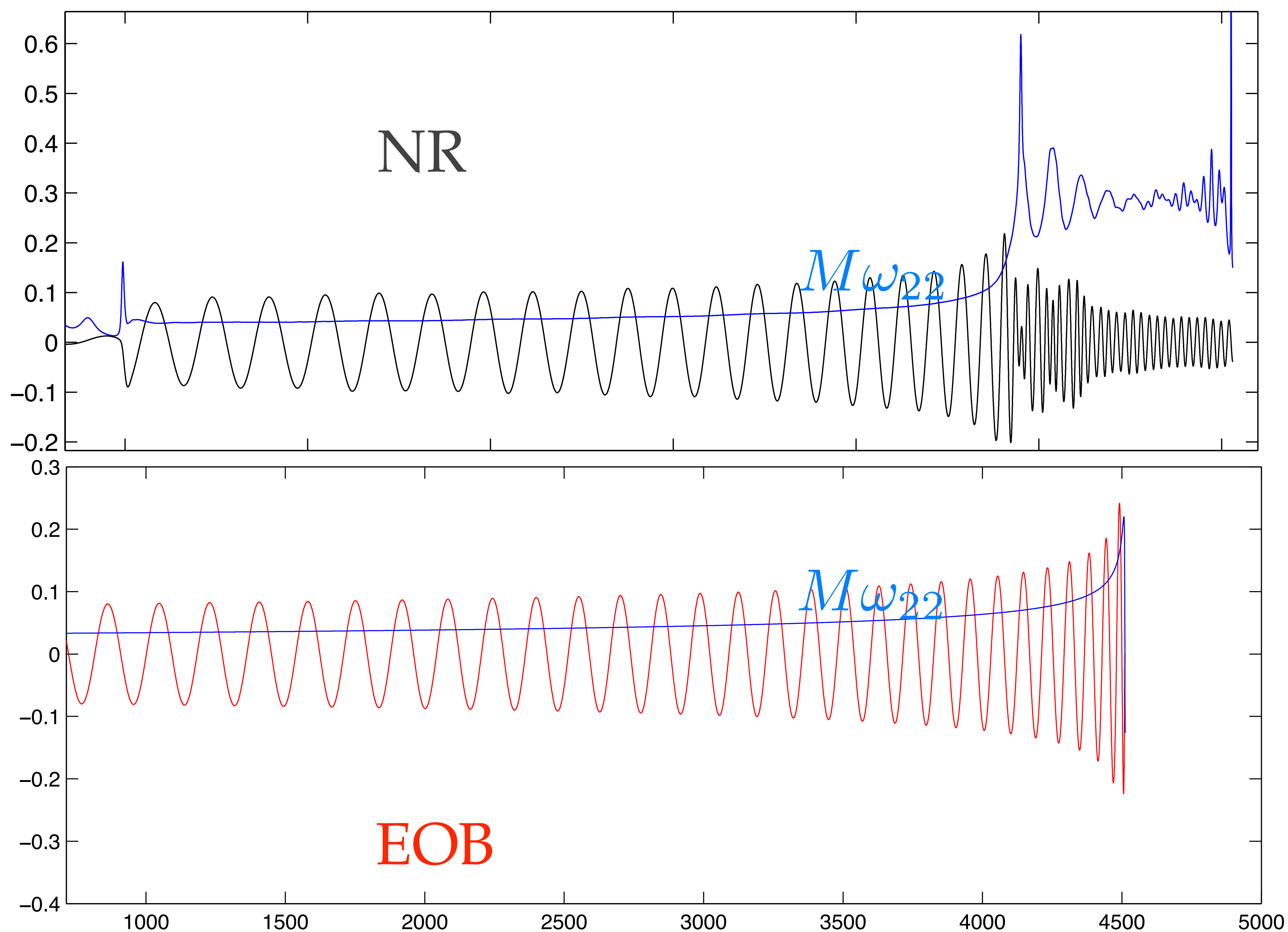
$$A^{\rm tidal} = \sum_{\ell\geq 2} -\kappa_\ell^T u^{2\ell+2}\hat{A}_\ell^{\rm tidal}(u)$$

"Newtonian" (LO) part
+ PN corrections (NLO, NNLO, ...)

NLO & NNLO tidal PN corrections known analytically
[Bini, Damour& Faye 2011]

$$\hat{A}_2^{\rm tidal} = 1 + \frac{5}{4}u + \frac{85}{14}u^2$$

LOVE IS WHEN EOBTIDAL MEETS NR



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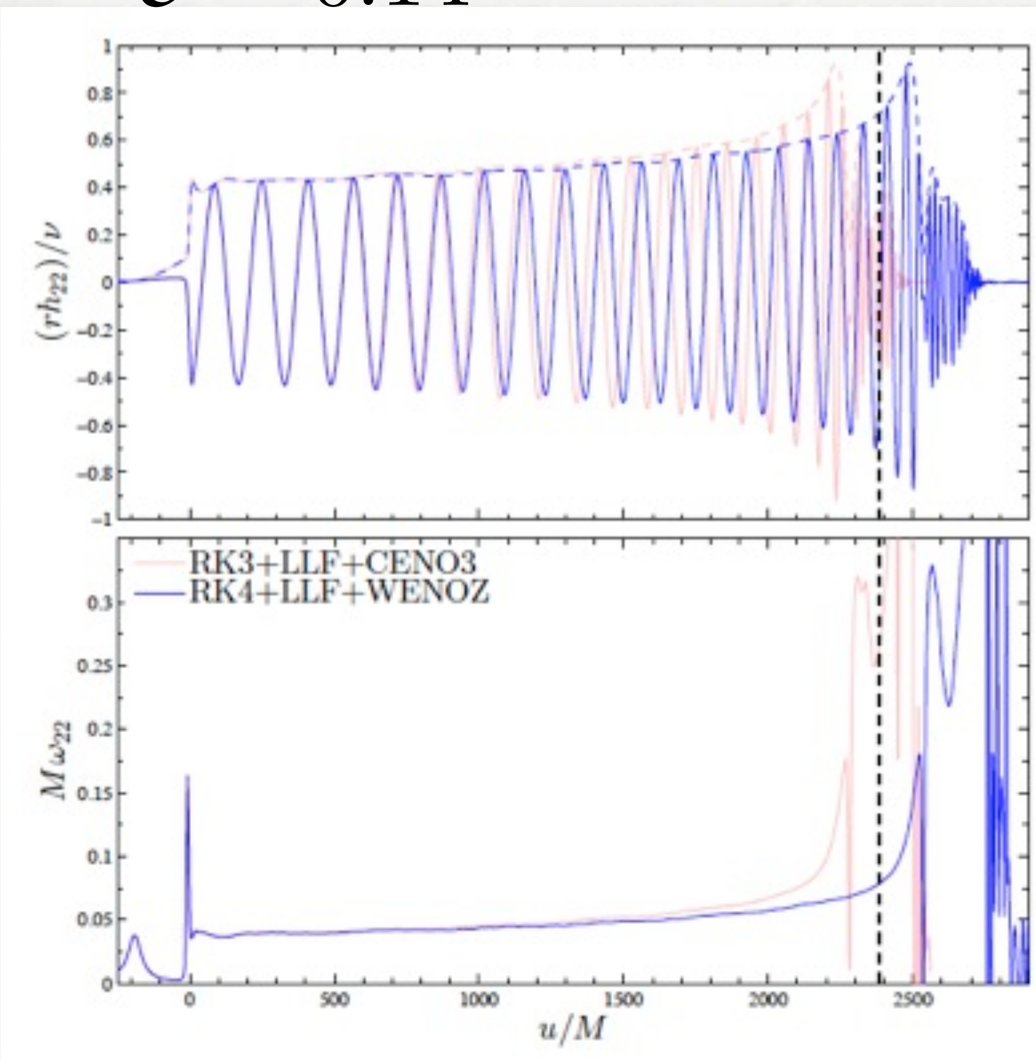
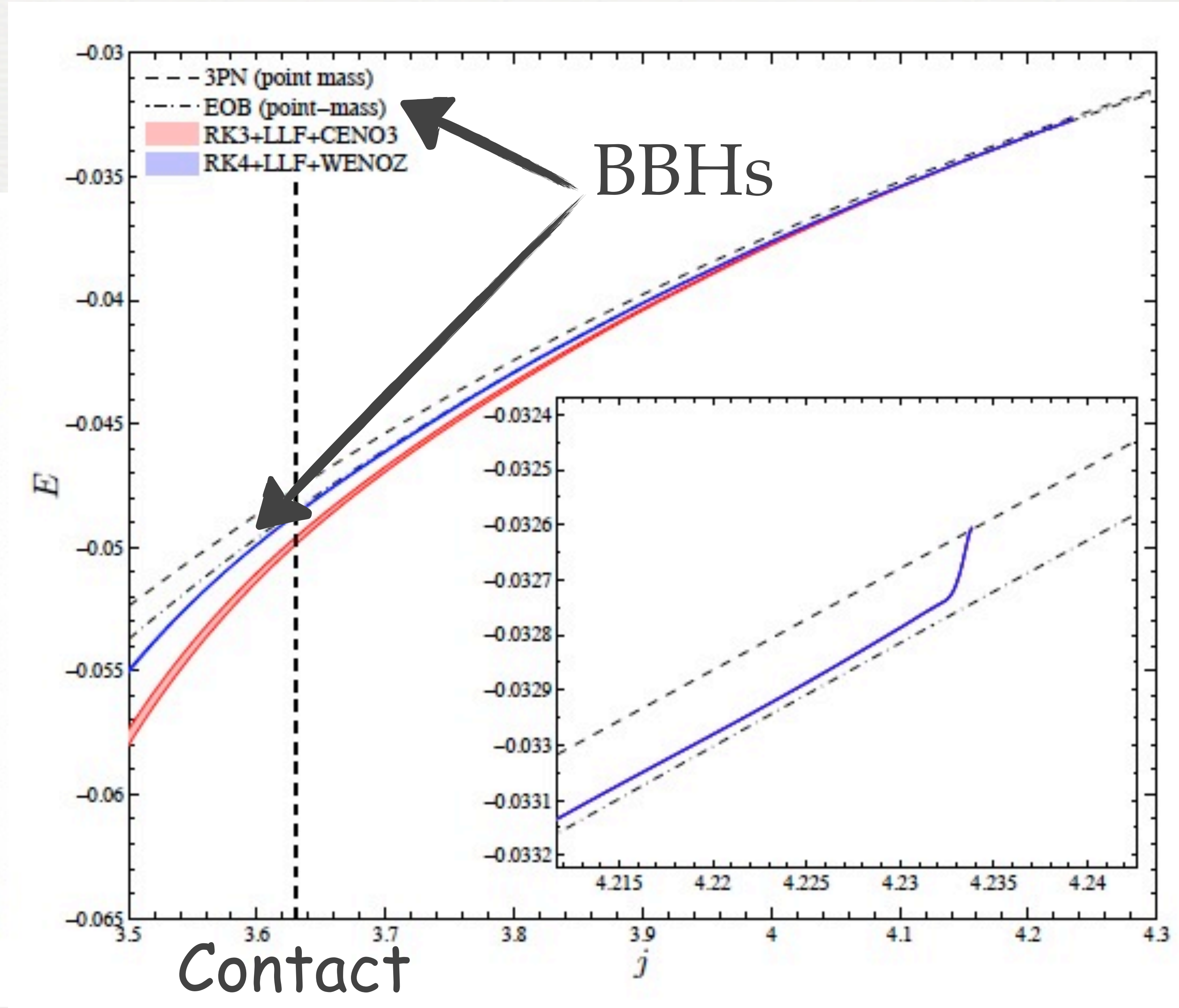
TIDAL EFFECTS IN NR: HOWTO

S. Bernuzzi, AN, Thierfelder & Bruegmann.,
PRD 86 (2012) 044030

– polytropic EOS

– $\mathcal{C} = 0.14$





Hydro (HRSC): 3rd-order flux reconstruction vs 5-th order flux reconstruction

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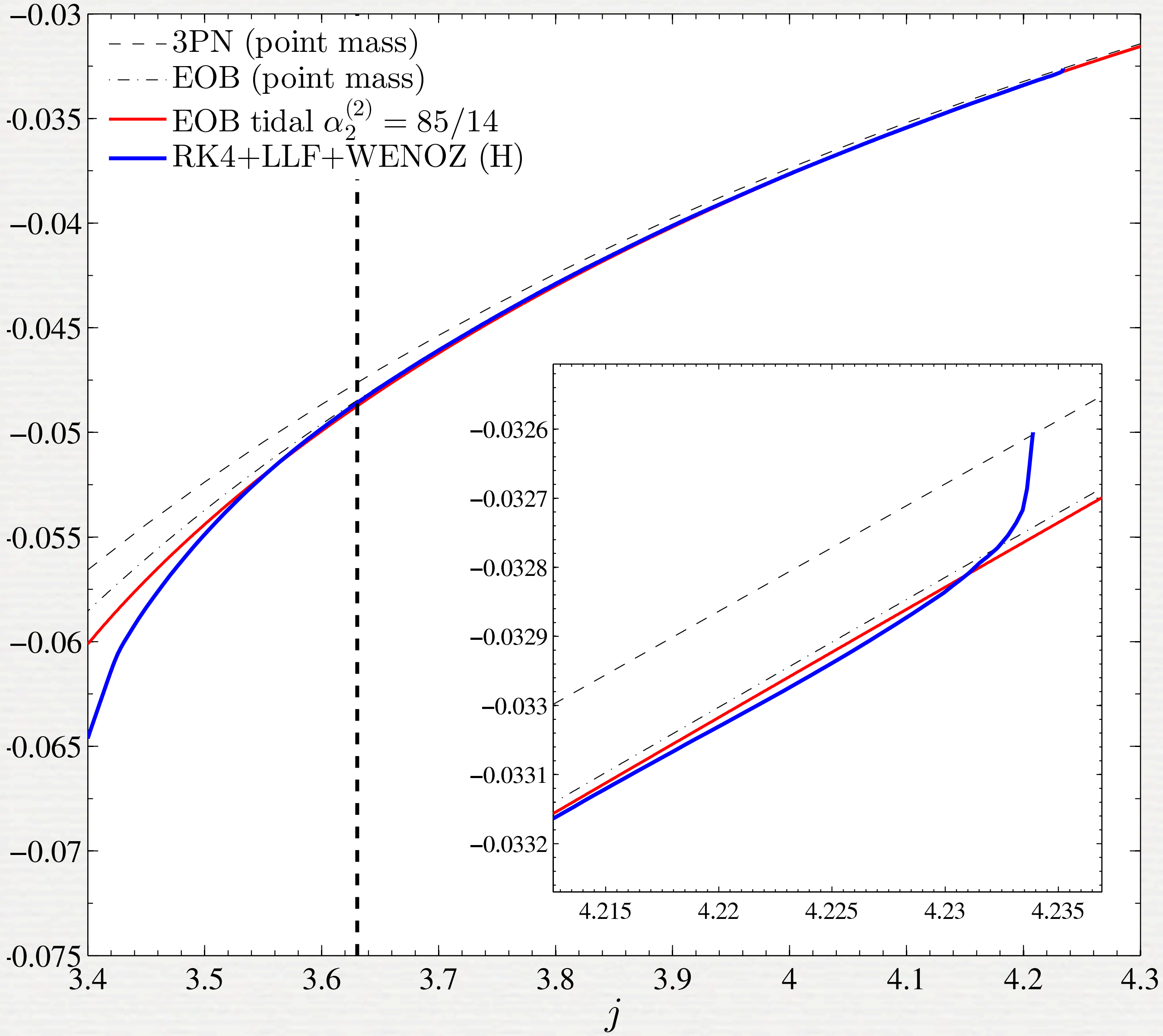
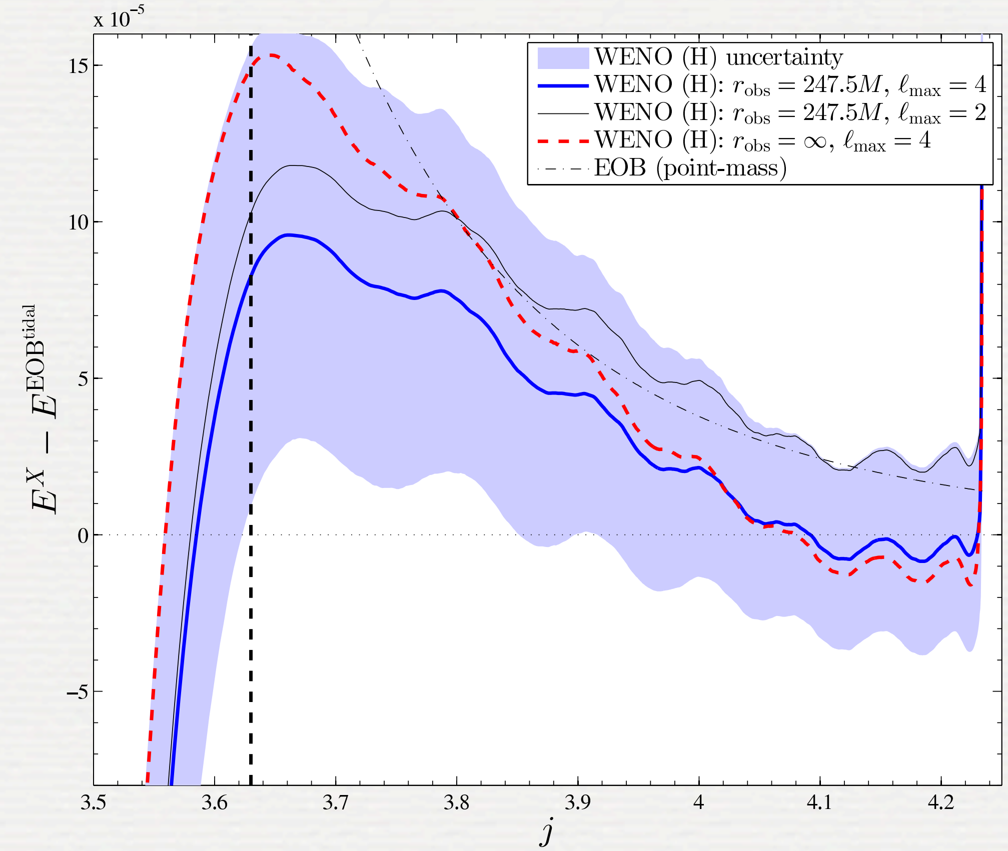
BNS DYNAMICS: BINDING ENERGY





Qualitative (and semi-quantitative) consistence NR-EOB up to contact (same for phasing)

Bernuzzi,Nagar et al. 2012

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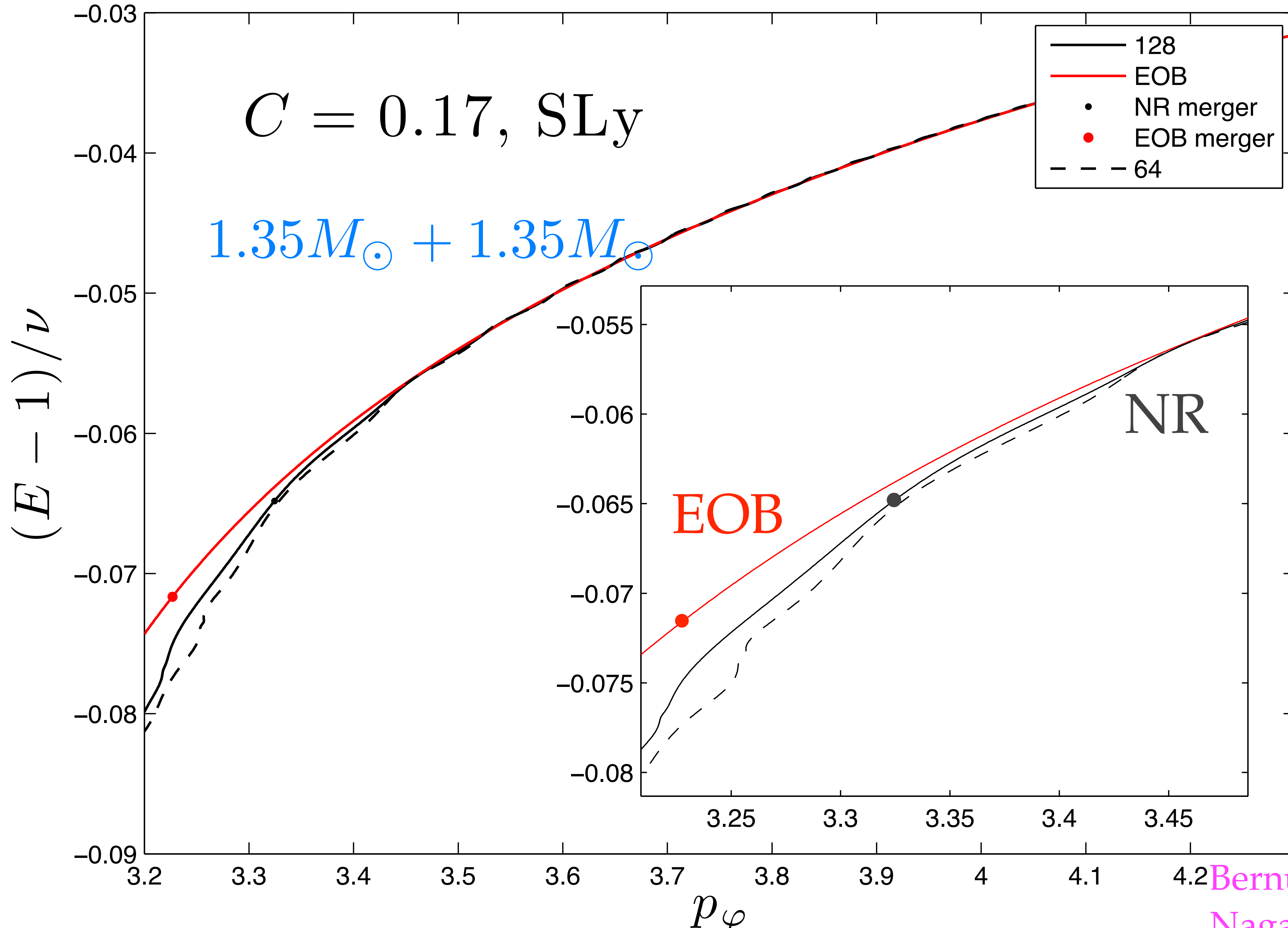
NEW SIMULATIONS

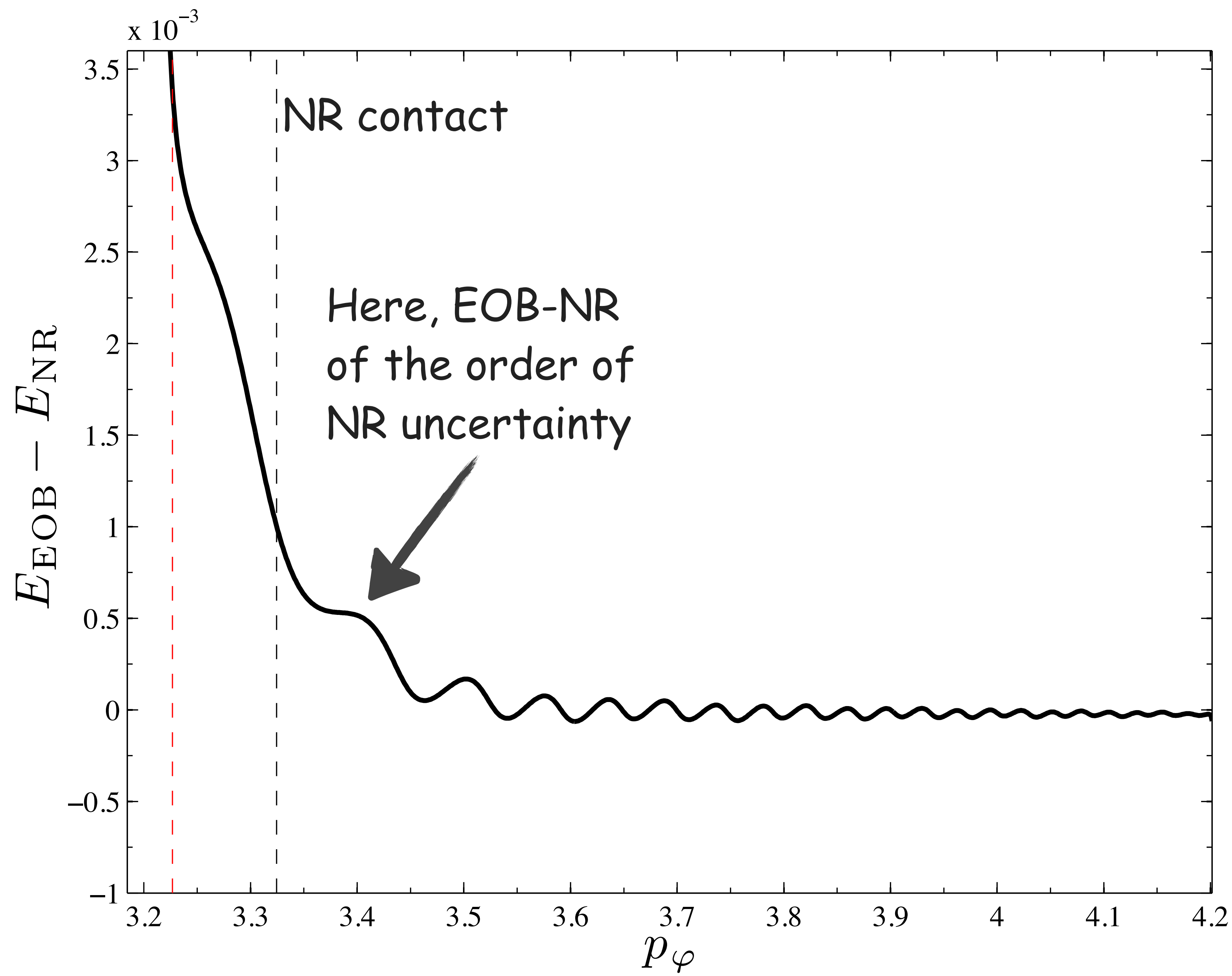
Z4c formulation of EE (Bernuzzi&Hilditch 2010)

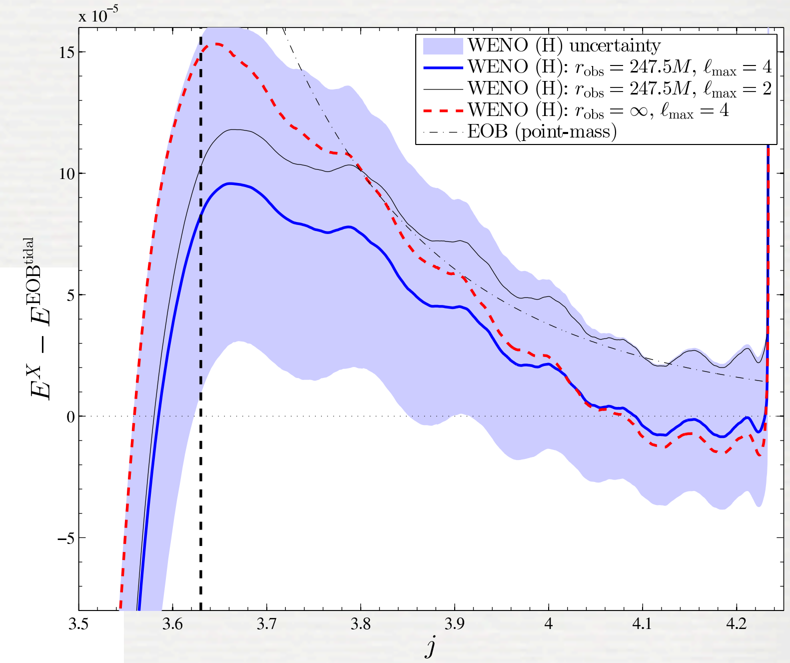


Bernuzzi, Dietrich, Nagar in prep.

THE DIFFERENCE

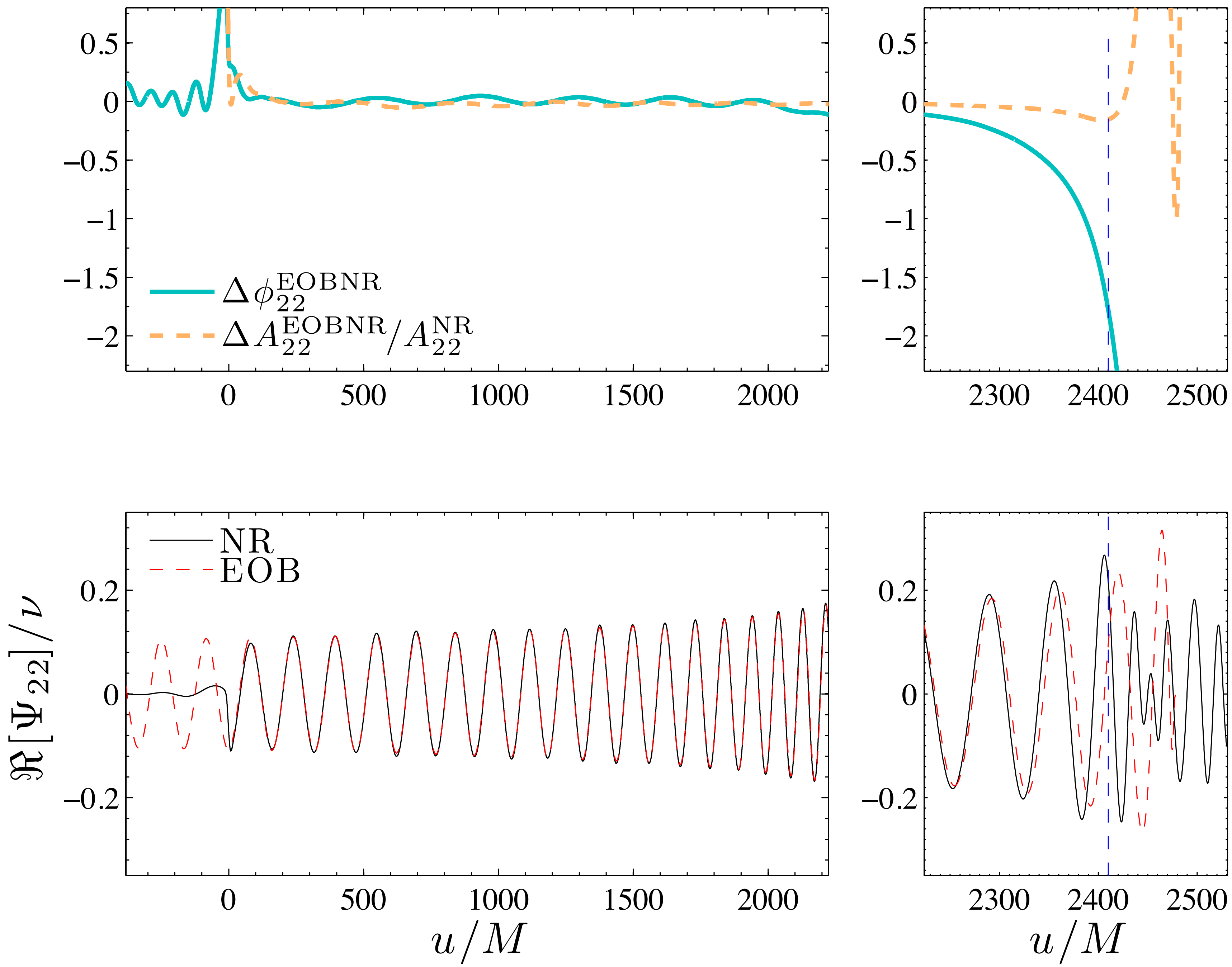
Excellent agreement
(essentially) up to NR merger





Bernuzzi, Dietrich,
Nagar in prep.

WAVEFORM

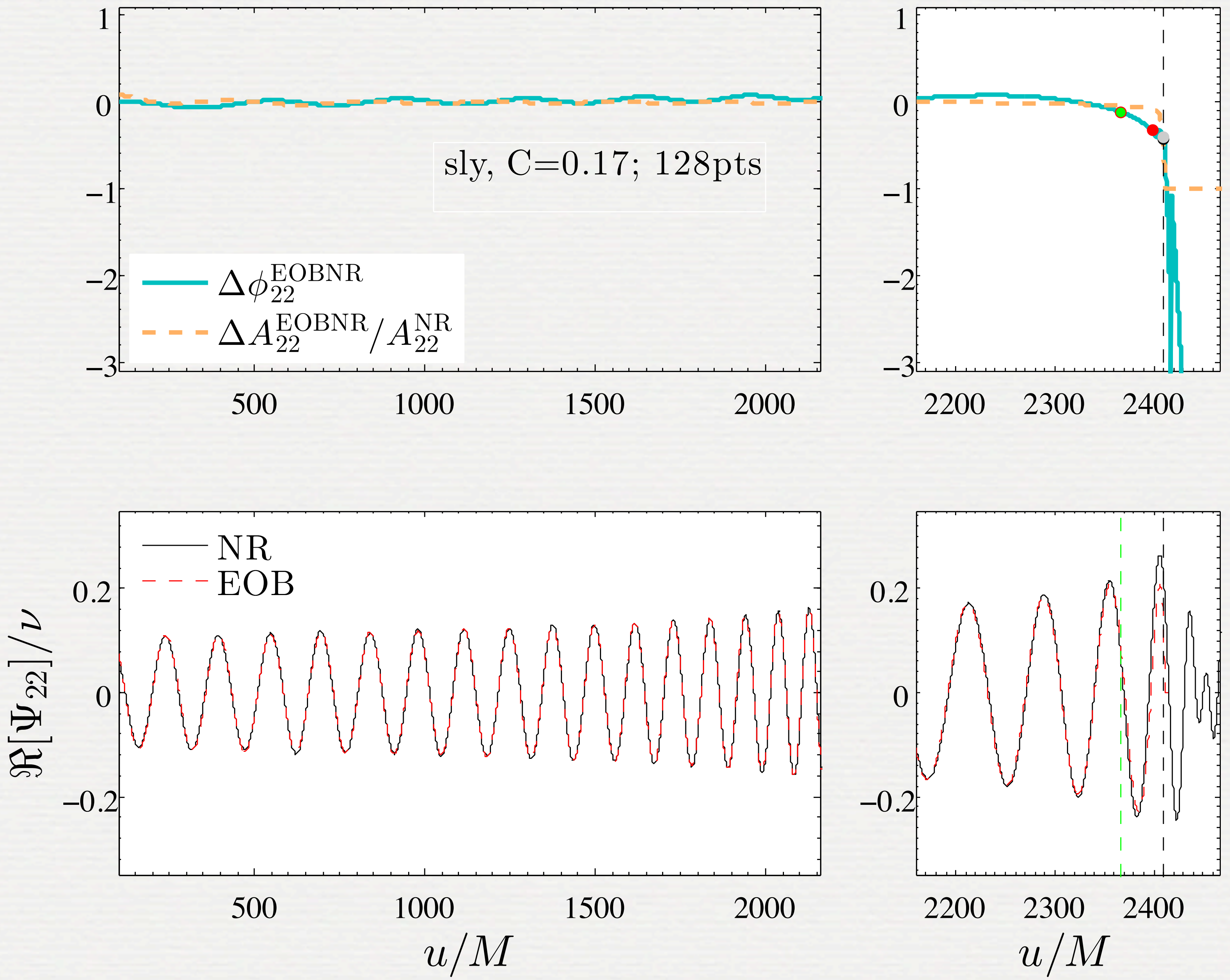


Agreement up to 1.5 orbits before merger (up to contact)

NR (actual!) strong-field tides are not excluded (uncertainties?)

IMPROVEMENTS

Light-ring behavior + GSF knowledge [Bini&Damour2014]



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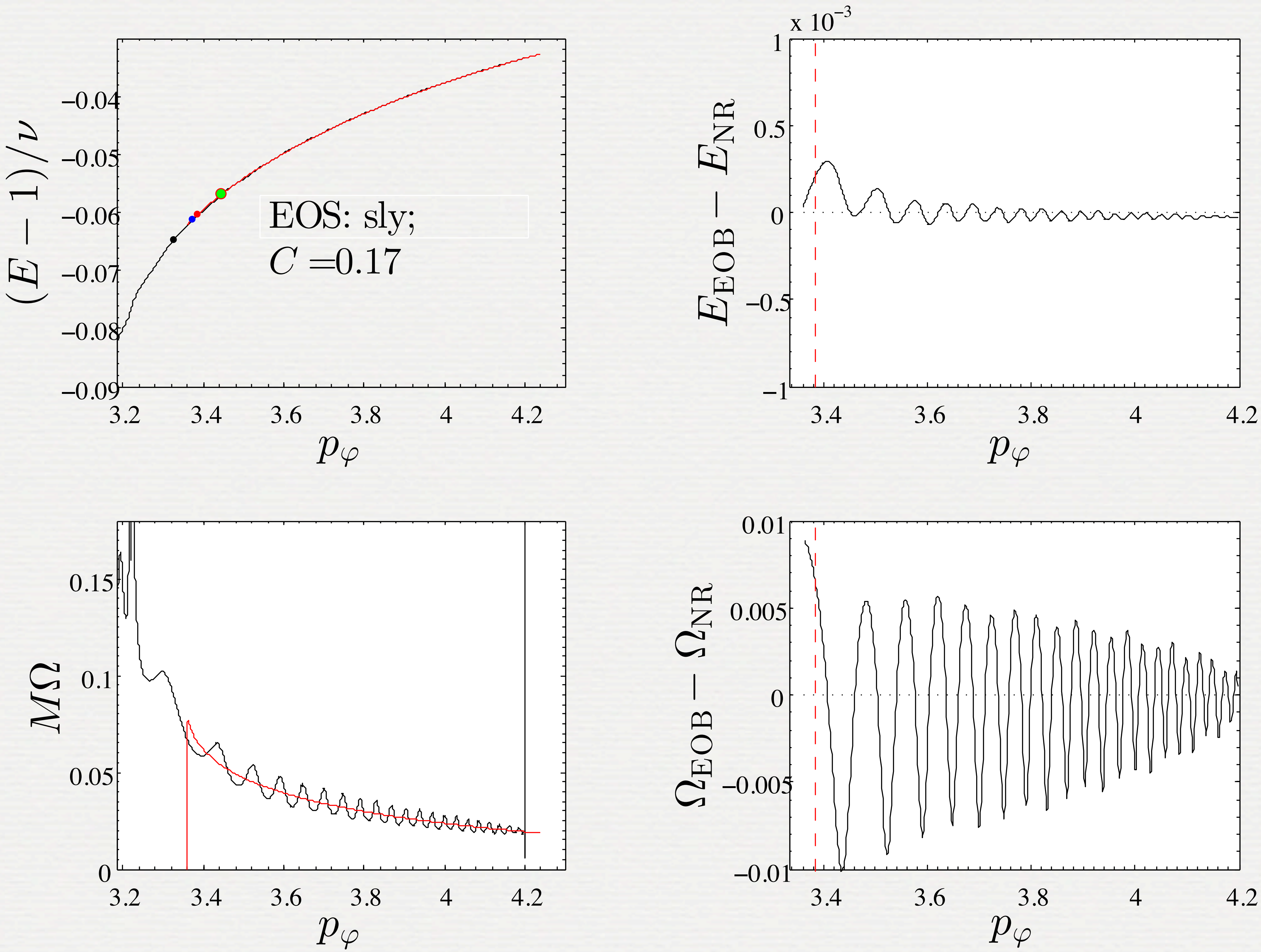
IMPROVEMENTS



BNS TAKE-AWAY

1. Tidal effects are incorporated in EOB formalism. Adding spin is trivial (not in NR!)

2. NR waveform data are quantitatively consistent with the EOB analytical predictions. Numerical uncertainties, though smaller than in the past, do not allow us yet to make out-of-questions assertions on possible amplifications of tidal effects in the late inspiral, after contact

3. The EOB theory easily allows to check the soundness of NR data. The simple theoretical understanding of BNS merger gives a paradigmatic example

3. Nowadays, the EOB model is the MORE ROBUST AND EFFICIENT METHOD to produce BNS templates waveforms up to contact with the hope of measuring the Love numbers, and thus the EOS, from the late-inspiral phase. Eventual (NR-driven) amplifications of tidal effects are not relevant for detection; if actual, they can be effectively incorporated in the model.

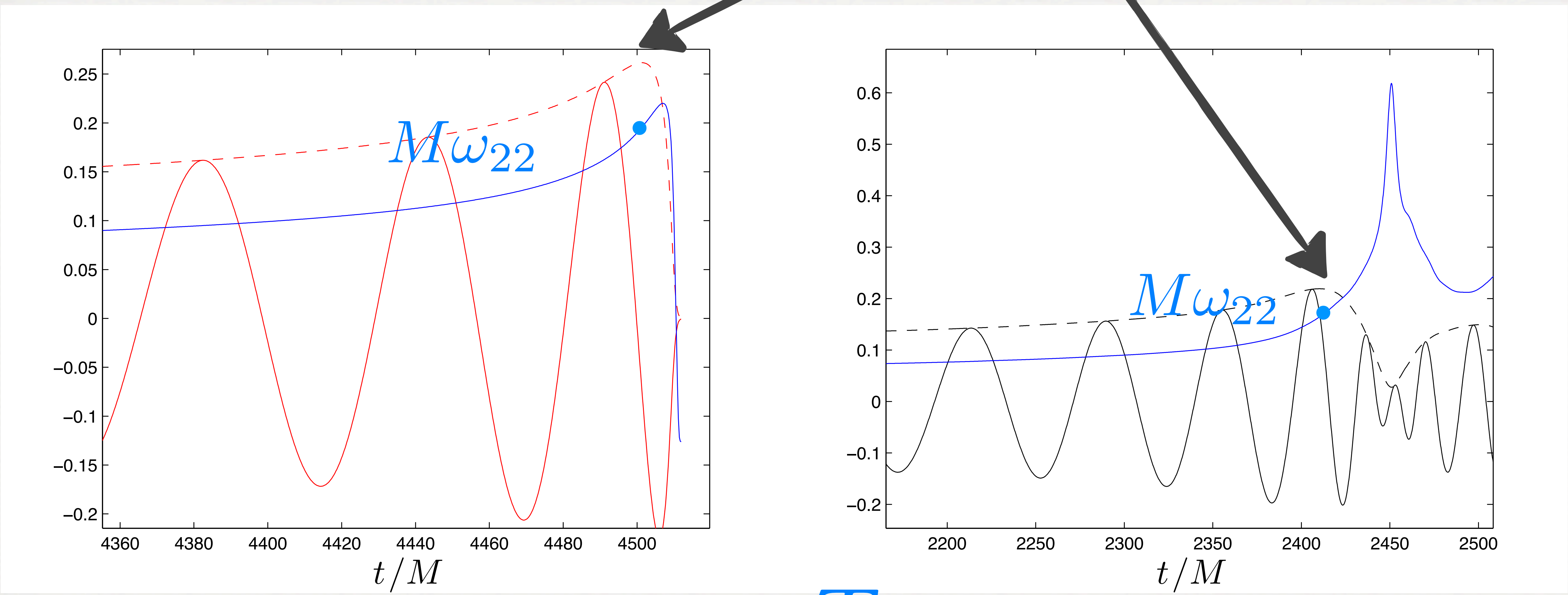
4. Further NR improvements (less eccentricity, more accuracy) are certainly useful but not essential (contrary to BBHs) for DA purposes

5. EOB-based BNS templates should be implemented NOW in DA pipelines

EOS QUASI-UNIVERSALITY OF BNS MERGER

S. Bernuzzi, **AN**, S. Balmelli, T. Dietrich and M. Ujevic, PRL 112 (2014) 201101

Take GW frequency and binding energy@mrg; various EOS.



$$\kappa_2^T$$

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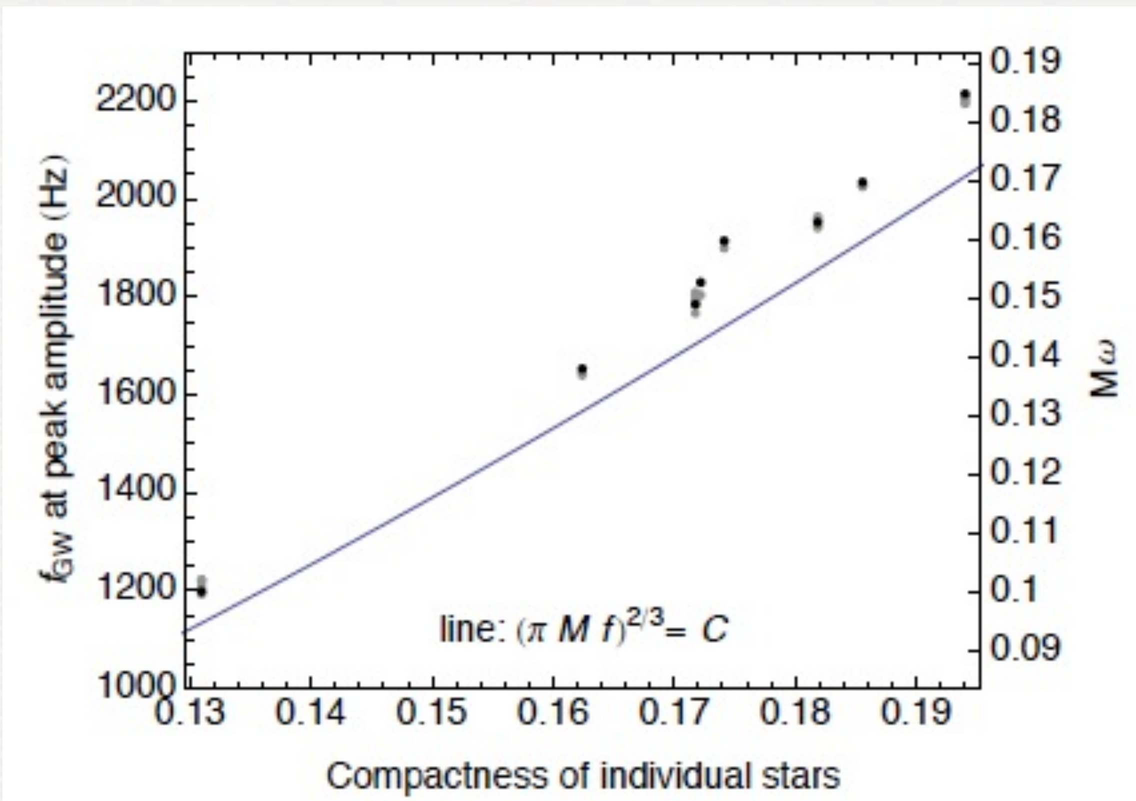
EOS QUASI-UNIVERSALITY OF BNS MERGER

Matter effects on binary neutron star waveforms

Jocelyn S. Read,[1,2] Luca Baiotti,[3,4] Jolien D. E. Creighton,[5] John L. Friedman,[5] Bruno Giacomazzo,[6] Koutarou Kyutoku,[5] Charalampos Markakis,[7,9] Luciano Rezzolla,[8] Masaru Shibata,[4] and Keisuke Taniguchi[10]



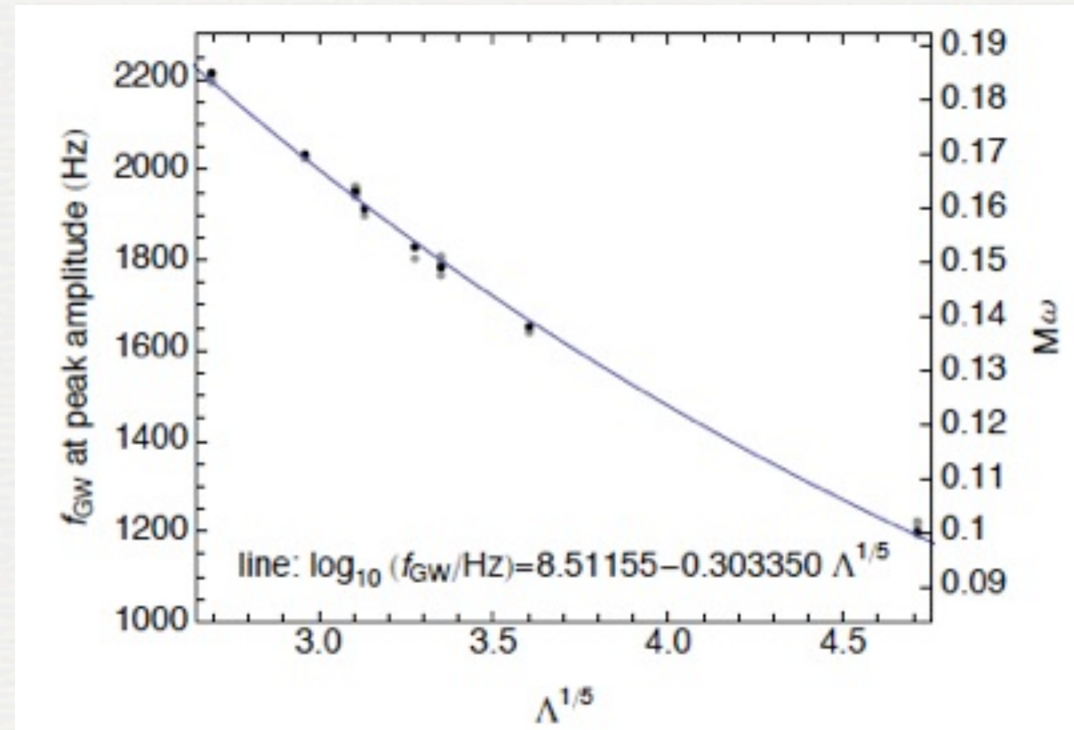


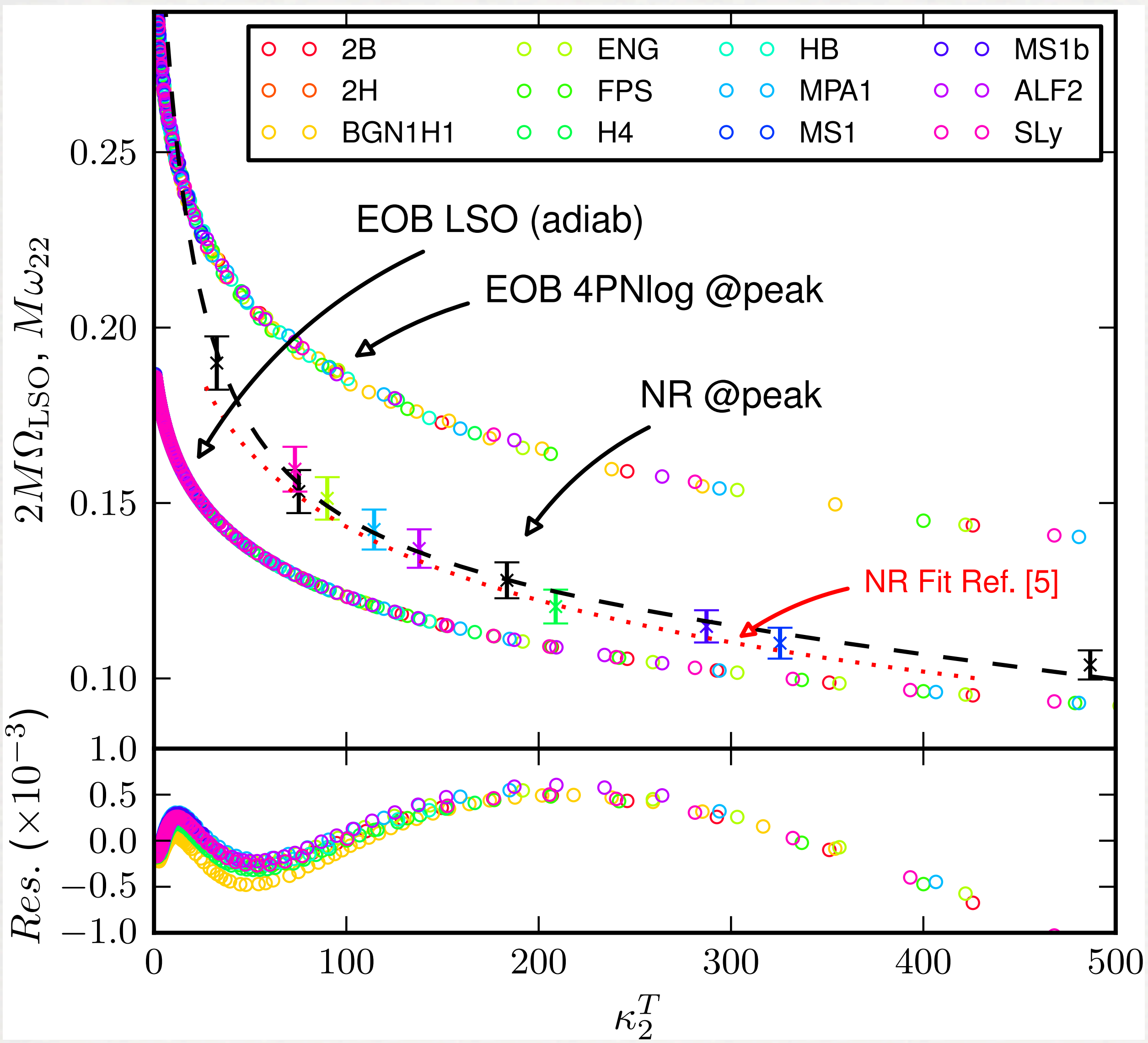
FIG. 4. Instantaneous gravitational-wave frequency at the point of peak amplitude, as a function of the tidal parameter $\Lambda^{1/5}$ (bottom panel) and as a function of individual star compactness C (top). For each model, the highest-resolution simulation for a given EOS is plotted in black, lower-resolution simulations in grey. The $x = (\pi M f)^{2/3} = C$ relation used in [15] to characterize merger frequency is shown in the compactness plot. An empirical fit using $\Lambda^{1/5}$ is shown in the bottom plot; the frequency of merger is more tightly correlated with Λ than with compactness/radius.

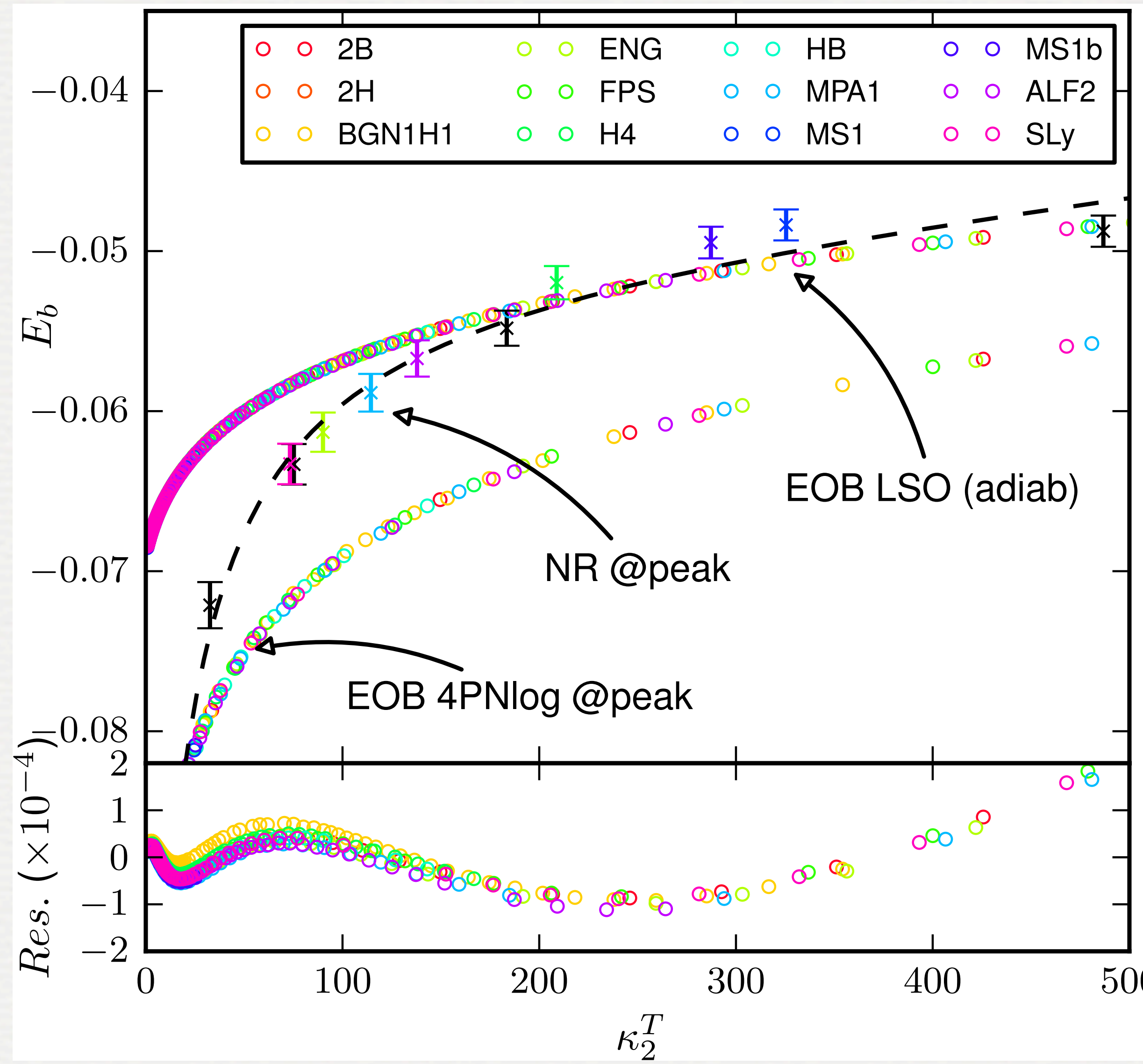
$$\Lambda = \frac{2}{3} k_2 \frac{1}{\mathcal{C}^5}$$

empirical fit using $\Lambda^{1/5}$????

EOS QUASI-UNIVERSALITY OF BNS MERGER

Simple EOS-universal behavior: measure the frequency, constrain the EOS





$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$$X_{A,B} \equiv M_{A,B}/M$$