Holographic phase space and black holes as renormalization group flows

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Outline

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c-Functions

- c-functions measure degrees of freedom along RG flows Zamolodchikov '86, Cappelli, Friedan Latorre '90

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle = \frac{\pi}{3}\int_0^\infty \mathrm{d}\mu c(\mu)\int \frac{d^2p}{(2\pi)^2}e^{ipx}\frac{(g_{\mu\nu}p^2-p_\mu p_\nu)(g_{\rho\sigma}p^2-p_\rho p_\sigma)}{p^2+\mu^2}$$

- Positivity of spectral measure implies

$$c_{UV}\equiv\int_0^\infty \mathrm{d}\mu\, c(\mu) \qquad \geq \qquad c_{IR}\equiv\lim_{\epsilon\to 0}\int_0^\epsilon \mathrm{d}\mu\, c(\mu)$$

- At fixed points $c(\mu)=c\delta(\mu)$: the c-function matches the CFT central charge.
- Monotonicity $\Rightarrow$ irreversibility.

Cardy's conjecture

- In higher d, more anomaly coefficients:

$$\langle T^a_{\ a} \rangle \simeq (-1)^{d/2} A \times (\text{Euler density}) + \sum B_i \times (\text{Weyl contractions}) + \nabla(\ldots)$$

- Conjecture: c-function involves Euler anomaly A Cardy '88.
- Recently proven in four dimensions. Komargodski, Schwimmer '11.
- No proof for general d. Can holography help?

Holography and RG flow

- AdS/CFT: a geometrization of RG flow.
- Scale $\rightarrow$ extra dimension r (non-trivial).
- Example: relevant flows in $N=4$ SYM $\Rightarrow$ domain wall backgrounds

$$ds^2 = \mathrm{d}r^2 + e^{2A(r)}(-\mathrm{d}t^2 + \mathrm{d}x^2)$$

with

$$A(+\infty) = r/L_1, \qquad A(-\infty) = r/L_2, \qquad L_1 > L_2$$

- Shrinking of AdS radius corresponds to loss of degrees of freedom.

Can we make this precise?

Holographic c-functions

- Einstein equations determine: Girardello et al '98, Freedman et al '99.

$$(d-1)A''(r) = (T_t^t - T_r^r) = -(\rho + p_r)$$

- Null energy condition $\Rightarrow A''(r) \leq 0$. Define the c-function

$$c(r) = \frac{c_0}{l_P^{d-2}(A')^{d-2}} \quad \Rightarrow \quad c_{AdS} \propto (L_{AdS}/l_P)^{d-2} \, .$$

- Radial dependence related to field theory cut-off. Simple argument suggests:

Polchinski, Heemskerk '10

$$\Lambda \simeq e^A A'(r)$$

- Exact relation unknown.

Lovelock theories of gravity

- To distinguish different anomalies need more general gravity theory - higher derivatives!
- Generically introduces ghosts, complicated equations. Special choice: Lovelock theory.

$$\mathcal{L} \simeq R - 2\Lambda + E_{2k}$$

with E^{2k} the $(2k)$ dimensional Euler densities.

- Nice properties!
  - Non-ghosty vacua.
  - Linearized EOM are 2-derivative.
  - Exact black hole solutions exist.

Plan of attack 1.

What is the plan?

- Construct a c-function for Lovelock theories of gravity.
- Extra parameters will allow us to determine what the c stands for (i.e. the Euler anomaly).
- Strategy is to consider equations of motion and reconstruct c-function from there.

Plan of attack 2.

- Construct a “c-function” for black hole backgrounds:

$$ds^2 = -\left(\kappa + \frac{r^2}{L^2} f(r)\right) \frac{dt^2}{f_\infty} + \frac{L^2 dr^2}{\kappa + \frac{r^2}{L^2}\, g(r)} + r^2 (\mathrm{d}\Sigma_\kappa^{d-2})^2,$$

- Domain wall solutions are special case.
- **Motivation:** black hole horizons appear to have “emergent” conformal symmetry, as in extremal black hole geometries containing an AdS_2 factor. Carlip
- Suggests such geometries describe intriguing RG flows between CFT’s of different dimensionality.

Equations of motion

- Action for Lovelock theories:

$$S = \frac{1}{l_P^{d-2}} \int d^d x \sqrt{-g} \left(R - 2\Lambda + \sum_{k=2}^{K} n_k c_k E^{2k} \right) + S_{\text{matter}}$$

tt equation:

$$\frac{\mathrm{d}}{\mathrm{d}r} \left(r^{d-1} \Upsilon[g] \right) = -\frac{L^2 \, l_P^{d-2}}{d-2} \, r^{d-2} T_t^t$$

with

$$\Upsilon[g] \equiv \sum c_k g^k = 1 - g + c_2 g^2 + \dots$$

- In the absence of matter, $f = g$ and exact black hole solutions can be found by solving a *polynomial* equation!

$$\Upsilon[g] = \frac{m_0}{r^{d-1}}$$

$m_0 \simeq$ mass.

Equations of motion 2.

- Integrating the equation we find

$$\Upsilon[g] = \frac{L^d\, l_P^{d-2}}{d-2} \frac{\int_{r_0}^{r} \mathrm{d}r'\, (r'/L)^{d-2} \rho(r')}{r^{d-1}} \equiv \frac{L^d}{(d-2)V_{d-2}} l_P^{d-2} \frac{M(r)}{r^{d-1}}.$$

- $\Upsilon[g] \simeq \psi$, the "Newtonian" potential.
- The tt equation can be rewritten

$$\left(-\Upsilon'[g]\right) \frac{\mathrm{d}g}{\mathrm{d}r} = \frac{2L^d}{d-2} \frac{\mathrm{d}\Psi}{\mathrm{d}r}.$$

- The gravitational field tells us about the direction in which g is decreasing.

The N-function

- Start from the flow equation:

$$\left(-\Upsilon'[g]\right)\frac{\mathrm{d}g}{\mathrm{d}r}=\frac{2L^d}{d-2}\frac{\mathrm{d}\Psi}{\mathrm{d}r}.$$

- Define the $\mathcal{N}$ function:

$$\boxed{\mathcal{N}(r)=\frac{1}{g^{\frac{d-2}{2}}}\left(\sum_{k=1}^{K}\frac{(d-2)k}{d-2k}\,c_k(-g)^{k-1}\right).}$$

- The flow equation becomes

$$\boxed{\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}r}=\left(\frac{L}{\sqrt{g}}\right)^d\left(-\frac{\mathrm{d}\Psi}{\mathrm{d}r}\right)}$$

- **Important result**: it describes the flow of $\mathcal{N}$, in terms of local gravitational field.

Interlude: Euler anomaly

- Conformal anomaly in even-dimensional CFT's:

$$\langle T^a_{\ a} \rangle \simeq (-1)^{d/2} A \times \text{(Euler density)} + \sum B_i \times \text{(Weyl contractions)} + \nabla(\ldots)$$

- Computed holographically by Skenderis, Henningson '98.
- The A coefficient can be extracted from on-shell Lagrangian.

  Imbimbo,Schwimmer,Theisen '99

- How does A relate to the $\mathcal{N}$ function?

Interlude: Euler anomaly

- Start with action:

$$S = \int d^d x \sqrt{-g}(\mathcal{L}_g + \mathcal{L}_{\text{matter}}),$$

$$\mathcal{L}_g = \sum_k L^{(k)}, \qquad (L^{(k)} \text{ has } k \text{ curvatures})$$

and the equation of motion

$$-2\nabla_a \nabla_b X_{acbd} + X^{aecf} R_{aef}{}^{d} + \frac{1}{2} g^{cd} \mathcal{L}_g + \frac{\partial \mathcal{L}}{\partial g_{cd}} = T^{cd}_{\text{matter}}$$

with

$$X_{abcd} = \frac{\delta S}{\delta R^{abcd}}$$

Interlude: Euler anomaly

$$-2\nabla_a \nabla_b X_{acbd} + X^{aecf} R_{aef}{}^{d} + \frac{1}{2} g^{cd} \mathcal{L}_g + \frac{\partial \mathcal{L}}{\partial g_{cd}} = T^{cd}_{\text{matter}}$$

- Assume we have an AdS background. In these circumstances we must have $T^{cd} = -\frac{1}{2} g^{cd} \mathcal{L}_m$, and all covariant derivatives vanish. Taking the trace of the equation of motion above we find

$$\sum_k \left(kL_k + \frac{d}{2} L_k - 2kL_k) \right) + \frac{d}{2} \mathcal{L}_m = 0$$

$$\Rightarrow \qquad X^{abcd} R_{abcd} = \sum kL_k = \frac{d}{2} \left(\mathcal{L}_g + \mathcal{L}_m \right)$$

Interlude: Euler anomaly

- In AdS

$$X^{abcd} = (g^{ac}g^{bd} - g^{bc}g^{ad})X^{rt}{}_{rt},$$

and therefore we conclude

$$\mathcal{L}_g + \mathcal{L}_m = \frac{4}{d} R \frac{\delta S}{\delta R^{rt}{}_{rt}}$$

- The Euler anomaly is given by:

$$A = \frac{1}{2} \frac{\partial \mathcal{L}_g}{\partial R_{abcd}} \epsilon_{ab}\epsilon_{cd} \Big|_{\text{boundary}}.$$

where $\epsilon_{rt} = \sqrt{-g_{rr}g_{tt}}$ with all other components zero is a spacelike surface binormal.

- This is very similar to Wald's black hole entropy formula:

$$S_{\text{BH}} = -2\pi \oint_{\text{horizon}} \sqrt{h} \frac{\partial \mathcal{L}_g}{\partial R_{abcd}} \epsilon_{ab}\epsilon_{cd}$$

Euler anomaly for Lovelock theories

- For Lovelock theories of gravity, we get

$$A = \left(\frac{L}{l_P}\right)^{d-2} \frac{1}{f_\infty^{\frac{d-2}{2}}} \left(\sum_{k=1}^{K} \frac{(d-2)k}{d-2k}\, c_k (-f_\infty)^{k-1}\right)$$

- This should be compared to the $\mathcal{N}$ function

$$\mathcal{N}(r) = \frac{1}{g^{\frac{d-2}{2}}} \left(\sum_{k=1}^{K} \frac{(d-2)k}{d-2k}\, c_k (-g)^{k-1}\right).$$

Clearly then we have

$$\mathcal{N}(\infty) = \left(\frac{L}{l_P}\right)^{d-2} A$$

$\mathcal{N}$ as a c-function

- More generally, $\mathcal{N}$ correctly captures the Euler anomaly in AdS background. The flow equation sets its monotonicity:

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}r} = \left(\frac{L}{\sqrt{g}}\right)^d \left(-\frac{\mathrm{d}\Psi}{\mathrm{d}r}\right)$$

- We impose the null energy condition, $\rho + p \geq 0$. However in RG flow backgrounds we have

$$p_r = -\frac{M(r)/V_{d-2}}{r^{d-1}}.$$

- This implies

$$\rho + p \geq 0 \quad \Rightarrow \quad M(r) \leq 0 \quad \Rightarrow \quad -\frac{\mathrm{d}\Psi}{\mathrm{d}r} \geq 0$$

  Therefore $\mathcal{N}(r)$ monotonously decreases from UV to IR in RG flow backgrounds, and equals the Euler anomaly at AdS fixed points: it is a holographic c-function.

Black holes as RG flows

- The $\mathcal{N}$ function was defined for black hole backgrounds.
- If no matter present:

$$\Psi = -l_P^{d-2}\left(\frac{r_0}{r}\right)^{d-1}\Upsilon[g(r_0)], \qquad g(r_0) = -\kappa L^2/r_0^2$$

- $\mathcal{N}(r)$ is monotonously *increasing* from UV to IR!
- **Explanation**: gravitational field is now pointing in the “right” direction, since the matter has positive energy density.
- In general, $\mathcal{N}$ has no well-defined monotonicity.

- At the horizon, $\mathcal{N}$ is related to black hole entropy:

$$S = \left(\frac{L}{l_P}\right)^{d-2} V_{d-2}(-\kappa)^{\frac{d-2}{2}}\mathcal{N}(r_0)$$

Black holes as RG flows

- $\kappa = -\mathbf{1}$
  $\mathcal{N}(r)$ monotonously increases from the boundary to the horizon where

$$S_{\text{BH}} = 4\pi V_{d-2} \left(\frac{L}{l_P}\right)^{d-2} \mathcal{N}(r_0)$$

- N function nicely interpolates between the A anomaly in the UV and the black hole entropy in the IR.

Black holes as RG flows

- $\kappa = \mathbf{0}$
  $\mathcal{N}(r)$ monotonously increases from the boundary to the horizon where it diverges. Regulating by setting $\kappa = \left(\frac{l_P}{L}\right)^2 \Rightarrow g(r_0) = \frac{l_P^2}{r_0^2}$, then

$$S_{\text{BH}} = 4\pi V_{d-2}\mathcal{N}(r_0)$$

- Dramatic increase from $\mathcal{O}(1)$ to order $\mathcal{O}(L/l_P)^{d-2}$ in the number of effective field theory degrees of freedom as one approaches the black hole horizon.

Black holes as RG flows

- $\kappa = \mathbf{1}$
  $\mathcal{N}$ diverges at $g = 0$, which is outside black hole horizon. Expressions become imaginary or negative there, depending on d. We now have

$$S_{\text{BH}} = 4\pi V_{d-2} \left(\frac{L}{l_P}\right)^{d-2} |\mathcal{N}(r_0)|.$$

- $\mathcal{N}$ is perfectly finite at the horizon.
- Why is there a divergence ?!

Outline

Entropy and $\mathcal{N}$

- $\mathcal{N}$ is connected to entropy and Euler anomaly - both given by a Wald formula.
- Consider a radial foliation defined by

$$n_r = \sqrt{g_{rr}}, \qquad m_t = \sqrt{-g_{tt}},$$

$$h_{ab} = g_{ab} - n_a n_b + m_a m_b,$$

- Computing the Wald formula on a radial surface leads to

$$S = -2\pi V_{d-2}\sqrt{h}\frac{\partial \mathcal{L}}{\partial R_{abcd}}\epsilon_{ab}\epsilon_{cd} = 4\pi\, V_{d-2}\, \left(\frac{L}{l_P}\right)^{d-2} \sum_{k=1}^{K} \frac{(d-2)k}{d-2k} c_k (-g)^{k-1}$$

Entropy and $\mathcal{N}$

- We can therefore write:

$$\boxed{\mathcal{N} = \frac{S}{4\pi\Omega_{\text{eff}}},}$$

- If $\mathcal{N}$ is a number of degrees of freedom and S is an entropy, then Ω_{eff} is an effective phase space, whose expression is:

$$\Omega_{\text{eff}} = \left(\frac{L}{l_P}\right)^{d-2} \left(\frac{r}{L^2}\, g(r)\right)^{d-2} V_{d-2}.$$

- An analogy would be with black holes where

$$S = c_S \times N^2 \times V_{d-2} \times T^{d-2}$$

- Clearly in this case S is the product of a phase space volume by a number of degrees of freedom, c_S.

Information content and phase space

- In our formula, S is not an entropy but rather an information content: it counts the possible states of some closed region of spacetime to which we have no access.
- Ω_{eff} is an effective single particle phase space volume. In RG flow backgrounds it takes the form

$$\Omega_{\text{eff}} = \left(\frac{L}{l_P}\right)^{d-2} \times V_{d-2} \times \left(e^A A'\right)^{d-2}$$

- We can identity as the momentum space cut-off:

$$\Lambda_{\text{eff}} = e^A A'.$$

- This agrees with a proposal of Polchinski and Heemskerk.

Holographic phase space

- For the general case, if we compute $K_{ab} = \nabla_a n_b$ we get

$$K_{ij} = -\frac{h_{ij}}{L}\sqrt{\frac{L^2}{r^2}\kappa + g(r)}.$$

Then it follows that

$$\Omega_{\text{eff}} = \left(\frac{L}{l_P}\right)^{d-2} \oint d^{d-2}x\sqrt{h}\sqrt{\text{det}\left(K_i^k K_k^j - \kappa\,\delta_i^j \frac{L^2}{r^2}\right)}$$

- Notice that the above has the structure $\simeq \Lambda^2 - m^2$, precisely as expected if we are to interpret Ω_{eff} as counting states.
- Suggests momentum cut-off is connected to canonical radial momentum of metric:

$$\Lambda^{d-2} = \sqrt{h}\,\text{det}\left(K_i^j\right).$$

- One can also rewrite Ω_{eff} in terms of curvature using

$$K_i^k K_k^j - \kappa\,\delta_i^j \frac{L^2}{r^2} = R_{ik}{}^{jk},$$

Relation to Verlinde's work

- The flow equation for $\mathcal{N}$ can now be rewritten as:

$$\frac{\mathrm{d}S}{\Omega_{\text{eff}}} = \left(\frac{L}{\sqrt{g}}\right)^d \mathrm{d}\Psi + \mathcal{N}\,\mathrm{d}\log(\Omega_{\text{eff}})$$

- This is reminiscent of an equation found by Verlinde relating the depletion of entropy per bit to the variation of the Newtonian potential.
- If we make a virtual variation of the mass keeping the phase space fixed, one can show that the above becomes (not-trivial!)

$$\frac{\mathrm{d}S}{\Omega_{\text{eff}}} = \left(\frac{L}{\sqrt{g}}\right)^d \mathrm{d}\Psi.$$

- This puts Verlinde's relation on a firm footing. However, our interpretation is different: what he calls a number of bits, we claim to be a phase space volume.

$\mathcal{N}$ in three dimensions

- For new massive gravity we have the action

$$S = \frac{1}{l_P}\int d^3x\sqrt{-g}\left(R + 2 + 4\lambda\left(R_{ab}^2 - \frac{4}{3}R^2\right)\right)$$

- Such theories support a holographic c-function. Also, exact black hole solutions can be found with metric

$$ds^2 = -\frac{r^2 g(r)}{L^2 f_\infty}\,dt^2 + \frac{L^2 dr^2}{r^2 g(r)} + \frac{r^2}{L^2}dx^2,$$

- $\mathcal{N}$-function defined as

$$\mathcal{N} = \frac{S}{4\pi\Omega_{\text{eff}}} = \frac{1+2\lambda f_\infty}{\sqrt{g}} = c \times \frac{l_P}{L}\sqrt{\frac{f_\infty}{g}}.$$

- Flow of $\mathcal{N}$ is trivial (unlike Lovelock theories): $\mathcal{N}$ provides connection between UV (central charge) and IR (black hole entropy).
- Analogous results hold for other 3d gravity theories Paulos '10.

Outline

Holographic phase space

- Susskind: number of degrees of freedom holographically associated to a region is area of region divided by Planck length.
- Our interpretation: the above gives not degrees of freedom, but phase space volume.
- Definition:

$$\Omega = \oint_{\partial M} \frac{dA}{l_P^{d-2}},$$

- This is *not* equal to Ω_{eff} we defined previously! Try to connect later on.

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$$\Omega = \oint_{\partial M} \frac{dA}{l_P^{d-2}},$$

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- In Poincaré patch of AdS:

$$\Omega = \left(\frac{L}{l_P}\right)^{d-2} V_{d-2} \left(\frac{r}{L^2}\right)^{d-2}.$$

- Equals product of gauge configurations, coordinate volume and momentum space volume.

Degrees of freedom

- Can define a number of degrees of freedom:

$$N_{\text{dof}} \equiv \frac{S}{4\pi\Omega} = 2\frac{\partial\mathcal{L}}{\partial R_{rt}{}^{rt}}.$$

- This matches a proposal for the surface density of degrees of freedom of Padmanabhan.
- In black hole backgrounds get

$$N_{\text{dof}} = \sum_k \frac{(d-2)k}{d-2k} c_k g^k.$$

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- This is not equal to $\mathcal{N}$, and hence does not satisfy a nice flow equation.
- In the following, it is useful to work with the proper radial distance:

$$\beta = \int dr \sqrt{g_{rr}}$$

- For large r, $\beta \simeq \log r$. If r is energy scale, β counts number of coarse grainings along RG flow - it is natural quantity parametrizing flow even in other geometries.

Phase space for AdS-Schwarzschild

- AdS-Schwarzschild black hole solution:

$$ds^2 = \frac{L^2\, dr^2}{r^2 g(r)} + \frac{r^2}{L^2}\left(-g(r)dt^2 + d\mathbf{x}^2\right)$$

with $g(r) = 1 - (r_0/r)^4$.

- In terms of the proper distance β:

$$r = r_0 \sqrt{\cosh(2\beta/L)}.$$

- The phase space volume corresponding to a given direction, say x:

$$\Omega_x = \frac{L}{l_P} \times R \times \pi T \times \sqrt{\cosh(2\beta)}$$

- Matches the partition function of an anyon harmonic oscillator BoschiFilho:1994an.
- Suggestive of an equivalence between a classical microcanonical partition function or phase space volume at a given cut-off, and a canonical partition function at an inverse temperature β related to this cut-off.

Thermodynamics and curvature

- Pushing the thermodynamic analogy further, compute mean energy and energy squared:

$$\langle E\rangle = -\frac{d\log\Omega}{d\beta} = -\sqrt{g_{rr}}\,\frac{d\log(\sqrt{h})}{dr} = h^{ab}K_{ab}$$

$$\langle E^2\rangle = \frac{1}{\Omega}\frac{d^2\Omega}{d\beta^2} = -R_{abcd}n^a n^c h^{bd}.$$

- "Thermodynamic" quantities turn out to have a simple relation to natural geometric quantities in this formalism.
- We can write $\langle E\rangle$ as the sum of three separate contributions

$$\langle E\rangle = -\sum_{i=1}^{d-2}\frac{d\log\Omega_i}{d\beta} \equiv -\sum_{i=1}^{d-2}\mathcal{E}_i$$

- $\mathcal{E}_i$ are the average energies along each direction

More on $\mathcal{E}_i$

- $\mathcal{E}_i$ tells us how much the logarithm of the phase space volume is changes when the RG parameter β changes.
- In empty AdS, β and $\mathcal{E}_i$ are proportional: RG flow is parameterized by *scale*
- In general, $\mathcal{E}_i$ is non-trivially related to β: natural RG parameter is *not* scale.
- For black holes
$$\mathcal{E}_i = \frac{\sqrt{g}}{L}.$$
- Close to horizon β parameter is going to zero, but the phase space volume Ω is becoming a constant.
- Quantum correlations also vanish for scales larger than $1/T$. This suggests that β might generically be related to correlations and not scale.

Relating Ω and Ω_{eff}

- Two different notions of holographic phase space:

$$\frac{\Omega_{\text{eff}}}{\Omega} = \left(\frac{\Lambda_{\text{eff}}}{\Lambda}\right)^{d-2}$$

For planar black holes we can write

$$\Lambda_{\text{eff}} = \Lambda \times \sqrt{g} = \Lambda(L\mathcal{E}).$$

and therefore

$$\mathcal{N} = \frac{N_{\text{dof}}}{(L\mathcal{E})^{d-2}},$$

- Holographic phase space is naturally defined by Ω; number of degrees of freedom in terms of $\mathcal{N}$. More work is necessary to establish precise connection!

What about the divergence?

- Puzzle in this work: the divergence of $\mathcal{N}$.

$$\Omega_{\text{eff}} \quad = \quad \left(\frac{L}{l_P}\right)^{d-2} \oint d^{d-2}x\sqrt{h}\,\sqrt{\det\left(K_i^k K_k^j - \kappa\,\delta_i^j\,\frac{L^2}{r^2}\right)}$$

- Divergence occurs when Ω_{eff} vanishes. This occurs when we reach the gap scale.
- At same point spatial curvature becomes positive. Geometry looks more like flat space black hole then.
- Divergence might signal transition in the nature of holographic degrees of freedom - entanglement entropy calculations suggest flat space dual is non-local theory.
- N^2 increase in degrees of freedom could signal appearance of such a non-local theory.

Conclusions and open questions

- We have found a function $\mathcal{N}$ which captures holographic degrees of freedom.
- Monotonicity controlled by local gravitational field.
- Provides holographic c-function in RG flow backgrounds.
- Interpolates between central charge and entropy in black hole backgrounds.

Conclusions and open questions

- Through $\mathcal{N}$, arrived at a notion of effective holographic phase space.
- Momentum cut-off agrees with previous proposal in the literature.
- Connection of this proposal with a more standard one (area in Planck units) is not completely established.
- Interpretation as phase space leads to a thermodynamic analogy for geometric quantities in AdS solutions

Conclusions and open questions

- Is S related to entanglement entropy in momentum space?
- Can the interpretation of β as a temperature be made precise, and is this connected to entanglement?
- Does a local version of the flow equation exist? What plays the role of radial coordinate in general?
- Study of RG flows in non-trivial states pretty much undeveloped.
- Connection with entanglement renormalization methods? Vidal '05