Using NR to explore fundamental physics and astrophysics

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Bures-sur-Yvette, 04/11/10

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numrel@aei

Plan of the talk

The goals of numerical relativity *vacuum spacetimes *nonvacuum spacetimes
Recent developments in binary BHs *final spin *final recoil
Recent developments in binary NSs

★equal-mass, with/without magnetic field
★unequal-mass, nonzero magnetic field

NR: ie when everything else fails

Numerical relativity (NR) solves Einstein equations in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory. We build codes which we consider as "**theoretical laboratories**".

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$$

 $p = p(\rho, \epsilon, \ldots)$.

 $T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{em}} + \dots$

(field eqs: 6 + 6 + 3 + 1)

(cons. en./mom.: 3+1)

 $\nabla_{\mu}(\rho u^{\mu}) = 0 , \qquad (\text{cons. of}$

(cons. of baryon no : 1)

In vacuum space times the theory is complete and a simulation is limited only by the truncation error.

(EoS: 1 + ...)

 $\nabla^*_{\nu} F^{\mu\nu} = 0,$ (Maxwell eqs. : induction, zero div.)

NR: ie when everything else fails

Numerical relativity (NR) solves Einstein equations in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory. We build codes which we consider as "**theoretical laboratories**".

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} \quad \text{(field eqs: 6+6+3+1)}$$

 $\nabla_{\mu} T^{\mu\nu} = 0$, (cons. en./mom. : 3+1)

 $\nabla_{\mu}(\rho u^{\mu}) = 0$, (cons. of baryon no : 1)

$$p = p(\rho, \epsilon, ...) . \quad (\text{EoS}: 1 + ...)$$
$$\nabla_{\nu}^{*} F^{\mu\nu} = 0, \quad (\text{Maxwell eqs.}: \text{ induction, zero } \epsilon)$$
$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{em}} + ...$$

In non-vacuum space times a simulation is only as good as our models. It's our approximation to "reality".

Sometimes crude but it can be improved: microphysics for the EOS, magnetic fields, viscosity, radiation transport ,...

Binary Black Holes

Koppitz et al. PRL 2007 Pollney et al., PRD 2007 LR et al, 2007, ApJ LR et al, 2008 ApJL LR et al, 2009 PRD LR, CQG 2009

500

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Barausse, LR, ApJL 2009 Reisswig et al., PRD 2009 Reisswig et al., PRL 2009 Reisswig et al., CQG, 2009 Pollney et al., PRD 2009 Pollney et al., 2009

Palenzuela et al., PRL 2009 Moesta et al., PRD 2010 Palenzuela et al. PRD, 2010 Zanotti et al., A&A 2010

RePsid

ImPsid

In vacuum the Einstein equations reduce to $R_{\mu\nu}=0$ How difficult can that be?

100



Animation by Kaehler, Reisswig, LR







All the information is in the waveforms • used in matched filtering techniques (data analysis) • compute the physical/ astrophysical properties of the merger (kick, final spin, etc.) Modelling the final state Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes Before the merger...

 M_1, \vec{S}_1

 \dot{L} orbital angular mom.

 M_2, \vec{S}_2

Modelling the final state Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes

After the merger...

LR et al, 2007 LR et al, 2008 LR et al, 2008 LR, 2009 Barausse, LR 2009



Buonanno et al. 2007 Boyle et al, 2007 Boyle et al, 2008 Tichy & Marronetti, 2008 Kesden, 2008 Lousto et al. 2009 van Meter et al. 2010 Kesden et al. 2010

The final BH has 3 specific properties: mass, spin, recoil. Their knowledge is important for astrophysics and cosmology • A lot of work, especially at the AEI, has gone into mapping the initial configuration to the final one without the need of performing a simulation.

• We can predict with **% precision** the **magnitude** and **direction** of the final **spin** as well as the **magnitude** of the **kick** for arbitrary binaries.

Using a number assumptions derived from PN theory we have derived an **algebraic** expression for the **final spin** vector

$$\begin{aligned} |\boldsymbol{a}_{\text{fin}}| &= \frac{1}{(1+q)^2} \left[|\boldsymbol{a}_1|^2 + |\boldsymbol{a}_1|^2 q^4 + 2|\boldsymbol{a}_2| |\boldsymbol{a}_1| q^2 \cos \alpha + \\ & 2\left(|\boldsymbol{a}_1| \cos \beta + |\boldsymbol{a}_2| q^2 \cos \gamma \right) |\boldsymbol{\ell}| q + |\boldsymbol{\ell}|^2 q^2 \right]^{1/2}, \quad \boldsymbol{L} \end{aligned}$$
where

$$|\boldsymbol{\ell}| = \frac{s_4}{(1+q^2)^2} \left(|\boldsymbol{a}_1|^2 + |\boldsymbol{a}_2|^2 q^4 + 2|\boldsymbol{a}_1| |\boldsymbol{a}_2| q^2 \cos \alpha \right) + \boldsymbol{\alpha} \boldsymbol{S}_2$$
$$\left(\frac{s_5\nu + t_0 + 2}{1+q^2} \right) \left(|\boldsymbol{a}_1| \cos \beta + |\boldsymbol{a}_2| q^2 \cos \gamma \right) + 2\sqrt{3} + t_2\nu + t_3\nu^2$$

Note that the final spin is fully determined in terms of the 5 coefficients s_4 , s_5 , t_0 , t_2 , t_3 which can be computed via numerical simulations. The agreement with data is at % level!

LR et al, 2007, LR et al, 2008, LR et al, 2008, LR, 2009, Barausse, LR 2009

Unequal-mass, aligned binaries

The resulting expression is $(\nu = M_1 M_2 / (M_1 + M_2)^2)$ $a_{\text{fin}}(a, \nu) = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + t_1 \nu + t_2 \nu^2 + t_3 \nu^3$

Numerical data



How to produce a Schwarzschild bh...

Is it possible to produce a Schwarzschild bh from the merger of two Kerr bhs?



How to flip the spin...

In other words: under what conditions does the final black hole spin a direction which is opposite to the initial one?



large spins for comparable masses

 $a_{\text{fin}}(a,\nu) a < 0$ Spin-flips are possible if: • initial spins are antialigned with orbital angular mom. small spins for small mass ratios

Spin-up or spin-down?... Similarly, another basic question with simple answer: does the merger generically spin-up or spin-down?



Just find solutions for:

$$a_{\text{fin}}(a,\nu) = a$$

Clearly, the merger of aligned BHs statistically, leads to a **spin-up**. Note however that for very high spins, the merger actually leads to a spin down: no naked singularities are expected.

Modelling the final state

•final spin vector

•final recoil velocity

Campanelli et al, 2006 Campanelli et al, 2007 Baker et al, 2008 Gonzalez et al, 2007 LR et al, 2007 Hermann et al, 2007 Buonanno et al. 2007 LR et al, 2007 Boyle et al, 2007 Marronetti et al, 2007 LR et al, 2007 Boyle et al, 2008 Baker et al, 2008 Lousto et al, 2008 Tichy & Marronetti, 2008 Kesden, 2008 Barausse, LR, 2009 Lousto et al. 2009 van Meter et al. 2010

Understanding the recoil

At the end of the simulation and unless the spins are equal, the final black hole will acquire a recoil velocity: aka "kick".

The emission of GWs is beamed and thus asymmetrical: the linear momentum radiated at an angle will not be compensated by the momentum after one orbit.

A simple mechanic analogue is offered by a rotary sprinkler

Consider a sequence of spinning BHs in which one of the spins is held fixed and the other one is varied in amplitude



What we know (now) of the kick $v_{\text{kick}} = v_m e_1 + v_{\perp} (\cos(\xi)e_1 + \sin(\xi)e_2) + v_{\parallel}e_3$ where

$$\begin{aligned} v_m &\simeq A\nu^2 \sqrt{1 - 4\nu(1 + B\nu)} \\ v_\perp &\simeq c_1 \frac{\nu^2}{(1+q)} \left(q a_1^{\parallel} - a_2^{\parallel} \right) + c_2 \left(q^2 (a_1^{\parallel})^2 - (a_2^{\parallel})^2 \right) \\ v_\parallel &\simeq \frac{K_1 \nu^2 + K_2 \nu^3}{(1+q)} \left[q a_1^{\perp} \cos(\phi_1 - \Phi_1) - a_2^{\perp} \cos(\phi_2 - \Phi_2) \right] \end{aligned}$$

mass asymmetry $\lesssim 150 \mathrm{km/s}$

spin asymmetry; contribution off the plane

spin asymmetry; contribution in the plane

 $\lesssim 450 \mathrm{km/s}$ $\lesssim 3500 \mathrm{km/s}$

> LR 2008 (review) van Meter et al. 2010

However, there is more than just the final recoil velocity



Understanding the anti-kick

LR, Macedo, Jaramillo, PRL 2010

The basic idea:

• At coalescence a single deformed BH is formed, i.e. a BH with an anisotropic (i.e. non-axisymmetric) distribution of mean curvature.

• Asymptotically all of this curvature must be radiated to leave a Kerr (or Schwarzschild) BH

• The emission of the distorted BH (i.e. what sometimes appears as the anti-kick) will reflect the anisotropic distribution of the curvature, which will therefore dictate the directionality of the recoil (holographic view).

A useful example: head-on collision of unequal-mass nonspinning BHs



The computed BH recoil is shown in the right panel and indicates a positive acceleration and then a negative one Consider two unequalmass nonspinning BHs moving along the z-axis



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Robinson-Trautman spacetimes



The RT spacetime is the class of solutions of the vacuum Einstein equations admitting a congruence of null geodesics which are hypersurface orthogonal, shear-free but with expansion. The asymptotic state is Schwarzschild BH

In other words, a RT spacetime can be seen as an isolated nonspherical white hole emitting GWs. Modulo the fact that the apparent horizon shrinks rather than expand (i.e. it's a past AH) it is a valuable tool to study radiation in nonlinear regimes Using ID "reminiscent" of a head-on collision of BHs, we have looked at the evolution the horizon curvature and of the recoil.



The intrinsic curvature is different at the N-S poles and is radiated exponentially fast. When the curvature is uniform across the horizon, the acceleration stops and the recoil reaches its final value

head-on collision of unequal-mass nonspinning BHs



We are extending the analysis to the headon collision of unequalmass BHs



head-on collision of unequal-mass nonspinning BHs



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Binary Neutron Stars

Baiotti, Giacomazzo, LR, PRD (2008); Baiotti, Giacomazzo, LR, CQG (2009); Giacomazzo, LR, Baiotti, MNRAS (2009); LR, et al (CQG 2010); Giacomazzo, LR, Baiotti, PRD (2010)



Why investigate binary neutron stars?

We know they exist (as opposed to binary BHs) and are among the strongest sources of GWs
We expect them related to SGRBs: energies released are huge: 10⁵⁰⁻⁵¹ erg. Equivalent to what released by the whole Galaxy over ~ 1 year:





Despite decades of observations no self-consistent model has yet been produced to explain them.
Unique examples of complex micro/macro physics:

The two-body problem: GR

Modelling binary black holes (BHs) and binary neutron stars (BNSs) is very different and not because the eqs are different

In the case of BHs we know what to expect:

BH + BH -----> BH + gravitational waves (GWs)

In the case of NSs the question is more **subtle** because in general the merger will lead to an hyper-massive neutron star (HMNS), namely a self-gravitating object in metastable equilibrium:

NS + NS -----> HMNS + GWs + ...? ----> BH + GWs

It's in the intermediate stage that all the physics and complications are; the rewards are however high (GRBs, nuclear physics, etc).

Cold vs Hot EOSs

Simplest example of a **"cold"** EOS is the polytropic EOS. This isentropic: internal energy (temperature) increases/ decreases only by mechanical work (compression/expansion)

$$p = K \rho^{\Gamma}, \qquad \epsilon = \frac{K \rho^{\Gamma-1}}{\Gamma-1}$$

Simplest example of a **"hot"** EOS is the ideal-fluid EOS. This non-isentropic in presence of shocks: internal energy (i.e. temperature) can increase via shock heating.

$$p = \rho \epsilon (\Gamma - 1), \quad \partial_t \epsilon = \dots$$

A cold EOS is optimal for the inspiral; a hot EOS is essential after the merger. Take them as extremes of possible behaviours

Animations: Kaehler, Giacomazzo, LR



Baiotti, Giacomazzo, LR (PRD 2008, CQG 2008)

Cold EOS: high-mass binary $M=1.6\,M_{\odot}$



Density [gicm"3]

Matter dynamics high-mass binary



soon after the merge the torus is formed and undergoes oscillations

Waveforms: cold EOS

high-mass binary



first time the full signal from the formation to a bh has been computed

"merger HMNS BH + *torus" Quantitative differences are produced by: differences induced by the gravitational MASS:*a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

Animations: Kaehler, Giacomazzo, LR



Cold EOS: low-mass binary





Matter dynamics high-mass binary low-mass binary



soon after the merge the torus is formed and undergoes oscillations

long after the merger a BH is formed surrounded by a torus

Waveforms: cold EOS high-mass binary low-mass binary



first time the full signal from the formation to a bh has been computed

development of a bar-deformed NS leads to a long gw signal

"merger HMNS BH + torus" Quantitative differences are produced by: - differences induced by the gravitational MASS: a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time - differences induced by the EOS ("cold" or "hot"): a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

Animations: Kaehler, Giacomazzo, Rezzolla

T[ms] - 0.00 T[M] - 0.00

Hot EOS: high-mass binary $M=1.6\,M_{\odot}$

0.0



Waveforms: hot EOS high-mass binary low-mass binary



the high internal energy (temperature) of the HMNS prevents a prompt collapse the HMNS evolves on longer (radiation-reaction) timescale

Imprint of the EOS: hot vs cold



After the merger a BH is produced over a timescale comparable with the dynamical one After the merger a BH is produced over a timescale larger or much larger than the dynamical one







Andersson et al. (GRG 2009)



collapse to BH and ringdown



- "merger HMNS BH + torus" Quantitative differences are produced by: - differences induced by the gravitational MASS: a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time - differences induced by the EOS ("cold" or "hot"): a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later - differences induced by MASS ASYMMETRIES: tidal disruption before merger; may lead to prompt BH - differences induced by MAGNETIC FIELDS: the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse - differences induced by RADIATIVE PROCESSES:
 - radiative losses will alter the equilibrium of the HMNS

Unequal-mass binaries

In contrast to binary black holes, binary neutron stars do not show large variations in the mass ratio.



M	M_2	
1.44	1.38	B1913+16
1.33	1.34	B1534+12
1.33	1.25	J0737-3039
1.40	1.18	J1756-2251
1.36	1.35	B2127+11C
1.35	1.26	J1906+0746
1.62	1.11	J1811-1736
1.56	1.05	J1518+4904
1.14	1.36	J1829+2456

Are these small (!) mass asymmetries important? For black holes they would hardly matter

D. Gondek-Rosinska (2009)



Animations: Giacomazzo, Koppitz, LR

Total mass : $3.37 M_{\odot}$; mass ratio :0.80;



* the torii are generically more massive
* the torii are generically more extended
* the torii tend to stable quasi-Keplerian configurations
* overall unequal-mass systems have all the ingredients
needed to create a GRB

Torus properties: size



Note that although the total mass is very similar, the unequal-mass binary yields a torus which is about ~ 4 times larger and ~ 200 times more massive

Torus properties: density

spacetime diagram of rest-mass density along x-direction



equal mass binary: note the periodic accretion and the compact size; densities are not very high

unequal mass binary: note the continuous accretion and the very large size and densities (temperatures)

Torus properties: unequal-masses

M _T (M _{sun})			0.4
It's much	harder	to produ	ice tori
of such	large mai	co prode	realistic
	hipppioc	SSCS WILLI	realistic
	Dinaries.		
Prospec	ts tor mo	baelling C	JKBS
trom BN	vss are p	romising	
	3.6	0.75	0.7 0.65 0.6
	3.8 1 0.95 0.	9 0.85 0.8 0.70 q	

Model	$M_{\rm total}$	q	$M_{\rm torus}$
	(M_{\odot})		(M_{\odot})
M3.6q1.00	3.558	1	0.0010
M3.7q0.94	3.680	0.94	0.0100
M3.4q0.91	3.404	0.91	0.0994
M3.4q0.80	3.375	0.80	0.2088
M3.5q0.75	3.464	0.75	0.0802
M3.4q0.70	3.371	0.70	0.2116

The torus mass decreases with the mass ratio and with the total mass; at lowest order:

 $M_{\text{tor}}(q, M_{\text{tot}}) = (M_{\text{max}} - M_{\text{tot}}) [c_1(1-q) + c_2]$

where M_{max} is the maximum (baryonic) mass of the binary and c_1 , c_2 are coefficients computed from the simulations.

Extending the work to MHD

NSs have large magnetic fields but these have been traditionally neglected. It is natural to ask:

- can we detect B-fields during the inspiral?
- can we detect B-fields after the merger?
- how do B-fields influence the dynamics of the tori?

This is not easy but can be done: relativistic hydrodynamics is extended to *ideal-MHD* (infinite conductivity).
The B-fields are initially contained inside the stars: ie no magnetospheric effects.
We have considered 12 binaries (low/high mass) with MFs: B = 0, 10⁸, 10¹⁰, 10¹², 10¹⁴, 10¹⁷ G

Animations: Koppitz, Giacomazzo, LR





Typical evolution for a magnetized binary (hot EOS) $M=1.65\,M_{\odot},\ B=10^{10}\,{\rm G}$

9 15 log(rho)[g/cm²]



Waveforms: comparing against magnetic fields



Compare against very strong and no B-field: • the post-merger evolution is different for all masses; strong Bfields delay the collapse to BH • the evolution in the inspiral is also different for such large B-fields However, mismatch is too small for present detectors: influence of B-fields on the inspiral is cannot be detected!

AFTER

MERGER

AFTER BH FORMATION

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Autorised

At the end of the simulation the Bfield in the torus is mostly toroidal and has reached values of $\sim 10^{15}$ G.A poloidal component is dominant instead along the axis where the Lorentz factor is ~ 4 . Correct physical conditions for launching a jet

This is the first evidence that from generic initial data it is possible to obtain a configuration to explain the central engine of gamma-ray bursts

Conclusions

* Evolution BBHs is under control and interfaced with DA (NINJA+, NR-AR, etc). Higher precision is needed, small mass ratios and a better understanding of the nonlinear dynamics

*With simple EOSs have reached possibly the most complete description of BNSs from the inspiral, merger, collapse to BH. Can draw this picture with/without B-fields, equal and unequal masses.

*GWs from BNSs are much complex/richer than from BBHs: can be the Rosetta stone to decipher the NS interior.

*Magnetic fields unlikely to be detected during the inspiral but important after the merger (amplified by dynamos/instabilities)

*Much remains to be done to model realistically BNSs, both from a microphysical point of view (EOS, neutrino emission, etc) and a from a macrophysical one (instabilities, etc.)