## Using NR to explore fundamental

## physics and astrophysics

## Luciano Rezzolla



Albert Einstein Institute, Potsdam, Germany
Dept. of Physics and Astronomy, Louisiana State Univ. Louisiana, USA

Institut des Hautes Études Scientifiques
Fondation reconnue d'utilité publique


## Plan of the talk

- The goals of numerical relativity
* vacuum spacetimes
*nonvacuum spacetimes
- Recent developments in binary BH
* final spin
*final recoil
- Recent developments in binary NSs
*equal-mass, with/without magnetic field
*unequal-mass, nonzero magnetic field


## NR: ie when everything else fails

Numerical relativity (NR) solves Einstein equations in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory. We build codes which we consider as "theoretical laboratories".

$$
R_{\mu \nu}-\frac{1}{2} g_{\mu \nu} R=8 \pi T_{\mu \nu}
$$

(field eqs : $6+6+3+1$ )
(cons. en./mom. : $3+1$ )
(cons. of baryon no : 1)

$$
p=p(\rho, \epsilon, \ldots) . \quad(\operatorname{EoS}: 1+\ldots)
$$

In vacuum space times the theory is complete and a simulation is limited only by the
truncation error.
(Maxwell eqs. : induction, zero div.)

$$
T_{\mu \nu}=T_{\mu \nu}^{\mathrm{fluid}}+T_{\mu \nu}^{\mathrm{em}}+\ldots
$$

## NR: ie when everything else fails

Numerical relativity (NR) solves Einstein equations in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory. We build codes which we consider as "theoretical laboratories".

In non-vacuum space times a simulation is only as good as our
$\nabla_{\mu} T^{\mu \nu}=0$,
(cons. en./mom. : $3+1$ )
$\nabla_{\mu}\left(\rho u^{\mu}\right)=0$,
(cons. of baryon no: 1)
$p=p(\rho, \epsilon, \ldots) . \quad(\operatorname{EoS}: 1+\ldots)$
models. It's our
approximation to
"reality".
Sometimes crude but
it can be improved:
microphysics for the
$\nabla_{\nu}^{*} F^{\mu \nu}=0, \quad$ (Maxwell eqs. : induction, zero div.) EOS , magnetic fields,
viscosity, radiation
$T_{\mu \nu}=T_{\mu \nu}^{\text {fluid }}+T_{\mu \nu}^{\mathrm{em}}+\ldots$
transport ,...

## Binary Black Holes



In vacuum the Einstein equations reduce to

## $R_{\mu \nu}=0$

How difficult orat that be?


All the information is in the waveforms

- used in matched filtering techniques (data analysis)
- compute the physical/ astrophysical properties of the merger (kick, final spin, etc.)


## Modelling the final state

Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes Before the merger...

$\uparrow \vec{L}$ orbital angular mom.

## Modelling the final state

Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes

After the merger...

LR et al, 2007
LR et al, 2008
LR et al, 2008
LR, 2009
Barausse, LR 2009

Buonanno et al. 2007
Boyle et al, 2007
Boyle et al, 2008
Tichy \& Marronetti, 2008 Kesden, 2008
Lousto et al. 2009
van Meter et al. 2010
Kesden et al. 2010

The final BH has 3 specific properties: mass, spin, recoil. Their knowledge is important for astrophysics and cosmology

- A lot of work, especially at the AEI, has gone into mapping the initial configuration to the final one without the need of performing a simulation.
- We can predict with \% precision the magnitude and direction of the final spin as well as the magnitude of the kick for arbitrary binaries.

Using a number assumptions derived from PN theory we have derived an algebraic expression for the final spin vector

$$
\begin{aligned}
& \qquad\left|\boldsymbol{a}_{\mathrm{fin}}\right|=\frac{1}{(1+q)^{2}}\left[\left|\boldsymbol{a}_{1}\right|^{2}+\left|\boldsymbol{a}_{1}\right|^{2} q^{4}+2\left|\boldsymbol{a}_{2}\right|\left|\boldsymbol{a}_{1}\right| q^{2} \cos \alpha+\right. \\
& \text { where } \\
& \left.\qquad|\boldsymbol{\ell}|=\frac{s_{4}}{\left(1+q^{2}\right)^{2}}\left(\left|\boldsymbol{a}_{1}\right| \cos \beta+\left|\boldsymbol{a}_{2}\right|^{2}+\mid q^{2} \cos \gamma\right)|\boldsymbol{\ell}| q+\left.\left|\boldsymbol{\boldsymbol { a } _ { 2 }}\right|^{2}\right|^{2} q^{2}\right]^{1 / 2} \\
& \left.\qquad\left(\frac{s_{5} \nu+t_{0}+2}{1+q^{2}}\right)\left(\left|\boldsymbol{a}_{1}\right|\left|\boldsymbol{a}_{2}\right| q^{2} \cos \alpha\right)+\cos _{\alpha} \beta+\left|\boldsymbol{a}_{2}\right| q^{2} \cos \gamma\right)+2 \sqrt{3}+t_{2} \nu+t_{3} \nu^{2}
\end{aligned}
$$

Note that the final spin is fully determined in terms of the 5 coefficients $s_{4}, s_{5}, t_{0}, t_{2}, t_{3}$ which can be computed via numerical simulations. The agreement with data is at \% level!

## Unequal-mass, aligned binaries

The resulting expression is $\left(\nu=M_{1} M_{2} /\left(M_{1}+M_{2}\right)^{2}\right)$

$$
a_{\mathrm{fin}}(a, \nu)=a+s_{4} a^{2} \nu+s_{5} a \nu^{2}+t_{0} a \nu+t_{1} \nu+t_{2} \nu^{2}+t_{3} \nu^{3}
$$

Numerical data


Analytic expression

EMRL: extreme mass-ratio limit

The functional dependence is simple enough that a low-order polynomial is sufficient

## How to produce a Schwarzschild bh...

Is it possible to produce a Schwarzschild bh from the merger of two Kerr bhs?


Find solutions for:
$a_{\mathrm{fin}}(a, \nu)=0$
Unequal masses and spins
antialigned to the orbital ang. mom. are necessary

Isolated Schwarzsanild ${ }^{0.05}$ b likely${ }^{\text {aresult }}$ of a simllar merger!

## How to flip the spin...

In other words: under what conditions does the final black hole spin a direction which is opposite to the initial one?


Find solutions for:

$$
a_{\mathrm{fin}}(a, \nu) a<0
$$

Spin-flips are possible if:

- initial spins are antialigned with orbital angular mom.
- small spins for small mass ratios
- large spins for comparable masses


## Spin-up or spin-down?...

Similarly, another basic question with simple answer: does the merger generically spin-up or spin-down?


Just find solutions for:

$$
a_{\text {fin }}(a, \nu)=a
$$

Clearly, the merger of aligned BHs statistically, leads to a spin-up. Note however that for very high spins, the merger actually leads to a spin down: no naked singularities are expected.

## Modelling the final state

## -final spin vector

## -final recoil velocity

Campanelli et al, 2006
Campanelli et al, 2007
Baker et al, 2008
Gonzalez et al, 2007
LR et al, 2007
Hermann et al, 2007
Buonanno et al. 2007
LR et al, 2007
Boyle et al, 2007
Marronetti et al, 2007

LR et al, 2007
Boyle et al, 2008
Baker et al, 2008
Lousto et al, 2008
Tichy \& Marronetti, 2008
Kesden, 2008
Barausse, LR, 2009
Lousto et al. 2009
van Meter et al. 2010

## Understanding the recoil

At the end of the simulation and unless the spins are equal, the final black hole will acquire a recoil velocity: aka "kick".

The emission of GWs is beamed and thus asymmetrical: the linear momentum radiated at an angle will not be compensated by the momentum after one orbit.

A simple mechanic analogue is offered by a rotary sprinkler


Consider a sequence of spinning BH in which one of the spins is held fixed and the other one is varied in amplitude
ro: $\uparrow \downarrow\left(a_{1} / a_{2}=-4 / 4\right)$
$r 2: \uparrow \downarrow\left(a_{1} / a_{2}=-2 / 4\right)$
r4: $\uparrow$.
$\left(a_{1} / a_{2}=-0 / 4\right)$
$r 6: \uparrow \uparrow \quad\left(a_{1} / a_{2}=2 / 4\right)$
$r 8: \uparrow \uparrow\left(a_{1} / a_{2}=4 / 4\right)$


## What we know (now) of the kick

$$
v_{\text {kick }}=v_{m} \boldsymbol{e}_{1}+v_{\perp}\left(\cos (\xi) \boldsymbol{e}_{1}+\sin (\xi) \boldsymbol{e}_{2}\right)+v_{\|} \boldsymbol{e}_{3}
$$ where

$$
\begin{aligned}
& v_{m} \simeq A \nu^{2} \sqrt{1-4 \nu}(1+B \nu) \\
& v_{\perp} \simeq c_{1} \frac{\nu^{2}}{(1+q)}\left(q a_{1}^{\|}-a_{2}^{\|}\right)+c_{2}\left(q^{2}\left(a_{1}^{\|}\right)^{2}-\left(a_{2}^{\|}\right)^{2}\right) \\
& v_{\|} \simeq \frac{K_{1} \nu^{2}+K_{2} \nu^{3}}{(1+q)}\left[q a_{1}^{\perp} \cos \left(\phi_{1}-\Phi_{1}\right)-a_{2}^{\perp} \cos \left(\phi_{2}-\Phi_{2}\right)\right]
\end{aligned}
$$

## mass asymmetry $\lesssim 150 \mathrm{~km} / \mathrm{s}$

spin asymmetry; contribution off the plane $\lesssim 450 \mathrm{~km} / \mathrm{s}$
spin asymmetry; contribution in the plane $\lesssim 3500 \mathrm{~km} / \mathrm{s}$

However, there is more than just the final recoil velocity
no: $\uparrow \downarrow\left(a_{1} / a_{2}=-4 / 4\right)$
$r 2: \uparrow \downarrow\left(a_{1} / a_{2}=-2 / 4\right)$
$r 4: \uparrow \quad\left(a_{1} / a_{2}=-0 / 4\right)$
$r 6: \uparrow \uparrow \quad\left(a_{1} / a_{2}=2 / 4\right)$
rB: $\uparrow \uparrow$
$\left(a_{1} / a_{2}=4 / 4\right)$
why do BHt "anti-kick"?


## Understanding the anti-kick <br> LR, Macedo, Jaramillo, PRL 2010

## The basic idea:

- At coalescence a single deformed BH is formed, i.e. a BH with an anisotropic (i.e. non-axisymmetric) distribution of mean curvature.
- Asymptotically all of this curvature must be radiated to leave a Kerr (or Schwarzschild) BH
-The emission of the distorted BH (i.e. what sometimes appears as the anti-kick) will reflect the anisotropic distribution of the curvature, which will therefore dictate the directionality of the recoil (holographic view).


## A useful example: head-on collision of unequal-mass nonspinning BHs



The computed BH recoil is shown in the right panel and indicates a positive acceleration and then a negative one

Consider two unequalmass nonspinning BHs moving along the $z$-axis


## A useful example: head-on collision of unequal-mass nonspinning BHs



The computed BH recoil is shown in the right panel and indicates a positive acceleration and then a negative one

Consider two unequalmass nonspinning BH moving along the $z$-axis


# A useful example: head-on collision of unequal-mass nonspinning BHs 



The computed BH recoil is shown in the right panel and indicates a positive acceleration and then a negative one

Consider two unequalmass nonspinning BH moving along the $z$-axis


## Robinson-Trautman spacetimes



The RT spacetime is the class of solutions of the vacuum Einstein equations admitting a congruence of null geodesics which are hypersurface orthogonal, shear-free but with expansion.The asymptotic state is Schwarzschild BH

In other words, a RT spacetime can be seen as an isolated nonspherical white hole emitting GWs. Modulo the fact that the apparent horizon shrinks rather than expand (i.e. it's a past AH ) it is a valuable tool to study radiation in nonlinear regimes

Using ID "reminiscent" of a head-on collision of BHs, we have looked at the evolution the horizon curvature and of the recoil.


$$
\begin{aligned}
& K(u, \Omega) \equiv Q^{2}\left(1+\nabla_{\Omega}^{2} \ln Q\right) \\
& \partial_{u} Q(u, \Omega)=-Q^{3} \nabla_{\Omega}^{2} K(u, \Omega) /\left(12 M_{\infty}\right)
\end{aligned}
$$



The intrinsic curvature is different at the $\mathbf{N}$ - $\mathbf{S}$ poles and is radiated exponentially fast. When the curvature is uniform across the horizon, the acceleration stops and the recoil reaches its final value

## head-on collision of unequal-mass nonspinning BHs



We are extending the analysis to the headon collision of unequalmass BH


## head-on collision of unequal-mass nonspinning BHs



We are extending the analysis to the headon collision of unequalmass BH


## head-on collision of unequal-mass nonspinning BHs



We are extending the analysis to the headon collision of unequalmass BH


## Binary Neutron Stars

Baiotti, Giacomazzo, LR, PRD (2008); Baiotti, Giacomazzo, LR, CQG (2009); Giacomazzo, LR, Baiotti, MNRAS (2009); LR, et al (CQG $2010)$; Giacomazzo, LR, Baiotti, PRD (20I0)


## Why investigate binary neutron stars?

- We know they exist (as opposed to binary BHs ) and are among the strongest sources of GWs
- We expect them related to SGRBs: energies released are huge: $10^{50-51}$ erg. Equivalent to what released by the whole Galaxy over ~ lyear:

- Despite decades of observations no self-consistent model has yet been produced to explain them.
- Unique examples of complex micro/macro physics:


## The two-body problem: GR

Modelling binary black holes ( BH s) and binary neutron stars (BNSs) is very different and not because the eqs are different In the case of BH s we know what to expect:
$\mathrm{BH}+\mathrm{BH} \longrightarrow \mathrm{BH}+$ gravitational waves (GWs)
In the case of NSs the question is more subtle because in general the merger will lead to an hyper-massive neutron star (HMNS), namely a self-gravitating object in metastable equilibrium:
$\mathrm{NS}+\mathrm{NS} \longrightarrow \mathrm{HMNS}+\mathrm{GWs}+\ldots ? \longrightarrow \mathrm{BH}+\mathrm{GWs}$
It's in the intermediate stage that all the physics and complications are; the rewards are however high (GRBs, nuclear physics, etc).

## Cold vs Hot EOSs

Simplest example of a "cold" EOS is the polytropic EOS. This isentropic: internal energy (temperature) increases/ decreases only by mechanical work (compression/expansion)

$$
p=K \rho^{\Gamma,} \quad \epsilon=\frac{K \rho^{\Gamma-1}}{\Gamma-1}
$$

Simplest example of a "hot" EOS is the ideal-fluid EOS. This non-isentropic in presence of shocks: internal energy (i.e. temperature) can increase via shock heating.

$$
p=\rho \epsilon(\Gamma-1), \quad \partial_{t} \epsilon=\ldots
$$

A cold EOS is optimal for the inspiral; a hot EOS is essential after the merger. Take them as extremes of possible behaviours

Animations: Kaehler, Giacomazzo, LR
I[ms] - 0.00
Baiotti, Giacomazzo, LR (PRD 2008, CQG 2008)
$T[M]=0.00$

Cold EOS: high-mass binary


## Matter dynamics

high-mass binary


## Waveforms: cold EOS

high-mass binary

first time the full signal from the formation to a bh has been computed

- differences induced by the gravitational MASS: a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

T[ms] $=0.00$

Animations: Kaehler, Giacomazzo, LR

$T M]=0.00$

## Cold EOS: low-mass binary

$$
M=1.4 M_{\odot}
$$

## Matter dynamics

high-mass binary
low-mass binary

soon after the merge the torus is formed and undergoes oscillations

long after the merger a BH is formed surrounded by a torus

## Waveforms: cold EOS

high-mass binary low-mass binary


first time the full signal from the formation to a bh has been computed
development of a bar-deformed NS leads to a long gw signal

- differences induced by the gravitational MASS:
a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- differences induced by the EOS ("cold" or "hot"): a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

Animations: Kaehler, Giacomazzo, Rezzolla
I[ms] - 0.00

$[\mathrm{M}]=0.00$


Hot EOS: high-mass binary

## $M=1.6 M_{\odot}$

## Waveforms: hot EOS

high-mass binary
low-mass binary

the high internal energy (temperature) of the HMNS prevents a prompt collapse
the HMNS evolves on longer (radiation-reaction) timescale

## Imprint of the EOS: hot vs cold



After the merger a BH is produced over a timescale comparable with the dynamical one

After the merger a BH is produced over a timescale larger or much larger than the dynamical one

## Imprint of the EOS: frequency domain

Andersson et al. (GRG 2009)



Imprint of the EOS: frequency domain
Andersson et al. (GRG 2009)


Imprint of the EOS: frequency domain
Andersson et al. (GRG 2009)


Imprint of the EOS: frequency domain
Andersson et al. (GRG 2009)


Imprint of the EOS: frequency domain
Andersson et al. (GRG 2009)


With sufficiently sensitive detectors, GWs will work ${ }_{10}$ as the Rosetta stone to decipher the NS interior

## "merger $\rightarrow \mathrm{HMNS} \longrightarrow \mathrm{BH}+$ torus"

Quantitative differences are produced by:

- differences induced by the gravitational MASS:
a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- differences induced by the EOS ("cold" or "hot"):
a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later
- differences induced by MASS ASYMMETRIES:
tidal disruption before merger; may lead to prompt BH
- differences induced by MAGNETIC FIELDS:
the angular momentum redistribution via magnetic braking or
MRI can increase/decrease time to collapse
- differences induced by RADIATIVE PROCESSES:
radiative losses will alter the equilibrium of the HMNS


## Unequal-mass binaries

In contrast to binary black holes, binary neutron stars do not show large variations in the mass ratio.

D. Gondek-Rosinska (2009)

| $M_{1}$ | $M_{2}$ |  |
| :---: | :---: | :---: |
| 1.44 | 1.38 | B1913+16 |
| 1.33 | 1.34 | B1534+12 |
| 1.33 | 1.25 | Jo737-3039 |
| 1.40 | 1.18 | $J 1756-251$ |
| 1.36 | 1.35 | B2127+11C |
| 1.35 | 1.26 | $J 1906+0746$ |
| 1.62 | 1.11 | $J 1811-1736$ |
| 1.56 | 1.05 | $J 1518+4904$ |
| 1.14 | 1.36 | J1829+2456 |

Are these small (!) mass asymmetries important?
For black holes they would hardly matter

## Total mass : $3.37 M_{\odot} ; \quad$ mass ratio :0.80;



* the torii are generically more massive
* the torii are generically more extended
* the torii tend to stable quasi-Keplerian configurations overall unequal-mass systems have all the ingredients


## Torus properties: size





Note that although the total mass is very similar, the unequal-mass binary yields a torus which is about $\sim 4$ times larger and $\sim 200$ times more massive

## Torus properties: density

spacetime diagram of rest-mass density along $x$-direction

equal mass binary: note the periodic accretion and the compact size; densities are not very high

unequal mass binary: note the continuous accretion and the very large size and densities (temperatures)

## Torus properties: unequal-masses

${ }^{M_{r}\left(M_{m e n}\right)}$ -

It's much harder to produce tori of such large masses with realistic BH-NS binaries.
Prospects for modelling GRBs from BNSs are promising


| Model | $M_{\text {total }}$ <br> $\left(M_{\odot}\right)$ | $q$ | $M_{\text {torus }}$ <br> $\left(M_{\odot}\right)$ |
| :---: | :---: | :---: | :---: |
| M3.6q1.00 | 3.558 | 1 | 0.0010 |
| M3.7q0.94 | 3.680 | 0.94 | 0.0100 |
| M3.4q0.91 | 3.404 | 0.91 | 0.0994 |
| M3.4q0.80 | 3.375 | 0.80 | 0.2088 |
| M3.5q0.75 | 3.464 | 0.75 | 0.0802 |
| M3.4q0.70 | 3.371 | 0.70 | 0.2116 |

The torus mass decreases with the mass ratio and with the total mass; at lowest order:

$$
\widetilde{M}_{\mathrm{tor}}\left(q, M_{\mathrm{tot}}\right)=\left(M_{\max }-M_{\mathrm{tot}}\right)\left[c_{1}(1-q)+c_{2}\right]
$$

where $M_{\max }$ is the maximum (baryonic) mass of the binary and $c_{1}, c_{2}$ are coefficients computed from the simulations.

## Extending the work to MHD

NSs have large magnetic fields but these have been traditionally neglected. It is natural to ask:

- can we detect B-fields during the inspiral?
- can we detect B-fields after the merger?
- how do B-fields influence the dynamics of the tori?
- This is not easy but can be done: relativistic hydrodynamics is extended to ideal-MHD (infinite conductivity).
- The B-fields are initially contained inside the stars: ie no magnetospheric effects.
- We have considered 12 binaries (low/high mass) with MFs:

$$
B=0,10^{8}, 10^{10}, 10^{12}, 10^{14}, 10^{17} \mathrm{G}
$$



Typical evolution for a magnetized binary (hot EOS) $M=1.65 M_{\odot}, B=10^{10} \mathrm{G}$

Waveforms: comparing against magnetic fields


Compare against very strong and no B-field: -the post-merger evolution is different for all masses; strong Bfields delay the collapse to BH
-the evolution in the inspiral is also different for such large B-fields However, mismatch is too small for present detectors: influence of B-fields on the inspiral is cannot be detected!


INITIAL DATA


AFTER TWO ORBITS

## $\mathrm{t} \sim 17 \mathrm{~ms}$

AFTER MERGER





AFTER BH FORMATION


At the end of the simulation the Bfield in the torus is mostly toroidal and has reached values
of $\sim 10^{15} \mathrm{G}$. A
poloidal component is dominant instead along the axis where the Lorentz factor is $\sim 4$.
Correct physical conditions for launching a jet


This is the first evidence that from generic initial data it is possible to obtain a configuration to explain the central engine of gamma-ray bursts

## Conclusions

* Evolution BBHs is under control and interfaced with DA (NINJA+, NR-AR, etc). Higher precision is needed, small mass ratios and a better understanding of the nonlinear dynamics *With simple EOSs have reached possibly the most complete description of BNSs from the inspiral, merger, collapse to BH. Can draw this picture with/without B-fields, equal and unequal masses.
*GWs from BNSs are much complex/richer than from BBHs: can be the Rosetta stone to decipher the NS interior.
*Magnetic fields unlikely to be detected during the inspiral but important after the merger (amplified by dynamos/instabilities) *Much remains to be done to model realistically BNSs, both from a microphysical point of view (EOS, neutrino emission, etc) and a from a macrophysical one (instabilities, etc.)

