Highly Excited Strings

Dimitri Skliros (KCL & IHES)

IHES, Bures-sur-Yvette, 5 Nov 2014

based on work with:
E. Copeland, P. Saffin, M. Hindmarsh
(2×XXX 2015, PRL 2013, PRL 2011, PRD 2011)

Motivation

Highly Excited Strings (HES) are deeply rooted into the structure of string theory.

- they are related to the UV finiteness of string amplitudes;
- are energetically favourable at high energy densities (limiting Hagedorn temperature, . . .);[1]
- they provide a source of non-locality (desirable[2] e.g. in resolving information paradox);
- their properties may even lead to signatures unique to string theory (e.g. in context of cosmic superstrings[3])

---

[1] Deo, Jain, Tan (1987-1989); Tseytlin, Vafa (1991); Skliros, Hindmarsh (2008)

[2] Susskind (1993); Susskind (1995); Low, Polchinski, Susskind, . . . , (1997); Giddings (2007); Hartman, Maldacena (2013);. . .

[3] Sen (1998); Dvali & Vilenkin (2004); Copeland, Myers, Polchinski (2004); Hindmarsh (2011); DS, Copeland, Saffin (2013); . . .

HES as Black Holes I

A major challenge for any theory of quantum gravity is to provide:

(a) a microscopic interpretation of the Bekenstein-Hawking entropy
(b) to resolve the black hole (BH) information paradox

Currently a large amount of effort to address these, one approach being the fuzzball proposal, where:[4]

(a) quantum effects important at the would-be BH horizon;
(b) quantum matter that makes up the black hole is of order the horizon scale

And even more radical proposals (e.g. firewall proposal[5])

---

[4] Mathur, Turton (2014); Mathur (2009); Bena, Warner (2007); Skenderis, Taylor (2008);... Chen, Michel, Polchinski, Puhm (2014)

[5] Almheiri, Marolf, Polchinski, Sully (2013)

HES as Black Holes II

Ultrarelativistic scattering of D-branes leads to copious production of (open) HES[6] (velocity-dependent correction to open string mass). So expect enhanced string production as late-time in-falling observers are strongly boosted in near horizon[7]

Inline with earlier suggestions that[8] HES effectively spread out on the horizon relative to external observer

In absence of RR charges a single[9] HES the most likely BH microstate

$\Rightarrow$ An explicit handle on quantum HES should settle these speculations.

---

[6] McAllister, Mitra (2004)

[7] Silverstein (2014)

[8] Susskind (1993); Susskind (1995); Low, Polchinski, Susskind, . . . , (1997); Giddings (2007); Hartman, Maldacena (2013);. . .

[9] Susskind (1993); Horowitz, Polchinski (1997); Damour, Veneziano (2000); . . .

HES as Cosmic Strings I

Renewed interest in cosmic strings (CS) in recent years (warped compactifications, brane inflation, ...)

Compactifications of string theory lead to many potential cosmic string candidates:[10]

- F-strings
- D-strings
- (p, q)-strings
- wrapped D-branes
- solitonic strings
- electric and magnetic flux tubes

⋮

[10] Sarangi, Tye (2002); Dvali, Vilenkin (2004); Copeland, Myers, Polchinski (2004); Polchinski (2006); Copeland, Kibble (2009); Sakellariadou (2009); Hindmarsh (2011); Banks, Seiberg (2011)

HES as Cosmic Strings II

General consensus on large scale evolution[11]

String inter-commutations[12] and string decay[13] play a fundamental role in the cosmological relevance of CS

Strongest signal from CS: gravitational wave bursts from string with cusps may be detectable in near future for string models with $G\mu \geq 10^{-13}$ (LIGO2,LISA)[14]

However, back-reaction effects (which can play a crucial role) neglected

---

[11] Vilenkin, Albrecht-Turok, Allen-Shellard, Hindmarsh, Urrestilla, Copeland, Kibble, Steer, Sakellariadou, Avgoustidis, Bevis

[12] Shellard '86, Jackson-Jones-Polchinski '04, Achúcarro-Putter '06

[13] Chialva-Iengo-Russo (2003-06); Skliros, Copeland Saffin (2013)

[14] Damour,Vilenkin '00,'01, Siemens,Olum '03, Blanco-Pillado,Olum, Binetruy et al, ...

Effective Theory Description

One may discuss HES in terms of EFTs, e.g.:[15]

$$S_{\rm eff} = \frac{1}{16\pi G_D}\int d^D x\sqrt{-G}\, e^{-2\Phi}\Big(R_{(D)} + 4(\nabla\Phi)^2 - \frac{1}{12}\,H^2_{(3)} + \dots\Big)$$
$$- \mu\int_{S^2}\partial X^\mu \wedge \bar\partial X^\nu\big(G_{\mu\nu} + B_{\mu\nu}\big) + \dots$$

Although adequate for certain purposes, these do not crucial stringy features,[16] such as:

- couplings to infinite set of oscillator states
- inherently QM processes (such as string intercommutations)
- break down in UV and at small scales

---

[15] where Φ, $G_{\mu\nu}$ and $H_{(3)}$ are the dilaton, spacetime metric and 3-form field strength, $H = dB$, respectively

[16] Tseytlin (1990); Dabholklar, Harvey (1989)

HES in String Theory

Going beyond EFTs . . .

Would like to phrase the above in terms of available tools in perturbative string theory: quantum vertex operators and associated string amplitudes

Although string computations with HES are non-trivial, new efficient tools appropriate for HES now available, making computations with HES tractable and efficient[17]

⇒ Trick is to consider strings in a coherent state basis[18] ..

[17] Skliros, Copeland, Saffin (PRL 2013)
[18] Hindmarsh, Skliros (PRL 2011); Skliros, Hindmarsh (PRD 2011)

Overview of Talk

- Coherent vertex operator construction of HES
- Generic two-point amplitudes (at fixed-loop momenta) and duality (on $\mathbb{R}^{D-1,1} \times T^{26-D}$)
- Example: decay rates and power associated to massless emission for special class of HES
- Effective field theory limit and α' corrections

Context

We will be working within simplest non-trivial superstring context,

$$I = \frac{1}{2\pi\alpha'} \int d^2z\, \partial X^M \bar{\partial} X^N G_{MN} + \dots,$$

in absence of RR charges, where '...' denote fermions (and other background fields) that won't be relevant for the basic stringy picture. Here $X : \Sigma \to \mathcal{M}$ denote worldsheet embeddings into spacetime $\mathcal{M}$. We assume that topologically $\mathcal{M} = \mathbb{R}^{D-1,1} \times T^{D_{\rm tot}-D}$, with $D = \#$non-compact dimensions:

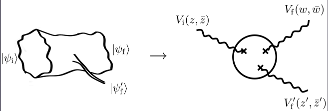
$$ds^2 = e^{2A(Y_0)} \eta_{\mu\nu} dX^\mu dX^\nu + e^{-2A(Y_0)} \eta_{ab} dY^a dY^b$$

leading to effective string tension:

$$\mu = \frac{e^{2A(Y_0)}}{2\pi\alpha'}$$

Vertex Operators

Asymptotic states described by vertex operators, $V(z,\bar{z})$:



Basic interaction is splitting and joining of open or closed strings
$V(z,\bar{z})$ must be composed of fields present ($X^M, g_{\alpha\beta}$):

$$V(z,\bar{z}) = \sum_i P_i\left[\partial^{\#}X\right] e^{ik_{(i)}\cdot X(z)}\ \bar{P}_i\left[\bar{\partial}^{\#}X\right] e^{i\bar{k}_{(i)}\cdot X(\bar{z})}$$

Q: For what choice of polynomials, $P_i, \bar{P}_i$, and momenta, $k_i, \bar{k}_i$ will $V(z,\bar{z})$ represent a HES?

Answer most elegantly expressed in terms of **coherent vertex operators** ...

Coherent States in QM

Consider harmonic oscillator Hamiltonian,

$$\hat{H} = \omega\left(a^\dagger a + \frac{1}{2}\right), \qquad \text{with} \qquad [a, a^\dagger] = 1 \quad \text{and} \quad a|0\rangle = 0,$$

$a^\dagger$, a are creation and annihilation operators. Coherent states are eigenstates of the annihilation operator, a,

$$a|\lambda\rangle = \lambda|\lambda\rangle, \qquad \text{with} \qquad |\lambda\rangle = \exp\left(\lambda a^\dagger - \lambda^* a\right)|0\rangle,$$

which therefore have classical expectation values

$$\langle x(t)\rangle = \frac{1}{\sqrt{2}}\left(\lambda^* e^{i\omega t} + \lambda e^{-i\omega t}\right), \qquad \text{with} \qquad \frac{d^2}{dt^2}\langle x(t)\rangle = -\omega^2\langle x(t)\rangle.$$

Note presence of continuous quantum numbers: λ

QM→ Strings

If we make use of operator-fields correspondence, $\alpha^{\mu}_{-n} \leftrightarrow \partial^n X^{\mu}$,

$$\alpha^{\mu}_{-n} \cong \frac{i}{(n-1)!}\,\partial^n X^{\mu}(z), \qquad \text{and} \qquad |0,0;k\rangle \sim e^{ik\cdot X(z,\bar{z})},$$

H.O.	Strings
$\lvert 0\rangle$	$\lvert 0,0;k\rangle$
$a^{\dagger}, a$	$\alpha^{\mu}_{-n}, \alpha^{\mu}_{n}$
$[a, a^{\dagger}] = 1$	$[\alpha^{\mu}_{n}, \alpha^{\nu}_{m}] = n\eta^{\mu\nu}\delta_{n+m,0}$
$\lvert\lambda\rangle = \exp\big(\lambda a^{\dagger}\big)\lvert 0\rangle$	$\lvert V\rangle = \exp\Big(\sum_n \lambda_n \cdot \alpha_{-n}\Big)\lvert 0;k\rangle$

$\dots|V\rangle$ not physical state unless we break covariance. . .

. . . in closed string theory, eigenstates of α_n, $\tilde{\alpha}_n$ do not even exist[19] (unless X^- compact), so need more general definition . . .

[19] Hindmarsh, Skliros (2011)

Coherent Vertex Operators

Definition of closed string coherent state:

(a) is specified by a (possibly infinite) *set of continuous* labels $(\lambda, \bar{\lambda})$, which may be associated to the left- and right-moving modes;

(b) produces a resolution of unity,

$$\mathbb{1} = \sum\!\!\!\!\!\!\int_{\dots} d\lambda d\bar{\lambda} |\lambda, \bar{\lambda}; \dots \rangle \langle \lambda, \bar{\lambda}; \dots |,$$

so that the $|\lambda, \bar{\lambda}; \dots \rangle$ span the string Hilbert space. The dots "$\dots$" denote possible additional quantum numbers;

(c) transforms correctly under all symmetries of the string theory

Coherent Vertex Operators

Construction of coherent vertex operators: define DDF operators,[20]

$$A_n^i = \frac{1}{2\pi}\oint dz\, \partial_z X^i\, e^{inq\cdot X(z)}, \qquad \bar{A}_n^i = \frac{1}{2\pi}\oint d\bar{z}\, \partial_{\bar{z}} X^i\, e^{inq\cdot X(\bar{z})},$$

with $q^2 = 0$, $q \cdot A_n = 0$ and $[A_n^i, A_m^j] = n\delta^{ij}\delta_{n+m,0}$.

Generic states of the form:

$$|\xi; k\rangle = \xi_{i\ldots j;k\ldots l} A^i_{-n_1} \ldots A^j_{-n_g} \bar{A}^k_{-\bar{n}_1} \ldots \bar{A}^l_{-\bar{n}_h} e^{ip\cdot X(z,\bar{z})},$$

are physical when: $p^2 = 2$, $p \cdot q = 1$, and $N \equiv \sum_j n_j = \sum_j \bar{n}_j$, with momenta:

$$k = p - Nq, \qquad k^2 = 2 - 2N$$

[20] Del Giudice, Di Vecchia, Fubini 72; Ademollo, Del Guidice, Di Vecchia 74

Any linear superposition of such states will also be physical, so we consider in particular:[21]

$$V(z,\bar{z}) = C \int_0^{2\pi} đs \exp\left\{\sum_{n=1}^{\infty} \frac{1}{n} e^{ins} \lambda_n \cdot A_{-n}\right\}$$
$$\times \exp\left\{\sum_{m=1}^{\infty} \frac{1}{m} e^{-ims} \bar{\lambda}_m \cdot \bar{A}_{-m}\right\} e^{ip\cdot X(z,\bar{z})},$$

with $\int đs$ the level-matching condition and C a normalisation constant. V in one-to-one correspondence with classical solutions:

$$X^0(z,\bar{z}) = -iM \ln z\bar{z}, \qquad (M^2 = \sum_n |\lambda_n|^2 + \sum_m |\bar{\lambda}_m|^2 - 2)$$
$$X^i(z,\bar{z}) = \sum_n \frac{i}{n} \left(\lambda_n^i z^{-n} - \lambda_n^{*i} z^n\right) + \sum_m \frac{i}{m} \left(\bar{\lambda}_m^i \bar{z}^{-m} - \bar{\lambda}_m^{*i} \bar{z}^m\right),$$

These states, $V(z,\bar{z}) \simeq |\lambda, \bar{\lambda}; p, q\rangle$, satisfy all the above defining properties of a coherent state

[21] Hindmarsh & Skliros PRL (2011)

A rest frame only exists in an expectation value sense:

$$\langle \hat{p}^\mu \rangle \equiv M\delta^\mu_0, \qquad M^2 = \sum_n |\lambda_n|^2 + \sum_m |\bar{\lambda}_m|^2 - 2, \qquad M^2 \in [-2, \infty)$$

These strings have size, $\mathcal{R} \equiv \sqrt{\langle (\mathbf{X}(z,\bar{z}) - \mathrm{x})^2 \rangle}$, in the rest frame:

$$\mathcal{R}^2 = \sum_{n>0} \frac{1}{n^2}\Big(|\lambda_n|^2 + |\bar{\lambda}_n|^2 - 2\mathrm{Re}\big(\lambda_n \cdot \bar{\lambda}_n e^{-2in\tau_{\mathrm{M}}}\big)\Big)$$

Non-zero mode components, $S^{\mu\nu}$, of the angular momenta, $J^{\mu\nu} = L^{\mu\nu} + S^{\mu\nu}$, read:

$$\langle S^{ij} \rangle = \sum_{n>0} \frac{2}{n}\mathrm{Im}\big(\lambda_n^{*i}\lambda_n^j + \bar{\lambda}_n^{*i}\bar{\lambda}_n^j\big)$$

$$\langle S^{-i} \rangle = \sum_{n>0}\sum_{\ell\in\mathbb{Z}} \frac{\sqrt{2}}{nM}\mathrm{Im}\big(\lambda^*_{n-\ell}\cdot\lambda^*_\ell\,\lambda^i_n + \bar{\lambda}^*_{n-\ell}\cdot\bar{\lambda}^*_\ell\,\bar{\lambda}^i_n\big),$$

with all components involving the + directions equal to zero.

Dual Vertex Operators

Any classical string trajectory $X = X_L(z) + X_R(\bar{z})$, with $\partial\bar{\partial}X = 0$, and has a dual, defined by:[22]

$$\big(X_L(z), X_R(\bar{z})\big) \to \big(X_L(z), -X_R(\bar{z}^{-1})\big)$$

In the quantum theory, the X are mapped to coherent vertex operators and their *duals* are generated by:

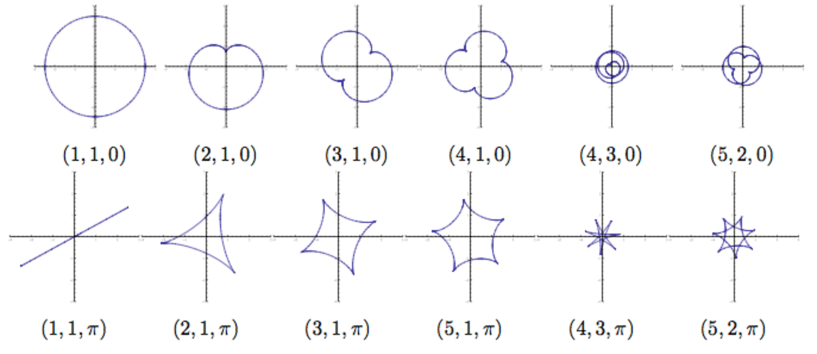
$$\lambda_n \to \lambda_n' = \lambda_n, \qquad \bar{\lambda}_n \to \bar{\lambda}_n' = (-)^n \bar{\lambda}_n^*, \qquad \text{for} \qquad n = 1, 2 \ldots$$

with $\lambda_n, \bar{\lambda}_n$ polarisation tensors of $V(z, \bar{z})$.

[22] Contrast with usual T-duality, $\big(X_L(z), X_R(\bar{z})\big) \to \big(X_L(z), -X_R(\bar{z})\big)$. Here dual directions non-compact, see e.g. Berkovits, Maldacena (2008)

Example

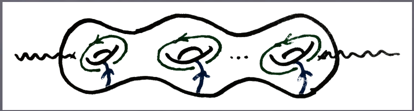
Some explicit classical string trajectories $(n, m, 0)$ (when only two harmonics, n, m, are present) and their duals (n, m, π).



Quantum Nature

To extract quantum properties of coherent vertex operators, $V(z, \bar{z})$, we need to compute amplitudes and relate these to associated observables.

The simplest non-trivial quantity to consider is the one-loop <u>two-point amplitude</u>; in general, $\mathcal{M} = \sum_h \mathcal{M}_h$,

$\mathcal{M}_h =$  ,

whose real and imaginary parts yield the mass shift[23] (due to self-gravity, etc.) and decay rates[24]:

$$\delta M^2 \sim \mathrm{Re}\mathcal{M}, \qquad \Gamma = \frac{1}{M}\mathrm{Im}\mathcal{M}$$

[23] Damour, Veneziano (2000)

[24] Chialva, Iengo, Russo (2003-06); . . .

2-Point Amlitudes (Notation & Conventions)

At genus $h = 1$ ($\mathcal{A}_1 = \frac{1}{2M}\delta^D(0)\mathcal{M}_1$):

$$\mathcal{A}_1 = \frac{1}{2}\int_{\mathcal{F}_1} d^2\tau \int \mathcal{D}(b,c,X)\, e^{-I} |(\mu, b)|^2 V^\dagger \hat{V}$$

where $V \equiv \int d^2 z V_{z\bar{z}}$ and $\hat{V} \equiv c^z \bar{c}^z V_{z\bar{z}}$ live in the cohomology of the BRST charge (and will be identified with coherent vertex operators), b, c are the $\mathrm{Diff}(\Sigma)$ ghosts, $\tau, \bar{\tau}$ is the modular parameter of the torus

Here $I = I_X + I_{\mathrm{ghosts}}$ and $\mu_z{}^{\bar{z}}$ a Beltrami differential (specifying the gauge slice).

To make the energy scales in the loops manifest (and to chirally factorize the amplitudes[25]) we fix the loop momenta by inserting,

$$1 = \int d^D\mathbb{P}\delta^D(\mathbb{P}-\hat{\mathbb{P}}), \qquad \hat{\mathbb{P}} \equiv \frac{1}{2\pi\alpha'}\int_{A_1}(\partial X - \bar{\partial}X),$$

integrate out b, c and slightly reorganise the various terms ($d\mathbf{M}_1 = \frac{1}{2}d^2\tau d^2z|\eta(\tau)|^4$):

$$\mathcal{A}_1 = \int d\!\!{}^-\!{}^D\mathbb{P}\int d\mathbf{M}_1\int \mathcal{D}X\, e^{-I_X}\,\delta\!\!\!{}^-{}^{Dh}\big(\mathbb{P}-\hat{\mathbb{P}}\big)\,V^\dagger_{z\bar{z}}V_{w\bar{w}},$$

Define:

$$\langle\!\langle V^\dagger_{z\bar{z}}V_{w\bar{w}}\rangle\!\rangle \equiv |\eta(\tau)|^{52}\int \mathcal{D}X\, e^{-I_X}\,\delta\!\!\!{}^-{}^{D}\big(\mathbb{P}-\hat{\mathbb{P}}\big)\,V^\dagger_{z\bar{z}}V_{w\bar{w}},$$

with $\eta(\tau)$ the Dedekind eta function.

[25] D'Hoker, Phong (1989)

The chiral splitting theorem[26] the ensures that:

$$\langle\!\langle V^{\dagger}_{z\bar{z}} V_{w\bar{w}} \rangle\!\rangle = i\delta(0) \sum_{N,M\in\mathbb{Z}^{d_c}} \int_0^{2\pi} ds\; \Phi(z|\tau)\bar{\Phi}(\bar{z},|\bar{\tau}),$$

where $\Phi(z|\tau)$ depends on the chiral moduli and the chiral halves of the asymptotic state quantum numbers.[27]

The sum over N, M is over instanton contributions associated to T^{d_c}, with $d_c = D_{\rm tot} - D$.

Q: So what is $\Phi(z|\tau)$ for different choice of coherent vertex operators?

---

[26] D'Hoker, Phong (1989)

[27] For mass eigenstates the s integral is trivial, whereas for coherent vertex operators it enforces level-matching (invariance under space-like shifts).

For $(1,1)$ *leading Regge* coherent vertex operators:[28]

$$V(z,\bar{z}) = :C \int_0^{2\pi} đs \exp\left(e^{is} i\zeta \cdot \partial_z X \, e^{-iq\cdot X(z)}\right) \times \exp\left(e^{-is} i\bar{\zeta} \cdot \partial_{\bar{z}} X \, e^{-iq\cdot X(\bar{z})}\right) e^{ip\cdot X(z,\bar{z})}:,$$

we find:[29]

$$\Phi(z|\tau) \equiv C\,\eta(\tau)^{-24} e^{\pi i \tau \mathbb{P}^2} E^{-2} e^{-2\pi i \mathbb{P}\cdot p\, z} \times \exp\left\{e^{is}|\lambda_1|^2 e^{2\pi i\, \mathbb{P}\cdot q\, z} E^2 \partial_z^2 \ln E\right\} \times I_0\left(2\sqrt{e^{is}|\mathbb{P}\cdot\lambda_1|^2 e^{2\pi i\, \mathbb{P}\cdot q\, z}(2\pi E)^2}\right),$$

where the $I_0(x)$ are modified Bessel functions and $E(z) = \vartheta_1(z|\tau)/\vartheta'(0|\tau)$ the prime form.

---

[28] $\zeta_\mu \equiv \lambda_1^i(\delta_\mu^i - p^i q_\mu)$, $M^2 = 2|\zeta|^2 - 2$, and $|\zeta| \in \mathbb{R}^+$.

[29] DS, Copeland, Saffin (2013)

For more general harmonics, (n, m), we find:[30]

$$\Phi(z|\tau) = C\,\eta(\tau)^{-24} e^{\pi i\tau\mathbb{P}^2} E^{-2} e^{-2\pi iz\,\mathbb{P}\cdot p} \times \exp\left\{ e^{ins}\frac{1}{n^2}|\lambda_n|^2\, e^{2\pi i(\mathbb{P}\cdot nq)z} E^{2n}\mathcal{D}_z^n\mathcal{D}_z^n \ln E \right\} \times I_0\Big(2\,e^{\frac{ins}{2}}\frac{1}{n}\,|\mathbb{P}\cdot\lambda_n|\,e^{\pi i(\mathbb{P}\cdot nq)z} 2\pi E^{p\cdot nq}\,\mathcal{S}_{n-1}\Big),$$

where,

$$\mathcal{D}_z^n \equiv \sum_{\ell=1}^{n}\frac{\mathcal{S}_{n-\ell}(a_s)}{(\ell-1)!}\,\partial_z^\ell, \qquad (1)$$

and the arguments of elementary Schur polynomials, $\mathcal{S}_{n-\ell}(a_s)$, are

$$a_s \equiv -\frac{n}{s!}\,\partial_z^s\mathcal{G}(z), \qquad \text{with} \qquad \mathcal{G}(z) \equiv -\ln|E(z)|^2 + 4\pi(\mathbb{P}\cdot q)\mathrm{Im}\,z.$$

[30] DS, Copeland, Saffin (to appear)

In fact, in the most general case of arbitrary polarisation tensors and in a general Lorentz frame,

$$\Phi(z|\tau) = C\eta(\tau)^{-24} \exp\left\{\pi i\tau\mathbb{P}^2 - 2\pi iz\mathbb{P}\cdot p\right\} E^{-2}$$

$$\times \exp\left\{-\sum_{n,m>0} e^{i(n+m)s}\frac{(-)^m\lambda_n^*\cdot\lambda_m}{nm} e^{\pi iz\mathbb{P}\cdot q\,(n+m)} E^{n+m}\mathcal{D}_z^n\mathcal{D}_z^m \ln E\right.$$

$$+\sum_{n,m>0} e^{i(n+m)s}\frac{\lambda_n^*\cdot\lambda_m^*}{2nm} e^{\pi iz\mathbb{P}\cdot q\,(n+m)} E^{n+m}\mathbb{S}_{n,m}$$

$$\left.+\sum_{n,m>0} e^{i(n+m)s}\frac{(-)^{n+m}\lambda_n\cdot\lambda_m}{2nm} e^{\pi iz\mathbb{P}\cdot q\,(n+m)} E^{n+m}\mathbb{S}_{n,m}\right\}$$

$$\times I_0\left(2i\sqrt{\sum_{n,m>0} e^{i(n+m)s} e^{\pi i\mathbb{P}\cdot q\,(n+m)z} E^{n+m} Y(\lambda_n) Y((-)^m\lambda_m^*)}\right)$$

with

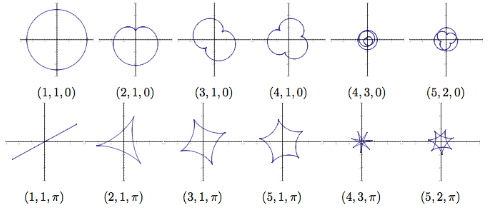
$$Y(\lambda_n) = (-)^n\Big(\frac{2\pi\mathbb{P}_l\cdot\lambda_n}{n}\,\mathcal{S}_{n-1}(a_s) + \frac{1}{n} ip\cdot\lambda_n\mathcal{D}_z^n \ln E - \frac{1}{n}\, ip\cdot\lambda_n\mathcal{S}_n(a_s)\Big)$$

Duality of 2-Point Amplitudes

Notice that all string 2-point amplitudes are invariant under:

$$\lambda_n \to \lambda'_n = (-)^n \lambda^*_n, \qquad \bar{\lambda}_n \to \bar{\lambda}'_n = \bar{\lambda}_n, \qquad \text{for} \qquad n = 1, 2 \ldots$$

$\rightarrow$ distinct string trajectories have the same decay rates and mass shifts!



Does this persist at higher loops? ...unclear, the quantity $(\mathbb{P}_I \cdot \lambda^*_n)\mathcal{D}^n_z \int^z \omega_I \, (\mathbb{P}_J \cdot \lambda_n)\mathcal{D}^n_w \int^w \omega_J$ that would appear in Bessel function in $\Phi_h(z, w|\Omega)$ only invariant for $h = 1$.

String Decay

Some History

A handful of references on (closed) HES string decay:

- Wilkinson, Turok, Mitchell (1990): leading Regge (bosonic) states, $\mathbb{R}^{25,1}$, (numerical), $\Gamma_{d=4} \propto L$ and $\Gamma_{d=26} \propto L^{-1}$
- Dabholkar, Mandal, Ramadevi (1998): higher genus bound on leading Regge Heterotic states, $\mathbb{R}^{3,1} \times T^6$, $\Gamma \lesssim M^{-1}$
- Iengo, Russo (2002-6); Chialva, Iengo, Russo (2004-5): leading Regge superstring states, $\mathbb{R}^{D-1,1} \times T^{10-D}$, (numerical),

$$\Gamma \sim G_D \mu^2 (M/\mu)^{5-D}, \qquad \mu = \frac{1}{2\pi\alpha'}$$

- Gutplerle & Krym (2006); leading Regge Heterotic states, $\mathbb{R}^{8,1} \times S^1$, (numerical)

$\vdots$

Some History

A handful of references on decay rates of HES:

- Wilkinson, Turok, Mitchell (1990): leading Regge (bosonic) states, $\mathbb{R}^{25,1}$, (numerical), $\Gamma_{d=4} \propto L$ and $\Gamma_{d=26} \propto L^{-1}$
- Dabholkar, Mandal, Ramadevi (1998): higher genus bound on leading Regge Heterotic states, $\mathbb{R}^{3,1} \times T^6$, $\Gamma \lesssim M^{-1}$
- Iengo, Russo (2002-6); Chialva, Iengo, Russo (2004-5): leading Regge superstring states, $\mathbb{R}^{D-1,1} \times T^{10-D}$, (numerical),

$$\Gamma \sim G_D \mu^2 (M/\mu)^{5-D}, \qquad \mu = \frac{1}{2\pi\alpha'}$$

- Gutplerle & Krym (2006); leading Regge Heterotic states, $\mathbb{R}^{8,1} \times S^1$, (numerical)

$\vdots$

String Decay Rates

From unitarity, $S^\dagger S = 1$, one can show that decay rates can be extracted (to leading order in g_s) from:

$$\Gamma = \frac{1}{M}\,\mathrm{Im}\int đ^D\mathbb{P}\,\mathcal{M}_1(\mathbb{P}),$$

which is of the form:

$$\Gamma = \frac{1}{M}\int đ^D\mathbb{P}\sum_{\{m_j,\,k^\mu\}} |\ldots|^2\,\delta(\mathbb{P}^2 + m_1^2)\,\delta\big((k-\mathbb{P})^2 + m_2^2\big)$$

with $m_1^2 = \left(\frac{N}{R}\right)^2 + \left(\frac{M'R}{2}\right)^2 + r + \bar{r} - 2$, $m_2^2 = \ldots$

For massless radiation (i.e. $m_1^2=0$) from $(1,1)$ vertices, in the IR the result ressums:[31]

$$\frac{d\Gamma}{d\Omega_{S^{D-2}}}\Big|_{m_1^2=0} = \sum_N \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}}\,\omega_N^{D-4-\delta} N^2$$
$$\Big[J'^2_N + \Big(\frac{1}{z^2}-1\Big)J_N^2 + \dots\Big]\Big[\bar{J_N}'^2 + \Big(\frac{1}{\bar{z}^2}-1\Big)\bar{J_N}^2 + \dots\Big]$$

where $J_N = J_N(Nz)$, $\bar{J_N} = J_N(N\bar{z})$, etc., and the frequency of emitted radiation,[32]

$$\omega_N = \frac{4\pi N}{L}, \qquad \text{with} \qquad N = 1, 2, \dots$$

Taking $\delta = 1$ yields a decay rate, $\delta = 0$ yields a power.

---

[31] DS, Copeland and Saffin (PRL 2013)

[32] Here $z=\sqrt{2}|\hat{\mathbb{P}}\cdot\hat{\lambda}_{\mathbf{1}}|$, $\bar{z}=\sqrt{2}|\hat{\mathbb{P}}\cdot\hat{\bar{\lambda}}_{\mathbf{1}}|$, the $J_{\boldsymbol{n}}(x)$ are Bessel and $M=\mu L$, $\mu = 1/(2\pi\alpha')$

Effective Description

Remarkably, the above was shown[33] to agree precisely with the effective theory,

$$S_{\rm eff} = \frac{1}{16\pi G_D}\int d^D x\sqrt{-G}\, e^{-2\Phi}\Big(R_{(D)} + 4(\nabla\Phi)^2 - \frac{1}{12}\,H_{(3)}^2 + \dots\Big)$$
$$- \mu\int_{S^2}\partial X^\mu \wedge \bar\partial X^\nu\big(G_{\mu\nu} + B_{\mu\nu}\big) + \dots,$$

where Φ, $G_{\mu\nu}$ and $H_{(3)}$ are the dilaton, spacetime metric and 3-form field strength, $H = dB$, respectively

(We plug classical solutions for X (from classical-CVO map) and compute perturbations in G, B and Φ)

---

[33] DS, Copeland and Saffin (PRL 2013)

Higher Harmonics

...the above correspondence acts as a guiding principle to write down the general result for arbitrary harmonics (n, m):[34]

$$\frac{d\Gamma}{d\Omega_{S^{D-2}}}\Big|_{m_1^2=0} = \sum_N \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}} \omega^{D-4-\delta} (Nuwg)^2$$
$$\left[J'^2_{Nw}(A) + \left((Nw/A)^2 - 1\right) J^2_{Nw}(A)\right]$$
$$\left[J'^2_{Nu}(\bar{A}) + \left((Nu/\bar{A})^2 - 1\right) J^2_{Nu}(\bar{A})\right]$$

with $n \equiv gu$, $m \equiv gw$, integers and u, w relatively prime.
(g can be interpreted as a winding number: $M \sim g\mathcal{R}/\alpha'$, with $\mathcal{R}$ determined by *dynamics*.)

---

[34] Here $A = Nw\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_{\boldsymbol{n}}|$, $\bar{A} = Nu\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_{\boldsymbol{m}}|$

α' corrections

The UV region of the emission spectrum is particularly important, as, e.g., this is where the characteristic cosmic string cusp signal is, which according to classical effective theory computations[35] leads to the strongest GW signal:[36]

$$\frac{d\Gamma}{d\Omega_{S^{D-2}}}\Big|_{m_1^2=0} = \sum_N \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}} \omega^{D-4-\delta} N^2$$

$$\left[J'^2_N + \left(\frac{1}{z^2} - 1\right) J^2_N - (-)^N \frac{\omega}{M} J_N J'_N z + \dots\right]$$

$$\left[\bar{J}_N'^2 + \left(\frac{1}{\bar{z}^2} - 1\right) \bar{J}_N^2 - (-)^N \frac{\omega}{M} \bar{J}_N \bar{J}_N' \bar{z} + \dots\right]$$

The corrections become important when $\omega \sim \sqrt{M/\sqrt{\alpha'}}$, long before the cutoff $\omega \sim M$.

[35] Damour, Vilenkin (2001)

[36] Skliros, Copeland, Saffin (2013)

Summary

- Discussed construction of generic covariant coherent vertex operators and their classical analogues
- Explicit expression for generic two-point function (at fixed-loop momenta) (on $\mathbb{R}^{D-1,1} \times T^{26-D}$) $\rightarrow$ novel duality
- Analytically computed decay rates and powers associated to massless emission for special class of IHES states in IR
- Found effective field theory that reproduces the leading terms of these decay rates and powers
- Computed UV corrections, which can become very significant in the UV (where the interesting cusp radiation signal is).

Chiral Splitting Theorem

To prove chiral splitting theorem, use point splitting to write a generic amplitude in the form:

$$\Big\langle\!\Big\langle \prod_{j=1}^{\mathcal{I}} \left(D_j X^{\mu_j} + T_j^{\mu_j}\right) e^{i\int J\cdot X} \Big\rangle\!\Big\rangle,$$

for generic X-independent operators $\{D_j, T_j, J\}$.

- Exponentiate delta functions, $\delta(\mathbb{P}-\hat{\mathbb{P}}) = \int dy e^{iy(\mathbb{P}-\hat{\mathbb{P}})}$
- Expand $X = X_{\rm cl} + \tilde{X}$ and integrate out $\tilde{X}$ with propagator

$$G(z,w) = -\ln|E(z,w)|^2 + 2\pi\,\mathrm{Im}\int_w^z \omega_I \,(\mathrm{Im}\Omega)^{-1}_{IJ}\,\mathrm{Im}\int_w^z \omega_J$$

- Poisson-resum on integers $M \in \mathbb{Z}^{d_c h}$ of $X_{\rm cl}$
- Make use of (quasi-)periodicity properties of prime form, $E(z,w)$, and $\oint_{A_I}\omega_J = \delta_{IJ}$, $\oint_{B_I}\omega_J = \Omega_{IJ}$

To evaluate $\langle\!\langle \dots \rangle\!\rangle$, for $X : \Sigma \to \mathbb{R}^{D-1,1} \times T^{26-D}$:

- if $X \in \mathbb{R}^{D-1,1}$:

$$X = x + \tilde{X}, \qquad x = \mathrm{const}$$

- if $X \in T^{26-D}$:

$$X = x + \gamma_I z + \bar{\gamma}_I \bar{z} + \tilde{X},$$

$$\oint_{A_I} dX^a_{\mathrm{cl}} = (2\pi N_I R)^a, \quad \oint_{B_I} dX^a_{\mathrm{cl}} = (2\pi M_I R)^a,$$

with $\gamma_I, \bar{\gamma}_I$ determined from the latter; $N, M \in \mathbb{Z}^{d_c h}$, and $\tilde{X}$ denote quantum fluctuations.

The result is the following.
Drop contact terms and the theorem is proven:

$$\left\langle\!\!\left\langle \prod_{j=1}^{\mathcal{I}} (D_j X^{\mu_j} + T_j^{\mu_j})\ e^{i\int J\cdot X} \right\rangle\!\!\right\rangle = i(2\pi)^D \delta^D\Big(\int J\Big)(g_D^2 \alpha'(2\pi)^{26})^{h-1}$$

$$\sum_{k=0}^{\lfloor \mathcal{I}/2 \rfloor} \sum_{\pi\in S_{\mathcal{I}}/\sim} \prod_{l=1}^{k} \Big\{ -\eta^{\mu_{\pi(2l-1)}\mu_{\pi(2l)}} (DD\ln|E|^2)_{\pi(2l-1)\pi(2l)}\Big)\Big\}$$

$$\prod_{q=2k+1}^{\mathcal{I}} \Big\{ i4\pi \mathbb{P}_M^{\mu_{\pi(q)}} D_{\pi(q)} \mathrm{Im}\int^{z_{\pi(q)}} \omega_M - i\int J^{\mu_{\pi(q)}} (D\ln|E|^2)_{\pi(q)} + T_{\pi(q)}^{\mu_{\pi(q)}} \Big\}$$

$$\sum_{N,M\in\mathbb{Z}^{d_c h}} \left| \exp\Big\{ \pi i \mathbb{P}_I^{\mu} \Omega_{IJ} \mathbb{P}_{J\mu} + i2\pi \mathbb{P}_I \cdot \int d^2z J(z,\bar{z}) \int^z \omega_I \Big\} \right|^2$$

$$\times \exp\Big\{ \frac{1}{2} \int d^2z \int d^2z' J(z,\bar{z}) \cdot J(z',\bar{z}') \ln|E(z,z')|^2 \Big\}$$

. . . The result is quite complicated

However, when asymptotic states are identified with coherent vertex operators the result simplifies dramatically, especially at genus $h = 0$ or 1

In particular, for coherent vertex operators the sum over k and sum over permutations can be carried out explicitly