Antigravity and the Big Crunch/Big Bang Transition

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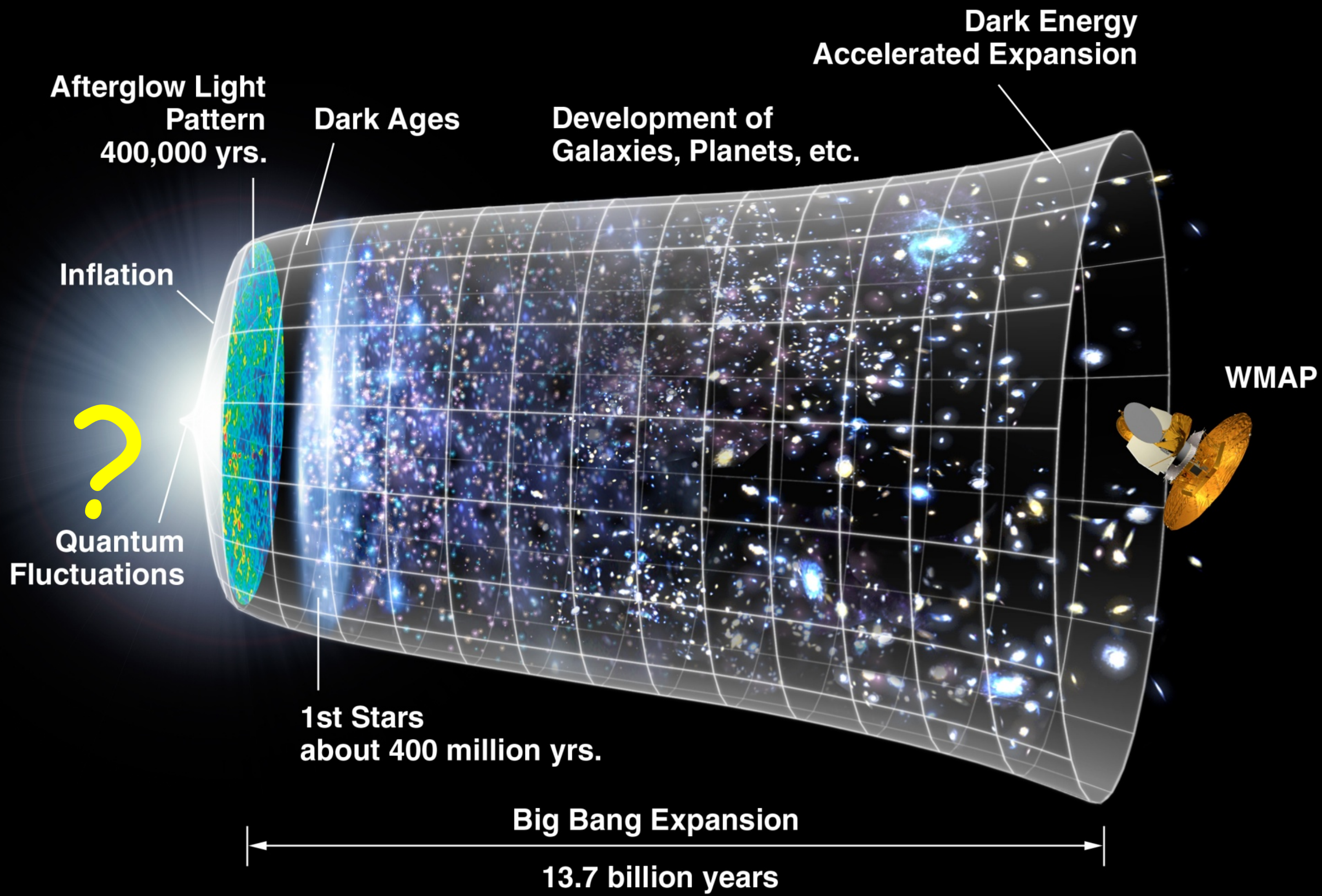
- a proposal for continuing time through cosmological singularities

I. Bars, S-H Chen, P. Steinhardt, NT
arXiv/0365509 (gr-qc): today!

Concordance Model



Success!

fluctuation level:

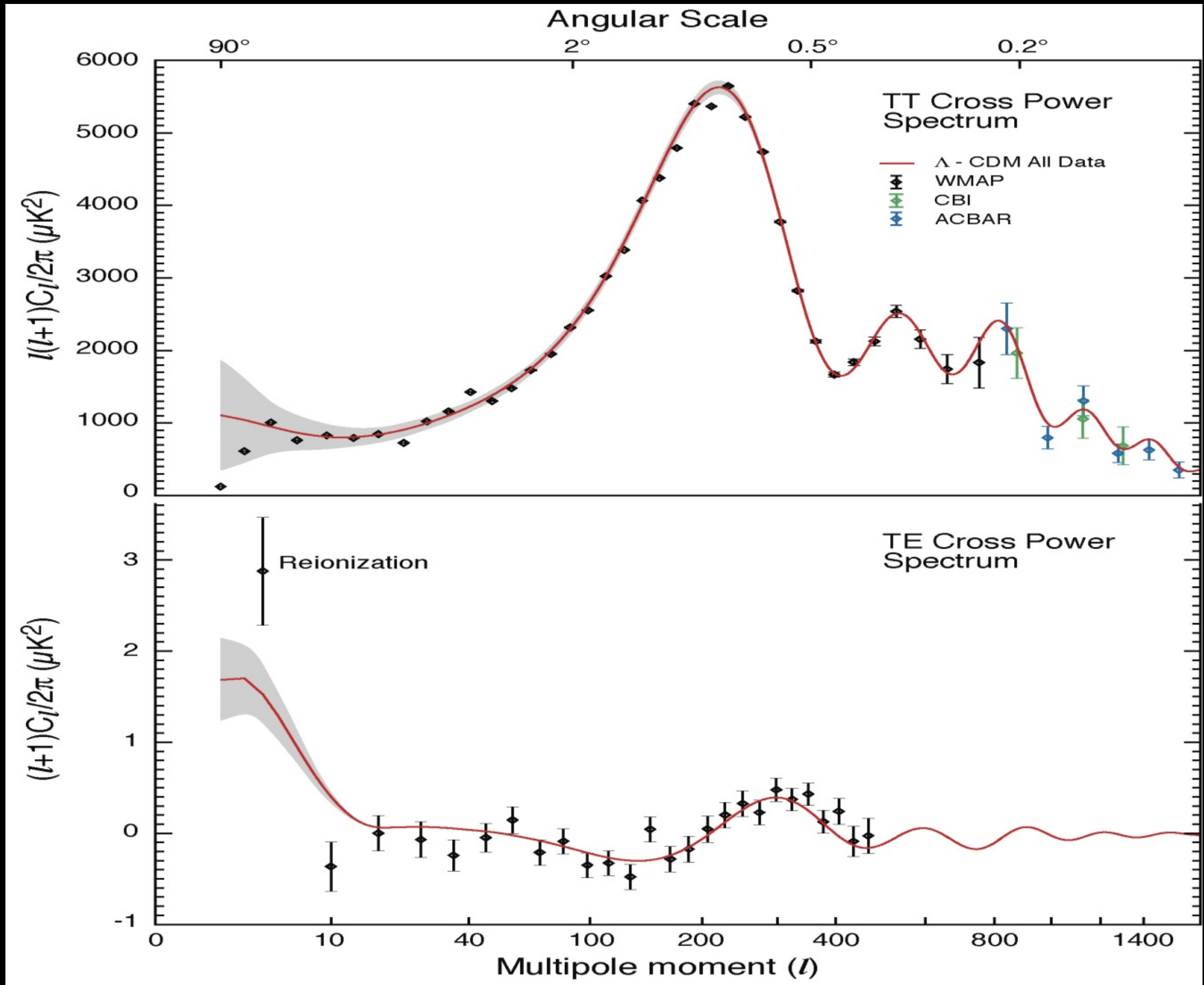
temperature

Z'eldovich, Peebles+Yu 70's

Bond+Efstathiou 80's

polarization

Coulson, Crittenden, NT 94



100 10 1 .1 ($l = 2\pi/\theta$)

Angle on Sky (Degrees)

good evidence for ...

nearly flat FRW universe:

$\Omega_\Lambda : \Omega_{CDM} : \Omega_B : \Omega_\nu : \Omega_\gamma \sim 0.7 : 0.25 : 0.05 : 0.003 : 0.0003$

primordial perturbations

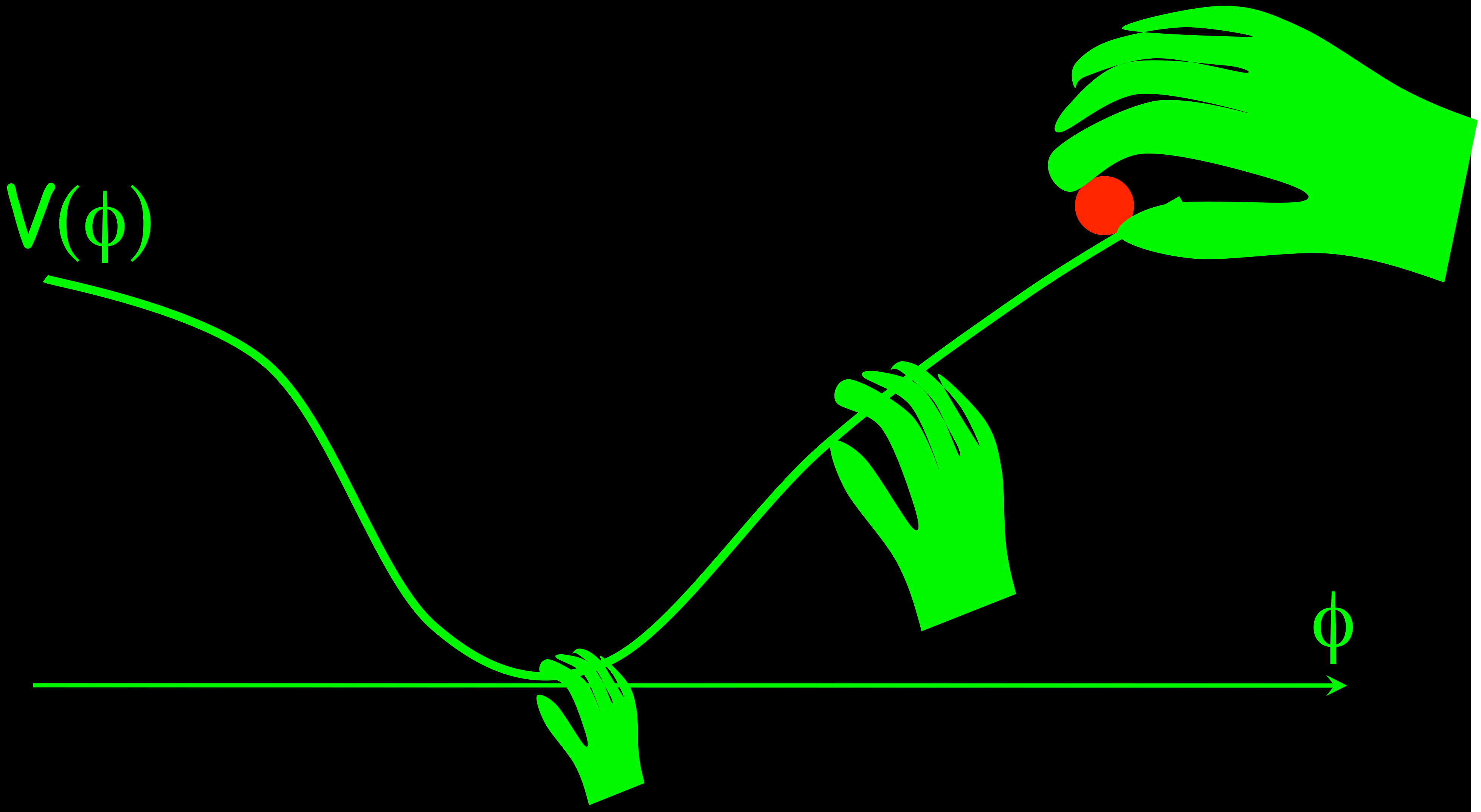
* linear
* growing mode
* nearly scale-invariant
* nearly "adiabatic"
* nearly Gaussian

universe is: *geometrically astonishingly simple*
compositionally complex

The inflationary paradigm has several basic conceptual difficulties

inflation

* initial conditions
* fine-tuned potentials
* $\Lambda \sim 10^{-120}$; $\Lambda_I \sim 10^{-15}$



* eternal inflation
  "anything that can happen will happen:
  and it will happen an infinite number
  of times" A. Guth, 2002
* string landscape: measure problem

-> reliance on anthropic arguments

- see P. Steinhardt, Sci Am 304, 36, 2011
NT, http://pirsa.org/11070044

Inflation is based on the idea that the big bang singularity was the beginning.

But this may contradict unitarity.

What if the singularity was instead a bounce from a pre-bang universe?

An attractive cyclic universe scenario then becomes feasible.

The “big” puzzles:

- flatness, homogeneity and isotropy
- origin of perturbations

are solved via a pre-big bang period of ultra-slow contraction with an equation of state $w=P/\rho>>1$.

Since $\rho \sim a^{-(1+w)}$ rises rapidly as a->0 this nearly homogeneous and isotropic component* - rapidly dominates as the universe contracts to a “big crunch.”

Quantum fluctuations can generate scale-invariant, Gaussian, adiabatic perturbations.

*e.g. a scalar field with a steep negative potential.

For this scenario to be viable, we have to understand whether the universe can bounce from a "crunch" into a "bang."

We shall try to do this largely using classical GR-scalar theory: we do not yet know how to properly include quantum corrections.

Our method is to introduce a new gauge symmetry – Weyl symmetry – allowing us to move the problem of $\det(g_{\mu\nu})$ vanishing to a sector where it appears milder. The field space in the "lifted" theory is larger and Newton's constant is not necessarily positive.

A certain Weyl-invariant quantity passes analytically through the singularity, causing M_{PL} to vanish momentarily*, and G_N to briefly become negative.

We shall take this seriously, and study the resulting dynamics. We find the antigravity phase is brief, and the universe quickly recovers normal gravity.

Through a combination of analytic continuation and symmetry arguments we shall argue the outcome is unique: a completely predictable bounce.

*and hence Weyl symmetry to be restored.

Starting point: Einstein-scalar gravity

$$S = \int d^4x\sqrt{-g}[\frac{1}{2\kappa^2}R(g) - \frac{1}{2}(\partial\sigma)^2 - V(\sigma)]$$

Initial conditions: nearly homogenous, isotropic, flat universe with small perturbations.

As long as V(σ) is bounded, it becomes negligible as singularity nears.

Kinetic energy of scalar σ dominates, removes mixmaster chaos, ensures smooth ultralocal (locally Kasner) dynamics

Belinski+Khalatnikov+Lifshitz,Anderson+Rendall

In the final approach to the singularity, scalar kinetic energy density, scaling as $\sim a^{-6}$, dominates over anisotropies (also $\sim a^{-6}$), radiation ($\sim a^{-4}$), matter ($\sim a^{-3}$), pot energy($\sim a^{0}$).

We use Bianchi IX as an illustration:

$$ds^2 = a_E^2(\tau)\left(-d\tau^2 + ds_3^2\right)$$

$$ds_3^2 \;=\; e^{-2\sqrt{2/3}\kappa\alpha_1} d\sigma_z^2 + e^{\sqrt{2/3}\kappa\alpha_1}\left(e^{\sqrt{2}\kappa\alpha_2} d\sigma_x^2 + e^{-\sqrt{2}\kappa\alpha_2} d\sigma_y^2\right)$$

($d\sigma_{x,y,z}$ are $SU(2)$ left-invariant one-forms)

$\alpha_{1,2}$ parameterise the anisotropy

Generic solutions with anisotropy ($\alpha_{1,2}$)

$$\frac{\dot{a}_E^2}{a_E^4} = \frac{\kappa^2}{3}\left[\frac{\dot{\sigma}^2 + \dot{\alpha_1}^2 + \dot{\alpha_2}^2}{2a_E^2} + V(\sigma) + \frac{\rho_r}{a_E^4}\right] - \frac{Kv(\alpha_1, \alpha_2)}{a_E^2}$$

$$\frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} = -\frac{\kappa^2}{3}\left[\frac{\dot{\sigma}^2 + \dot{\alpha_1}^2 + \dot{\alpha_2}^2}{a_E^2} - V(\sigma) + \frac{\rho_r}{a_E^4}\right],$$

$$\frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E\dot{\sigma}}{a_E^3} + V'(\sigma) = 0,$$

$$\frac{\ddot{\alpha}_1}{a_E^2} + 2\frac{\dot{a}_E\dot{\alpha}_1}{a_E^3} - \frac{6K}{\kappa^2 a_E^2}\partial_{\alpha_1} v(\alpha_1, \alpha_2) = 0,$$

$$\frac{\ddot{\alpha}_2}{a_E^2} + 2\frac{\dot{a}_E\dot{\alpha}_2}{a_E^3} - \frac{6K}{\kappa^2 a_E^2}\partial_{\alpha_2} v(\alpha_1, \alpha_2) = 0.$$

Near singularity, reduce to:

$$\frac{\dot{a}_E^2}{a_E^4} = \frac{\kappa^2}{3}\left[\frac{\dot{\sigma}^2 + \dot{\alpha_1}^2 + \dot{\alpha_2}^2}{2a_E^2} + \frac{\rho_r}{a_E^4}\right],$$

$$\ddot{\sigma} + 2\frac{\dot{a}_E}{a_E}\dot{\sigma} = 0; \qquad \ddot{\alpha}_i + 2\frac{\dot{a}_E}{a_E}\dot{\alpha}_i = 0$$

following from the effective action:

$$\int d\tau \left\{\frac{1}{2e}\left[-\frac{6}{\kappa^2}\dot{a}_E^2 + a_E^2(\dot{\sigma}^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2)\right] - e\rho_r\right\}$$

Our approach: "lift" Einstein-scalar to a Weyl-invariant theory

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}\left((\partial\phi)^2-(\partial s)^2\right)+\frac{1}{12}(\phi^2-s^2)R\right]$$

- add scalar ghost plus new gauge symmetry:

  $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $\phi \to \Omega^{-1}\phi$, $s \to \Omega^{-1}s$

  (original motivation: brane picture/2T physics)
- gravitational trace anomaly cancels
- global O(1,1) symmetry*: $\phi'^2 - s'^2 = \phi^2 - s^2$:

* a closely related classical, approximate, shift symmetry appears in string theory - at tree level in g_s, but to all orders in α'

Special quantity: Weyl and O(1,1)-invariant:

$$\chi \equiv \frac{\kappa^2}{6}(-g)^{\frac{1}{4}}(\phi^2 - s^2)$$ ($a_E^2 = |\chi|$)

- obeys Friedmann-like equation:

$$\dot{\chi}^2 = \frac{2\kappa^2}{3}\left(p^2 + 2\rho_r \chi\right)$$ $$p \equiv \sqrt{p_\sigma^2 + p_1^2 + p_2^2}$$

- analytic at kinetic-dominated cosmological singularities

Gauges:

1.Einstein gauge $\phi^2 - s^2 = 6\kappa^{-2}$:

$$\phi_E = \pm(\sqrt{6}/\kappa)\cosh(\kappa\sigma/\sqrt{6})$$
$$s_E = (\sqrt{6}/\kappa)\sinh(\kappa\sigma/\sqrt{6})$$

2. "Supergravity-like" gauge $\phi = \phi_0 = \text{const}$:

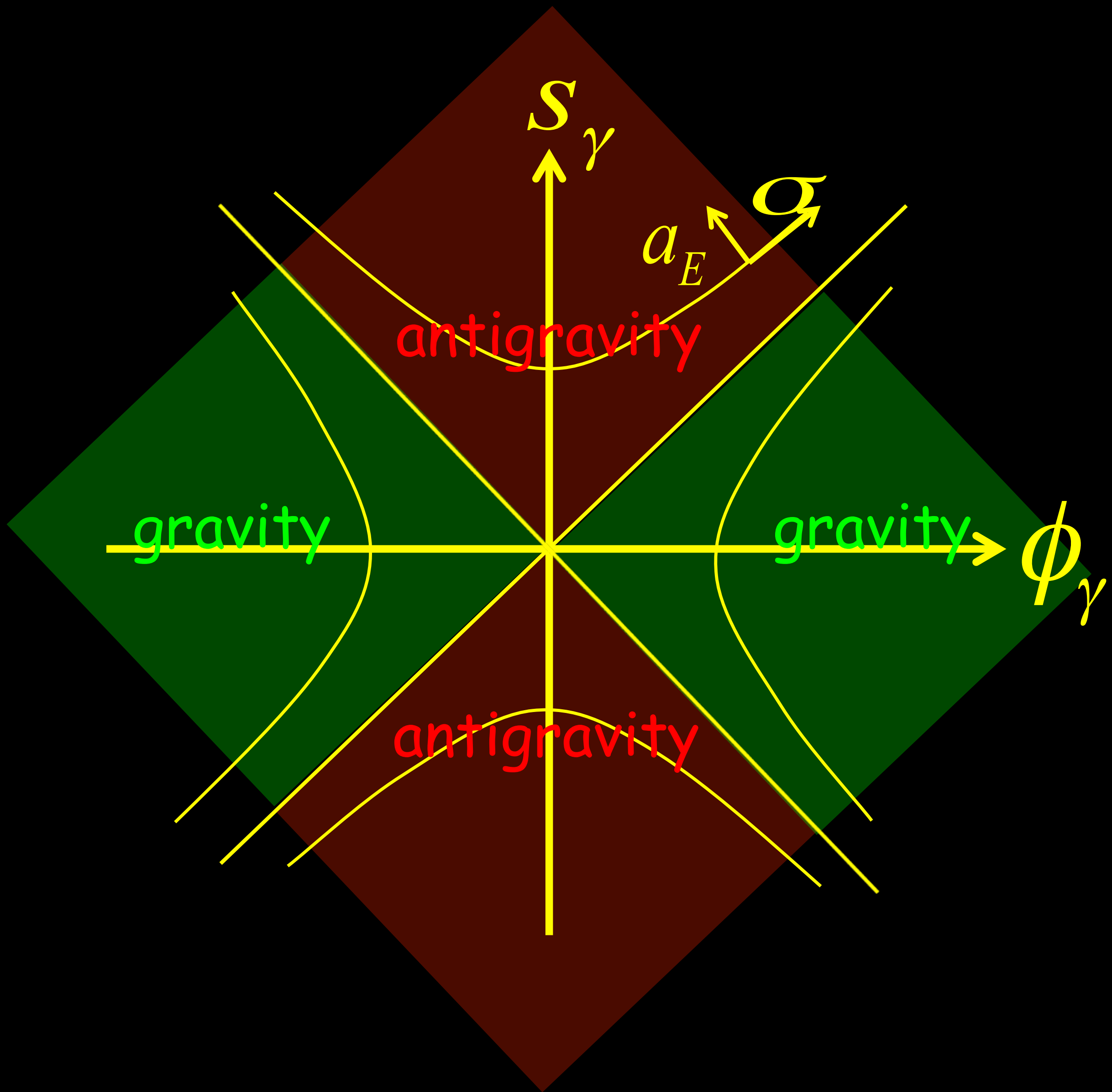
$$\int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial s)^2 + \frac{1}{12}(\phi_0^2 - s^2)R\right]$$

-cf N=1 SUGRA models (e.g. S.Weinberg QFT III)

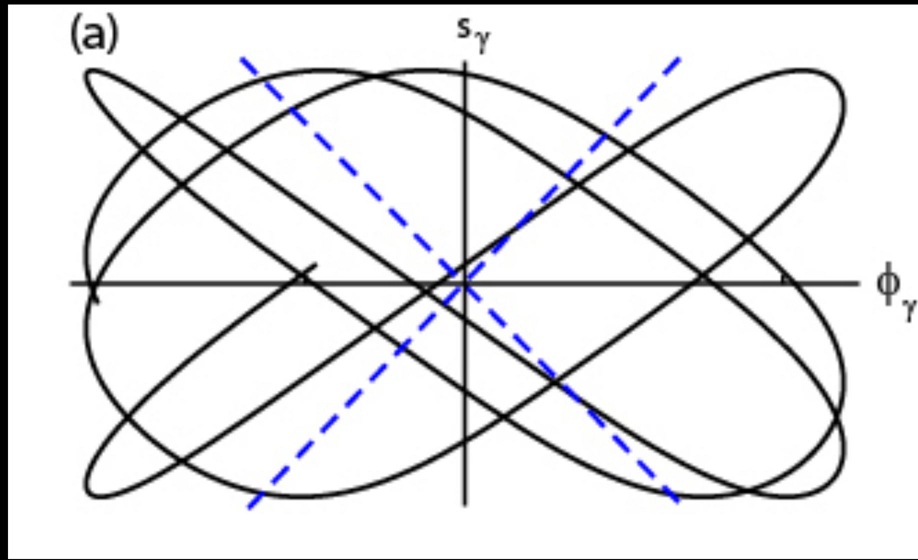
3. "γ–gauge": Detg = -1:

$$\int d\tau\left[-\frac{1}{2}\dot{\phi}_\gamma^2 + \frac{1}{2}\dot{s}_\gamma^2 + \frac{\kappa^2}{12}(\phi_\gamma^2 - s_\gamma^2)(\dot{\alpha}_1^2 + \dot{\alpha}_2^2)\right]$$

Weyl- extended superspace

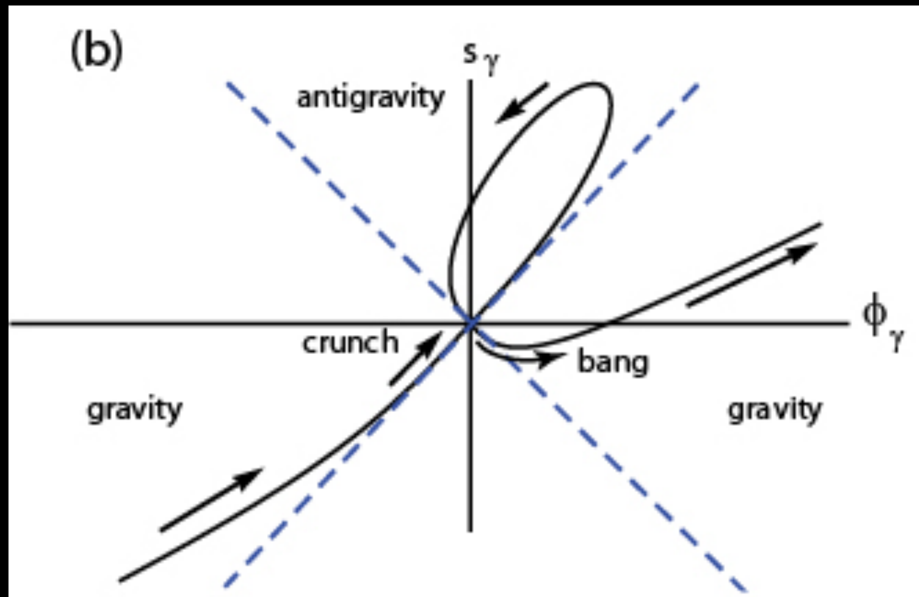


Isotropic case:
$\alpha_1=\alpha_2=0$

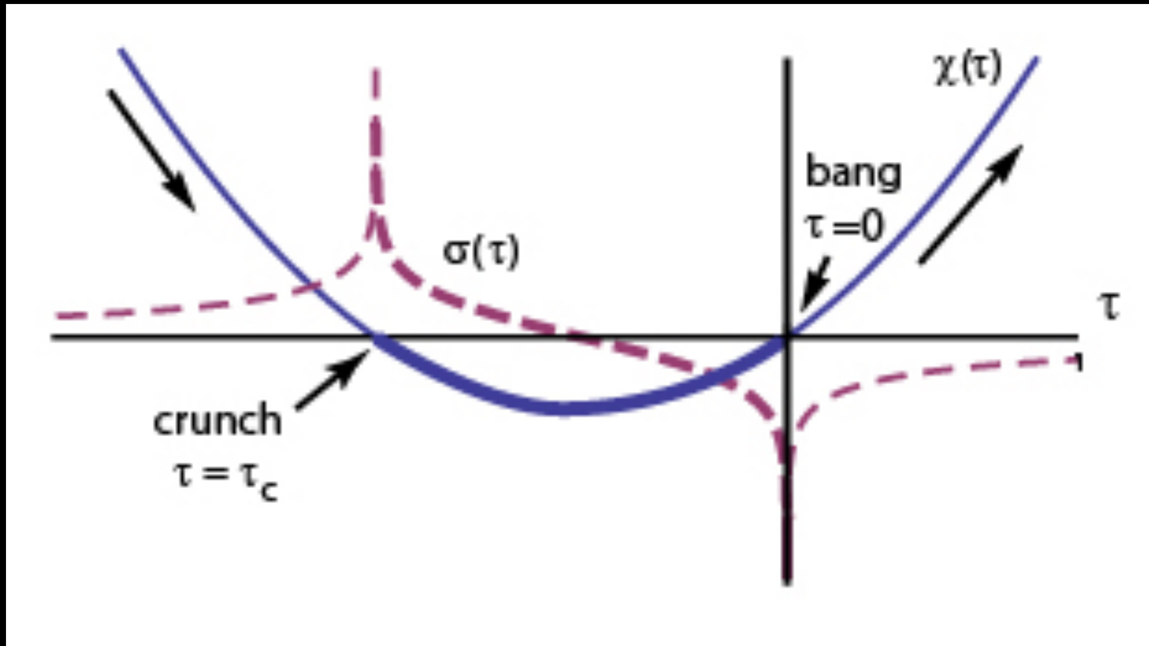


Generic case w/anisotropy:

Weyl restored at gravity/antigravity transition



Solution with radiation only



$$\chi(\tau) = 2\overline{\tau}(p + \rho_r \overline{\tau})$$

$$\overline{\tau} = \kappa\tau/\sqrt{6}$$

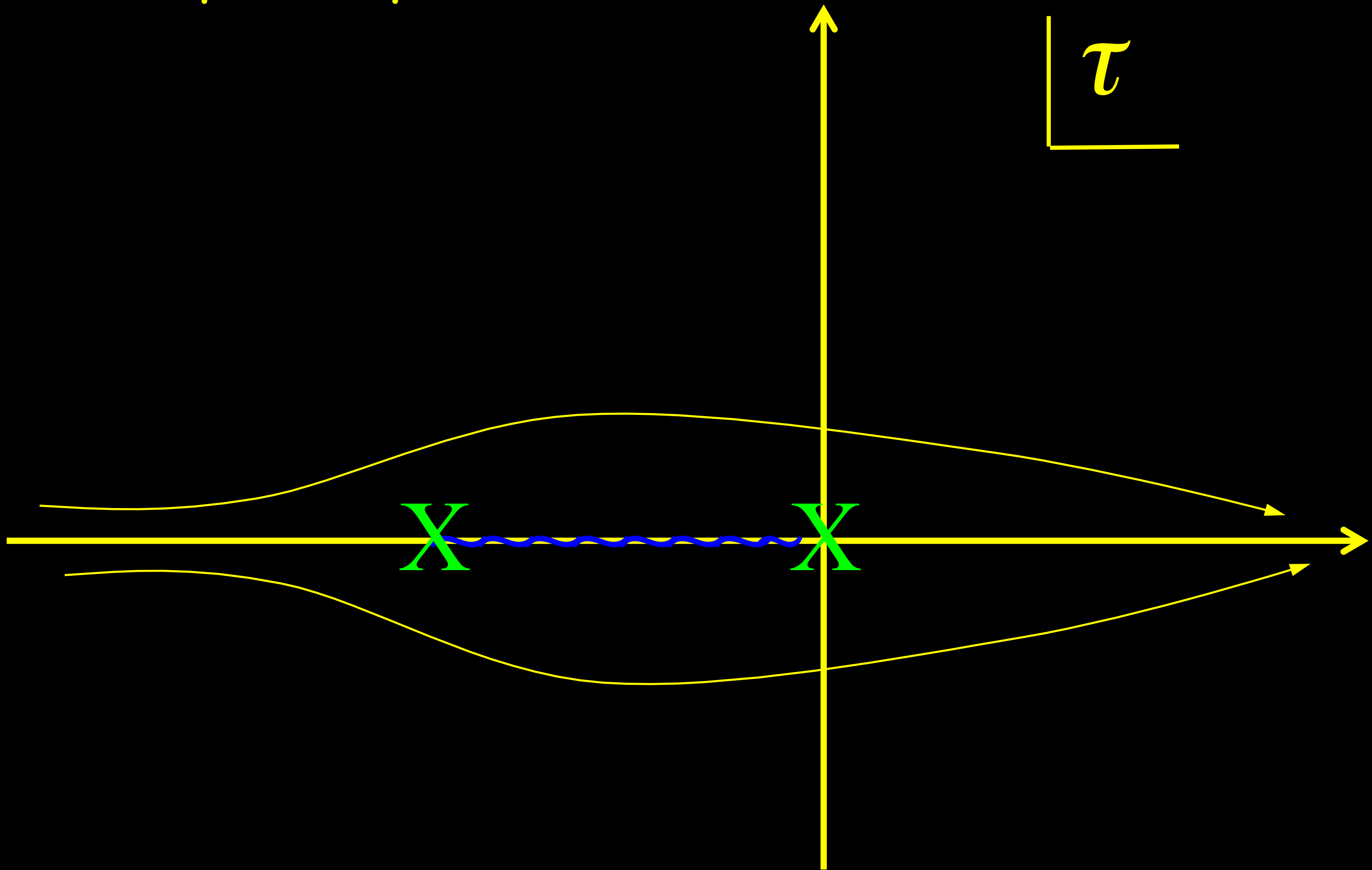
$$\frac{\kappa}{\sqrt{6}}\sigma(\tau) = \frac{p_\sigma}{2p}\ln\left|\frac{\overline{\tau}}{T(p+\rho_r\overline{\tau})}\right|$$

, sim^y $\alpha_{1,2}$

Uniqueness of solution

1. Analytic continuation
2. Asymptotic symmetries
3. Stationary points of action

1: unique extension of σ, $\alpha_{1,2}$ around singularities in complex τ–plane



2. Asymptotic symmetries

Recall: $$\int d^4x\sqrt{-g}\left[\frac{1}{2}\left((\partial\phi)^2-(\partial s)^2\right)+\frac{1}{12}(\phi^2-s^2)R\right]$$

Define: $$\alpha_0 \equiv \kappa^{-1}\sqrt{3/2}\ln|\chi|, \qquad \alpha_3 \equiv \sigma$$

Effective action becomes:

$$\int d\tau\left\{\frac{\chi}{2e}\left[-\dot{\alpha_0}^2+\dot{\alpha}_1^2+\dot{\alpha}_2^2+\dot{\alpha}_3^2)\right]-e\rho_r\right\}$$

Effect of last term negligible as χ vanishes. ->massless particle on a conformally flat background. Invariant under ...

Special Conformal Group

$$p_\mu = \chi \eta_{\mu\nu} \dot{\alpha}^\mu / e$$

$$M_{\mu\nu} = \alpha_\mu p_\nu - \alpha_\nu p_\mu$$

$$D = \alpha^\mu p_\mu$$

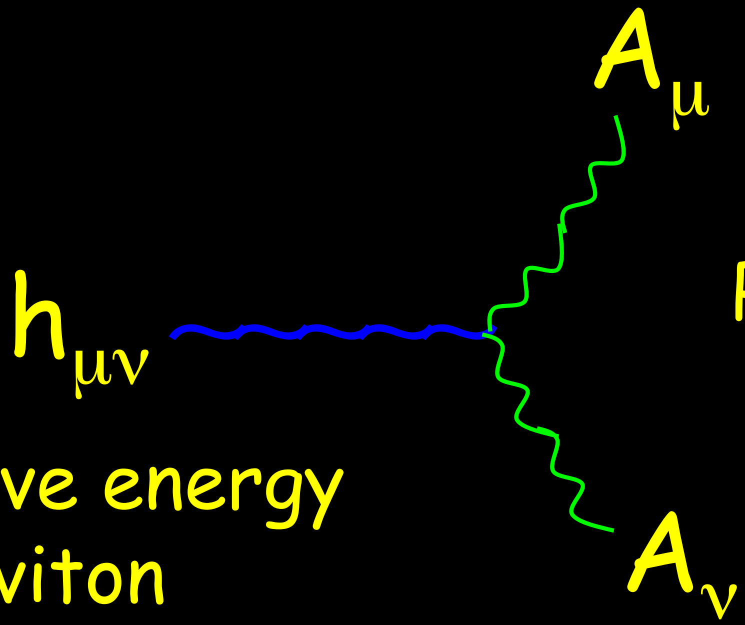
$$K_\mu = \alpha^2 p_\mu - 2\alpha_\mu \alpha . p$$

- -asymptotically conserved, and thus finite at singularity
- -analytically continuing χ, and matching SCG generators uniquely fixes the solution

3. Stationary point of Action

action finite: calculation varying all parameters governing passage across singularity shows action is stationary only on this solution

Is vacuum unstable in antig. region?

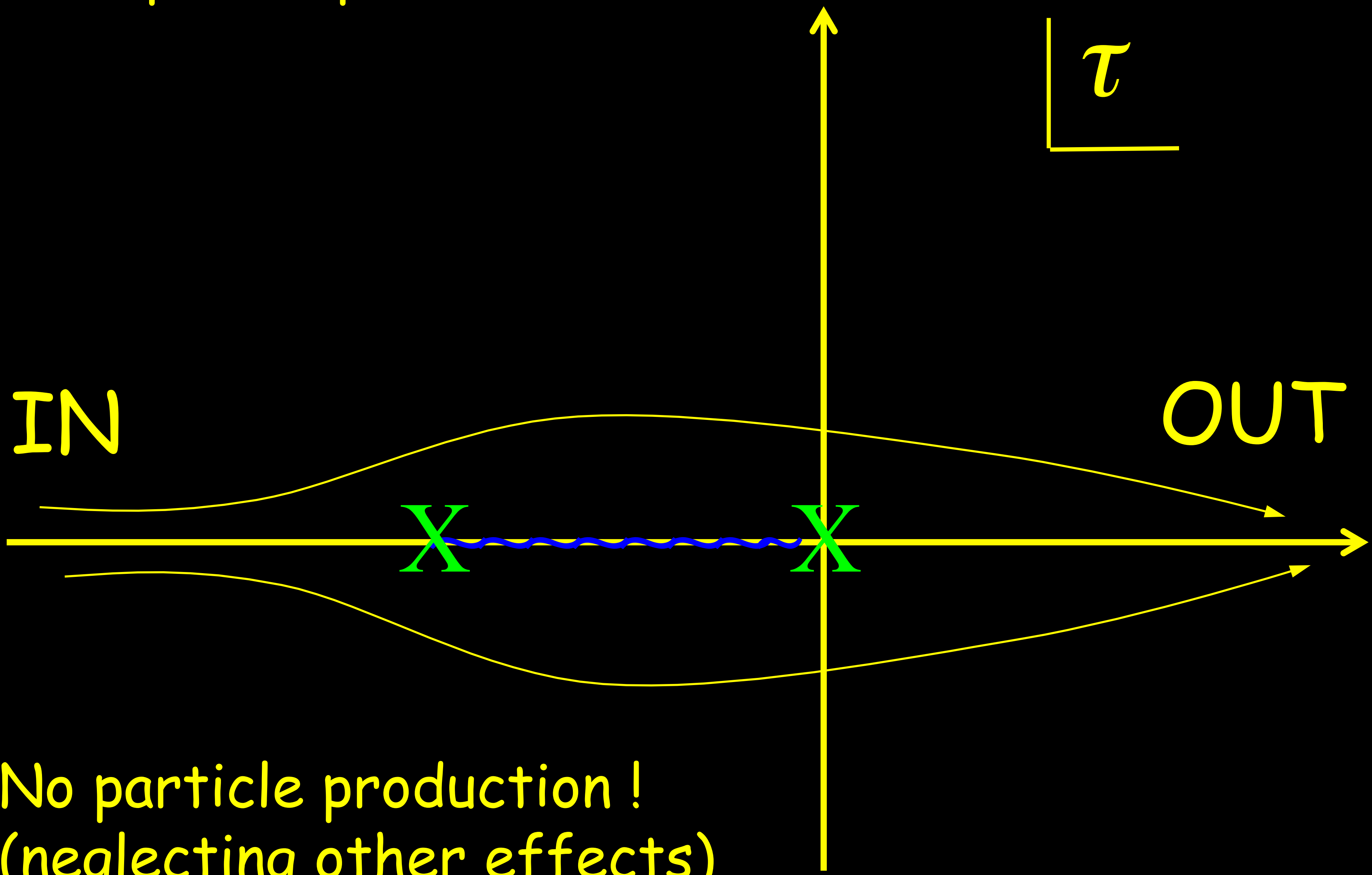


Positive energy photons

Negative energy graviton

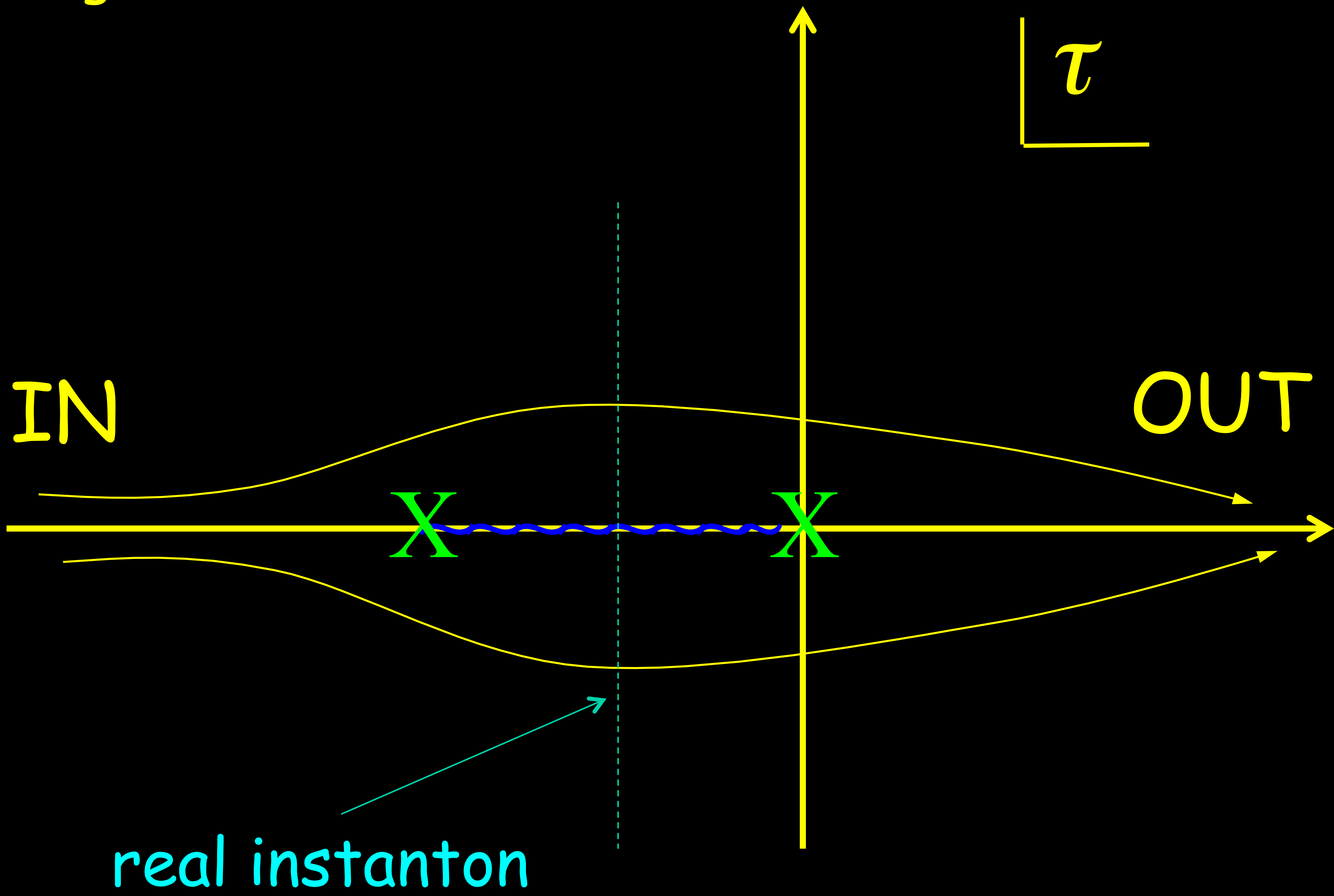
No: grav. vac in = grav. vac out

unique extension around singularities in complex τ-plane



No particle production !
(neglecting other effects)

In fact, there is a Euclidean instanton defining the global vacuum state



1. Stable in UV due to analyticity

2. Any particle production only shortens antigravity phase: proper time spent in the antigravity loop is

$$\int_{\tau_c}^{0} a_E(\tau)d\tau = \sqrt{3}\pi p^2/(4\kappa\rho_r^{\frac{3}{2}})$$

We have studied the same problem in the Wheeler-de Witt equation for (ultralocal) quantum gravity in the M_{PL}->0 limit

The conclusion is the same: there was a brief antigravity phase between the crunch and the bang.

Conclusions

* There seems to be a more-or-less unique way to continue 4d GR-scalar theory through cosmological singularities.

* Most surprisingly, it involves a brief antigravity phase.

* Does it agree w/ fully quantum approaches?
  (eg using holography: Craps/Hertog/NT)

Thank you!