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An adapted coordinate system for light-signal-based cosmology

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INTRODUCTION

- Most of cosmology is based on observing & interpreting light (or light-like) signals.

- Such signals travel on null geodesics lying on our past light cone (PLC).

- In a FLRW space-time it's easy to define our PLC and describe geodesics therein.

- In the presence of inhomogeneities our PLC and its null geodesics become messy.

- Q: Can we simplify our life by a suitable choice of coordinates?

- And, if yes: What can we do with them?

OUTLINE

- The GLC gauge & its properties
- Light-cone averaging in GLC coordinates
- Average & dispersion in the Hubble diagram for a "realistic" Universe
- Lensing in GLC coordinates?

If time allows

- Gravitational radiation from massless particle collisions (A. Gruzinov&GV, 1409.4555, gr-qc)

The geodetic light cone (GLC) gauge

(Gasperini, Marozzi, Nugier & GV, 1104.1167)

An almost fully gauge-fixed variant of the "observational coordinates" of G. Ellis et al.

The metric w.r.t. the coordinates (τ, w, θ^a):

$$\boxed{ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\theta^a - U^a dw)(d\theta^b - U^b dw) \quad ; \quad a, b = 1, 2}$$

$$g_{\mu\nu} = \begin{pmatrix} 0 & -\Upsilon & \vec{0} \\ -\Upsilon & \Upsilon^2 + U^2 & -U_b \\ \vec{0}^T & -U_a^T & \gamma_{ab} \end{pmatrix} \quad , \quad g^{\mu\nu} = \begin{pmatrix} -1 & -\Upsilon^{-1} & -U^b/\Upsilon \\ -\Upsilon^{-1} & 0 & \vec{0} \\ -(U^a)^T/\Upsilon & \vec{0}^T & \gamma^{ab} \end{pmatrix}$$

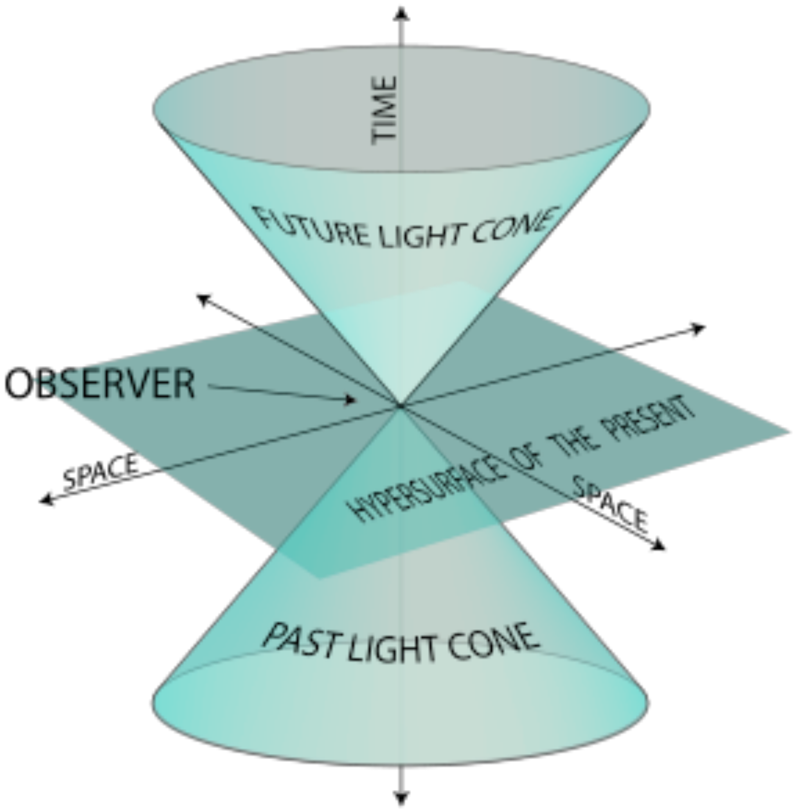
Flat-FRW limit $(a(\eta)d\eta = dt, \eta = \text{conformal time})$:

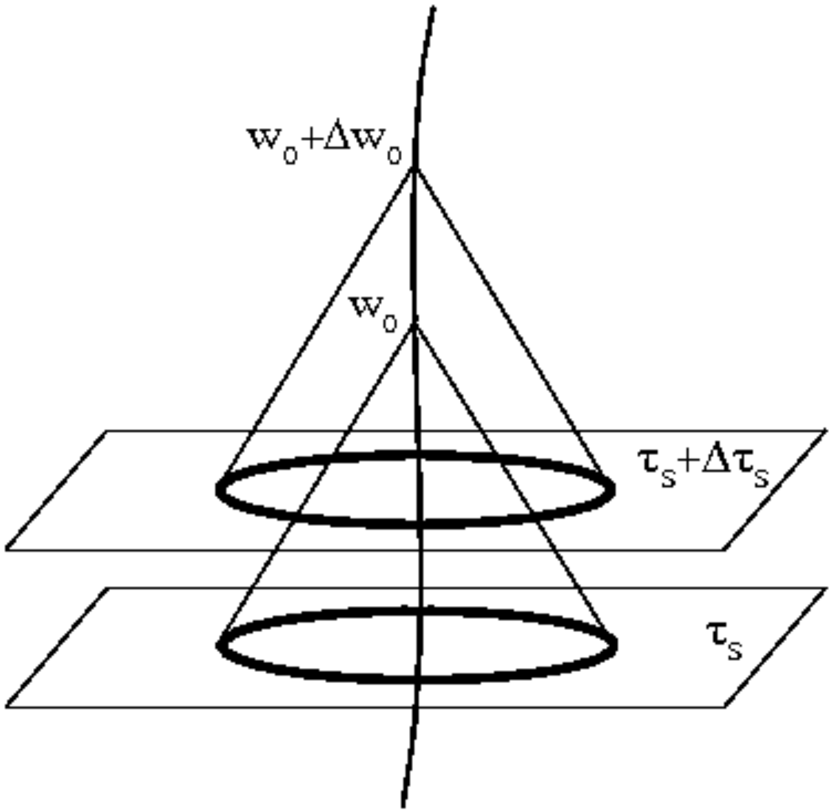
$$\begin{aligned} \tau &= t, & w = r + \eta, & & \Upsilon = a(t) \\ U^a &= 0, & \gamma_{ab} d\theta^a d\theta^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2). \end{aligned}$$

Generic properties of GLC coordinates

- $w = (<)\ w_0$ defines our past light cone (causal past)
- w = constant hypers. provide a null-foliation
- τ can be identified with synchronous-gauge time
- Static geodetic observers in SG have $u_\mu = \partial_\mu \tau$
- Photons travel at fixed w and θ^a :

$$k_\mu = \partial_\mu w \Rightarrow \dot{x}^\mu \sim \delta^\mu_\tau$$


TIME
FUTURE LIGHT CONE
OBSERVER
SPACE
HYPERSURFACE OF THE PRESENT
SPACE
PAST LIGHT CONE


$w_0+\Delta w_0$
w_0
$\tau_S+\Delta\tau_S$
τ_S

Other nice properties of the GLCG

1. A simple expression for the redshift z

In FRW cosmology z is simple (& factorizes) in terms of entries of the standard FRW metric

$$(1+z_s) = \frac{a(\eta_o)}{a(\eta_s)}$$

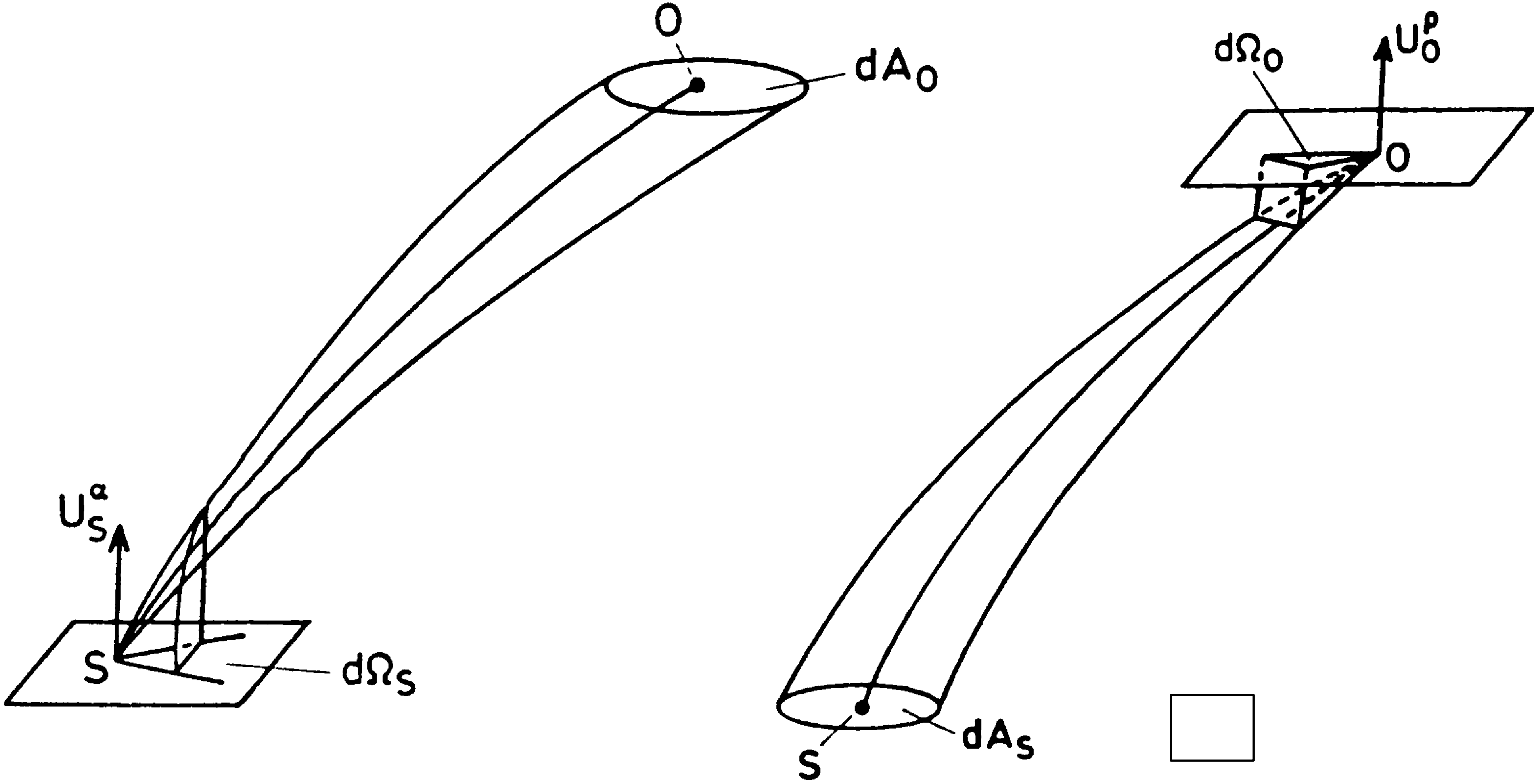
In the GLC gauge this property remains true:

$$(1+z_s) = \frac{(k^\mu u_\mu)_s}{(k^\mu u_\mu)_o} = \frac{\Upsilon_o}{\Upsilon_s}\frac{(u_\tau)_s}{(u_\tau)_o} \rightarrow \frac{\Upsilon_o}{\Upsilon_s}$$

Ratio depends in general depends from the θ^a coordinates

2. An exact & factorized expression for the Jacobi Map

(Fanizza, Gasperini, Marozzi, GV, 1308.4935)



From Schneider, Ehlers & Falco

Recall deviation equation for null geodesics:

$$\boxed{\nabla_\lambda^2 \xi^\mu = R_{\alpha\beta\nu}{}^\mu k^\alpha k^\nu \xi^\beta \quad ; \quad \nabla_\lambda \equiv k^\alpha \nabla_\alpha}$$

projected along the Sachs basis:

$$s^\mu_A (A = 1, 2) \ ; \ s^\mu_A u_\mu = s^\mu_A k_\mu = 0 \ ; \ g_{\mu\nu} s^\mu_A s^\nu_B = \delta_{AB} \ ; \boxed{\xi^A = \xi^\mu s^A_\mu}$$

$$\Pi^\mu_\nu \nabla_\lambda s^\nu_A = 0 \quad ; \quad \Pi^\mu_\nu = \delta^\mu_\nu - \frac{k^\mu k_\nu}{(u^\alpha k_\alpha)^2} - \frac{k^\mu u_\nu + u^\mu k_\nu}{u^\alpha k_\alpha}$$

$$\boxed{\frac{d^2 \xi^A}{d\lambda^2} = R^A_B \xi^B \ ; \ \frac{d}{d\lambda} \equiv k^\mu \partial_\mu \ ; \ R^A_B \equiv R_{\alpha\beta\nu\mu} k^\alpha k^\nu s^\beta_B s^\mu_A}$$

$$\boxed{\frac{d^2\xi^A}{d\lambda^2} = R^A_B\,\xi^B \;;\; \frac{d}{d\lambda} \equiv k^\mu\partial_\mu \;;\; R^A_B \equiv R_{\alpha\beta\nu\mu}k^\alpha k^\nu s^\beta_B s^\mu_A}$$

Def. of J: $\xi^A(\lambda_s) = J^A_B(\lambda_s,\lambda_o)\left(\frac{k^\mu\partial_\mu\xi^B}{k^\nu u_\nu}\right)_o$ $J^A{}_B$ obeys:

$$\frac{d^2}{d\lambda^2}J^A_B(\lambda,\lambda_o) = R^A_C J^C_B \;;\; J^A_B(\lambda_o,\lambda_o) = 0 \;;\; \frac{d}{d\lambda}J^A_B(\lambda_o,\lambda_o) = \delta^A_B\,(k^\nu u_\nu)_o$$

FGMV: exact expression for J in GLCG!

$$\boxed{J^A_B(\lambda,\lambda_o) = s^A_a(\lambda)\left\{\left[\left(\frac{k^\mu\partial_\mu s}{k^\mu u_\mu}\right)^{-1}\right]^a_B\right\}_{\lambda=\lambda_o}} \;;\; \boxed{s^A_a s^A_b = \gamma_{ab}}$$

Again (bi)local and factorized ($s_a{}^A$ = zweibeins for γ_{ab})
in this gauge (NB: expression is NOT covariant!)

3. Area & luminosity distance (d_A, d_L)

(Ben-Dayan, Gasperini, Marozzi, Nugier & GV, 1202.1247 & FGMV 1308.4935)

Much easier if one has the Jacobi map!

$$d_A^2 = \det\left(J^A_B(\lambda_s,\lambda_o)\right) = \frac{\sqrt{\gamma(\lambda_s)}}{\det\left(u_\tau^{-1}\partial_\tau s^B_b\right)_{\lambda=\lambda_o}} \; ; \; \gamma \equiv \det\gamma_{ab}$$

$$\det\left(u_\tau^{-1}\partial_\tau s^B_b\right)_{\lambda=\lambda_o} = \frac{1}{4}\left[\det\left(u_\tau^{-1}\partial_\tau \gamma^{ab}\right)\gamma^{3/2}\right]_o$$

Using residual gauge freedom in GLCG:

$d_A^2 = \dfrac{\sqrt{\gamma}}{\sin\theta}$ & finally: $d_L = (1+z)^2 d_A$

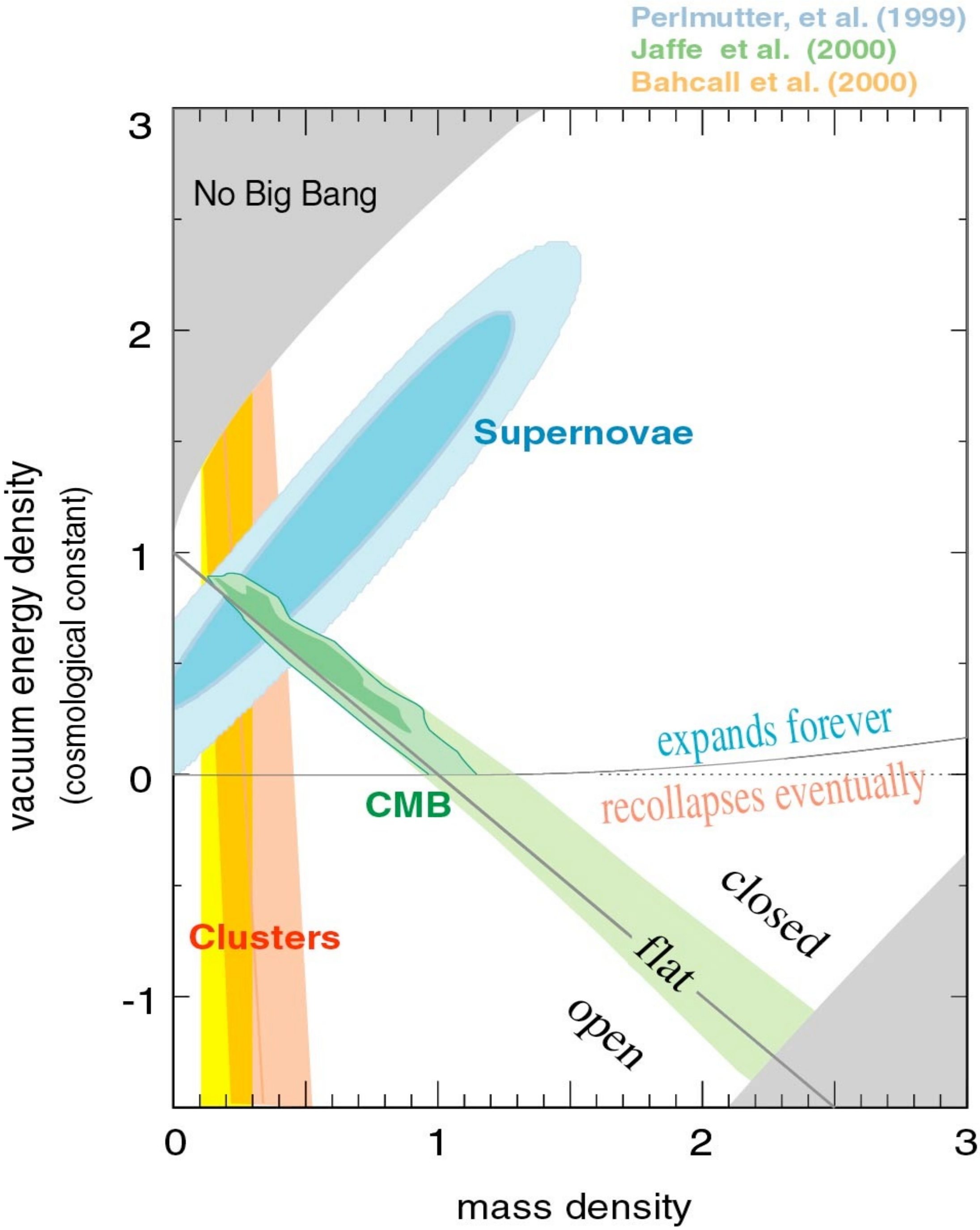
I: The inhomogeneous Hubble diagram in a realistic cosmology

For a complete summary of our (and related) work see: F. Nugier's thesis: 1309.65420

The concordance model:

3 sets of data pointing at Dark Energy

Cosmic Concordance



Two arguments for DE are based on inhomogeneities/structures
The 3rd (SNIa) ignores them completely!

Basic tool: the famous Hubble diagram of redshift vs. luminosity-distance

A short reminder (for FLRW)

Definition of luminosity distance d_L:

$$\Phi = \frac{L}{4\pi d_L^2}$$

where L is the absolute luminosity and Φ the flux.

For FLRW: $1 + z(t) = \frac{a_0}{a(t)}$ $\qquad q_0 \equiv -\frac{a\ddot{a}}{\dot{a}^2}(t = t_0)$

For a spatially flat ΛCDM Universe (for simplicity):

$$d_L^{FLRW}(z) = \frac{1+z}{H_0}\int_0^z \frac{dz'}{[\Omega_{\Lambda 0} + \Omega_{m0}(1+z')^3]^{1/2}}$$

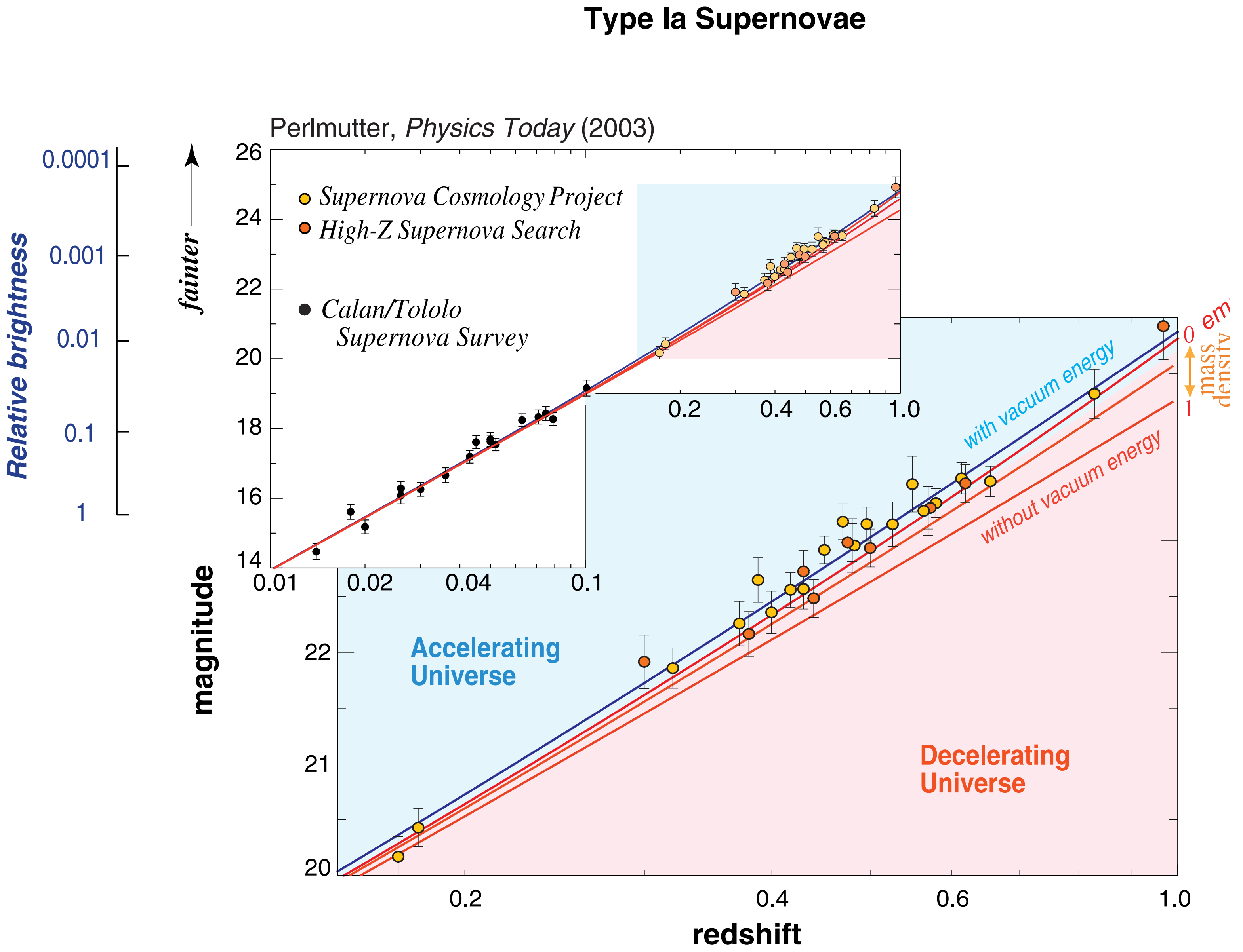
If expanded to 2nd order in z:

$$d_L(z) = H_0^{-1}\left[z + \frac{1}{2}(1-q_0)z^2 + O(z^3)\right]$$

In FLRW cosmology: $q_0 = \frac{4\pi G(\rho_0 + 3p_0)}{3H_0^2} = \frac{1}{2}(\Omega_{m,0} - 2\Omega_{\Lambda,0})$

Hubble law beyond linear order => information about eq. of state!

Using Type Ia supernovae as standard candles: evidence for negative q_0, DE...



Perlmutter, *Physics Today* (2003)

The Universe is fairly homogeneous only on very large scales (> few 100 Mpc?).

Q: What's the effect of smaller scale inhomogeneities?

A. Not obvious! Averages of physical quantities do not obey the homogeneous EEs (Buchert & Ehlers, ...).

There are extra, so-called "backreaction", terms. This "averaging problem" has been a rather hot topic in recent years.

Hopes have been raised that inhomogeneities might "explain" cosmic acceleration and give a natural resolution of the famous coincidence (why now?) problem (Buchert, Rasanen, Kolb-Matarrese-Riotto...)

Too optimistic (given other evidence for DE)? Yet still important to take inhomogeneities into account for (future) precision cosmology and/or for testing the concordance model itself.

Most of previous work deals with spatial averages and with formal definitions of acceleration...

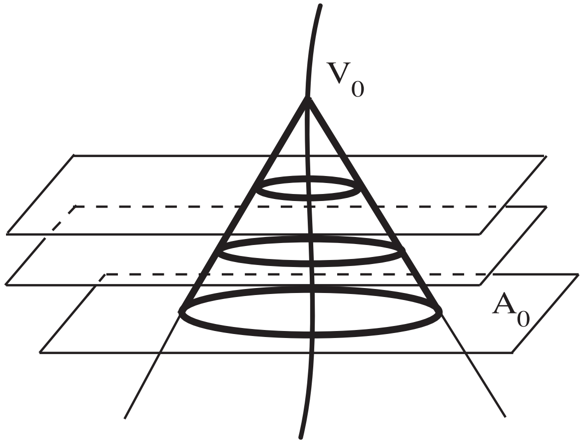
Not clear what's the relation between such averages and the averaged d_L-z relation (Hubble diagram)

We therefore looked at how to average directly that relation.

Gauge-invariant light-cone averages

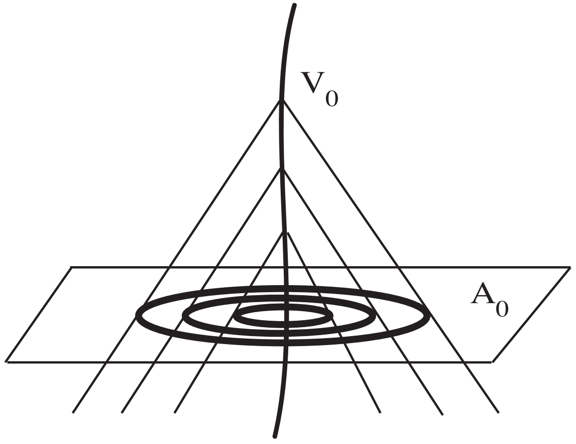
(Gasperini, Marozzi, Nugier & GV, 1104.1167)

truncated light cone



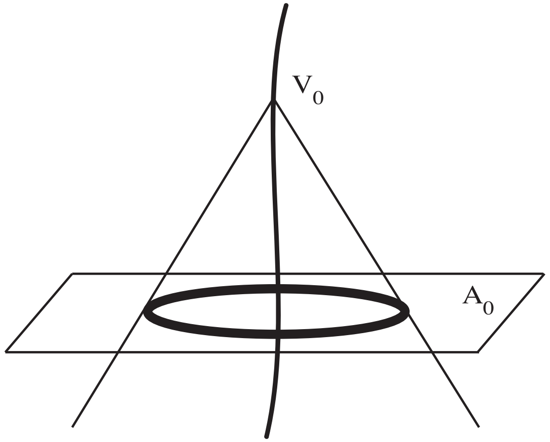
(a) $I(1; V_0; A_0)$

causally connected sphere



(b) $I(1; A_0; V_0)$

Relevant for this talk



2-sphere embedded
in the light cone

(c) $I(1; V_0, A_0; -)$

WHAT'S THE CORRECT MEASURE?

(G. Marozzi and G. Veneziano, in preparation)

An important issue (that came out after!) is whether one should weight the physical quantity (e.g. d_L^{-2}) with a non trivial averaging measure.

In our SNe papers we took as measure the proper area of the 2-D surface element.

Justified if the proper number density of SNe is constant on a fixed-z hypersurface.

Then our procedure gives the measured average!

For CMB on the last-scattering surface one may argue that the correct measure is simply the solid angle at the observer.

Averaging the flux at 2nd-order

(BGMNV,1207.1286, 1302.0740; BGNV,1209.4326)

Considering $< \Phi > \sim <d_L^{-2}>$ (not $<d_L>^{-2}$) simplifies life further. In GLCG (w/ our measure):

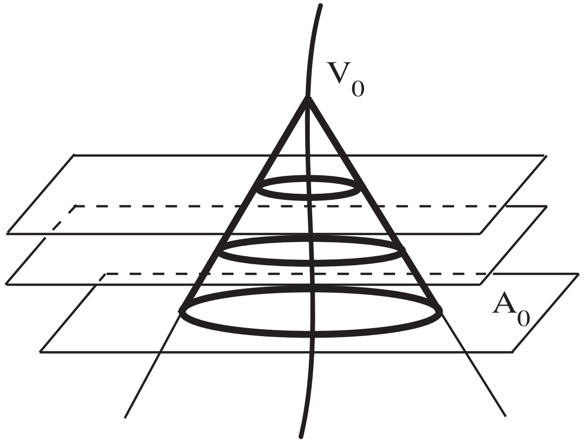
$$\langle d_L^{-2}\rangle(z_s, w_0) = (1+z_s)^{-4}\left[\int \frac{d^2\theta}{4\pi}\gamma^{\frac{1}{2}}(w_0, \tau_s(z_s, \theta^a), \theta^b)\right]^{-1}$$

where $\tau_s(z_s, \theta^a)$ is the solution of:

$$(1+z_s) = \frac{\Upsilon(w_0, \tau_0, \theta^a)}{\Upsilon(w_0, \tau_s, \theta^a)}$$

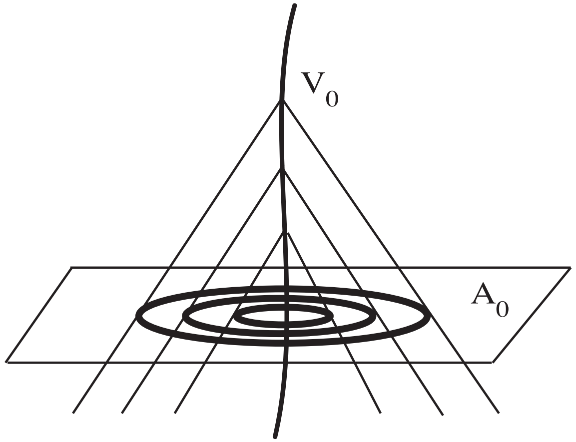
Intersection of $w = w_0$ and $z = z_s$ hypersurfaces is a 2-surface (topologically a sphere) on which SNe of given redshift z_s are located.

truncated light cone



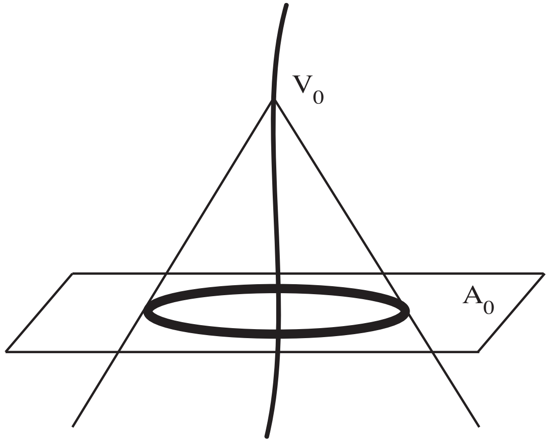
(a) $I(1;V_0;A_0)$

causally connected sphere



(b) $I(1;A_0;V_0)$

Relevant for this talk



2-sphere embedded
in the light cone

(c) $I(1;V_0,A_0;-)$

This is exact: can be used for any specific (fixed-geometry) inhomogeneous model (e.g. LTB with us at center)

A more realistic (and Copernican) model is the one produced by inflation: a stochastic background of perturbations with statistical isotropy and homogeneity.
Vanishing effects at 1st order, need 2nd order (at least)

Unfortunately, perturbations are normally studied in other gauges (e.g. Newtonian or Poisson): we need to find the coordinate transformation up 2nd order (quite a lot of work, see F. Nugier's thesis, yet easier than starting directly in the Poisson Gauge, see e.g. Bernardeau, Bonvin, Vernizzi 0911.2244).

The calculation proceeds in two steps:

1. Calculation of d_L^{-2} to 2nd order in the Poisson gauge (BGNV,1209.4326) via coordinate transformation. Independent result by Umeh, Clarkson & Maartens (1207.2109, 1402.1933) being compared to ours (G. Marozzi, 1406.1135: some errors in both?).

2. Performing the appropriate LC integrals both for computing the effect on different averages and on the corresponding dispersions. Part of the calculation is analytic, part is numerical using realistic power spectra (BGMNV,1302.0740).

See BGMNV 1207.1286 (prl) for a summary of both

$$\bar{\delta}^{(2)}_S(z_s,\tilde{\theta}^a)=\bar{\delta}^{(2)}_{path}+\bar{\delta}^{(2)}_{pos}+\bar{\delta}^{(2)}_{mixed}$$

$$\begin{aligned}
\bar{\delta}^{(2)}_{path} &= \Xi_s\Bigg\{-\frac{1}{4}\left(\phi^{(2)}_s-\phi^{(2)}_o\right)+\frac{1}{4}\left(\psi^{(2)}_s-\psi^{(2)}_o\right)+\frac{1}{2}\psi_s^2-\frac{1}{2}\psi_o^2-(\psi_s+J_2^{(1)})\partial_+Q_s \\
&+ \frac{1}{4}(\gamma_0^{ab})_s\partial_aQ_s\partial_bQ_s+Q_s\left(-\partial_+^2Q_s+\partial_+\psi_s\right)+\frac{1}{\mathcal{H}_s}\partial_+Q_s\,\partial_\eta\psi_s \\
&+ \frac{1}{4}\int_{\eta_o}^{\eta_s^{(0)-}}dx\ \partial_+\left[\phi^{(2)}+\psi^{(2)}+4\psi\ \partial_+Q+\gamma_0^{ab}\ \partial_aQ\ \partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a) \\
&- \frac{1}{2}\partial_a(\partial_+Q_s)\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\ \left[\gamma_0^{ab}\ \partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\Bigg\} \\
&- \frac{1}{2}\psi^{(2)}_s-\frac{1}{2}\psi_s^2-K_2+\psi_sJ_2^{(1)}+\frac{1}{2}(J_2^{(1)})^2+J_2^{(1)}\frac{Q_s}{\Delta\eta}-\frac{1}{\mathcal{H}_s\Delta\eta}\left(1-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\frac{1}{2}(\partial_+Q_s)^2 \\
&- \frac{2}{\mathcal{H}_s\Delta\eta}\psi_s\partial_+Q_s+\frac{1}{2}\partial_a\left(\psi_s+J_2^{(1)}+\frac{Q_s}{\Delta\eta}\right)\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\ \left[\gamma_0^{ab}\ \partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right) \\
&+ \frac{1}{4}\partial_aQ_s\partial_b\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\left[\gamma_0^{ab}\ \partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right) \\
&+ \frac{1}{16}\partial_a\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\ \left[\gamma_0^{bc}\ \partial_cQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\partial_b\left(\int_{\eta_o}^{\eta_s^{(0)-}}d\bar{x}\ \left[\gamma_0^{ad}\ \partial_dQ\right](\eta_s^{(0)+},\bar{x},\tilde{\theta}^a)\right) \\
&- \frac{1}{4\Delta\eta}\int_{\eta_o}^{\eta_s^{(0)-}}dx\ \left[\phi^{(2)}+\psi^{(2)}+4\psi\ \partial_+Q+\gamma_0^{ab}\ \partial_aQ\ \partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a) \\
&+ \frac{1}{\mathcal{H}_s}\partial_+Q_s\left\{-\partial_\eta\psi_s+\partial_r\psi_s+\frac{1}{\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\right\} \\
&+ Q_s\Bigg\{\partial_r\psi_s+\partial_+\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\frac{1}{(\eta_s^{(0)+}-x)^2}\int_{\eta_o}^{x}dy\Delta_2\psi(\eta_s^{(0)+},y,\tilde{\theta}^a)\right) \\
&+ \frac{1}{2\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\Bigg\} \\
&+ \frac{1}{16\sin^2\tilde{\theta}}\left(\int_{\eta_o}^{\eta_s^{(0)-}}dx\ \left[\gamma_0^{1b}\ \partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)^2, \qquad \text{(B.18)}
\end{aligned}$$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{pos} &= \frac{\Xi_s}{2}\Bigg\{ (\partial_r P_s)^2 + (\gamma_0^{ab})_s \partial_a P_s\, \partial_b P_s - \frac{2}{\mathcal{H}_s}\left(\partial_r P_s - \partial_r P_o\right)\left(\mathcal{H}_s \partial_r P_s + \partial_r^2 P_s\right) \\
&- \int_{\eta_{in}}^{\eta_s^{(0)}} d\eta' \frac{a(\eta')}{a(\eta_s^{(0)})}\partial_r \left[\phi^{(2)} - \psi^2 + (\partial_r P)^2 + \gamma_0^{ab}\partial_a P \partial_b P\right](\eta', \Delta\eta, \tilde{\theta}^a)\Bigg\} \\
&+ \frac{1}{2\mathcal{H}_s \Delta\eta}\Bigg\{ (\partial_r P_o)^2 + \lim_{r\to 0}\left[\gamma_0^{ab}\partial_a P \partial_b P\right] \\
&- \int_{\eta_{in}}^{\eta_o} d\eta' \frac{a(\eta')}{a(\eta_o)}\partial_r \left[\phi^{(2)} - \psi^2 + (\partial_r P)^2 + \gamma_0^{ab}\partial_a P \partial_b P\right](\eta', 0, \tilde{\theta}^a)\Bigg\} \\
&- \frac{1}{2\mathcal{H}_s \Delta\eta}\left(1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)(\partial_r P_s - \partial_r P_o)^2 \ , \qquad \text{(B.19)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{mixed} &= \Xi_s \Bigg\{ \partial_r P_s J_2^{(1)} - (\partial_r P_s - \partial_r P_o)\frac{1}{\mathcal{H}_s}\partial_\eta \psi_s - (\gamma_0^{ab})_s \partial_a Q_s \partial_b P_s \\
&+ \frac{1}{\mathcal{H}_s}\partial_+ Q_s \partial_r^2 P_s + Q_s \partial_r^2 P_s \\
&+ \frac{1}{2}\partial_a(\partial_r P_s - \partial_r P_o)\left(\int_{\eta_o}^{\eta_s^{(0)-}} dx \left[\gamma_0^{ab}\, \partial_b Q\right](\eta_s^{(0)+}, x, \tilde{\theta}^a)\right)\Bigg\} \\
&- \frac{1}{\mathcal{H}_s \Delta\eta}\left(\psi_o - \psi_s - J_2^{(1)}\right)\partial_r P_o + \frac{Q_s}{\Delta\eta}\partial_r P_s \\
&+ \frac{1}{\Delta\eta}(\partial_r P_s - \partial_r P_o)\left\{\frac{1}{\mathcal{H}_s}\left(1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_+ Q_s + \frac{2}{\mathcal{H}_s}\psi_s\right\} \\
&+ \frac{1}{\mathcal{H}_s}(\partial_r P_s - \partial_r P_o)\left\{\partial_\eta \psi_s - \partial_r \psi_s - \frac{1}{\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o} d\eta' \Delta_2 \psi(\eta', \eta_o - \eta', \tilde{\theta}^a)\right\}. \qquad \text{(B.20)}
\end{aligned}
$$

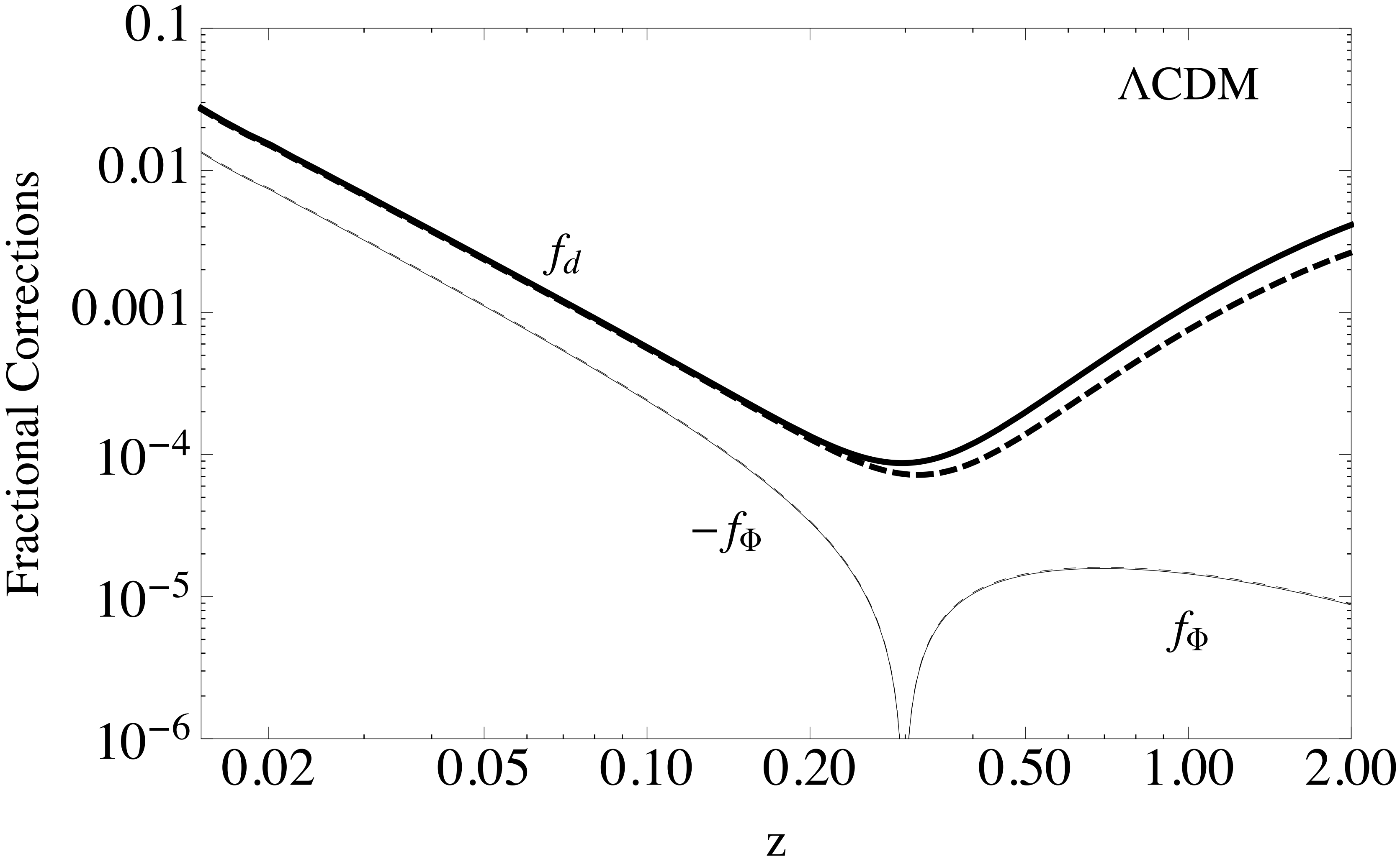
$$
P(\eta, r, \theta^a) = \int_{\eta_{in}}^{\eta} d\eta' \frac{a(\eta')}{a(\eta)}\phi(\eta', r, \theta^a) \quad , \qquad Q(\eta_+, \eta_-, \theta^a) = \int_{\eta_o}^{\eta_-} dx\, \frac{1}{2}\left(\psi + \phi\right)(\eta_+, x, \theta^a)
$$

Fortunately many terms are very small/negligible. The most important ones pick up some moments (2nd and 3rd at most) of the power spectrum.

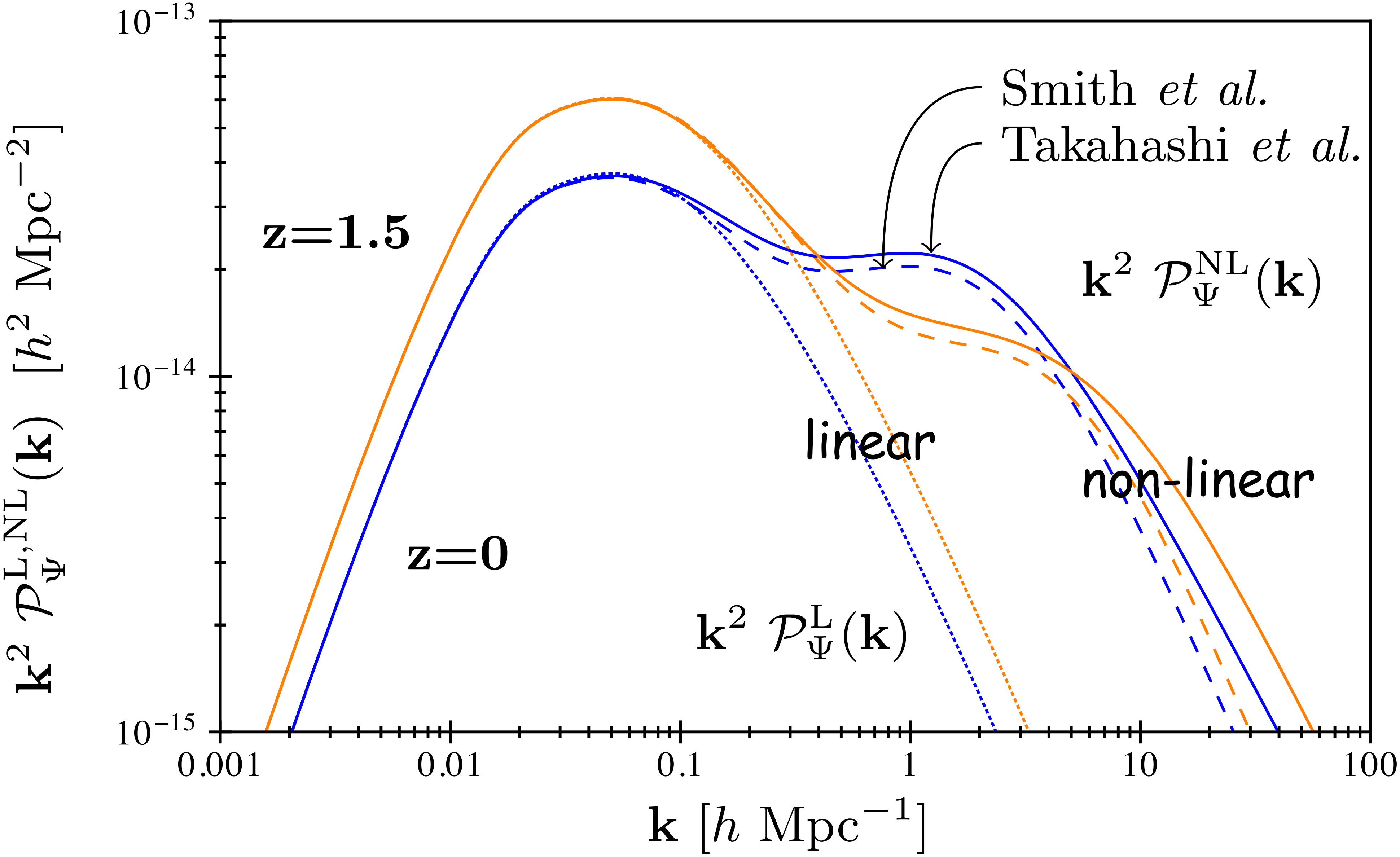
Their contribution is enhanced, relative to a very naive estimate of 10^{-10}, by powers of k^*/H_0, where k^* is a characteristic scale of the power spectrum.

Yet the overall effect is small...

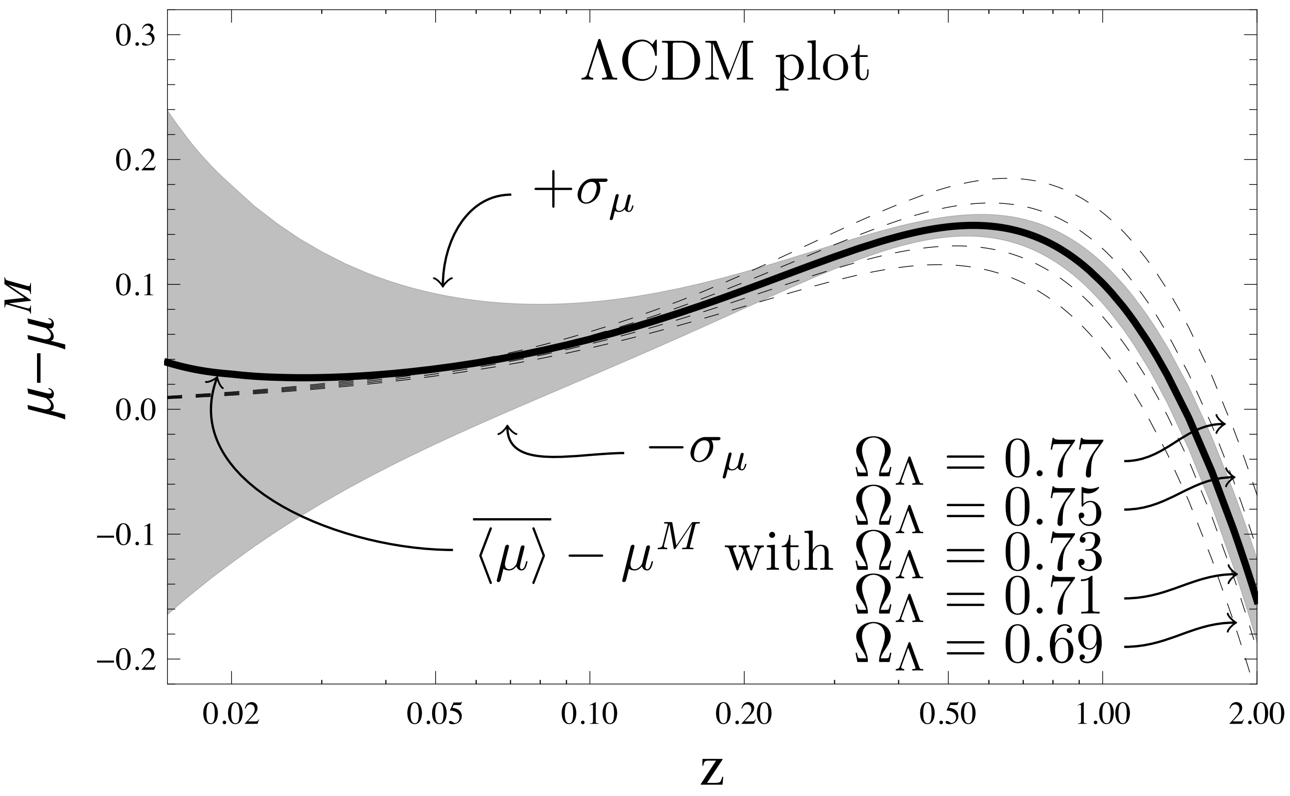
Different observables suffer different corrections (here w/out area measure)



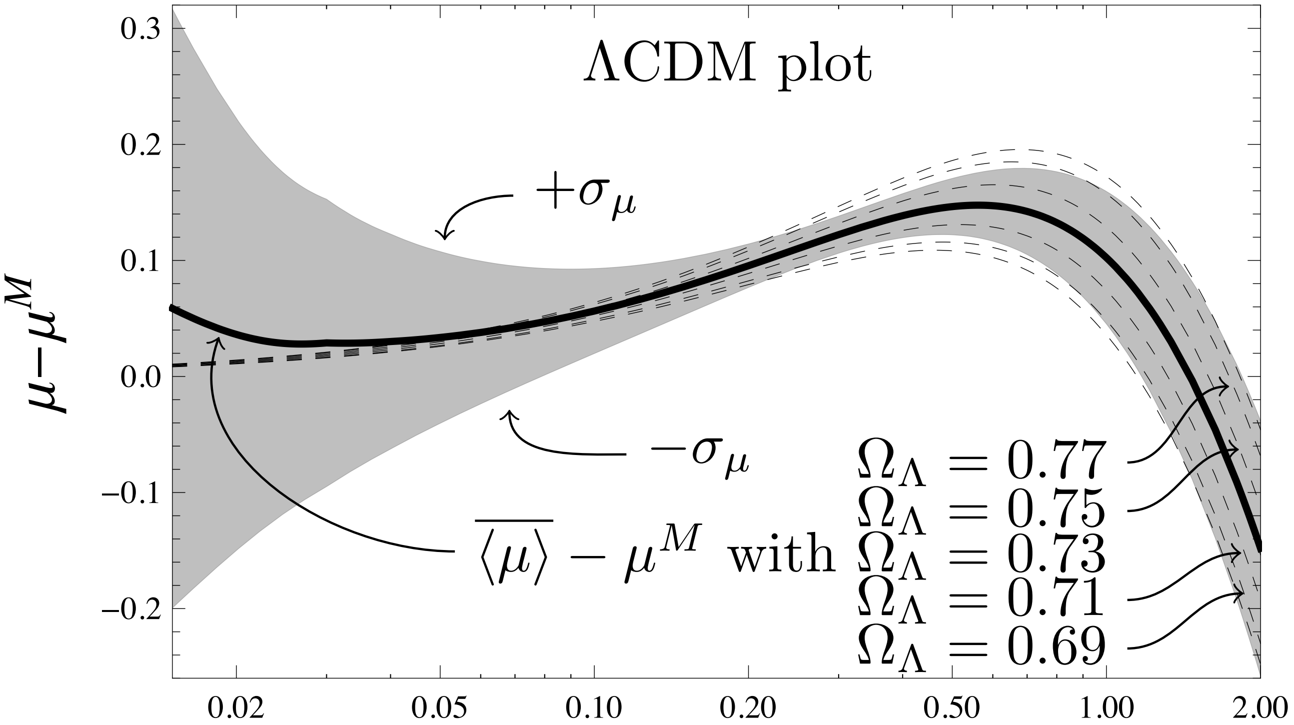
Results somewhat sensitive to the power spectrum used (but no IR or UV divergence)



Distance modulus

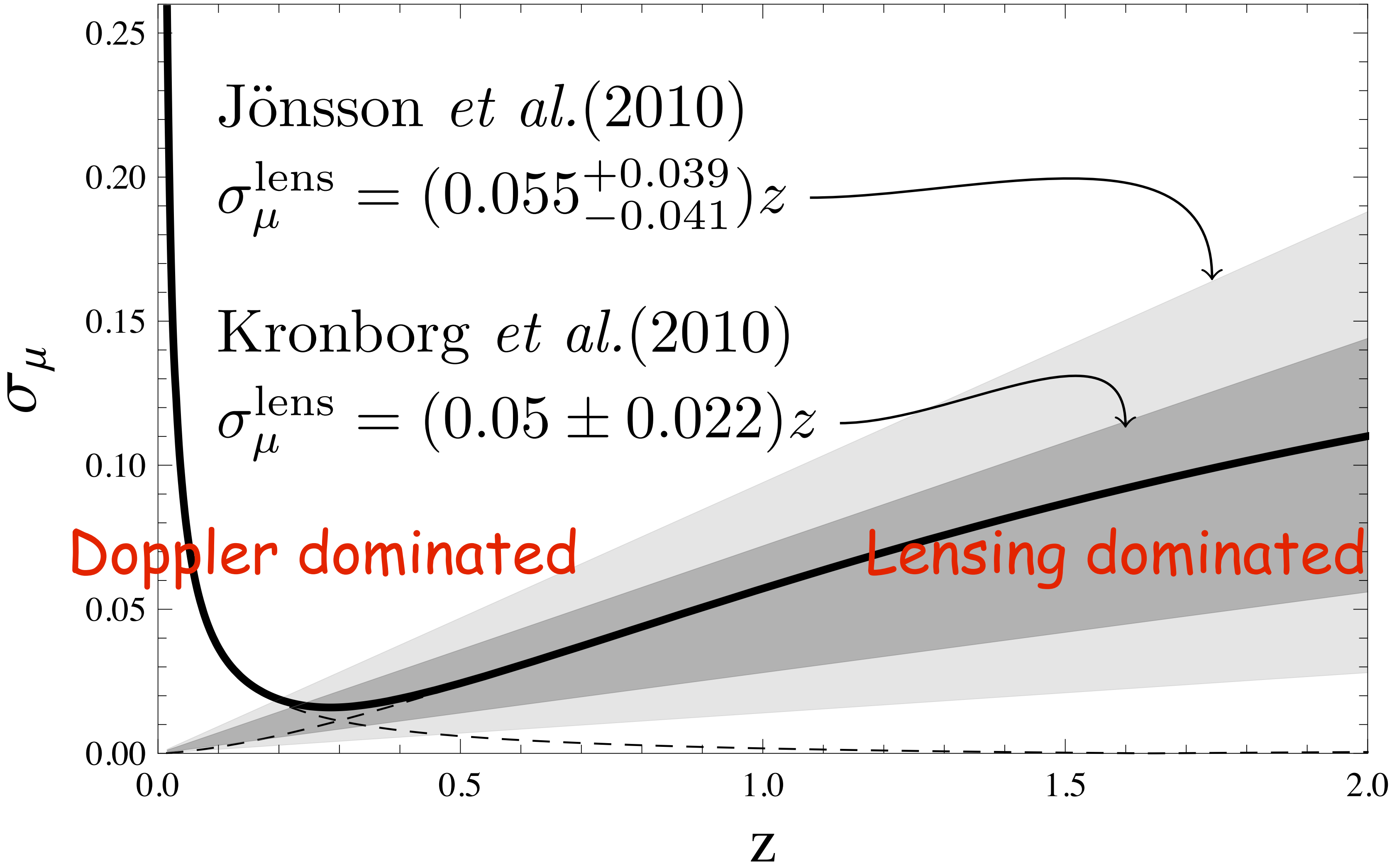


linear PS



non-linear PS

$$(\sigma_\mu^{\mathrm{obs}})^2 = (\sigma_\mu^{\mathrm{fit}})^2 + (\sigma_\mu^{z})^2 + \left[(\widehat{\sigma_\mu^{\mathrm{int}}})^2 + (\sigma_\mu^{\mathrm{lens}})^2\right]$$



Jönsson *et al.*(2010)

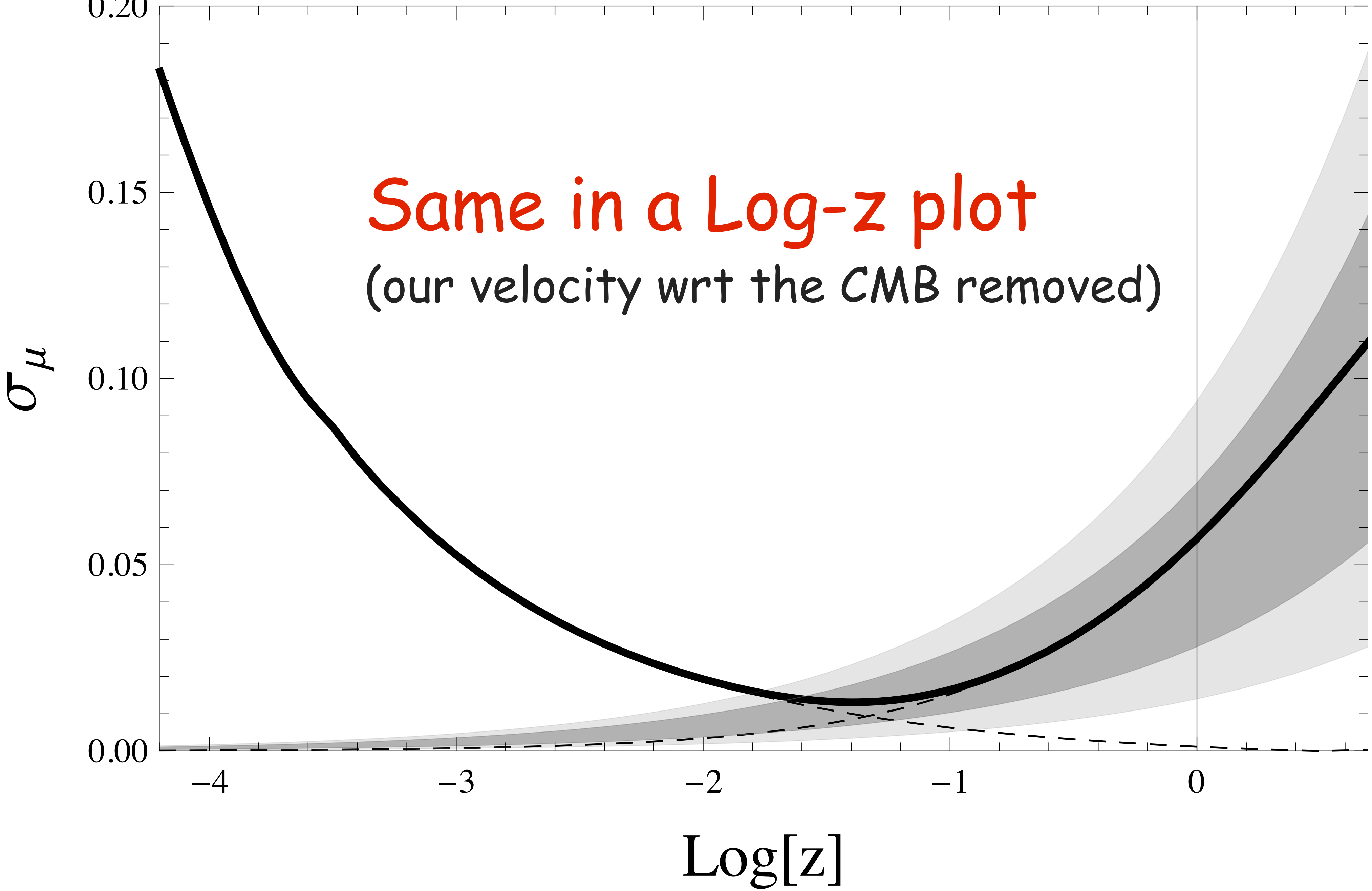
$$\sigma_\mu^{\mathrm{lens}} = (0.055^{+0.039}_{-0.041})z$$

Kronborg *et al.*(2010)

$$\sigma_\mu^{\mathrm{lens}} = (0.05 \pm 0.022)z$$

Doppler dominated

Lensing dominated



Same in a Log-z plot

(our velocity wrt the CMB removed)

Lensing dispersion is as in Betoule 1401.4064.
The (uncorrected) Doppler is a factor ~2 larger

Conclusions on DE application

Inhomogeneities (of a stochastic type) cannot mimic DE.

Averaging gives negligible corrections to the FLRW results.

In principle 10^{-4} precision attainable, however...

Effects on the variance/dispersion are much larger and may limit the determination of DE parameters (via SNIa data) to the few % level because of limited statistics.

II: GLC gauge and lensing?

(G.Fanizza and F. Nugier, 1408.1604 & work in progress)

Trying to make use of our simple, exact result on the Jacobi Map for gravitational lensing

The Jacobi map is a basic ingredient in gr. lensing (see "Gravitational Lensing" by Schneider, Ehlers & Falco). By its definition, J(s,o) connects lengths at the source to angles at the observer:

$$\xi_s^A = J_B^A(s,o)\left(\frac{k^\mu \partial_\mu \xi^B}{k^\nu u_\nu}\right)_o = J_a^A(s,o)\theta_o^a$$

Its determinant gives the so-called area distance:

$d_A{}^2 = dA_s/d\Omega_o = \det J$.

Another map, J(o,s), connects angles at the source to lengths at the observer:

$$\xi_o^A = J^A_B(o,s)\left(\frac{k^\mu \partial_\mu \xi^B}{k^\nu u_\nu}\right)_s = J^A_a(o,s)\theta^a_s$$

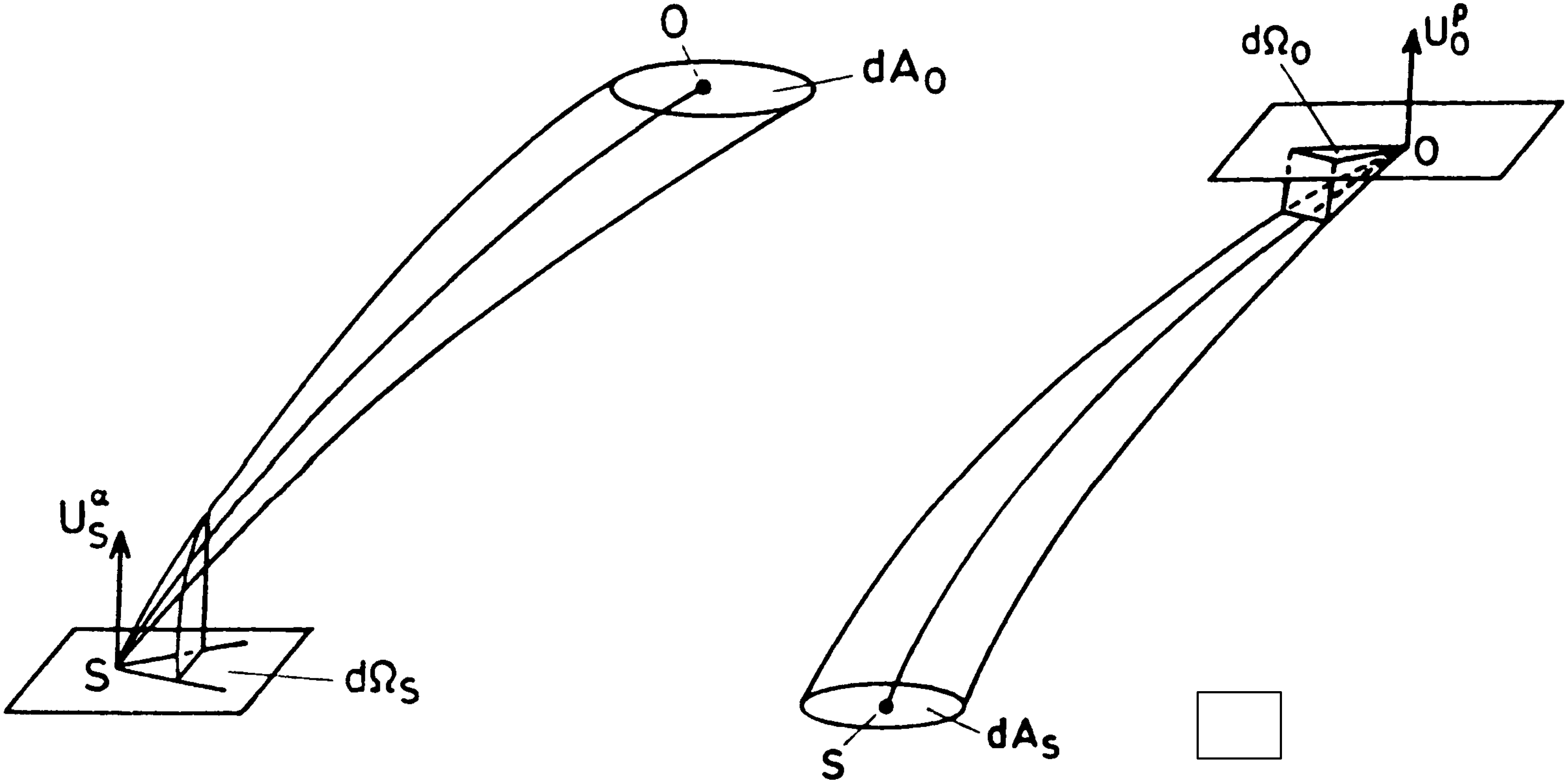
Its determinant gives the so-called corrected luminosity distance d'_L.
The two Jacobi maps (hence the two distances) are related by Etherington's (exact) reciprocity relation:

$$J(o,s) = -(1+z)\, J(s,o).$$

The (uncorrected) luminosity distance is given by:

$$d_L = (1+z)\, d'_L = (1+z)^2\, d_A$$

From Schneider, Ehlers & Falco

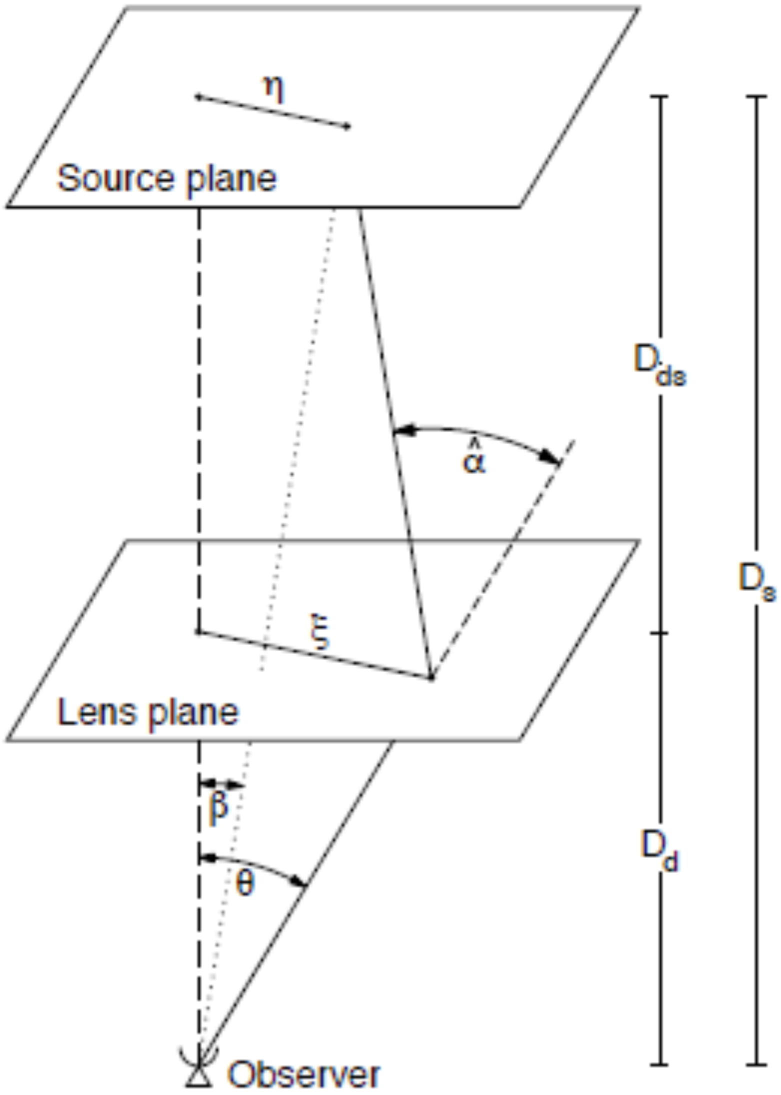


In the lensing literature one relates more often angles at the observers to angles at the source through the so-called (2x2) amplification matrix, containing both convergence κ and shear γ.

$$\mathcal{A} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

The total magnification μ is related to its determinant:

$$\mu^{-1} = \det \mathcal{A} = (1-\kappa)^2 - \gamma^2$$



Thin-lens example

$$\mathcal{A}_{ab} = \frac{\partial \beta^a}{\partial \theta^b} \rightarrow \frac{J^A_B(\lambda_s, \lambda_o)}{\bar{d}_A(\lambda_s)}$$

FLRW area distance

$$\mathcal{A} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

1. In the GLCG it should be possible to give the amplification matrix in a compact non-perturbative form directly from the known Jacobi map.

2. Improve treatment of gravitational lensing when a perturbative approach is inadequate, e.g. in the presence of caustics (points where rank (γ_{ab}) < 2);

3. Another quantity that can be studied is the deformation matrix $S^A{}_B$ (simply related to J). It contains the null expansion and shear. Its derivatives are related, through the EEs, to Ricci and Weyl focussing (cf. Raych. eqn.)

What else?

1. Give non perturbative arguments for the smallness (or otherwise?) of inhomogeneity effects on the z-Φ (z-d_L^{-2}) relation;

3. Set up Einstein's equations (at least in cosmological perturbation theory) directly in the GLCG (prel. investigations on H-constraint & Raychaudhuri eqn. encouraging, domain of dependence simple, ...)

4. ...Any suggestion?

Gravitational radiation from massless particle collisions

(A. Gruzinov & GV, 1409.4555)

A gravitational "energy crisis"?

(ACV 0712.1209, Wosiek & GV 0805.2973)

Within some (crude) approximations the graviton spectrum in a Transplanckian-E collision turned out to be:

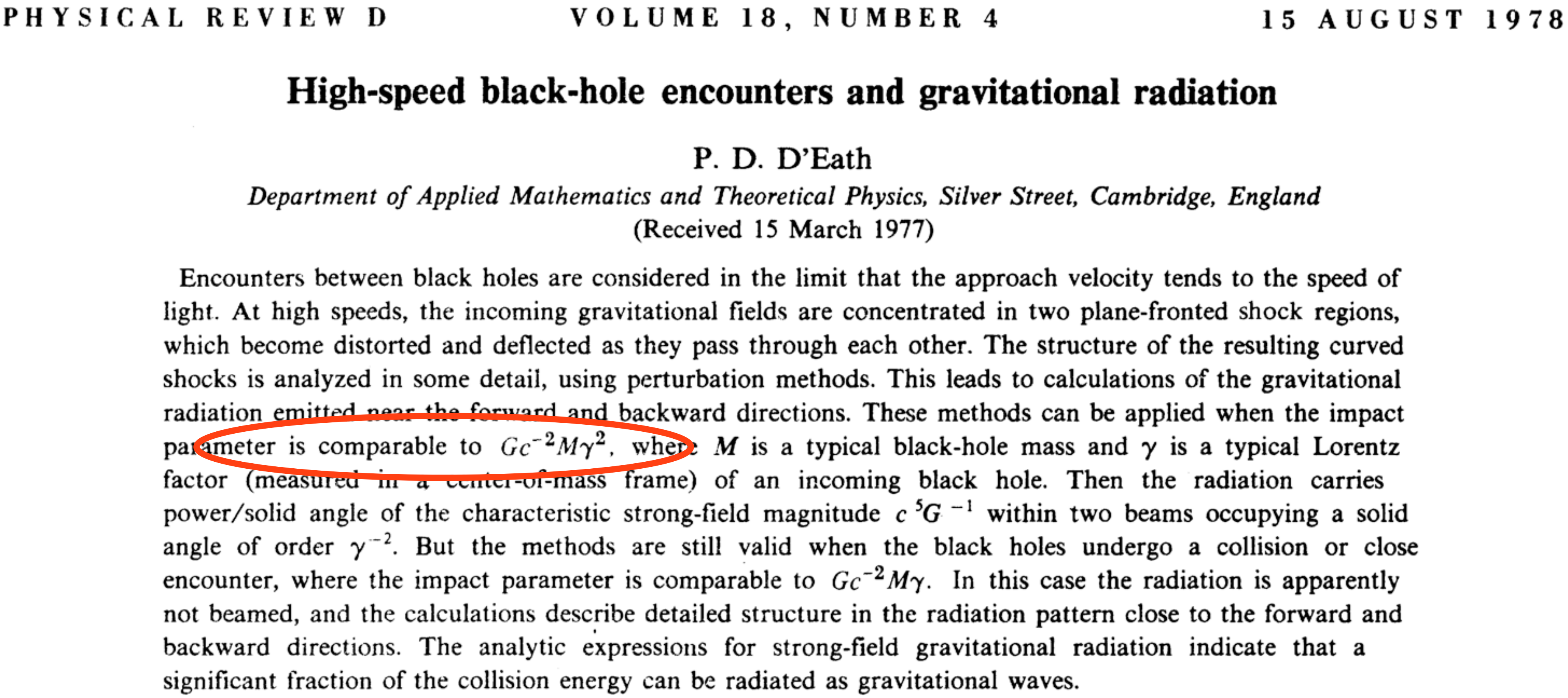
$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\ \frac{Gs}{\hbar}\frac{R^2}{b^2} >> 1$$

Accordingly, the fraction of energy emitted in GWs is O(1) already for b = b*>> R (i.e. for small deflection angle). Is this puzzling from a GR perspective? Given that spectrum is known to be flat @ small ω:

Q: What's the cutoff in ω for the GWs emitted in an ultra-relativistic small angle (b >> R) 2-body collision?

Possible answers for ω_c: 1/b, 1/R (my old guess), b/R^2, b^2/R^3 (ACV), γ/b (Gal'tsov et al, singular m=0 limit?), E/h (singular classical limit?)

GR's answer to this problem seems to be unknown...



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High-speed black-hole encounters and gravitational radiation

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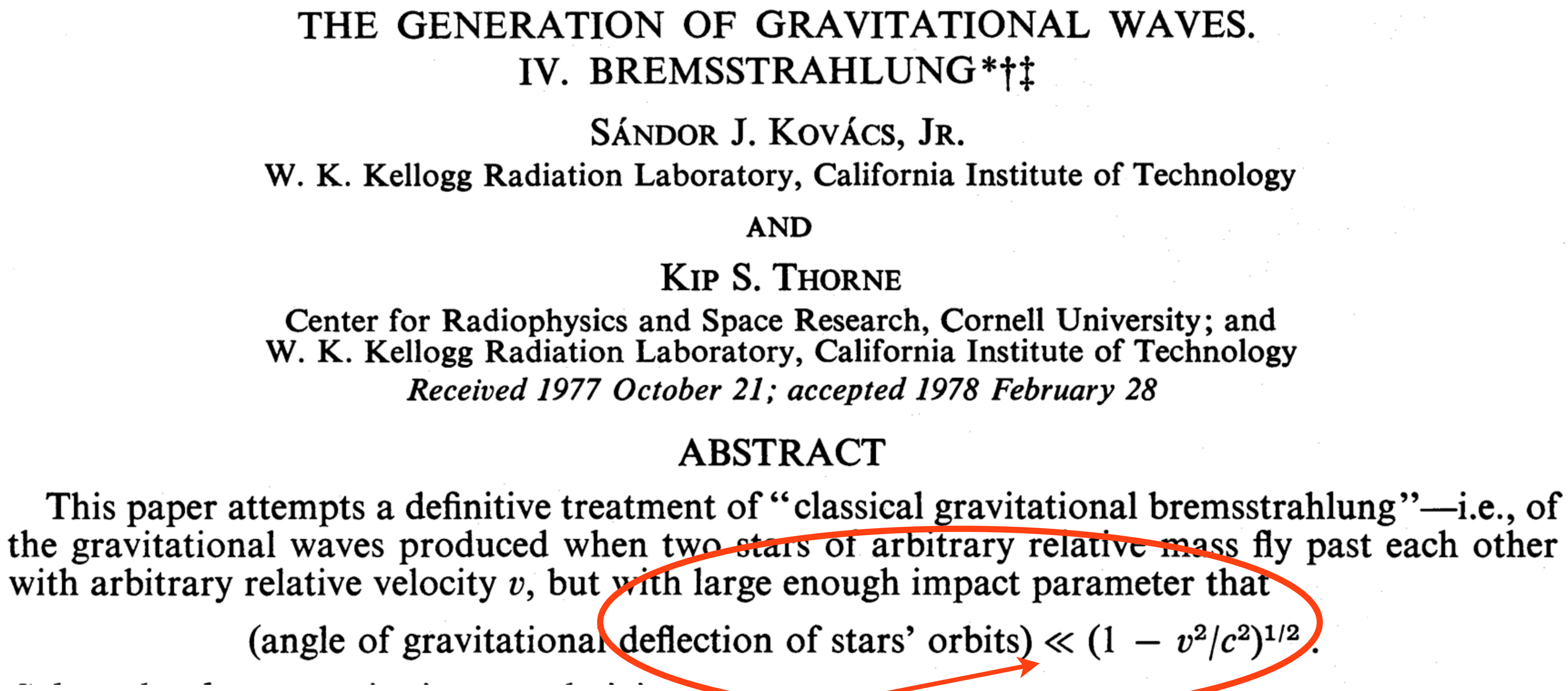
Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or close encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.



THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

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ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$(\text{angle of gravitational deflection of stars' orbits}) \ll (1 - v^2/c^2)^{1/2}.$$

For $\theta < 1/\gamma$ ($b > \gamma R$) agrees with GKST: $E^{GW}/E \sim \gamma\,\theta^3$

A long standing problem, also hard numerically

What's GR's answer for $\theta > 1/\gamma$?

Andrei Gruzinov and I (1409.4555/gr.qc) believe to know the answer at infinite

a bit tricky, but final result is simple.

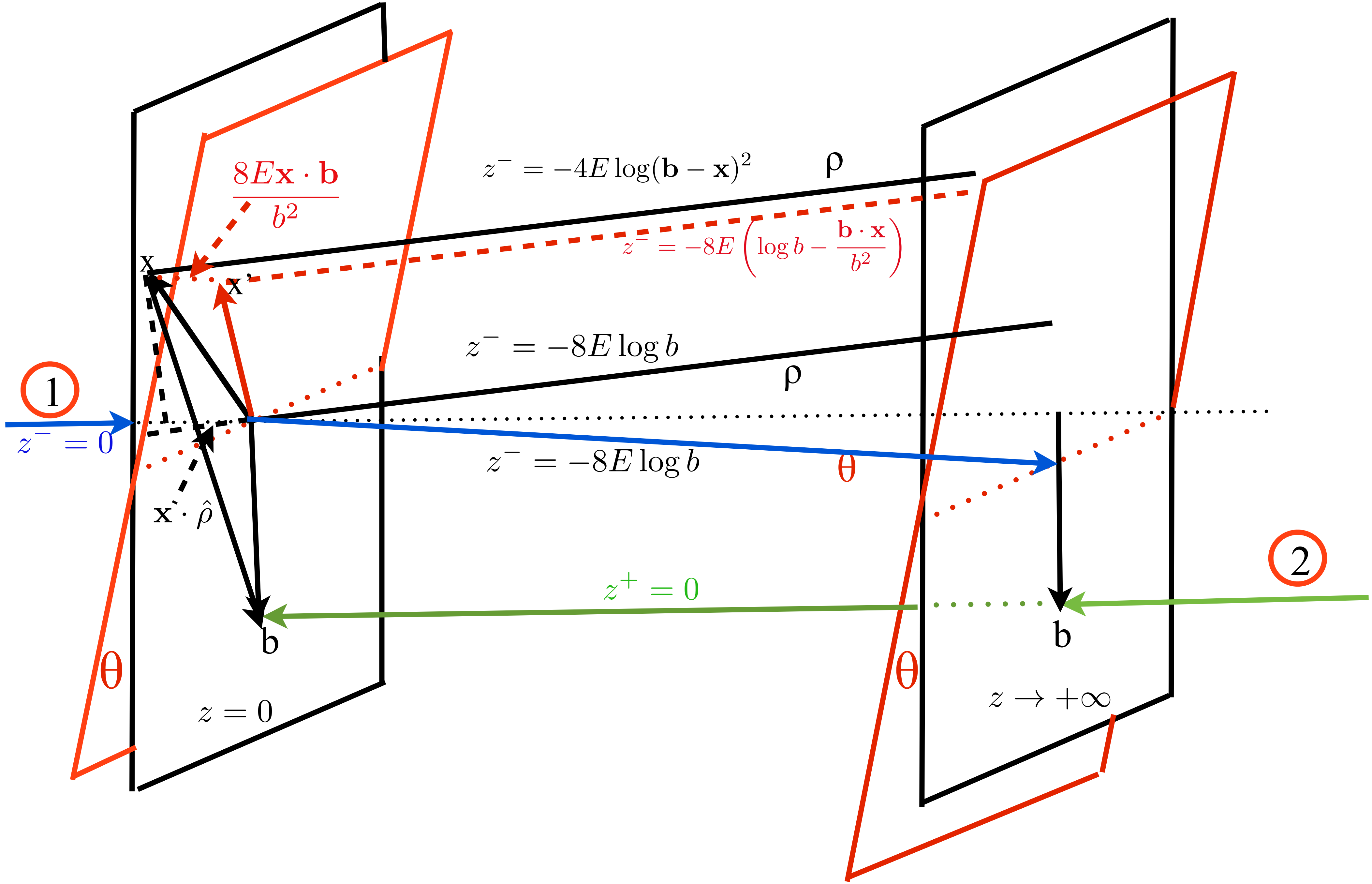
We found both the frequency and the angular distribution of the GW spectrum

Result obtained via Huygens principle in the Fraunhofer approximation to reconstruct the metric at future null infinity from the data on the collision surface.

NB: Subtracting the deflected shock wave (as in D'Eath's work) is crucial!

Rough i

(G = 1, E = R of previous slide)



AS shock wave metric, curvature, time delay

$$ds^2 = -dz^+dz^- + dx^2 + dy^2 - 4E\ln\frac{x^2+y^2}{\lambda^2}\delta(z^+)(dz^+)^2$$

$$R \equiv R_+ + iR_\times = 4E\delta(z^+)\frac{\zeta^2}{|\zeta|^4} \quad , \quad \zeta = x + iy$$

$$R_+ \equiv \frac{1}{2}(R_{+x+x} - R_{+y+y}), \quad R_\times \equiv R_{+x+y}$$

$$\delta z^+ = 0\ ;\ \delta z^- = -8E\ln(b/\lambda) \Rightarrow \delta t = -\delta z = -4E\ln(b/\lambda)\ ;\ z^\pm = t \pm z$$

General formula for GW

$$\frac{dE^{GW}}{du} = \frac{1}{4\pi}\int d^2\Omega|\partial_u C|^2$$

$$\frac{\partial^2}{\partial u^2}C = -rR \quad ; \quad r \to \infty$$

After FT and Huygens

$$\frac{dE^{GW}}{d\omega} = \frac{1}{2\omega^2}\int d^2\Omega\; r^2|\mathcal{R}_{\mathcal{I}^+}|^2$$

$$r\mathcal{R}(\mathbf{R}) = \frac{\omega}{2\pi i}\int d^2\mathbf{x}\;\mathcal{R}(\mathbf{x})e^{-i\omega u(\mathbf{x},\boldsymbol{\rho})}.$$

$$\mathcal{R}(\mathbf{x}) = \frac{4E}{2\pi}\frac{\zeta^2}{|\zeta|^4}$$

Putting everything together and subtracting deflected wave

$$E^{GW} = \frac{E^2}{2\pi^4}\int d^2\rho\; d\omega|c|^2,$$

$$c(\omega,\boldsymbol{\rho}) = \int d^2x\; e^{-i\omega\boldsymbol{\rho}\cdot\mathbf{x}}\cdot\frac{\zeta^2}{|\zeta|^4}\left(e^{i\omega\Delta z^-} - e^{i\omega\Delta z^-_{AS}}\right)$$

$$\frac{\Delta z^-}{4E} = -\ln\frac{(\mathbf{x}-\mathbf{b})^2}{\lambda^2};\; \frac{\Delta z^-_{AS}}{8E} = -\ln\frac{b}{\lambda} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}.$$

This can be written in its final form

$$\frac{dE^{GW}}{d\omega\ d^2\rho_s} = \frac{E^2}{2\pi^4}|c|^2\ ;\ \boldsymbol{\rho}_s = \boldsymbol{\rho} - 8E\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \boldsymbol{\rho}_s) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega \mathbf{x}\cdot\boldsymbol{\rho}_s}\left[e^{-iE\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy\ ;\ \Phi(\mathbf{x}) = 4\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + 8\frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

where ρ_s is the solid angle around (one of) the deflected trajectories and Re ζ^2 and Im ζ^2 correspond to the two physical polarizations.

The ω-spectrum is almost flat ($dE/d\omega \sim \log \omega$) up to $\omega \sim E^{-1}$ and at very small $\omega \sim b^{-1}$ reproduces the known "zero-frequency-limit" (Smarr 1977) based on the soft graviton limit (Weinberg, 1965):

$$\frac{dE^{GW}}{d\omega} \to \frac{2}{\pi}\ \theta^2 E^2\ \log(\theta^{-2})$$

At $\omega \sim E^{-1}$ there is a break in the spectrum, which becomes scale-invariant ($dE \sim d\omega/\omega$) producing an extra log in the "efficiency". Only logarithmic sensitivity to UV cutoff. With a reasonable guess on the latter we obtain:

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{\pi}\ \theta^2\ \log(\theta^{-2})$$

to leading-log accuracy.

We can also get the angular distribution.

The emerging picture is quite appealing: gravitons are mainly produced in two back-to-back cones of some typical angular size around the deflected trajectories. That size shrinks with increasing frequency: this is responsible for the $d\omega/\omega$ spectrum at $\omega > 1/E$.

Q: Can we get the same from our QFT diagrams?

A: Hopefully yes: Ciafaloni, Colferai & I (in progress) have some arguments about how that should work.

THANK YOU!