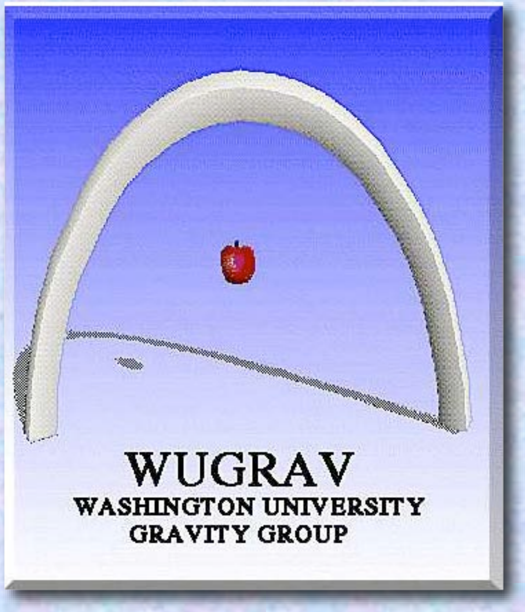
Testing General Relativity in the Strong-field Dynamical Regime



Clifford Will
Washington University, St. Louis

IHES
Bures-sur-Yvette, 26 May, 2011

Testing General Relativity in the Strong-field Dynamical Regime

20th century themes

- High precision technology (clocks, space)
- Frameworks for comparing and testing theories
- Theory-experiment synergy

21st century themes - Beyond Einstein

- Strong-field gravity
- Gravitational-waves
- Extreme-range gravity

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- Introduction - what is “strong”?
- Astrophysical tests
- Cosmic barbers: Are black holes really bald?
- Counting hair using gravitational waves
- Counting hair using SgrA*

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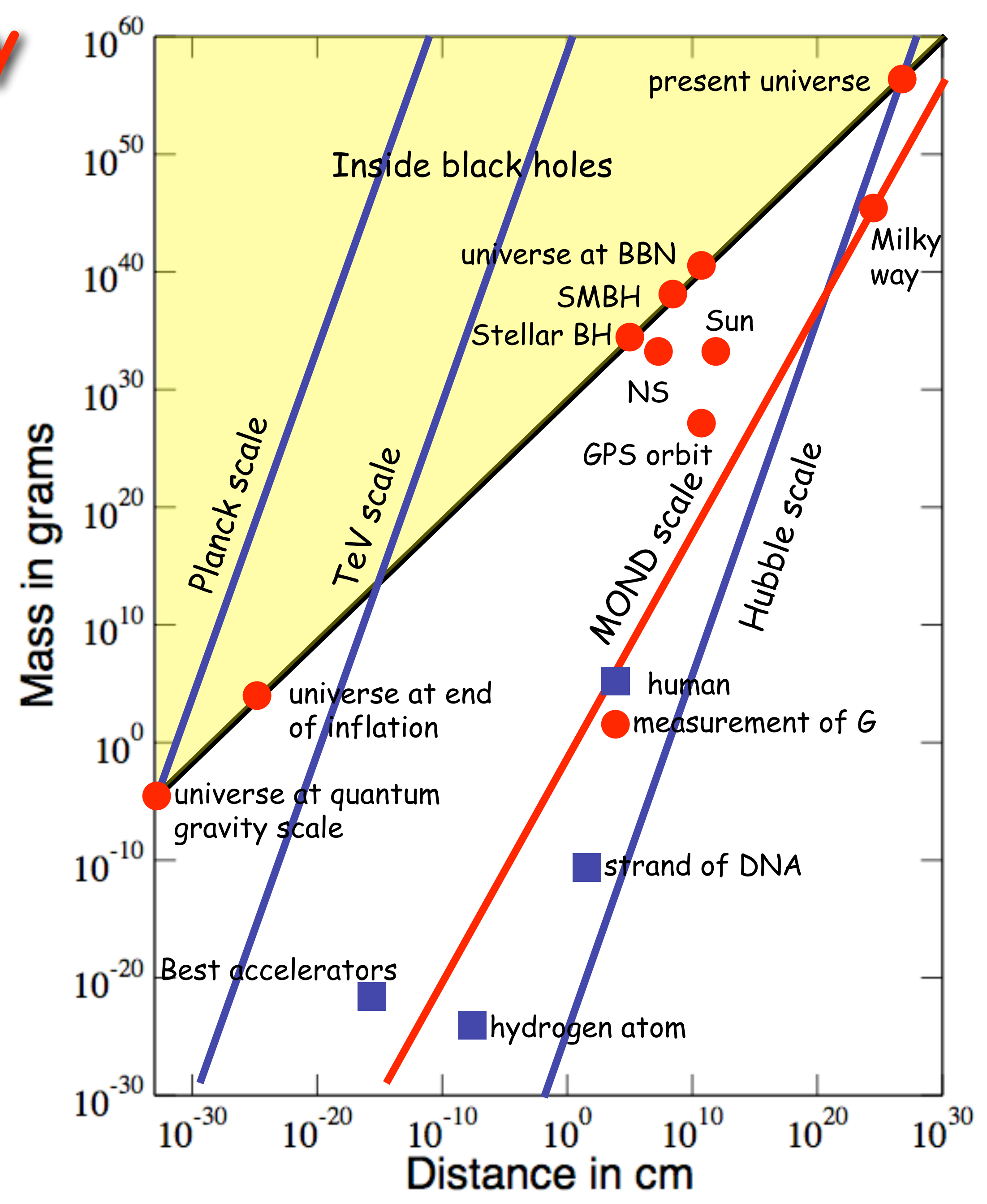
Strong Gravity
Weak Gravity

$$\frac{c^2 R}{GM} \sim 1$$ ▬

$$\frac{c^2 R^3}{GM} \sim \ell^2$$ ▬

$$\frac{GM}{R^2} \sim a_0$$ ▬



Adapted from original figure by CMW
Used in 1999 NRC Decadal Survey of Gravitational Physics
Used in *Gravity*, by James Hartle

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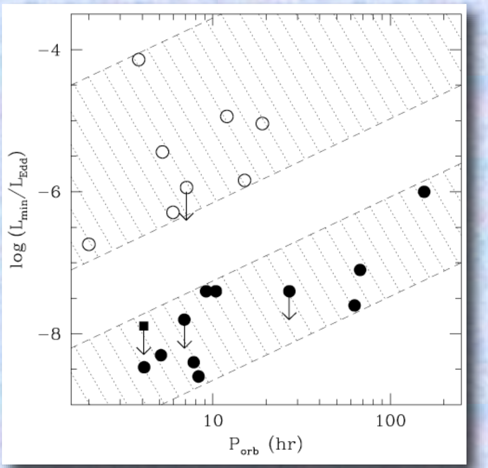
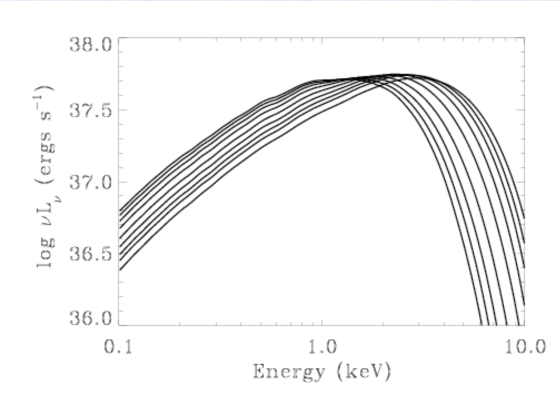
Astrophysical tests of strong gravity

Steady luminosity in LMXB:
BH or NS?



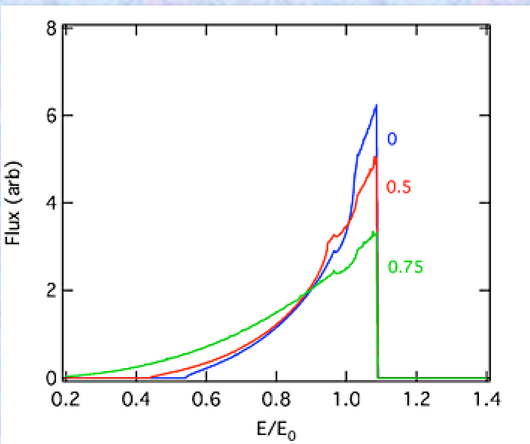
Narayan et al

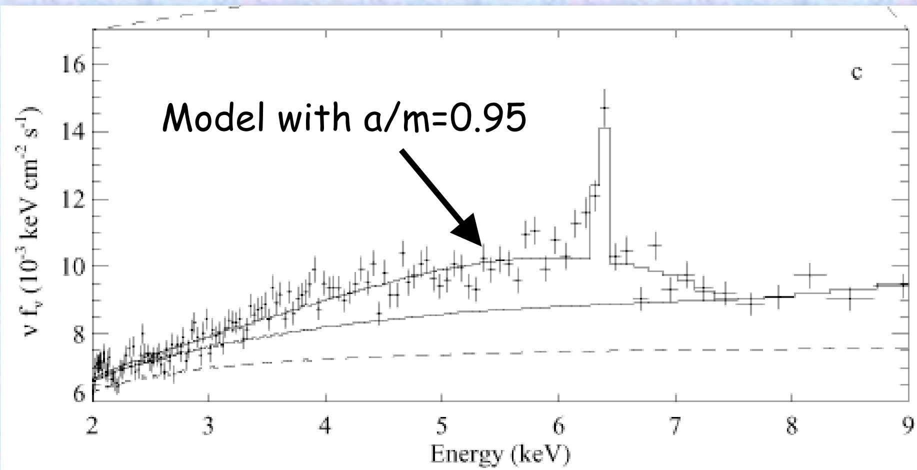
Accretion spectrum: radius of the ISCO?



a/m ~ 0.65 - 0.85 (GRS 1915+105,
4U 1543-47, GRO J1655-40)

Broadening of iron fluorescence lines in BH accretion





Model with a/m=0.95

SMBH in galaxy MCG-6-15-30 (Wilms et al 2001)

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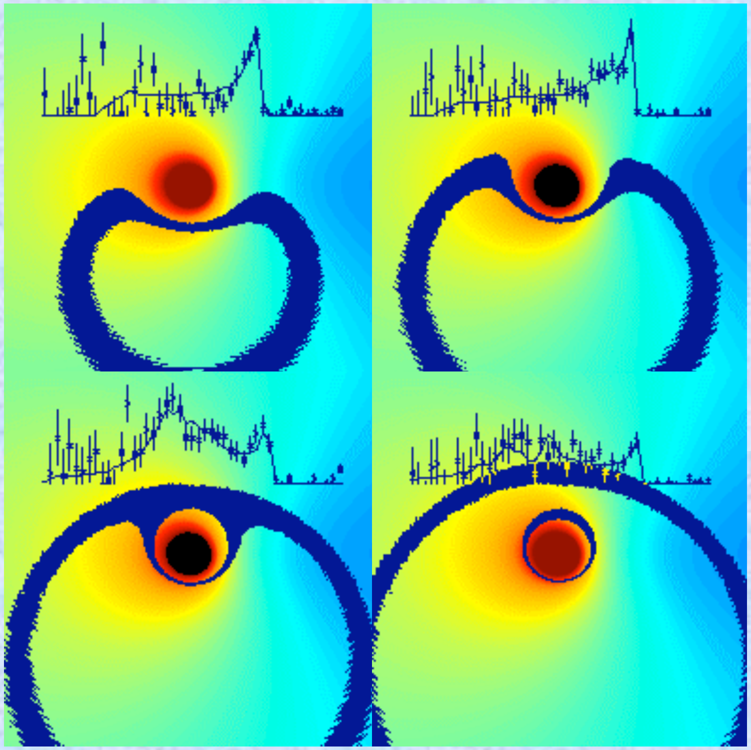
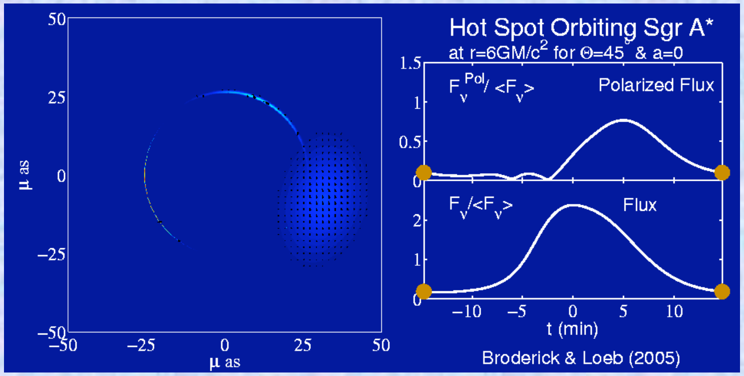
Astrophysical tests of strong gravity

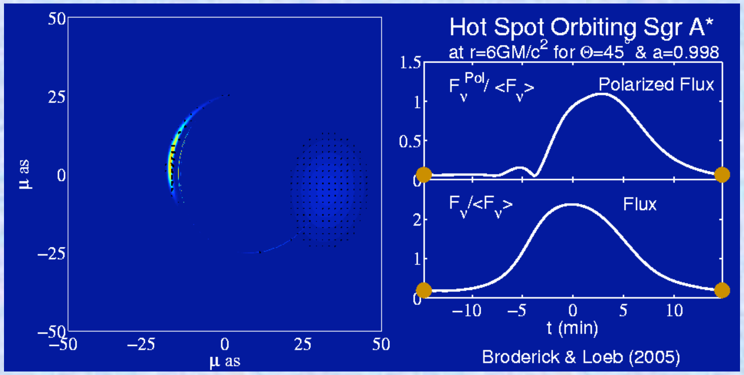


C. Reynolds, U. Md

- Evolution of Fe fluorescence lines during X-ray flare
- sensitive to M and J of BH
- IXO mission

- High resolution imaging of hot spot in accretion onto BH at Galactic Center
- 45° inclination
- a=0 and 0.998





Broderick & Loeb, CFA

See the "Living Review" by Dimitrios Psaltis - http://relativity.livingreviews.org/Articles/lrr-2008-9/

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Cosmic Barbers:
Are black holes really bald?

$1.6 \times 10^8\ M_{sun}$

J. Michell (1784):

If there should really exist in nature any bodies whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us... we could have no information from sight; yet if any other luminous bodies should happen to revolve about them we might still [infer] the existence of the central ones....

P. S. Laplace (1796):

... the attractive force of a heavenly body could be so large that light could not flow out of it.

Cosmic Barbers:
Are black holes really bald?

The 3 Stooges: Moe, Curly & Larry (1934 -46)

Rotating black holes in general relativity

The Schwarzschild solution (1916)

- unique static, spherical asymptotically flat vacuum solution
- matches smoothly to matter interior - star
- non-singular event horizon
- non-rotating black hole

The Kerr solution (1963)

- unique stationary axisymmetric, asymptotically flat vacuum solution with non-singular event horizon
- no reasonable fluid interior solution ever found
- rotating black hole if $J \leq GM^2/c$

External potentials of charge and mass distributions

Electromagnetism (axisymmetric body)

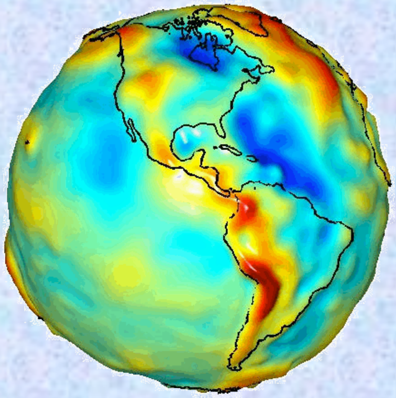
$$\Phi : \frac{e}{r} + \frac{DP_1(\cos\theta)}{r^2} + \frac{Q_2P_2(\cos\theta)}{r^3} + \ldots$$

$$A^i : \frac{\mu^i}{r^2} + \frac{M_2\tilde{P}_2{}^i(\cos\theta)}{r^3} + \ldots$$

Newtonian gravity (axisymmetric body)

$$U : \frac{M}{r} + \frac{Q_2P_2(\cos\theta)}{r^3} + \frac{Q_3P_3(\cos\theta)}{r^4} + \ldots$$

$$Q_\ell = MR^\ell j_\ell$$



Earth: $j_2 = 10^{-3}$, $j_3 = -2 \times 10^{-6}$, $j_4 = -1.5 \times 10^{-6}$, …

Grace, CHAMP: ……. j_{160}

Black holes have no hair

Exterior geometry of Kerr

$$g_{00} : \frac{M}{r} + \frac{Q_2 P_2(\cos\theta)}{r^3} + \frac{Q_4 P_4(\cos\theta)}{r^5} + \ldots$$

$$g_{0\varphi} : \frac{J}{r^2} + \frac{J_3 \tilde{P}_3(\cos\theta)}{r^4} + \frac{J_5 \tilde{P}_5(\cos\theta)}{r^6} + \ldots$$

No hair theorem

$$Q_\ell + iJ_\ell = M(ia)^\ell$$

$$Q_0 = M$$

$$J_1 = J$$

$$a = J/M$$

Hansen 1974

$$\boxed{Q_2 = -Ma^2 = -J^2/M}$$

$$\boxed{Q_{2\ell} = M\left(\frac{Q_2}{M}\right)^\ell}$$

Symmetries and conserved quantities

Symmetry:

$$x^\alpha \rightarrow x^\alpha + \xi^\alpha \text{ and } g_{\mu'\nu'}(x^{\alpha}{}') = g_{\mu\nu}(x^\alpha) \; \left(\text{or } L(x^{\alpha}{}') = L(x^\alpha)\right)$$

$$\xi_{\alpha;\beta} + \xi_{\beta;\alpha} = 0 \quad \text{Killing vector}$$

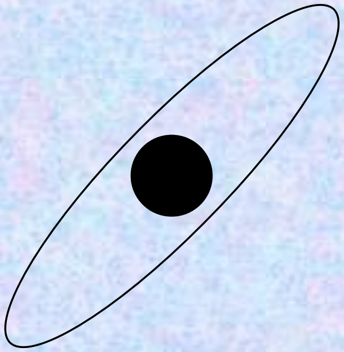
If **p** is tangent to a geodesic:

$$\vec{\xi} \bullet \vec{p} = \text{ constant } \left(\text{or } p_\alpha = \text{const}\right)$$

Schwarzschild:

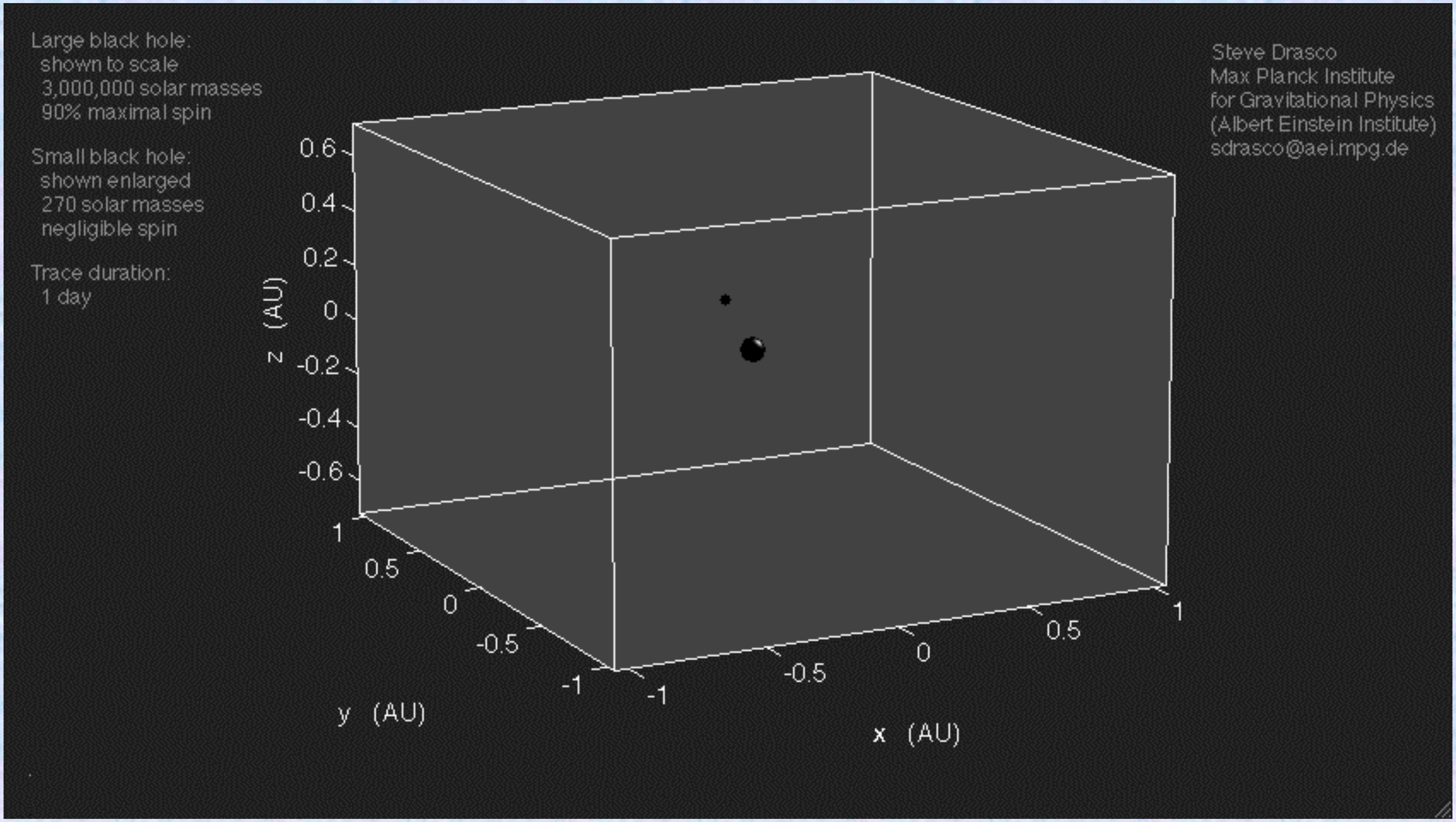
$$\xi_{(t)} \Rightarrow E$$

$$\left.\begin{aligned} \xi_{(\varphi)} &\Rightarrow L_z \\ \xi_{(1)} &\Rightarrow L_x \\ \xi_{(2)} &\Rightarrow L_y \end{aligned}\right\} \Rightarrow \text{orbital plane fixed}$$



Symmetries and conserved quantities

Kerr:
$$\xi_{(t)} \Rightarrow E$$
$$\xi_{(\varphi)} \Rightarrow L_z$$



Animation by Steve Drasco, JPL

The Carter constant of the motion

Hamilton-Jacobi methods (B. Carter 1968)

$$C = f(L^2, L_z^2, E^2, a, \cos\theta)$$

Killing tensor $\xi_{\alpha\beta}$:

$$\xi_{\alpha\beta;\gamma} + \xi_{\gamma\alpha;\beta} + \xi_{\beta\gamma;\alpha} = 0$$

$$\xi_{\alpha\beta}\, p^\alpha p^\beta = \text{constant}$$

Remark: geodesic motion in Kerr is completely integrable (reducible to quadratures)

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A Global Network of Interferometers

LIGO Hanford 4&2 km

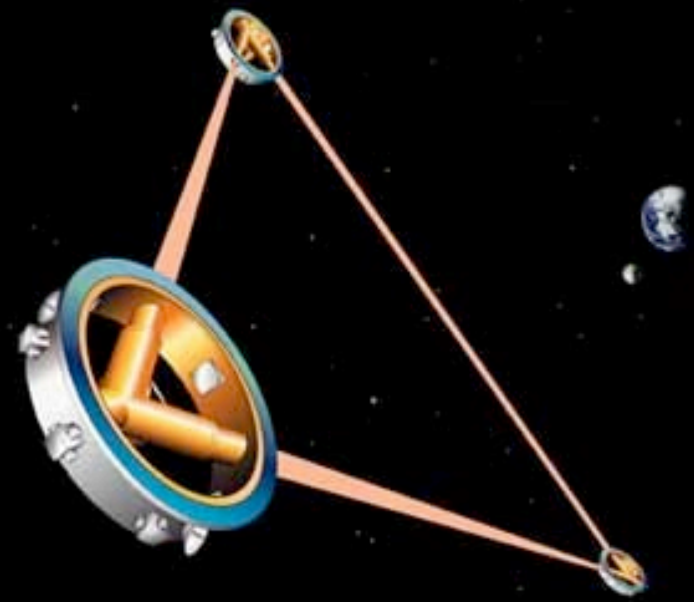
GEO Hannover 600 m

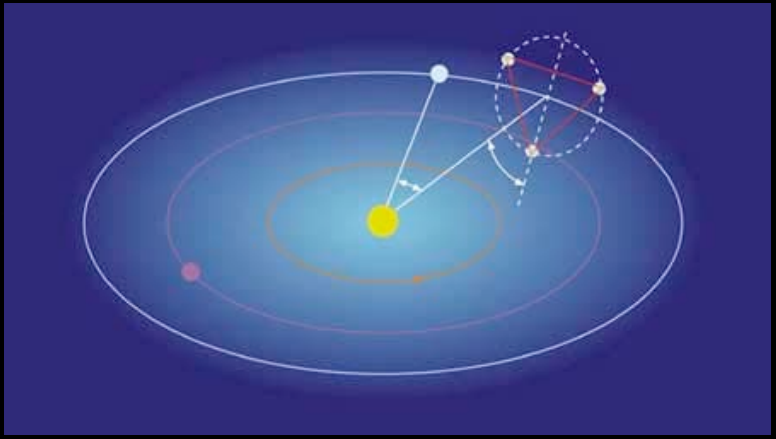
LIGO Livingston 4 km

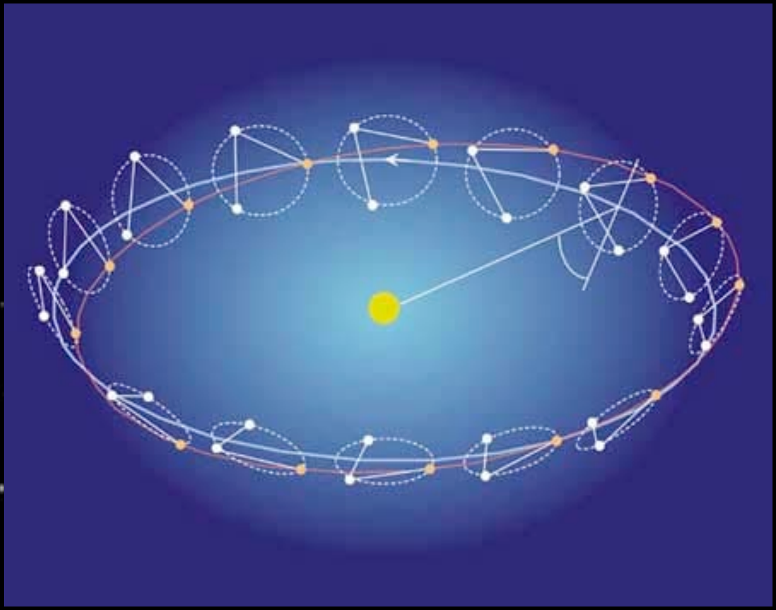
Virgo Cascina 3 km

TAMA Tokyo
300 m

LISA: a space interferometer for 2020







Inspiralling Compact Binaries - Strong Gravity GR Tests?

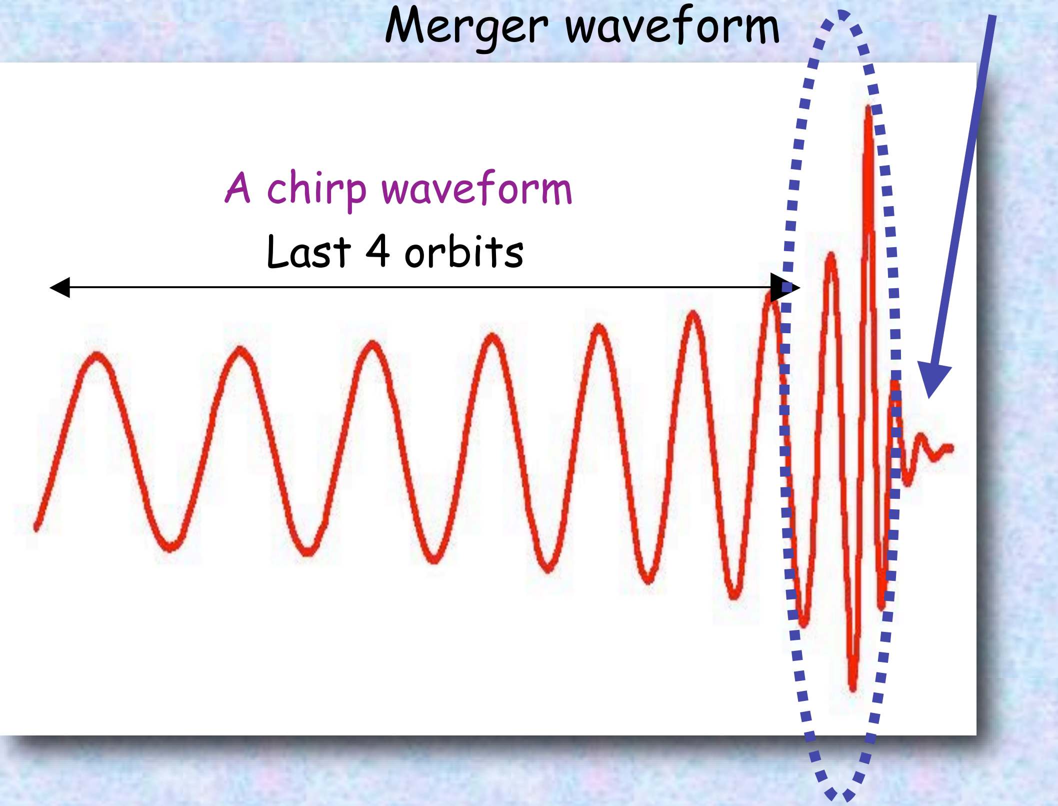
- Fate of the binary pulsar in 100 My
- GW energy loss drives pair toward merger

LIGO-VIRGO

- Last few minutes (10K cycles) for NS-NS
- 40 - 700 per year by 2014
- BH inspirals could be more numerous

LISA

- MBH pairs(10^5 - 10^7 M_s) in galaxies to large Z ~ 15
- EMRIs



Hair counting using GW from EMRIs

- EMRI: extreme mass-ratio inspiral
- GW source for LISA
- particle probes strong-field BH geometry

F. Ryan (1997)
Babak & Glampedakis (2006)
Hughes (2006)
Vigeland & Hughes (2009)

- accurate template waveforms needed

$$\xi_{(t)} \Rightarrow E \qquad \xi_{(\varphi)} \Rightarrow L_z \qquad \xi_{\alpha\beta} \Rightarrow C$$

- change of E, L_z calculable from flux to infinity
- no analogous flux known for C
- ad hoc or "kludge" approaches to find dC/dt
- post-Newtonian theory (Flanagan & Hinderer)
- "Capra program" to calculate local self force

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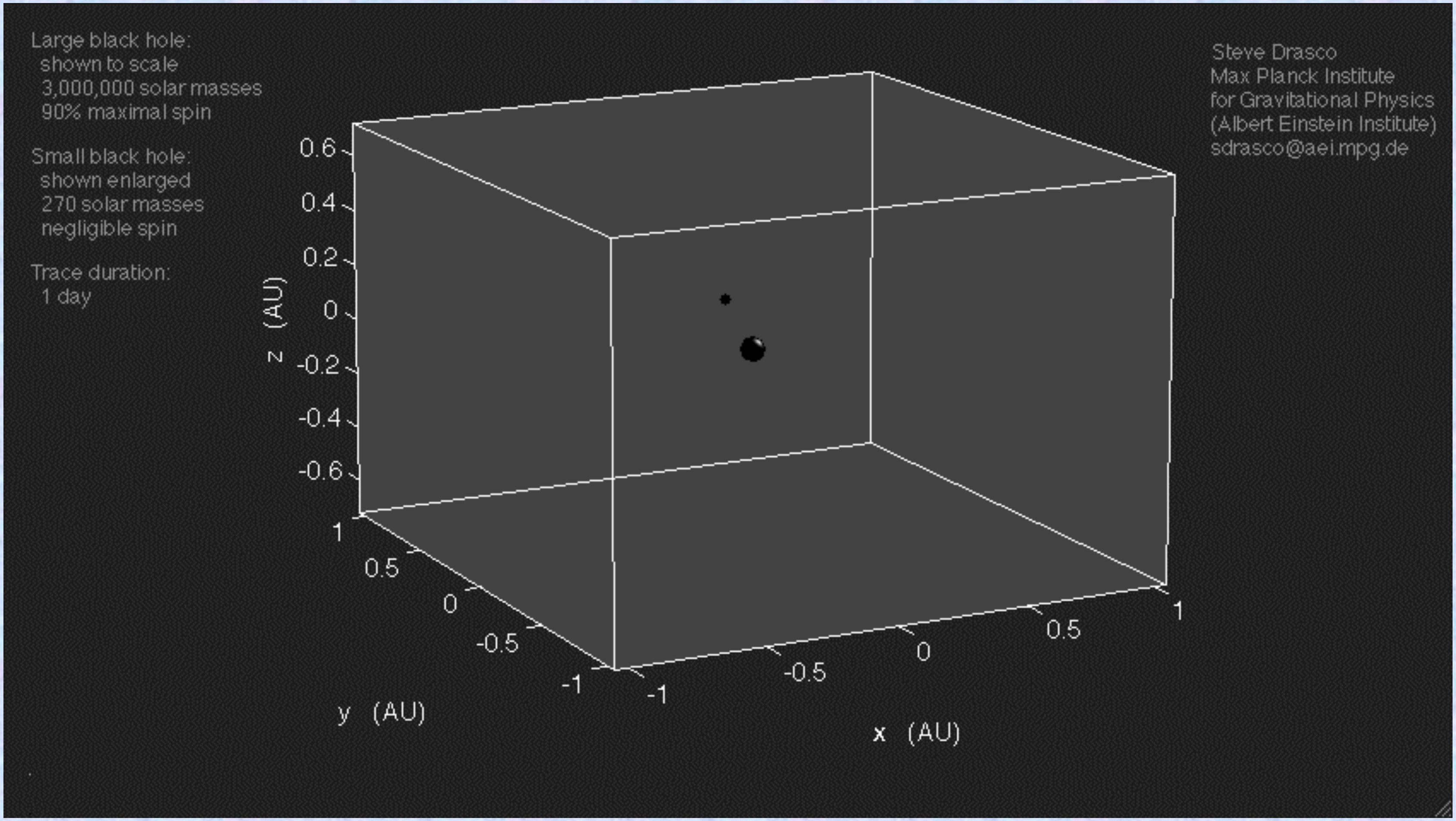
Hair counting using GW from EMRIs



Large black hole:
shown to scale
3,000,000 solar masses
90% maximal spin

Small black hole:
shown enlarged
270 solar masses
negligible spin

Trace duration:
1 day

Steve Drasco
Max Planck Institute
for Gravitational Physics
(Albert Einstein Institute)
sdrasco@aei.mpg.de

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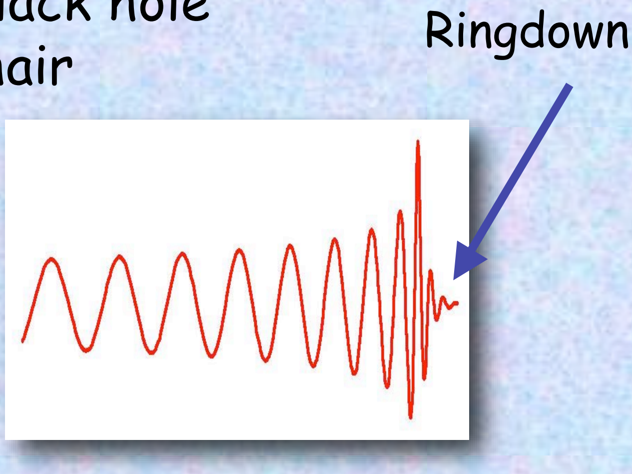
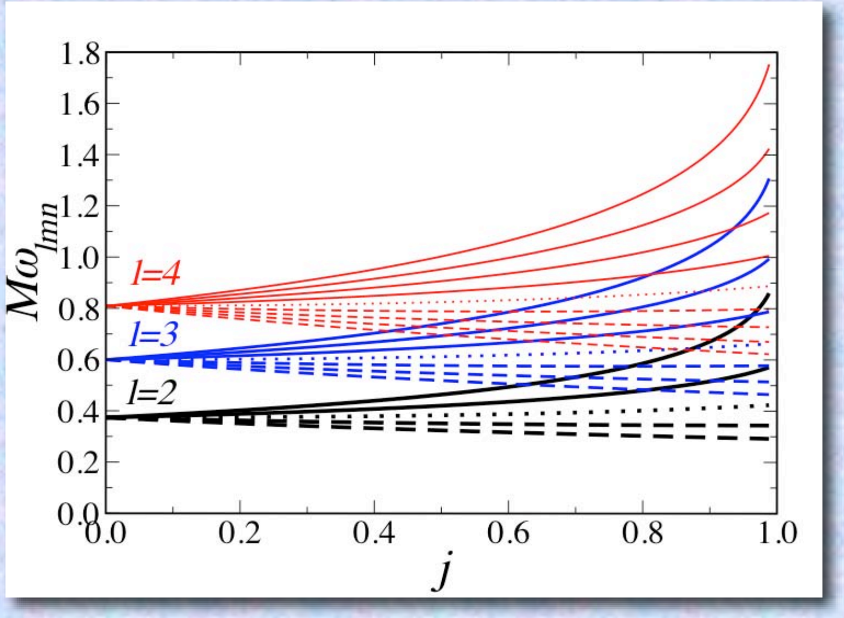
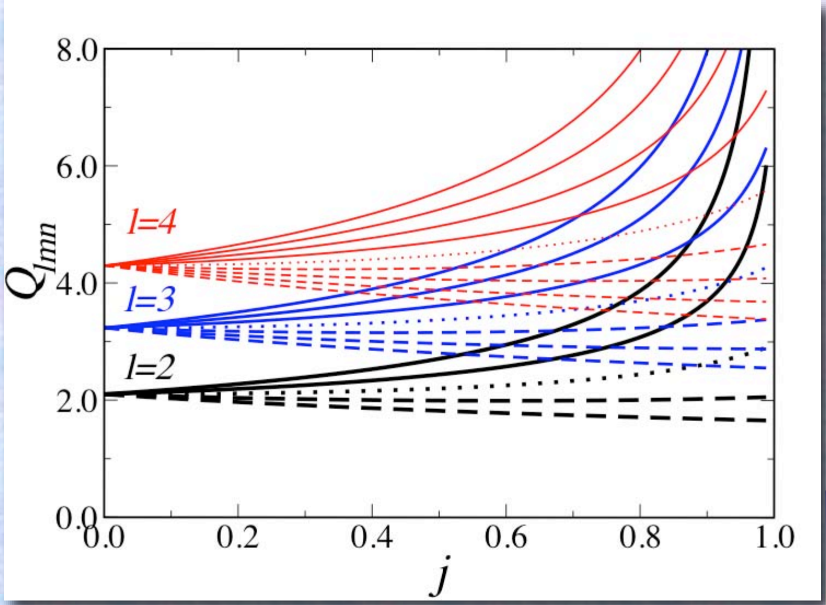
Temporary hair: Perturbed black holes

- collapse or merger produces distorted black hole
- hole radiates "ringdown" waves to shed hair
- final state a stationary Kerr black hole
- quasi-normal modes

$$\omega = \omega_{\ell mn} + i\left(\frac{\pi\omega_{\ell mn}}{2Q_{\ell mn}}\right)$$



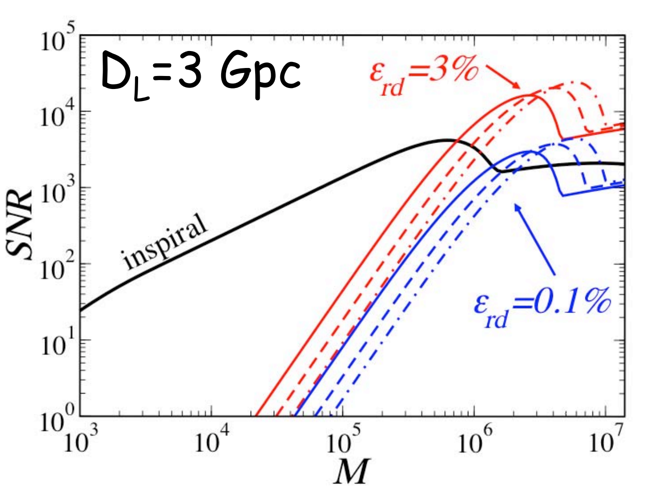


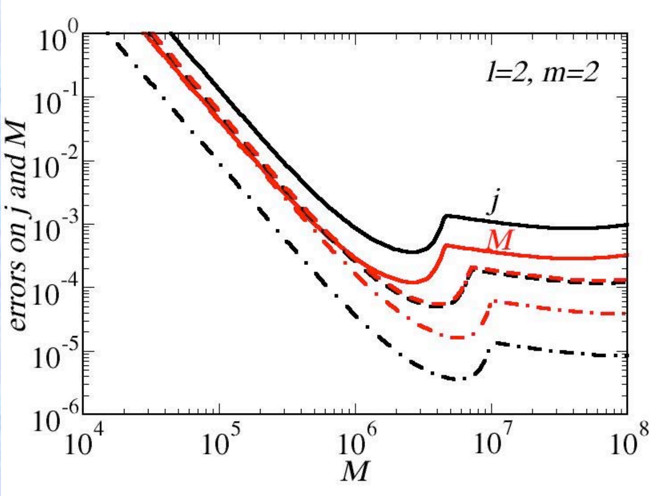


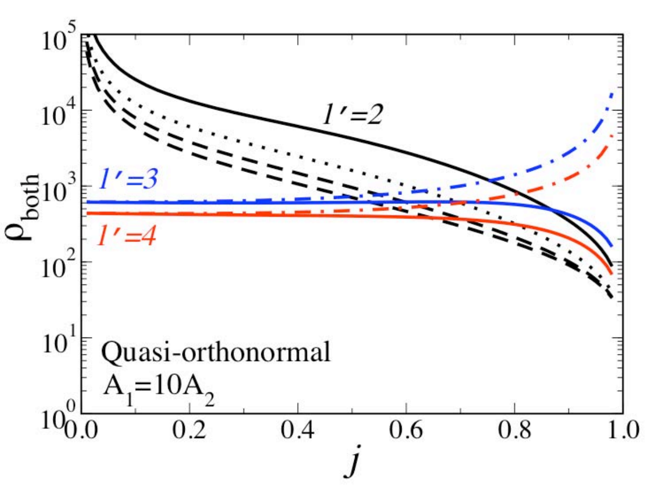
j = a/m

Hair counting using ringdown waves

- LISA will detect massive binary black hole inspirals to large Z
- SNR from ringdown waves is large for M > 10^5 M_{sun}
- M, j can be measured with high accuracy
- multimode detection needed to test no-hair theorems







Dreyer et al. (2004)
Berti, Cardoso & CMW (2006)

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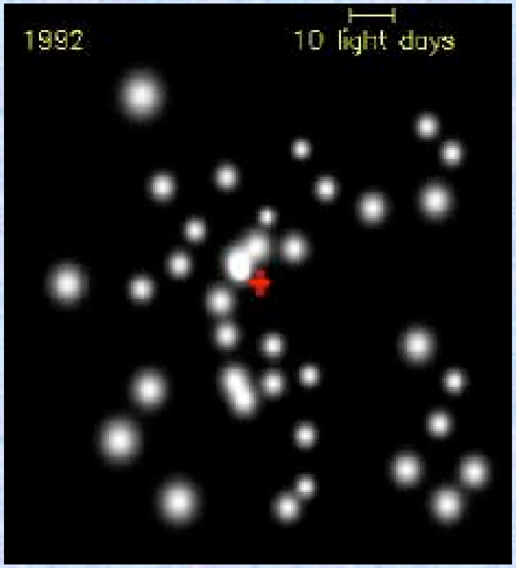
Counting hairs on the galactic center black hole SgrA*

- No hair theorems:

$$M_L + iJ_L = M(ia)^L$$

- $J = Ma;\ Q = -Ma^2$
- relativistic effects: periholion advance, redshift Doppler shifts, Shapiro delays
- Frame dragging (J) and quadrupole moment (Q) produce precessions of planes



SgrA* - a $3.6 \times 10^6\ M_{sun}$ rotating black hole

Jaroszynski (1998)
Fragile & Mathews (2000)
Rubilar & Eckart (2001)
Weinberg et al. (2005)
Zucker et al. (2006)
Kraniotis (2007)

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Counting hairs on the galactic center black hole SgrA*

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  $M_L + iJ_L = M(ia)^L$

- $J = Ma$; $Q = -Ma^2$
- relativistic effects:

  periholion advance, redshift

  Doppler shifts, Shapiro delays
- Frame dragging (J) and

  quadrupole moment (Q)

  produce precessions of planes

Orbital plane precessions as no-hair tests for SgrA*

	ω	Ω	i
M	✓		
J	✓	✓	✓*
Q	✓	✓	✓
dirt	✓	✓	✓

$$A_M = 6\pi \frac{M}{\bar{a}(1-e^2)}$$

$$A_J = 4\pi \frac{J}{M^2}\left(\frac{M}{\bar{a}(1-e^2)}\right)^{3/2}$$

$$A_Q = 3\pi \frac{Q}{M^3}\left(\frac{M}{\bar{a}(1-e^2)}\right)^2$$

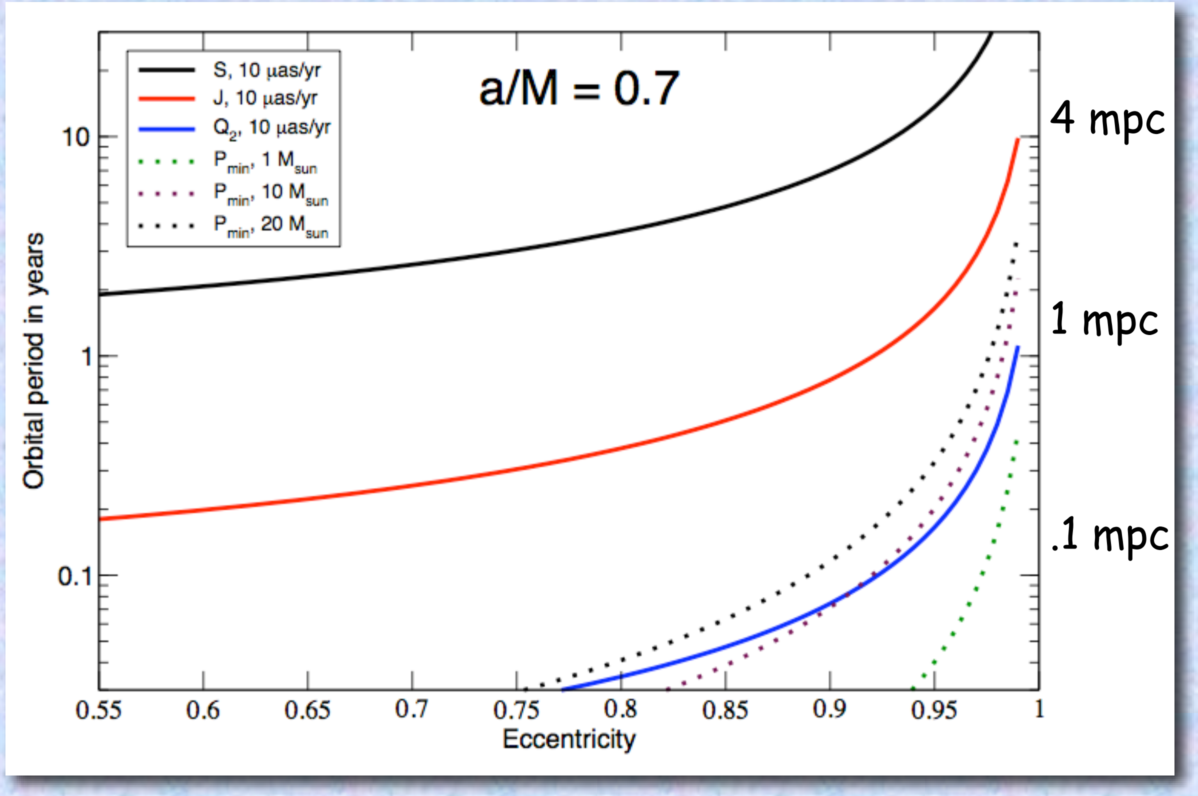
$a/M > 0.5$

$P \sim 0.1$ yr, $d < 10^{-3}$ pc, $e \sim 0.9$

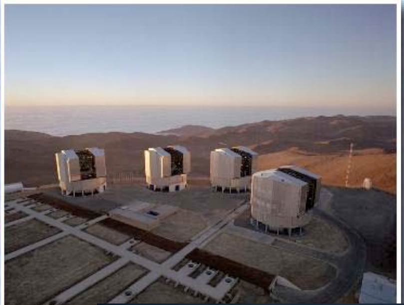
=> Precessions ~ 10 μas/yr

CMW, Ap J Lett. 647, L25 (2008)

The observational challenge

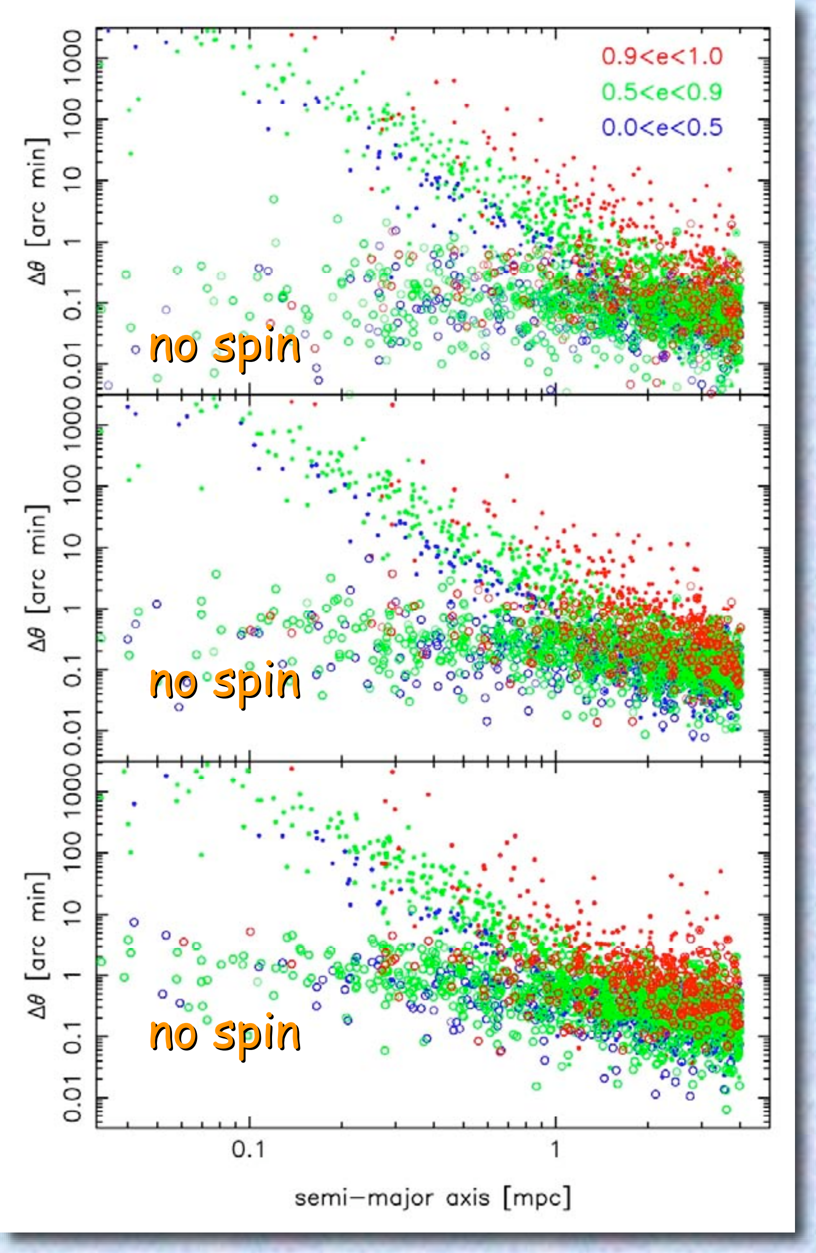


GRAVITY: near IR adaptive optics instrument for the Very Large Telescope Interferometer



ASTRA: extending the Keck interferometer

Effect of other stars/BH in the central mpc



10-year precession of orbital planes, for a/M = 1

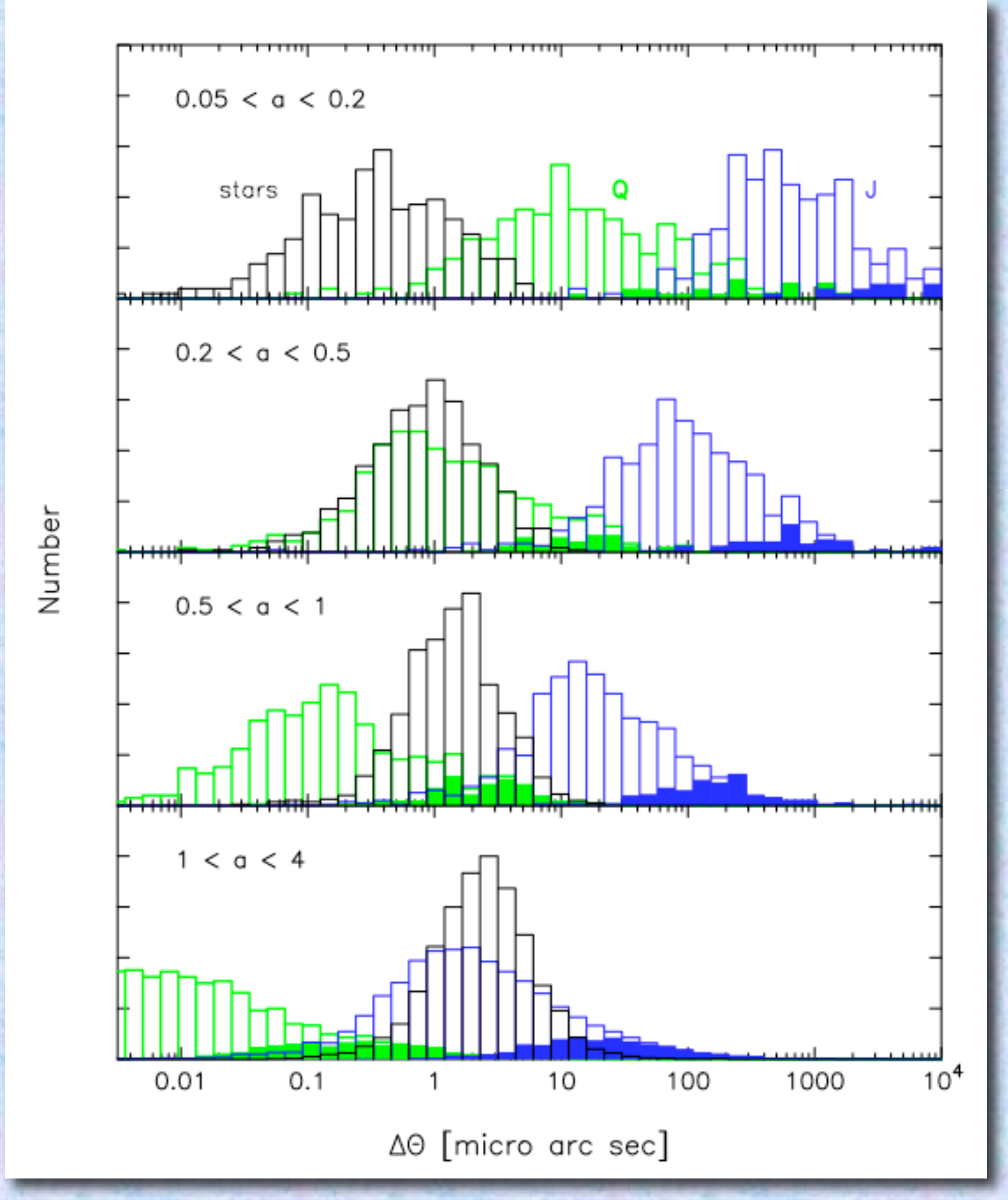
$M_{S/BH}$ (<1 mpc) = 10 M_{sun}

$M_{S/BH}$ (<1 mpc) = 30 M_{sun}

$M_{S/BH}$ (<1 mpc) = 100 M_{sun}

D. Merritt, T. Alexander,
S. Mikkola, CMW, arXiv:0911.4718
L. Sadeghian & CMW, in preparation

Effect of other stars/BH in the central mpc



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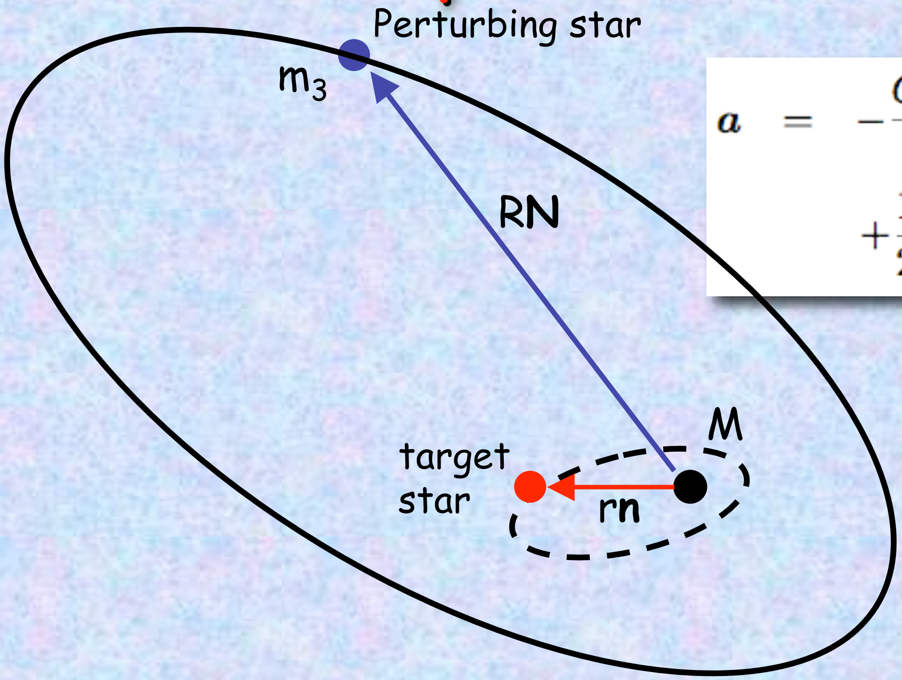
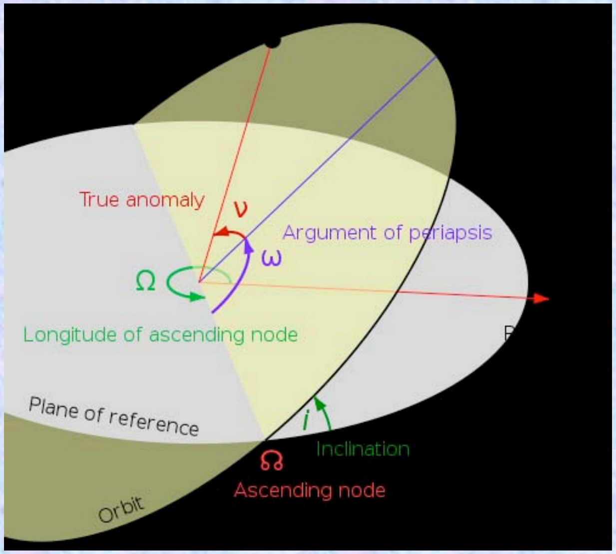
Orbital perturbations due to a third body



$$\boldsymbol{a} = -\frac{GM}{r^2}\left\{\boldsymbol{n} - \frac{m_3}{M}\frac{r^3}{R^3}\left[3(\boldsymbol{n}\cdot\boldsymbol{N})\boldsymbol{N} - \boldsymbol{n}\right] + \frac{1}{2}\frac{m_3}{M}\frac{r^4}{R^4}\left[15(\boldsymbol{n}\cdot\boldsymbol{N})^2\boldsymbol{N} - 6(\boldsymbol{n}\cdot\boldsymbol{N})\boldsymbol{n} - 3\boldsymbol{N}\right] + \dots\right\}$$

Perturbing acceleration:

$$\delta\boldsymbol{a} = \mathcal{R}\boldsymbol{n} + \mathcal{S}\boldsymbol{\lambda} + \mathcal{W}\boldsymbol{h}$$



$$\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{h}\left(\mathcal{S}\frac{p}{r} + \mathcal{R}e\sin f\right), \\
\frac{de}{dt} &= \frac{1-e^2}{h}\left(\mathcal{R}a\sin f + \frac{\mathcal{S}}{er}(ap - r^2)\right), \\
\frac{d\varpi}{dt} &= -\mathcal{R}\frac{p}{eh}\cos f + \mathcal{S}\frac{p+r}{eh}\sin f\,. \\
\sin i\frac{d\Omega}{dt} &= \mathcal{W}\frac{r}{h}\sin(\omega + f), \\
\frac{di}{dt} &= \mathcal{W}\frac{r}{h}\cos(\omega + f)\,.
\end{aligned}$$

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Orbital perturbations due to a third body

Time-averaged perturbation

$$\overline{\frac{dx}{dt}} \equiv \frac{1}{T}\int_0^T \frac{dx}{dt}\,dt = f(a,\, e,\, \omega,\, \Omega,\, i;\, a',\, e',\, \omega',\, \Omega',\, i')$$

$$\boxed{x = \{i,\, \Omega,\, \omega\}}$$

Average over a distribution of perturbing stars

$$\left\langle \overline{\frac{dx}{dt}} \right\rangle \equiv \int \overline{\frac{dx}{dt}} \mathcal{N}(a',\, e',\, \cancel{\omega',\, \Omega',\, i'})\, a'^2 da' de'^2 d\omega' \sin i' di' d\Omega'$$

because of spherical symmetry

Estimate discreteness effects via RMS change

$$\left\langle \overline{\frac{dx}{dt}} \right\rangle^2_{\rm RMS} \equiv \frac{1}{8\pi^2}\int \left(\overline{\frac{dx}{dt}}\right)^2 \tilde{\mathcal{N}}(a',\, e')\, a'^2 da' de'^2 d\omega' \sin i' di' d\Omega'$$

Orbital perturbations due to a third body

RMS change in inclination over one orbit (outer perturbing star)

$$P^2 \left\langle \frac{\overline{di}}{dt} \right\rangle^2_{\mathrm{RMS}} = \frac{3\pi^2}{40}\left(\frac{m_3}{M}\right)^2 \int \tilde{N}(a',e') \left(\frac{a}{a'}\right)^6 \frac{(2+6e^2+17e^4)}{(1-e^2)(1-e'^2)^3}\, a'^2\, da'\, de'^2$$

Normalized distribution in a', e':

$$\int \tilde{N}(a',e')\, a'^2\, da'\, de'^2 = N$$

Note

$$\langle \Delta i \rangle \propto \left(\frac{N m_3}{M}\right) N^{-1/2} \rightarrow 0 \text{ as } N \rightarrow \infty \text{ for fixed } N m_3$$

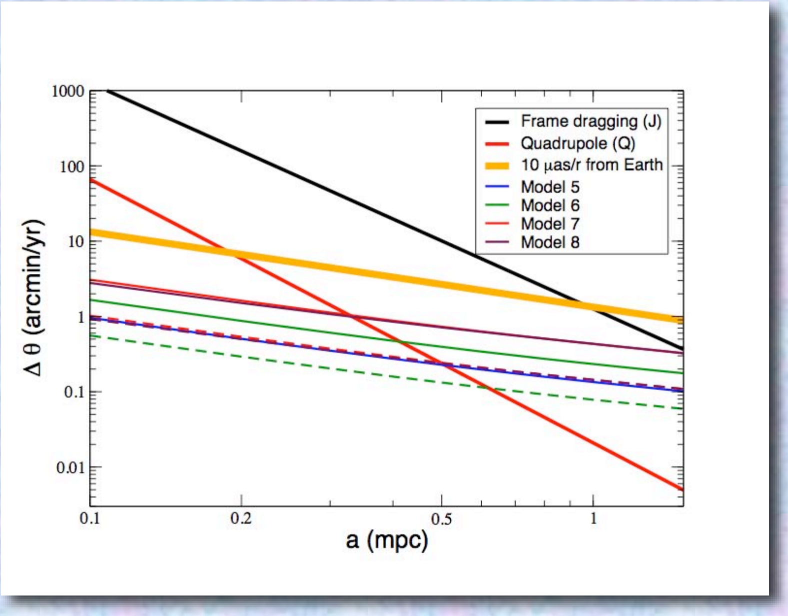
Stellar distribution assumptions: 1 M_{sun} stars & 10 M_{sun} BH

$$\tilde{N}(a',e') \propto a'^{-\gamma}(1-e'^2)^{-\beta}$$

$$R = N_{BH}/N_*$$

$$\gamma = \begin{cases} 0 & : \text{constant density} \\ 2 & : \text{mass segregated} \end{cases}$$

$$\beta = 1 - \sigma_t^2/\sigma_r^2 = 0 : \text{isotropy}$$



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Counting black hole hair at the galactic center

Future work:

- effects of tidal distortions at close approach to the BH
- covariance analysis of actual astrometric observations of N candidate stars
- effects of a dark matter distribution (Sadeghian & Ferrer)

Testing General Relativity in the Strong-field Dynamical Regime

- Introduction - what is “strong”?
- Astrophysical tests
- Cosmic barbers: Are black holes really bald?
- Counting hair using gravitational waves
- Counting hair using SgrA*

IHES 26 May, 2011

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