Nonlocal Cosmology

arXiv:1401.0254

arXiv:0705.0153 & 1307.6693 (Deser)

arXiv:0904.0961 (Deffayet)

Problem: What is making the universe accelerate?

- FLRW: $ds^2 = -dt^2 + a^2(t)\, d\boldsymbol{x} \cdot d\boldsymbol{x}$
  - ❖ $H(t) = \frac{\dot{a}}{a}$ ➔ $H_0 \sim 67 \frac{\text{km}}{\text{s-Mpc}}$
  - ❖ $q(t) \equiv -1 - \frac{\dot{H}}{H^2}$ ➔ $q_0 \sim -.54$
- General Relativity with $\frac{a_0}{a(t)} \equiv 1 + z$
  - ❖ $3H^2 = 3H_0^2 \left[\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_\Lambda\right]$
  - ❖ $-2\dot{H} - 3H^2 = 3H_0^2 \left[\tfrac{1}{3}\Omega_r (1+z)^4 + 0 - \Omega_\Lambda\right]$
- ΛCDM works
  - ❖ $\Omega_r \sim 8.5 \times 10^{-5}$, $\Omega_m \sim .306$, $\Omega_\Lambda \sim .692$
  - ❖ But why is $G\Lambda$ so small and why dominant NOW?

Scalar Quintessence Works

- $\mathcal{L} = -\frac{1}{2}\partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$
  - ❖ $3H^2 = 8\pi G[\frac{1}{2}\dot{f}^2 + V(f)]$
  - ❖ $-2\dot{H} - 3H^2 = 8\pi G[\frac{1}{2}\dot{f}^2 - V(f)]$
- Given $a(t)$ ➔ Reconstruct $V(\varphi)$
  - ❖ $-2\dot{H} = 8\pi G\dot{f}^2(t)$ ➔ $f(t) = f_0 \pm \int_0^t dt' \sqrt{\frac{-\dot{H}(t')}{4\pi G}}$
  - ❖ Monotonic ➔ $t[f]$
  - ❖ $\dot{H}(t) + 3H^2(t) = 8\pi G V$ ➔ $V(f) = \frac{\dot{H}(t[f]) + 3H^2(t[f])}{8\pi G}$
- But who ordered that?
  - ❖ Why is $\varphi(t, \boldsymbol{x}) \sim f(t)$ so homogeneous?
  - ❖ Why is $G^2 V(f) \sim 10^{-122}$ so small?
  - ❖ Why is there no observed scalar force?

$f(R)$ models don't really work

- $\mathcal{L} = \frac{f(R)\sqrt{-g}}{16\pi G}$
- Unique solution which gives ΛCDM is . . .
  - $f(R) = R - 2\Lambda$
  - Dunsby et al., arXiv:1005.2205
- Hence deviations occur even at 0th order!
- And there are other problems
  - Why now? ➔ new scales
  - New scalar DoF ➔ needs screening

Modifications of Gravity

- $f(R)$ only local, invariant, stable & $g_{\mu\nu}$-based
- Retain locality and sacrifice invariance
  - ❖Horava gravity
  - ❖Massive gravitons
- Retain invariance and sacrifice locality for:
  - ❖Summing QIR effects from primordial inflation
  - ❖Explaining late time acceleration w/o Dark Energy
  - ❖Explaining galactic structure w/o Dark Matter

Isaac Newton’s Take on Nonlocality

“that one body may act upon another at a distance thro’ a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophoical Matters a competent Faculty of thinking can ever fall into it.”

Was Newton too Harsh?

- I don't think so
  - ❖Fundamental theory is local
  - ❖But quantum effective field equations are not
  - ❖ $M = 0$ loops could give big IR corrections
- Primordial Inflation ➔ IR gravitons
  - ❖$N(t,k) = \left[\frac{Ha(t)}{2ck}\right]^2$ for EVERY wave vector
  - ❖Perhaps their attraction stops inflation
  - ❖Late time modifications from vacuum polarization
  - ❖Would affect large scales most
- But for now, just model-building

Late-Time Acceleration (arXiv:0705.0153 with Deser)

- Nonlocality via $\frac{1}{\Box}$ for $\Box \equiv \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)$
  - ❖ Retarded BC ➔ both $\frac{1}{\Box}$ and $\partial_t \frac{1}{\Box}$ vanish at $t = 0$
- Act it on R ➔ $X \equiv \frac{1}{\Box}R$ is dimensionless
- $\mathcal{L} = \frac{R[1+f(X)]\sqrt{-g}}{16\pi G}$
  - ❖ $f(X)$ the "nonlocal distortion function"
- Field equations: $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta G_{\mu\nu} = \left[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\right]\left(f(X) + \frac{1}{\Box}[Rf'(X)]\right)$$

$$+\left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}\right]\partial_\rho X\,\partial_\sigma\left(\frac{1}{\Box}[Rf'(X)]\right)$$

Field Equations Causal & Conserved

- Invariance implies conservation
- But variational symmetry precludes causality
  - Eg $S[q] = \int dt'\, q(t') \int dt''\, q(t'') G(t';t'')$
  - $\frac{\delta S[q]}{\delta q(t)} = \int dt' \,[G(t;t') + G(t';t)]\mathrm{q}(\mathrm{t}')$
- "Partial Integration Trick"
  - Make causal by changing $\left(\frac{1}{\square}\right)_{adv}$ to $\left(\frac{1}{\square}\right)_{ret}$
  - Conservation only requires $\square\left(\frac{1}{\square}\right) = 1$
- True derivation from Schwinger-Keldysh

Specialization to FLRW:

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x} \cdot d\boldsymbol{x}$$

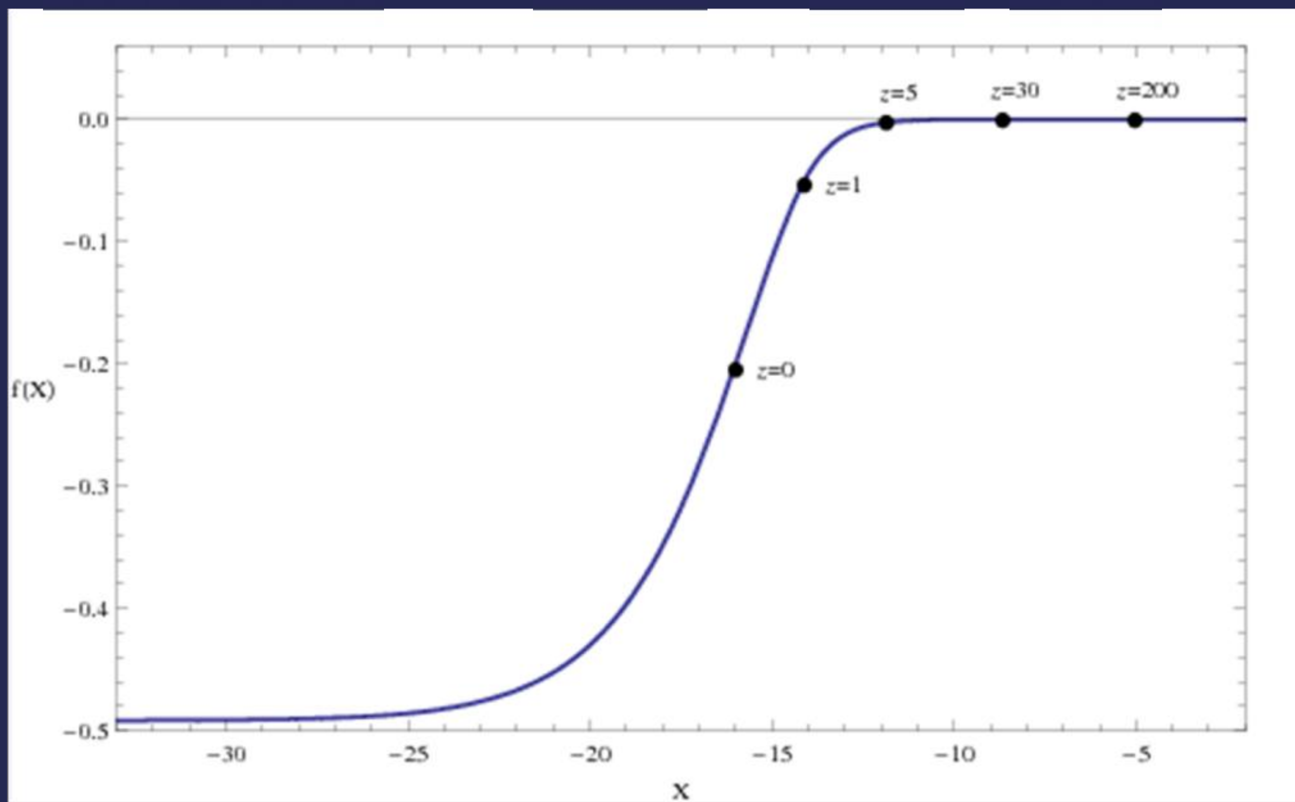
- $R = 6\dot{H} + 12H^2$
- $\left[\frac{1}{\Box} f\right](t) = -\int_0^t \frac{dt'}{a^3(t')} \int_0^{t'} dt''\, a^3(t'') f(t'')$
- Two Built-In Delays:
  - $R = 0$ during Radiation domination $\left(H = \frac{1}{2t}\right)$

    ➔No modification until $t_{\text{eq}} \sim 10^5$ years
  - $X = \frac{1}{\Box} R \sim -\frac{4}{3} \ln\left(\frac{t}{t_{\text{eq}}}\right)$ during Matter domination

    ➔$X \sim -15$ at $t \sim 10^{10}$ years

Reconstructing ΛCDM (arXiv:0904.0961 with Deffayet)

$$f(X) \approx \tfrac{1}{4}\left[\tanh\left(\frac{X}{3}+\frac{11}{2}\right)-1\right]$$



Screening

- Solar system a problem for $f(R)$ models
  - $R > 0$ for cosmology AND solar system
  - Need "screening mechanism" to suppress deviations inside solar system
- $f\left(\frac{1}{\Box}R\right)$ models avoid this problem
  - $\Box \sim -\partial_t^2 + \nabla^2$ ➔ $\frac{1}{\Box}$ provides a $\pm$ sign
    - $\frac{1}{\Box}R < 0$ for cosmology
    - $\frac{1}{\Box}R > 0$ for gravitationally bound systems
  - $f(X) = 0$ for $X > 0$ means NO solar system changes

Local Version Is Haunted (Nojiri & Odintsov, arXiv:0708.0924)

- $R\left[1+f\left(\frac{1}{\Box}R\right)\right] \rightarrow R[1+f(\phi)]+\xi[\Box\phi-R]$
  - Varying with respect to ξ enforces $\phi = R$
  - NB both scalars have 2 pieces of initial value data
- $\xi\Box\phi \rightarrow -\partial_\mu\xi\partial_\nu\phi g^{\mu\nu}$
  $= -\frac{1}{4}\partial_\mu(\xi+\phi)\partial_\nu(\xi+\phi)g^{\mu\nu} + \frac{1}{4}\partial_\mu(\xi-\phi)\partial_\nu(\xi-\phi)g^{\mu\nu}$
- $\xi - \phi$ has negative kinetic energy
- Mixing with gravity doesn't help

No new initial value data for the original nonlocal version

- Synchronous gauge: $ds^2 = -dt^2 + h_{ij}(t,\boldsymbol{x})d\boldsymbol{x}^i d\boldsymbol{x}^j$
- GR initial value data: $h_{ij}(0,\boldsymbol{x})$ & $\dot{h}_{ij}(0,\boldsymbol{x}) = 6 + 6$
  - 4+4 constrained fields
  - 2+2 dynamical gravitons
- NC initial value data ➔ count the ∂_t's
  - $R \sim \partial_t^2$ & $\frac{1}{\Box} \sim \frac{1}{\partial_t^2}$ ➔ $\frac{1}{\Box} R \sim (\partial_t)^0$
  - $\Delta G_{\mu\nu}$ has up to $\partial_t^2 \frac{1}{\Box}$
- Hence $h_{ij}(0,\boldsymbol{x})$ & $\dot{h}_{ij}(0,\boldsymbol{x})$, but what are they?

Initial Value Constraints Identical to General Relativity

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta G_{\mu\nu} = \left[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\right]\left(f(X) + \frac{1}{\Box}[R f'(X)]\right) + \left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - ½ g_{\mu\nu} g^{\rho\sigma}\right]\partial_\rho X\, \partial_\sigma\left(\frac{1}{\Box}[R f'(X)]\right)$$

- Retarded BC ➔ both $\frac{1}{\Box}$ & $\partial_t \frac{1}{\Box}$ vanish at $t = 0$
  - ❖ $f(X)$ also vanishes at $X = 0$
  - ❖ Only $\left[g_{\mu\nu}\Box - D_\mu D_\nu\right]\left\{f\left(\frac{1}{\Box}R\right) + \frac{1}{\Box}\left[R f'\left(\frac{1}{\Box}R\right)\right]\right\} \neq 0$
- Synchronous constraints ➔ ΔG_{00} and ΔG_{0i}
  - ❖ $g_{00}\Box - D_0 D_0 = ½ h^{ij} \dot{h}_{ij} \partial_t - \Delta$ ➔ 0 at $t = 0$
  - ❖ $g_{0i}\Box - D_0 D_i = -\partial_0 \partial_i + ½ h^{ik} \dot{h}_{ki} \partial_j$ ➔ 0 at $t = 0$

No Ghosts ➔ Check the ∂_t^2 Terms

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta G_{\mu\nu} = \left[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\right]\left(f(X) + \tfrac{1}{\Box}[Rf'(X)]\right) + \left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \tfrac{1}{2} g_{\mu\nu} g^{\rho\sigma}\right]\partial_\rho X\, \partial_\sigma\left(\tfrac{1}{\Box}[Rf'(X)]\right)$$

- Dynamical equations ➔ $G_{ij} + \Delta G_{ij} = 8\pi G T_{ij}$
  - ❖ $g_{ij}\Box - D_i D_j = h_{ij}\Box + O(\partial_t)$
  - ❖ $\Delta G_{ij} = 2h_{ij} R f'(X) + O(\partial_t)$
  - ❖ $R_{ij} = \tfrac{1}{2}\ddot{h}_{ij} + O(\partial_t)$ and $R = h^{kl}\ddot{h}_{kl} + O(\partial_t)$
- $G_{ij} + \Delta G_{ij}$ ➔ $\tfrac{1}{2}\left\{1 + f(X) + \tfrac{1}{\Box}[Rf'(X)]\right\} h_{ij}$ +Irrelevant
  - ❖ No graviton ever becomes a ghost
  - ❖ Still might have a potential energy instability

A problem with how the model reproduces ΛCDM without Λ

- For FLRW with slowly varying $H(t)$
  - ❖ $G_{\mu\nu} + \Delta G_{\mu\nu} \approx \left\{1 + f(X) + \frac{1}{\Box}[Rf'(X)]\right\} G_{\mu\nu} = 8\pi \;\; T_{\mu\nu}$
- This is effectively a time-varying Newton constant
  - ❖ $G_{eff}(t) = \frac{G}{1+f(X)+\frac{1}{\Box}[Rf'(X)]}$
  - ❖ Balances the Friedmann Eqn: $3H^2 \approx 8 \;\; G_{eff}(t) \times \frac{\rho_m}{a^3(t)}$
- But $G_{eff}(t)$ also strengthens the force of gravity
  - ❖ Not relevant for solar system
  - ❖ Should increase structure formation
  - ❖ Dodelson & Park have confirmed this, & it's bad

What Dodelson & Park Did (arXiv:1209.0836 & 1310.4329)

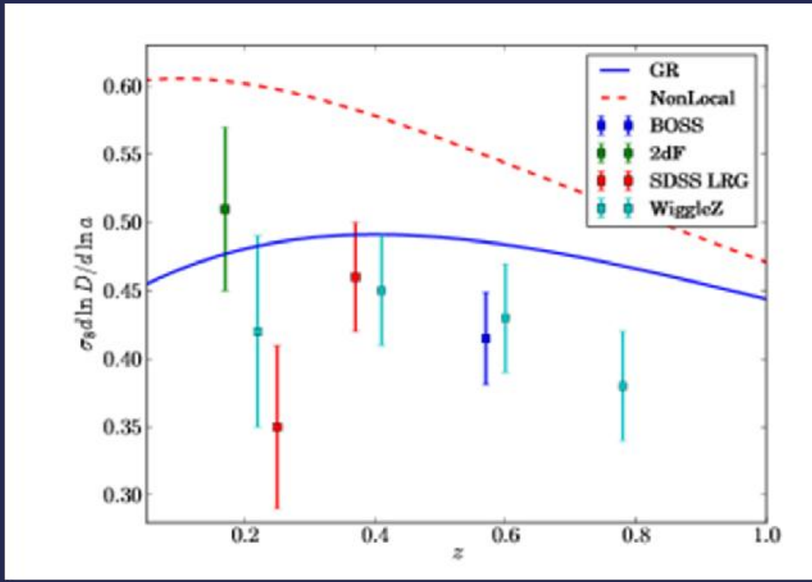
- Plane wave scalars: $\delta\left(G^{\mu}_{\nu} + \Delta G^{\mu}_{\nu}\right) = 8\pi G \delta T^{\mu}_{\nu}$
  - $ds^2 = -\left[1 + 2\Psi(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right]dt^2 + a^2(t)\left[1 + 2\Phi(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right]d\boldsymbol{x}\cdot d\boldsymbol{x}$
  - $\delta T^0_0 = \delta\rho,\ \delta T^i_j = 0$
  - $\delta G^0_0 = -\frac{2k^2}{a^2}\Phi - 6H\dot{\Phi} + 6H^2\Psi$
  - $\delta G^i_j = \frac{(k^i k^j - k^2\delta^{ij})}{a^2}[\Phi + \Psi] - 2\delta^{ij}\left[\ddot{\Phi} + H\left(3\dot{\Phi} - \dot{\Psi}\right) - (2\dot{H} + 3H^2)\Psi\right]$
- $\delta f(X) = f'(\bar{X})\,\delta X$
  - $\delta X = \int_0^t dt' G(t;t')\left[\frac{k^2}{a^2}(4\Phi + 2\Psi) + 6\ddot{\Phi} + 6H\left(4\dot{\Phi} - \dot{\Psi}\right) + \dot{\bar{X}}\left(3\dot{\Phi} - \dot{\Psi}\right)\right]$
  - $G(t;t') = -i\theta(t - t')a^3(t)[u(t)u^*(t') - u^*(t)u(t')]$
    - $\ddot{u} + 3H\dot{u} + \frac{k^2}{a^2}u = 0$ and $u\dot{u}^* - \dot{u}u^* = \frac{i}{a^3}$
    - WKB: $G(t;t') \rightarrow -\frac{\theta(t-t')a^2(t')}{ka(t)} sin\left[k\int_{t'}^{t}\frac{dt''}{a(t'')}\right]$

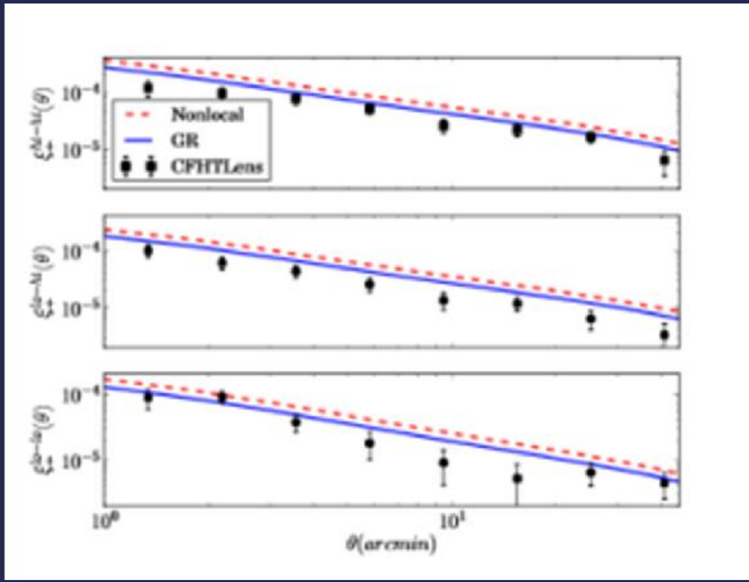
What Dodelson and Park Found

$$ds^2 = -\left[1 + 2\Psi(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right]dt^2 + a^2(t)\left[1 + 2\Phi(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right]d\mathbf{x}\cdot d\mathbf{x}$$

- Nonlocal Cosmology predicts:
  - ❖ $\Phi(t) \sim \Phi_{GR}$ throughout
  - ❖ $\Psi(t) \neq \Psi_{GR}$ by $z\sim 1.5$ and $\Psi(t_0)\sim 2\times\Psi_{GR}$
- Relevant data sets:
  - ❖ WiggleZ, 2dF, BOSS, SDSS LRG's (redshift space dist.)
  - ❖ CFHTLens (weak lensing)
- Preference of GR over Nonlocal Cosmology:
  - ❖ Redshift space distortions ➔ 7.8σ
  - ❖ Weak lensing ➔ 5.9σ
- Data favors a less highly evolved universe

Most data below BOTH Nonlocal Cosmology & General Relativity





Beyond $\Delta\mathcal{L} = \frac{R f(X)\sqrt{-g}}{16\pi G}$ for $X \equiv \frac{1}{\square} R$

- Model gives $G_{eff}(t)$ but we want $\Lambda_{eff}(t)$
- arXiv:0904.2368 (with Tsamis)
  - $\Delta T_{\mu\nu}[g] = (\rho + p) v_\mu v_\nu + p g_{\mu\nu}$
    - Fix $p[g] = -\frac{\Lambda}{8\pi G} + \Lambda^2 F(-G\Lambda X[g])$
    - Determine $\rho[g]$ and $v_\mu[g]$ by conservation
  - For ANY $F(z)$ which increases without bound
    - Universe inflates, then $\Delta T_{\mu\nu} = 0$ for radiation domination
    - But disaster at matter domination
- arXiv:1001.4929 (with Tsamis)
  - Change $F\left(-G\Lambda \frac{1}{\square}[R]\right)$ to $F\left(G \frac{1}{\square}[R u^\mu u^\nu R_{\mu\nu}]\right)$ $\quad u^\mu = \frac{-g^{\mu\nu}\partial_\nu X}{\sqrt{-g^{\alpha\beta}\partial_\alpha X \partial_\beta X}}$
  - FLRW: $u^\mu u^\nu R_{\mu\nu} = -3(\dot{H} + H^2) \sim -3H^2$ for inflation, $+\frac{3}{2}H^2$ for matter
- arXiv:1106.4984 (with Deffayet & Esposito-Farese)
  - Also need $u^\mu u^\nu R_{\mu\nu}$ for nonlocal MOND

Conclusions

- Nonlocal gravity not fundamental
  - Infrared QG corrections from primordial inflation
  - Purely phenomenological for now
- Simplest model based on $Rf\left(\frac{1}{\Box}R\right)$
  - Built-in delays explain cosmic coincidence
  - Simple $f(X)$ reproduces ΛCDM without Λ
  - But structure formation heavily favors GR
- Probably BETTER than GR with 2nd invariant
- Desirable properties
  - Perfect screening for gravitationally bound systems
  - No new degrees of freedom
  - Initial value constraints identical to GR
  - No kinetic energy instabilities