Quantum Gravity (?) from CDT

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4d QG regularized by CDT

Main goal (at least in 80ties) for QG

- Obtain the background geometry ($\langle g_{\mu\nu} \rangle$) we observe
- Study the fluctuations around the background geometry

What lattice gravity (CDT) offers:

- A non-perturbative QFT definition of QG
- A background independent formulation
- An emergent background geometry ($\langle g_{\mu\nu} \rangle$)
- The possibility to study the quantum fluctuations around this emergent background geometry.

Problems to confront for a lattice theory

(1) How to face the non-renormalizability of quantum gravity (this is a problem for any field theory of quantum gravity, not only lattice theories)

(2) Provide evidence of a continuum limit (where the continuum field theory has the desired properties)

(3) If rotation is performed to Euclidean signature, how does one deal with the unboundedness of the Euclidean Einstein-Hilbert action?

(4) If there exists no continuum field theory of gravity, can a lattice theory be of any use?

Effective QFT of gravity

We believe gravity exists as an effective QFT for $E^2 \ll 1/G$.

True for other non-renormalizable theories

Weak interactions $\mathcal{L} = \bar{\psi}\partial\psi + G_F \bar{\psi}(\cdot)\psi\bar{\psi}(\cdot)\psi$

Nonlinear sigma model $\mathcal{L} = (\partial\pi)^2 + \frac{1}{F_\pi^2}\frac{(\pi\partial\pi)^2}{1-\pi^2/F_\pi^2}$

Good for $E^2 \ll 1/G_F$ and $E^2 \ll F_\pi^2$.

Effective QFT of gravity

Lowest order quantum correction to the gravitational potential of a point particle:

$$\frac{G}{r} \to \frac{G(r)}{r}, \quad G(r) = G\left(1 - \omega\frac{G}{r^2} + \cdots\right), \quad \omega = \frac{167}{30\pi}.$$

The gravitational coupling constant becomes scale dependent and transferring from distance to energy we have

$$G(E) = G(1 - \omega G E^2 + \cdots) \approx \frac{G}{1 + \omega G E^2}.$$

Effective QFT of the electric charge

Same calculation in QED

$$\frac{e^2}{r} \to \frac{e^2(r)}{r}, \quad e^2(r) = e^2 \left(1 - \frac{e^2}{6\pi^2} \ln(m\,r) + \cdots \right), \quad m\,r \ll 1.$$

The electric charge is also scale dependent and has a Landau pole

$$e^2(E) = e^2 \left(1 + \frac{e^2}{6\pi^2} \ln(E/m) + \cdots \right) \approx \frac{e^2}{1 - \frac{e^2}{6\pi^2} \ln(E/m)}.$$

$$GE^2 \ll 1 \quad \rightarrow \quad G(E)E^2 \ll 1$$

BUT

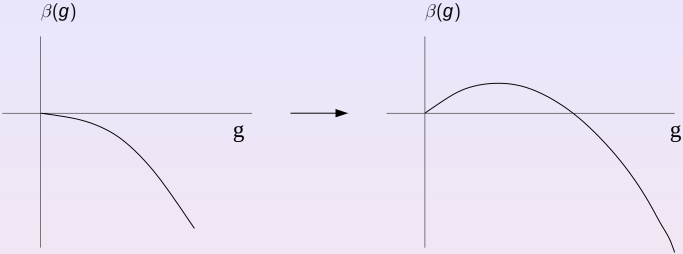
$$G(E)E^2 < 1 \quad (\ll 1\,?).$$

Suddenly seems as if quantum gravity has become an (almost) reliable quantum theory at all energy scales.

The behavior can be described the β function for QG. For the dimensionless coupling constant $\tilde{G}(E) = G(E)E^2$

$$E\frac{d\tilde{G}}{dE} = \beta(\tilde{G}), \quad \beta(\tilde{G}) = 2\tilde{G} - 2\omega\tilde{G}^2.$$

Two fixed points ($\beta(\tilde{G}) = 0$): $\tilde{G} = 0$ and $\tilde{G} = 1/\omega$.

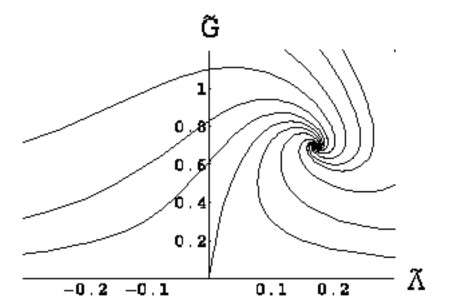


Generic situation for asymptotic free theories in d dimensions, extended to $d + \varepsilon$ dimensions.

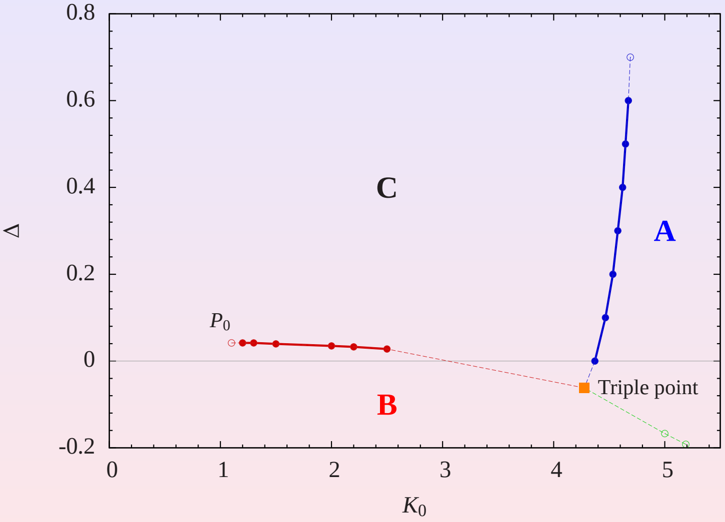
$$\beta(g) \to \varepsilon g + \beta(g)$$

The four-Fermi action, the nonlinear sigma model and QG are all renormalizable theories in 2d, with a negative β-function and have a $2+\varepsilon$ expansion. For QG first explored by Kawai et al.

Alternatively one can use the exact renormalization group approach. (Reuter et al., Litim, ...). Philosophy: asymptotic safety (Weinberg).



(2) Continuum limit ?



Defining the continuum limit in lattice field theory

Let the lattice coordinate be $x_n = a\,n$, a being the lattice spacing and $\mathcal{O}(x_n)$ an observable.

$$-\log\langle\mathcal{O}(x_n)\mathcal{O}(x_m)\rangle \sim |n-m|/\xi(g_0) + o(|n-m|).$$

$$\xi(g_0) \propto \frac{1}{|g_0 - g_0^c|^\nu}, \qquad \boxed{a(g_0) \propto |g_0 - g_0^c|^\nu}.$$

$$m_{ph}a(g_0) = 1/\xi(g_0), \qquad e^{-|n-m|/\xi(g_0)} = e^{-m_{ph}|x_n - x_m|}$$

$\langle\mathcal{O}(x_n)\mathcal{O}(y_m)\rangle$ falls off exponentially like $e^{-m_{ph}|x_n - y_m|}$ for $g_0 \to g_0^c$ when $|x_n - y_m|$, but not $|n-m|$, is kept fixed in the limit $g_0 \to g_0^c$.

How to define the equivalent of $\langle \mathcal{O}(x_n)\mathcal{O}(y_m)\rangle$ in a diffeomorphism invariant theory (F. David (1991))

$$\langle \phi\phi(R)\rangle \equiv \int \mathcal{D}[g_{\mu\nu}]\, e^{-S[g_{\mu\nu}]} \times$$
$$\iint \sqrt{g(x)}\sqrt{g(y)}\; \langle \phi(x)\phi(y)\rangle^{[g_{\mu\nu}]}_{matter}\; \delta(R - d_{g_{\mu\nu}}(x,y)).$$

$\langle \phi(x)\phi(y)\rangle^{[g_{\mu\nu}]}_{matter}$ denotes the correlator of the matter fields calculated for a fixed geometry, defined by the metric $g_{\mu\nu}(x)$.

It works in 2d Euclidean QG (Liouville gravity)

(3) Unboundedness of the Euclidean action

Already the discussion about continuum limit of the lattice theories hinted a rotation to Euclidean signature. The Einstein-Hilbert action is unbounded from below, caused by the conformal factor:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$S[g, \Lambda, G] = -\frac{1}{16\pi G} \int d^4\xi \sqrt{g} \Big(R - 2\Lambda \Big).$$

$$S[\tilde{g}, \Lambda, G] = -\frac{1}{16\pi G} \int d^4\xi \sqrt{g} \Big(\Omega^2 R + 6\partial^\mu \Omega \partial_\mu \Omega - 2\Lambda \Omega^4 \Big).$$

How is this dealt with ?.

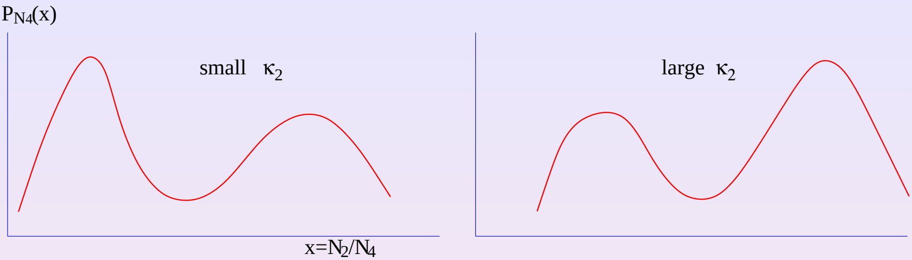
Using the lattice regularization called dynamical triangulations (DT) the Euclidean action is bounded for a fixed lattice spacing a and a fixed four-volume $V_4 = N_4 a^4$. However, for $a \to 0$ the unboundedness re-emerges.

$$S[T] = -\kappa_2 N_2(T) + \kappa_4 N_4(T), \quad c_1 < \frac{N_2}{N_4} \, (= x) < c_2.$$

The unbounded configurations corresponds to $x \approx c_2$. But are they important in the non-perturbative path integral ?

$$Z = \sum_T e^{-S[T]} = \sum_{N_4} e^{-k_4 N_4} \sum_{N_2} \mathcal{N}(N_2, N_4) e^{\kappa_2 N_2}$$

$$\mathcal{N}(N_2, N_4)\, e^{\kappa_2 N_2} = P_{N_4}(x), \quad \sum_x P_{N_4}(x) = f(N_4)\, e^{\kappa_4^c(\kappa_2) N_4}$$



$$P_{N_4}(x) \approx A\, e^{N_4\left(\kappa_4^c - \alpha(x-x_0)^2\right)} + \tilde{A}\, e^{N_4\left(\tilde{\kappa}_4^c - \tilde{\alpha}(x-\tilde{x}_0)^2\right)}.$$

$$k_2 \to \kappa_2 + \Delta\kappa_2, \quad \kappa_4^c \to k_4^c + \Delta\kappa_2 x_0, \quad \tilde{\kappa}_4^c \to \tilde{k}_4^c + \Delta\tilde{\kappa}_2\tilde{x}_0$$

Phase transition when $\kappa_4^c = \tilde{\kappa}_4^c$.

Do we know examples of such entropy driven phase transitions? Yes, the Kosterlitz-Thouless transition in the XY model. This Abelian 2d spin model has vortices with energy

$$E = \kappa \ln(R/a)$$

Saturating the partition function by single vortex configurations:

$$Z \equiv e^{-F/k_B T} = \sum_{\text{spin configurations}} e^{-E[\text{spin}]/k_B T} \approx \left(\frac{R}{a}\right)^2 e^{-[\kappa \ln(R/a)]/k_B T}.$$

$S = k_B \ln(\text{number of configurations})$ has the same functional form as the vortex energy. Thus

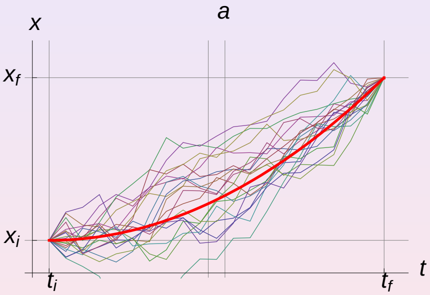
$$F = E - ST = (\kappa - 2k_B T) \ln(R/a)$$

Examples

- Lattice compact $U(1)$ gauge theory in 3 dimensions has confinement for all values of the coupling constant, due to lattice monopoles. It describes perfectly the non-perturbative physics of the Georgi-Glashow model, i.e. the physics below the scale of Higgs and the W-particle. The formula for the string tension is the same expressed in terms of lattice monopoles masses and continuum monopole masses.
- Lattice compact $U(1)$ gauge theory in 4 dimensions at the phase transition point describes the low energy physics of certain broken $\mathcal{N} = 1, 2$ supersymmetric field theories. In fact, one can use the supersymmetric symmetry breaking technology of Seiberg et al. scale matching to “post-dict” (unfortunately) the lattice critical exponents.

Basic tool: The path integral

Text-book example: non-relativistic particle in one dimension.



$$x(t) = \langle x(t)\rangle + y(t)$$
$$\langle |y| \rangle \propto \sqrt{\hbar/m\omega}$$

In QG we want $\langle x(t) \rangle$

$$\langle |y| \rangle \propto \sqrt{\hbar G}$$

Transition amplitude as a weighted sum over all possible trajectories. On the plot: time is discretized in steps a, trajectories are piecewise linear.

In a continuum limit $a \to 0$

$$G(\mathbf{x}_\mathrm{i}, \mathbf{x}_\mathrm{f}, t) := \int_{\text{trajectories}: \mathbf{x}_\mathrm{i} \to \mathbf{x}_\mathrm{f}} e^{iS[\mathbf{x}(t)]}$$

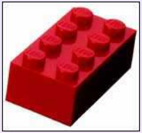
where $S[\mathbf{x}(t)]$ is a classical action.

The QG amplitude between the two geometric states

$$G(\mathbf{g}_\mathrm{i}, \mathbf{g}_\mathrm{f}, t) := \int_{\text{geometries}: \mathbf{g}_\mathrm{i} \to \mathbf{g}_\mathrm{f}} e^{iS[\mathbf{g}_{\mu\nu}(t')]}$$

To define this path integral we need a geometric cut-off a and a definition of the class of geometries entering.

showcasing piecewise linear geometries via building blocks:



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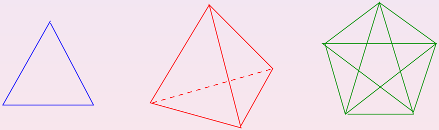
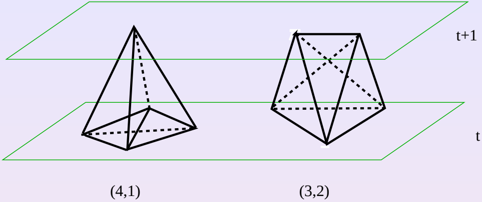
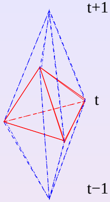


2d 3d 4d





CDT slicing in proper time. Topology of space preserved.

$$a_t^2 = -\alpha a_s^2, \qquad iS_L[\alpha] = -S_E[-\alpha]$$

$$S_E[-\alpha] = -(\kappa_0 + 6\Delta)N_0 + \kappa_4\left(N_4^{(2,3)} + N_4^{(1,4)}\right) + \Delta\left(N_4^{(2,3)} + 2N_4^{(1,4)}\right)$$

$$G(\mathbf{g}_\mathrm{i}, \mathbf{g}_\mathrm{f}, t) := \int_{\text{geometries: } \mathbf{g}_\mathrm{i} \to \mathbf{g}_\mathrm{f}} e^{iS[\mathbf{g}_{\mu\nu}(t')]}$$

$$= \lim_{a \to 0} \sum_{T: T_i^{(3)} \to T_f^{(3)}} \frac{1}{C_T} e^{iS_T}$$

$$G_E(\mathbf{g}_\mathrm{i}, \mathbf{g}_\mathrm{f}, t, \kappa_0, \kappa_4, \Delta) = \lim_{a \to 0} \sum_{T: T_i^{(3)} \to T_f^{(3)}} \frac{1}{C_T} e^{-S_E[T]}$$

$$\langle x_f | e^{i\hat{H}t} | x_i \rangle \to \langle x_f | e^{-\hat{H}\tau} | x_i \rangle$$

Scaling in the IR limit?

$$Z(\kappa_0,\kappa_4) = \sum_{N_4} e^{-\kappa_4 N_4}\, Z_{N_4}(\kappa_0),$$

where $Z_{N_4}(\kappa_0)$ is the partition function for a fixed number N_4 of four-simplices (we ignore Δ for simplicity), namely,

$$Z_{N_4}(\kappa_0) = e^{k_4^c N_4} f(N_4,\kappa_0)$$

We want to consider the limit $N_4 \to \infty$, and fine-tune $\kappa_4 \to \kappa_4^c$ for fixed κ_0. We expect the physical cosmological constant Λ to be defined by the approach to the critical point according to

$$\kappa_4 = \kappa_4^c + \frac{\Lambda}{16\pi G}\, a^4, \quad (\kappa_4 - \kappa_4^c)\, N_4 = \frac{\Lambda}{16\pi G}\, V_4, \quad V_4 = N_4 a^4,$$

How can one imagine obtaining an interesting continuum behavior as a function of κ_0? Assume $f(N_4, \kappa_0)$ has the form (numerical evidence)

$$f(N_4, \kappa_0) = e^{k_1(\kappa_0)\sqrt{N_4}}, \qquad \left\langle e^{-\frac{1}{G}\int_{V_4}\sqrt{g}R}\right\rangle = e^{c\frac{\sqrt{V_4}}{G}}.$$

$$Z(\kappa_4, \kappa_0) = \sum_{N_4} e^{-(\kappa_4-\kappa_4^c)N_4 + k_1(\kappa_0)\sqrt{N_4}}.$$

Search for κ_0^c with $k_1(\kappa_0^c) = 0$, with the approach to this point governed by

$$k_1(\kappa_0) \propto \frac{a^2}{G}, \quad \text{i.e.} \quad k_1(\kappa_0)\sqrt{N_4} \propto \frac{\sqrt{V_4}}{G}.$$

$$Z(\kappa_4, \kappa_0) \approx \exp\left(\frac{k_1^2(\kappa_0)}{4(\kappa_4 - \kappa_4^c)}\right) = \exp\left(\frac{c}{G\Lambda}\right),$$

as one would naïvely expect from Einstein's equations, with the partition function being dominated by a typical instanton contribution, for a suitable constant c.

UV scaling limit?

If we are close to the UV fixed point, we know that G will not be constant when we change scale, but $\hat{G}(a)$ will. Writing $G(a) = a^2 \hat{G}(a) \approx a^2 \hat{G}^*$,

$$\kappa_4 - \kappa_4^c = \frac{\Lambda}{G(a)} a^4 \approx \frac{\Lambda}{\hat{G}^*} a^2,$$

$$k_1(\kappa_0^c) = \frac{a^2}{G(a)} \approx \frac{1}{\hat{G}^*}.$$

The first of these relations now looks two-dimensional because of the anomalous scaling of $G(a)$! Nevertheless, the expectation value of the four-volume is still finite:

$$\langle V_4 \rangle = \langle N_4 \rangle \, a^4 \propto \frac{\kappa_1^2(\kappa_0^c)}{(\kappa_4 - \kappa_4^c)^2} \, a^4$$

Relation to asymptotic freedom

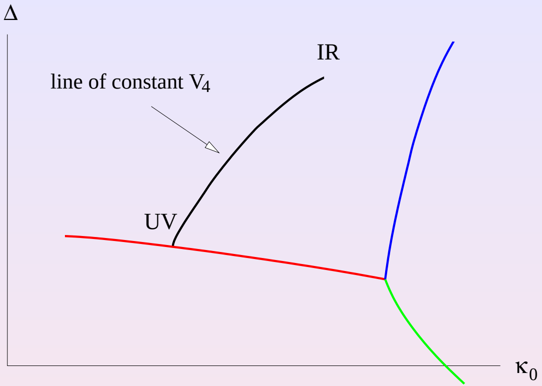
Assume now that we have a fixed point for gravity. The gravitational coupling constant is dimensionful, and we can write for the bare coupling constant

$$G(a) = a^2 \hat{G}(a), \quad a \frac{d\hat{G}}{da} = -\beta(\hat{G}), \quad \beta(\hat{G}) = 2\hat{G} - c\hat{G}^3 + \cdots .$$

The putative non-Gaussian fixed point corresponds to $\hat{G} \to \hat{G}^*$, i.e. $G(a) \to \hat{G}^* a^2$. In our case it is tempting to identify our dimensionless constant k_1 with $1/\hat{G}$, up to the constant of proportionality. Close to the UV fixed point we have

$$\hat{G}(a) = \hat{G}^* - K a^{\tilde{c}}, \quad k_1 = k_1^* + K a^{\tilde{c}}, \qquad \tilde{c} = -\beta'(\hat{G}^*).$$

Usually one relates the lattice spacing near the fixed point to the bare coupling constants with the help of some correlation length ξ.

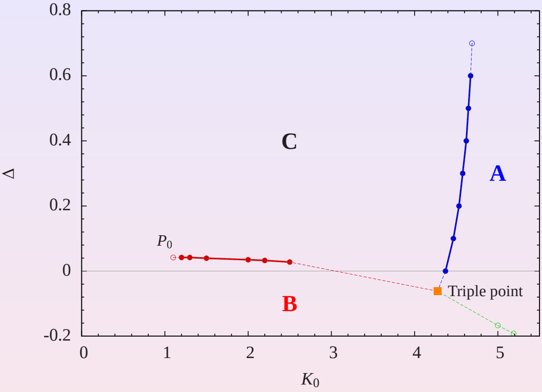


Consider $V_4 = N_4 a^4$ as fixed. It requires the fine-tuning of coupling constants.

$$k_1(N_4) = k_1^c - \tilde{K} N_4^{-\tilde{c}/4}.$$

How to determine $k_1(N_4)$?

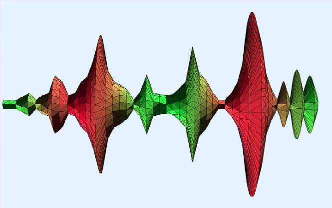
Phase diagram of CDT



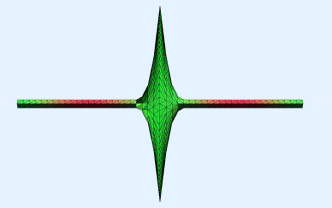
Lifshitz-like diagram....

Phase C: constant magnetization (constant 4d geometry)
Phase B: zero magnetization (no 4d geometry)
Phase A: oscillating magnetization (conformal mode ?)

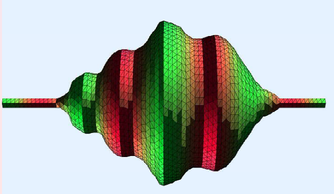
Volume distribution in (imaginary) time



- **Phase A**. The universe "oscillating" in time direction. The oscillation maybe reflecting the dominance of the conformal mode.

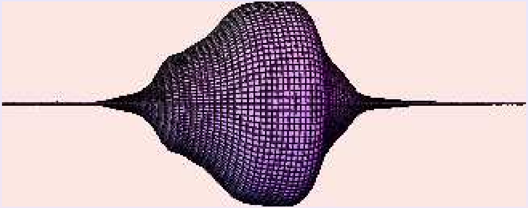


- **Phase B**. Compactification into a 3d Euclidean DT. Only minimal extension in the time direction.



- **Phase C**. Extended de Sitter phase. $d_H = 4$.

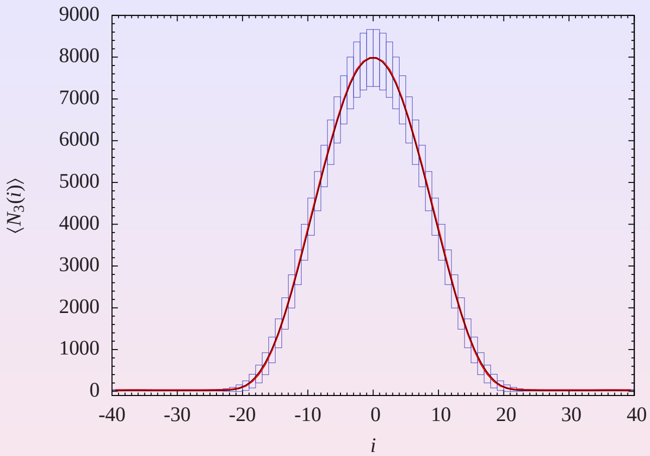
Snapshot of a typical configuration



A typical configuration. Distribution of a spatial volume $N_3(t)$ as a function of (imaginary) time t.

Quantum fluctuation around a semiclassical background?

Configuration consists of a "stalk" of the cut-off size and a "blob". Center of the blob can shift. We fix the "center of mass" to be at zero time.



$$\langle N_3(i) \rangle \propto N_4^{3/4} \cos^3 \left(\frac{i}{s_0 N_4^{1/4}} \right)$$

Minisuperspace model

The semiclassical distribution can be obtained from the minisuperspace effective action of Hartle and Hawking

$$S_{eff} = \frac{1}{24\pi G} \int dt \sqrt{g_{tt}} \left(\frac{g^{tt} \dot{V_3}^2(t)}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right),$$

The discretization of this action is (and we have reconstructed it from the date (the 3-volume–3-volume correlations))

$$S_{discr} = k_1 \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{k}_2 N_3^{1/3}(i) - \tilde{\lambda} N_3(i) \right),$$

$$G = \frac{a^2}{k_1} \frac{\sqrt{C_4}\, s_0^2}{3\sqrt{6}}.$$

Quantum fluctuations

The classical solution to the minisuperspace action is

$$\sqrt{g_{tt}}\, V_3^{cl}(t) = V_4\, \frac{3}{4B} \cos^3\left(\frac{t}{B}\right)$$

where $\tau = \sqrt{g_{tt}}\, t$, $V_4 = 8\pi^2 R^4/3$ and $\sqrt{g_{tt}} = R/B$.

Writing $V_3(t) = V_3^{cl}(t) + x(t)$ we can expand the action around this solution

$$S(V_3) = S(V_3^{cl}) + \frac{1}{18\pi G} \frac{B}{V_4} \int \mathrm{d}t\; x(t) \hat{H} x(t).$$

where the Hermitian operator $\hat{H}$ is:

$$\hat{H} = -\frac{\mathrm{d}}{\mathrm{d}t} \frac{1}{\cos^3(t/B)} \frac{\mathrm{d}}{\mathrm{d}t} - \frac{4}{B^2 \cos^5(t/B)},$$

In the quadratic approximation the volume fluctuations are:

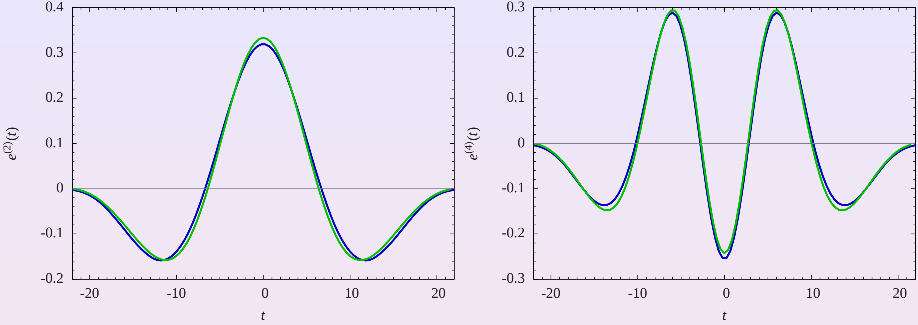
$$C(t,t') := \langle x(t)x(t')\rangle \sim \hat{H}^{-1}(t,t').$$

$\hat{C}$ and $\hat{H}$ have the same eigenfunctions.

$C(t,t')$ can be measured as

$$C(i,i') = \Big\langle \Big(N_3(i) - \langle N_3(i)\rangle\Big)\Big(N_3(i') - \langle N_3(i')\rangle\Big)\Big\rangle ,$$

and its eigenfunctions can be found and compared to the ones calculated from $\hat{H}$.



No parameters are put in ! (expect $t_i/B = i/s_0 N_4^{1/4}$)

We conclude that the quadratic approximation to the minisuperspace action describes the measured quantum fluctuations well.

Size of our Quantum universe

For a specific value of the bare coupling constants ($\kappa_0 = 2.2, \Delta = 0.6$) we have high-statistics measurements for N_4 ranging from 45.500 to 362.000 four-simplices.

Largest universe corresponds to approx. 10^4 hyper-cubes.

We have $G = \text{const.}\ a^2/k_1$ and we have measured k_1.

$$G \approx 0.23a^2, \quad \ell_P \approx 0.48a, \quad \ell_P \equiv \sqrt{G}.$$

From $V_4 = 8\pi^2 R^4/3 = C_4 N_4 a^4$, we obtain that

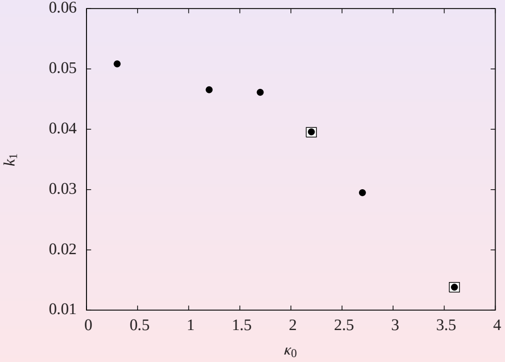
$$R = 3.1a$$

The linear size πR of the quantum de Sitter universes studied here lies in the range of 12-21 ℓ_P for the N_4 used.

Trans-Planckian ?

$$\ell_P = \sqrt{G} \propto \frac{a}{\sqrt{k_1(\kappa_0, \Delta)}} \quad \text{i.e.} \quad k_1(\kappa_0, \Delta) \to 0.$$

BUT IS IT POSSIBLE ?



Summary and perspectives

- We have obtained the (Euclidean) minisuperspace action from first principles. (The self-organized de Sitter space)
- We have an effective field theory of (something we call) QG down to a few Planck scales.

- Investigate a possible UV fixed point (points, the B-C line). Possibly Hořava-Lifshitz gravity.
- couple matter to the system and investigate cosmological implications.
- Measure the wave function of the universe

$$\langle x|\, e^{-t\hat{H}} |y\rangle \rightarrow \Psi_0(y)\Psi_0(x)\, e^{-tE_0}$$