Gravitational self force in extreme-mass-ratio binary inspirals

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Theory Meets Data Analysis
at Comparable and Extreme Mass Ratios
Perimeter Institute, June 2010

Conference summary by
Steve Detweiler
[arXiv 1009.2726, 15 September 2010]

...

As a member of the Capra community, I am pleased to report that we are reaching the end of a long, difficult adolescence. In the self-force portion of the meeting, a few serious meaningful applications of the gravitational self-force were described that allow for detailed comparisons among each other as well as with corresponding post-Newtonian analyses. The gravitational self-force has arrived.

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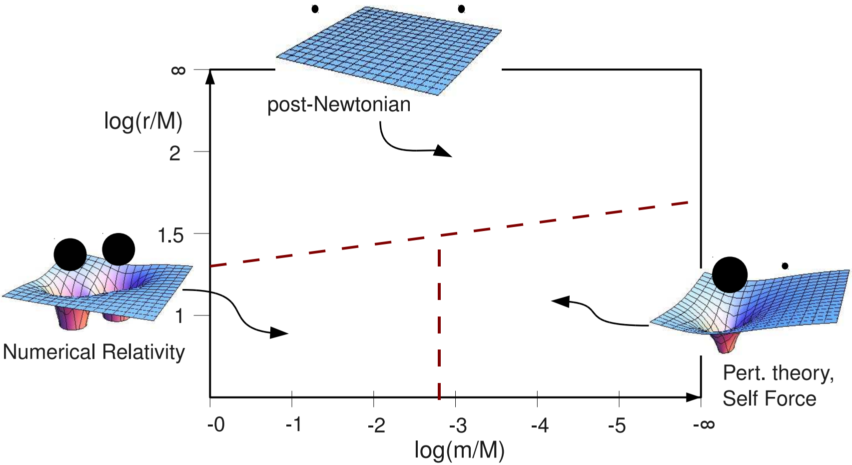
In this review:

- Motivation: EMRIs as sources for LISA
- Self force theory
- Implementation methods
- Conservative effects of the gravitational self force

2-body problem in relativity



EMRIs as probes of strong-field gravity

EMRI parameter extraction accuracies with LISA (SNR=30)

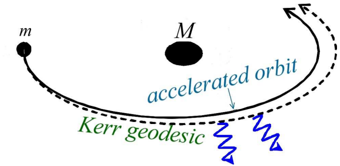
S/M^2	0.1	0.1	0.5	0.5	1	1
$e_{\rm LSO}$	0.1	0.3	0.1	0.3	0.1	0.3
$\mathbf{\Delta M/M}$	**2.6e−4**	**5.6e−4**	**2.7e−4**	**9.2e−4**	**2.8e−4**	**2.5e−4**
$\mathbf{\Delta(S/M^2)}$	**3.6e−5**	**7.9e−5**	**1.3e−4**	**6.3e−4**	**2.6e−4**	**3.7e−4**
$\mathbf{\Delta m/m}$	**6.8e−5**	**1.5e−4**	**6.8e−5**	**9.2e−5**	**6.1e−5**	**9.1e−5**
$\Delta(e_0)$	$6.3e{-}5$	$1.3e{-}4$	$8.5e{-}5$	$2.8e{-}4$	$1.2e{-}4$	$1.1e{-}4$
$\Delta(\cos\lambda)$	$6.0e{-}3$	$1.7e{-}2$	$1.3e{-}3$	$5.8e{-}3$	$6.5e{-}4$	$8.4e{-}4$
$\Delta(\Omega_s)$	$1.8e{-}3$	$1.7e{-}3$	$2.0e{-}3$	$1.7e{-}3$	$2.1e{-}3$	$1.1e{-}3$
$\Delta(\Omega_K)$	$5.6e{-}2$	$5.3e{-}2$	$5.5e{-}2$	$5.1e{-}2$	$5.6e{-}2$	$5.1e{-}2$
$\Delta[\ln(\mu/D)]$	$8.7e{-}2$	$3.8e{-}2$	$3.8e{-}2$	$3.7e{-}2$	$3.8e{-}2$	$7.0e{-}2$
$\Delta(t_0)\nu_0$	$4.5e{-}2$	$1.1e{-}1$	$2.3e{-}1$	$1.3e{-}1$	$2.5e{-}1$	$3.2e{-}2$

[LB & Cutler (2004)]

"Self force" description of the motion

Equations of motion

1. $mu^\beta \nabla_\beta u^\alpha = F^\alpha_{\rm self}$ $(\propto m^2)$
2. $\Box \bar{h}^{\rm ret}_{\mu\nu} + 2 R^{\alpha\ \ \beta}_{\ \mu\ \ \nu} \bar{h}^{\rm ret}_{\alpha\beta} = -16\pi T_{\mu\nu}$
3. $F^\alpha_{\rm self} = F^\alpha_{\rm self}(\bar{h}^{\rm ret}_{\alpha\beta}) =$?



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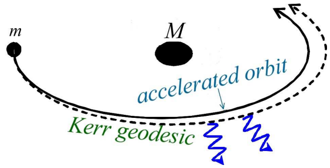


Challenges:

- regularization
- make sense of "point particle" in curved space
- self-interaction is not instantaneous in curved space ("tail" effect)
- self force (and orbit) are gauge dependent
- Lorenz-gauge condition dictates geodesic motion

Regularization:
Dirac's method and its failure in curved space

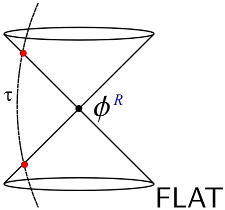
Decomposition of the EM vector potential for an electron in flat space:

$$A_\alpha^{ret} = \frac{1}{2}(A_\alpha^{ret} + A_\alpha^{adv}) + \frac{1}{2}(A_\alpha^{ret} - A_\alpha^{adv})$$

$$\equiv A_\alpha^S \qquad\qquad \equiv A_\alpha^R$$

Symmetric/Singular   Radiative/Regular

$$\rightarrow \quad F_{\text{self}}^\alpha = e\nabla^{\alpha\beta} A_\beta^R$$



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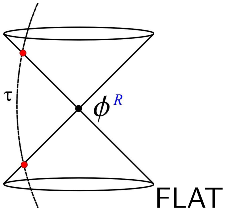
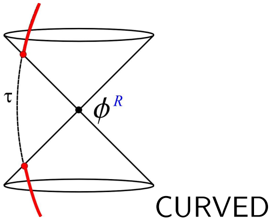


Difficulty: Local Radiative potential becomes non-causal in curved space!

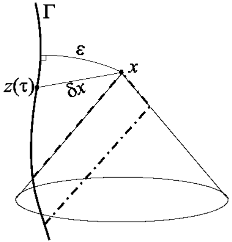


Regularization of the gravitational self-force

- Mino, Sasaki & Tanaka (1997):
  via Hadamard expansion + integration across in a thin worldtube
- Mino, Sasaki & Tanaka (1997), Poisson (2003), Pound (2010):
  via Matched Asymptotic Expansions
- Quinn & Wald (1997):
  via an axiomatic approach based on comparison to flat space
- Gralla& Wald (2008):
  by taking "far/near"-zone limits of a family of spacetimes
- Harte (2010):
  from generalized Killing fields

The gravitational self-force

$$F^{\alpha}_{\rm self} = \lim_{x\to z(\tau)} \nabla^{\alpha\mu\nu} h^{\rm tail}_{\mu\nu}$$
$$= \lim_{x\to z(\tau)} \nabla^{\alpha\mu\nu} \left(h^{\rm ret}_{\mu\nu} - h^{\rm dir}_{\mu\nu} \right)$$



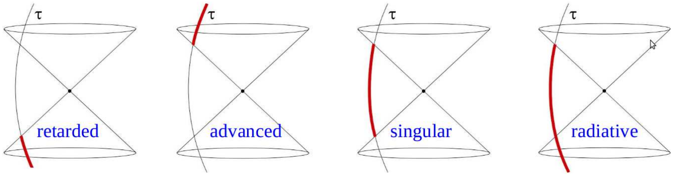
Detweiler–Whiting reformulation (2003)

Dirac-like decomposition of $h^{\rm ret}_{\alpha\beta}$ for a mass particle in curved space:

$$h^{ret}_{\alpha\beta} = \underbrace{\frac{1}{2}(h^{ret}_{\alpha\beta} + h^{adv}_{\alpha\beta}) - H_{\alpha\beta}}_{\equiv\, h^S_{\alpha\beta}} + \underbrace{\frac{1}{2}(h^{ret}_{\alpha\beta} - h^{adv}_{\alpha\beta}) + H_{\alpha\beta}}_{\equiv\, h^R_{\alpha\beta}}$$

Symmetric/Singular — Radiative/Regular

$$\rightarrow \; F^{\alpha}_{\rm self} = m\nabla^{\alpha\beta\gamma} h^R_{\beta\gamma}$$



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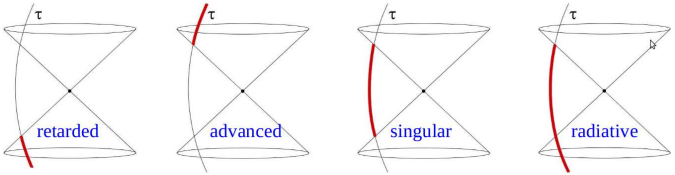

- $h^R_{\alpha\beta}$ is a vacuum solution of the Einstein equations.
  Interpretation: orbit is a geodesic of $g_{\alpha\beta} + h^R_{\alpha\beta}$.

Mode-sum method [LB & Ori (2000-2003)]

Define $F_{\mathrm{ret}/S} \equiv m\nabla h^{\mathrm{ret}/S}$ (as fields), then write

$$
\begin{aligned}
F_{\mathrm{self}} &= \left.(F_{\mathrm{ret}} - F_{\mathrm{S}})\right|_{\mathrm{p}} \\
&= \sum_{\ell=0}^{\infty} \left.\left(F^{\ell}_{\mathrm{ret}} - F^{\ell}_{S}\right)\right|_{\mathrm{p}} \quad (\ell\text{-mode contributions are finite}) \\
&= \sum_{\ell=0}^{\infty} \left[F^{\ell}_{\mathrm{ret}}(p) - AL - B - C/L\right] - \sum_{\ell=0}^{\infty} \left[F^{\ell}_{S}(p) - AL - B - C/L\right] \\
&= \sum_{\ell=0}^{\infty} \left[F^{\ell}_{\mathrm{ret}}(p) - AL - B - C/L\right] - D \quad (\text{where } L = \ell + 1/2)
\end{aligned}
$$

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\end{aligned}$$

- Regularization Parameters $A^{\alpha}, B^{\alpha}, C^{\alpha}, D^{\alpha}$ derived analytically for generic orbits in Kerr [LB & Ori (2003), LB (2009)].

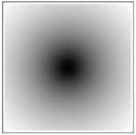
Implementations so far (geodesic orbits, no evolution yet)

year	Schwarzschild	Kerr
2000	static	
2000	head-on	
2001		static
2002	head-on	
2003	circular	
2007	eccentric	
2007	static	
2007	circular	
2009		circular-equatorial
2009	eccentric	
2010		eccentric-equatorial
2010		circular-inclined

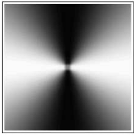
gravitational self force / scalar-field toy model

The gauge problem

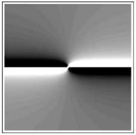
- Original regularization formulated in Lorenz gauge (div $\bar{h} = 0$).
  - Linearized Einstein equation takes a neat hyperbolic form
  - Particle singularity is "isotropic" and Coulomb-like



Lorenz gauge



Regge-Wheeler gauge



Radiation gauge

The gauge problem

- Original regularization formulated in Lorenz gauge (div $\bar{h} = 0$).
  - Linearized Einstein equation takes a neat hyperbolic form
  - Particle singularity is "isotropic" and Coulomb-like

- Unfortunately Lorenz-gauge equations are not easily amenable to numerical treatment.
- Options: Work out the singular gauge transformations, or develop methods to integrate the Lorenz-gauge equations.

Direct Lorenz-gauge implementation [LB & Lousto (2005)]

- Start with 10 coupled perturbation equations + 4 gauge conditions:

$$\Box\bar{h}_{\alpha\beta} + 2R^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta}\bar{h}_{\mu\nu} = -16\pi m \int_{-\infty}^{\infty} \frac{\delta[x^{\mu} - z^{\mu}(\tau)]}{\sqrt{-g}} u_{\alpha} u_{\beta} d\tau$$

$$Z_{\alpha} \equiv \nabla^{\beta}\bar{h}_{\alpha\beta} = 0$$

- Add "constraint damping" terms, $-\kappa t_{(\alpha} Z_{\beta)}$

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- Expand in tensor harmonics,
  $\bar{h}_{\alpha\beta} = \sum_{l,m}\sum_{i=1}^{10} h^{(i)lm}(r,t) Y^{(i)lm}_{\alpha\beta}$
  Obtain 10 coupled scalar-like eqs for $h^{(i)lm}(r,t)$

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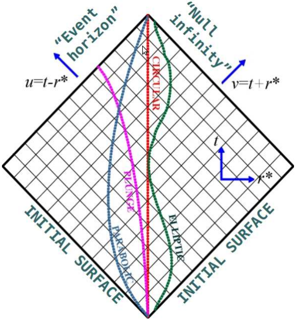
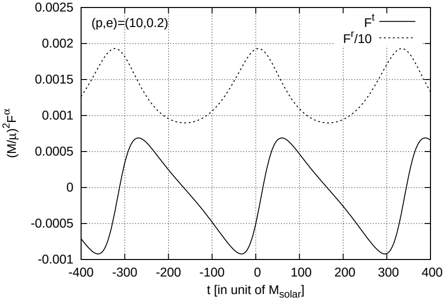
- Solve numerically using time-domain evolution in characteristic coordinates

- Use as input for the mode-sum formula

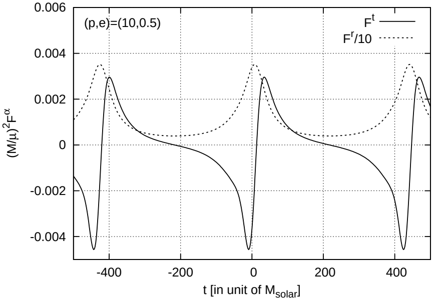


Gravitational self-force in Schwarzschild

$(p, e) = (10M, 0.2)$



$(p, e) = (10M, 0.5)$



Towards self force in Kerr: the Puncture method

$$\Box(\underbrace{h^{\rm ret} - h^{\rm punc}}_{h^{\rm Res}}) = S - \Box h^{\rm punc} \equiv S^{\rm Res}$$

$$F^{\rm self} = m \lim_{x \to z} \nabla h^{\rm Res}$$

- Does not rely on separability

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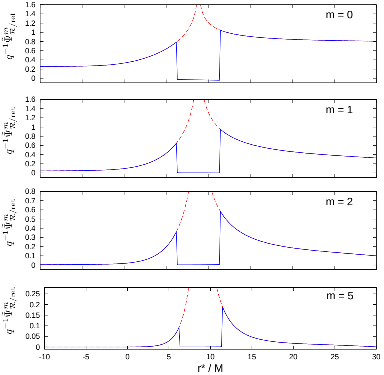
$$F^{\rm self} = m \lim_{x \longrightarrow z} \nabla h^{\rm Res}$$

- Does not rely on separability
- Can be implemented in
  - 1+1D [Vega & Detweiler (2007)]
  - 2+1D [LB & Golbourn (2007)]
    [Lousto & Nakano (2008)]
    [Dolan & LB (2010)]
  - 3+1D [Vega et al (2009)]



Conservative gauge-invariant effects of the self force

Conservative piece of the gravitational self force

$$F^{\rm self} \;=\; \underbrace{\frac{1}{2}\left(F^{\rm ret} - F^{\rm adv}\right)}_{F^{\rm diss}} + \underbrace{\frac{1}{2}\left(F^{\rm ret} + F^{\rm adv}\right)}_{F^{\rm cons}}$$

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Why study gauge-invariant conservative effects?

- secular effect on phase evolution
- tests of SF formalism & codes against PN theory
- strong-field calibration data for approximate analytic methods (EOB)
- inform development of “Kludge” orbital evolution schemes

1. The "red shift" invariant [Detweiler (2008)]

- The "red shift" invariant for circular orbits (Detweiler 2008):

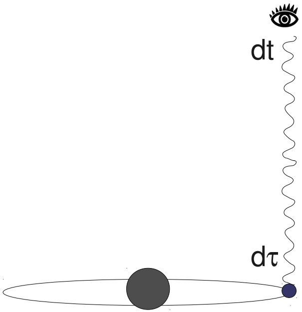
$$u^t \equiv \frac{dt}{d\tau}$$

  - ▶ $u^t(\Omega_\varphi)$ is gauge invariant.

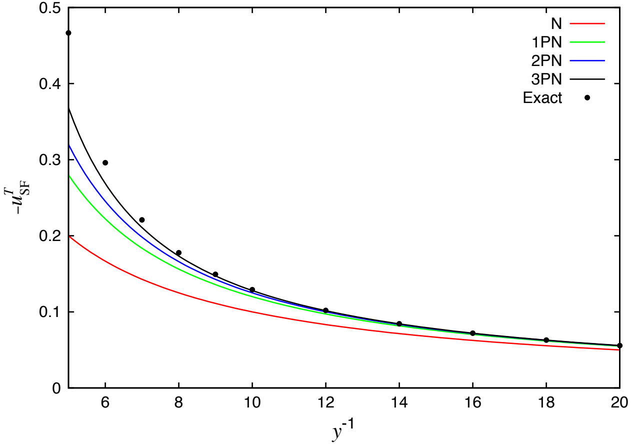
- Generalization to eccentric orbits (LB & Sago 2010):

$$\langle u^t \rangle \equiv \left\langle \frac{dt}{d\tau} \right\rangle_\tau = \frac{t \text{ period}}{\tau \text{ period}}$$

  - ▶ $\langle u^t \rangle(\Omega_\varphi, \Omega_r)$ is gauge invariant.



SF correction to the red shift function for circular orbits: comparison with PN



[Blanchet, Detweiler, Le Tiec and Whiting 2010]

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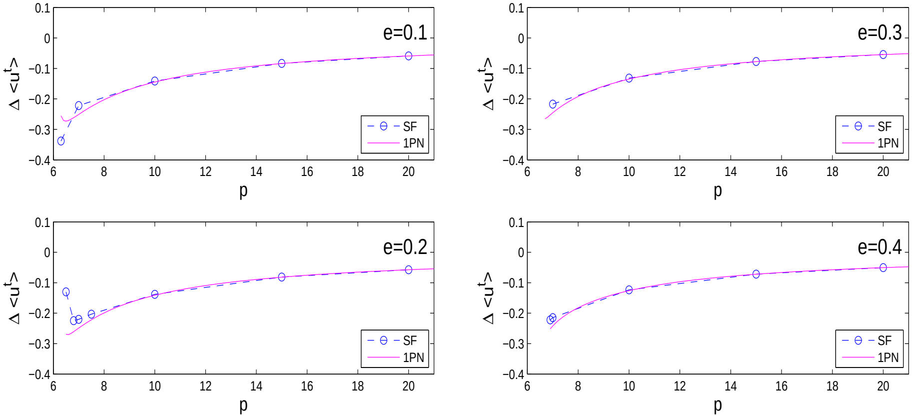


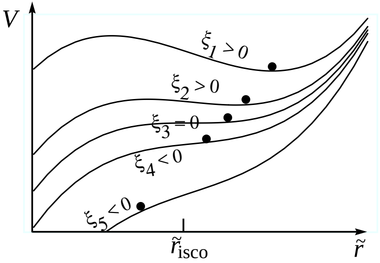
[LB, Le Tiec & Sago (preliminary)]

2. ISCO frequency as an accurate strong-field benchmark

[LB & Sago (2009)]

$$\Delta r_{isco} = -3.269 m (G/c^2)$$

$$\frac{\Delta\Omega_{isco}}{\Omega_{isco}} = 0.4870 m/M$$



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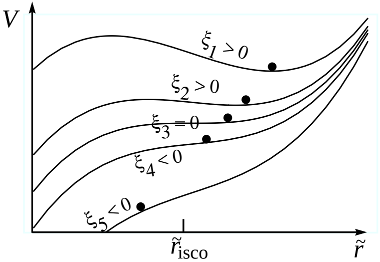
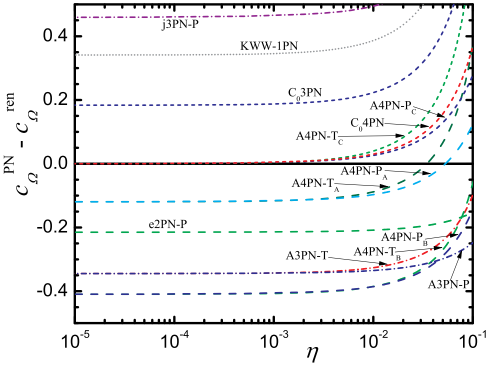


- used to break the degeneracy between the EOB parameters a_5 & a_6 [Damour 2010].
- used to inform an "empirical" formula for the remnant masses and spins in BBH mergers [Lousto et al 2010]
- used for an exhaustive comparative study of PN methods [Favata 2010]

ISCO shift as an accurate strong-field benchmark

Method	$c_\Omega^{\rm PN}$	Δ_{c_Ω}
A4PN-P$_A$	1.132	-0.0955
A4PN-T$_A$	1.132	-0.0955
$C_0$3PN	1.435	0.1467
e2PN-P	1.036	-0.1717
KWW-1PN	1.592	0.2726
A3PN-P	0.9067	-0.2754
A3PN-T	0.9067	-0.2754
A4PN-P$_B$	0.8419	-0.3272
A4PN-T$_B$	0.8419	-0.3272
j3PN-P	1.711	0.3671
j2PN-P	0.6146	-0.5088
KWW-S	0.5610	-0.5515
$C_0$2PN	0.5833	-0.5338
E_h3PN	0.4705	-0.6240
e3PN-P	2.178	0.7409
A2PN-P	0.2794	-0.7767
A2PN-T	0.2794	-0.7767
E_h2PN	0.0902	-0.9279
E_h1PN	-0.01473	-1.011
E_h-S	-0.05471	-1.044
HH-S	-0.1486	-1.119
j1PN-P	-0.1667	-1.133
KWW-2PN	-1.542	-2.232
j-P-S	-2.104	-2.682
KWW-3PN	4.851	2.877
HH-1PN	6.062	3.844
HH-2PN	-12.75	-11.19
HH-3PN	25.42	19.32



Results from M Favata 2010

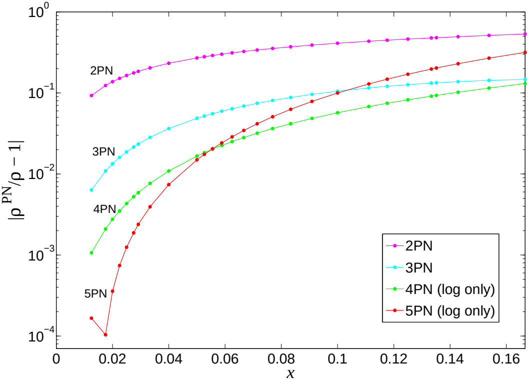
3. Precession effect for slightly eccentric orbits:

comparison with PN-calibrated EOB [LB, Damour & Sago 2010]

$$x = (M_{\rm total}\Omega_\varphi)^{2/3} = \frac{M_{\rm total}}{R}$$

$$\frac{\Omega_r^2}{\Omega_\varphi^2} = 1 - 6x + \frac{m}{M}\rho(x) + O\left(\frac{m}{M}\right)^2$$

$\rho(x)$ is gauge invariant



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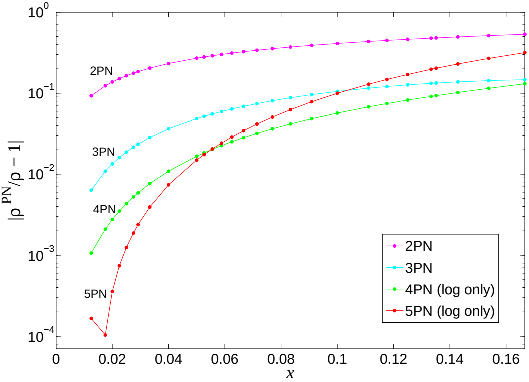


$$\rho^{PN} = \rho_2 x^2 + \rho_3 x^3 + (\rho_4^c + \rho_4^{\log} \ln x)x^4 + (\rho_5^c + \rho_5^{\log} \ln x)x^5 + O(x^6)$$

(• terms known analytically • terms not yet known)

4. Precession effect for slightly eccentric orbits:

strong-field calibration of EOB functions [LB, Damour & Sago 2010]

Is it possible to obtain a good global fit for $\rho(x)$ based on a minimal, "easy" set of SF data?

- Pink line is a 2-point Padé model

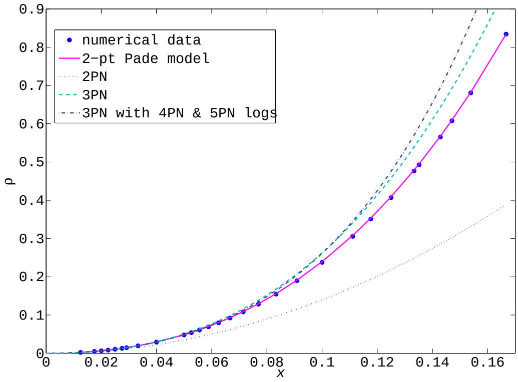
$$\rho_{\mathrm{pade2}}(x) = ax^2 \frac{1 + bx}{1 + cx + dx^2}$$

based only on
$\{\rho''(0), \rho'''(0)\}$ (from PN)
$\{\rho(1/6), \rho'(1/6)\}$ (from SF)

$$max\{|\rho_{\mathrm{pade2}} - \rho_{\mathrm{data}}|\} = 0.0024$$

- With a 3-pt Padé using
$\{\rho(\infty), \rho'(\infty), \rho(1/6), \rho(1/10)]$
this gets better still:

$$max\{|\rho_{\mathrm{pade3}} - \rho_{\mathrm{data}}|\} = 0.0002$$



What's next?

- More work on calibrating EOB (using marginally bound zoom-whirl orbits? equi-frequency separatrix?)
- Kerr codes, in both time and frequency domains
- More efficient numerical algorithms (mesh refinement, finite elements, improved initial conditions,...)
- Orbital evolution
- 2nd-order self force