A New Road to Massive Gravity?

Eric Bergshoeff

Groningen University

based on a collaboration with

Marija Kovacevic, Jose Juan Fernandez-Melgarejo, Jan Rosseel,
Paul Townsend and Yihao Yin

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Outline

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Why Higher-Derivative Gravity ?

Einstein Gravity is the unique field theory of interacting massless spin-2 particles around a given spacetime background that mediates the gravitational force

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a \left(R_{\mu\nu}{}^{ab}\right)^2 + b \left(R_{\mu\nu}\right)^2 + c\, R^2 \ :$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no massless spin 2 !

$\Rightarrow$ "New Massive Gravity"

Hohm, Townsend + E.B. (2009)

- Can this be extended to higher dimensions ?

Why Massive Gravity?

see talk by Deffayet

- Massive Gravity is an IR modification of Einstein gravity that describes a massive spin-2 particle via an explicit mass term

- modified gravitational force

$$V(r) \sim \frac{1}{r} \quad \rightarrow \quad V(r) \sim \frac{e^{-mr}}{r}$$

  - characteristic length scale $r = \frac{1}{m}$

- Cosmological Constant Problem

In the main part of this talk I will discuss

Higher-Derivative Gravity

At the end I will come back to

Massive Gravity

Outline

Underlying Trick

- Higher-Derivative Gravity theories can be constructed starting from Second-Order Derivative FP equations and solving for differential subsidiary conditions

- This requires fields with zero massless degrees of freedom

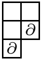
Massless Degrees of Freedom

cp. to Henneaux, Kleinschmidt and Nicolai (2011)

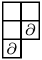
field $S \sim$ 

gauge parameters $\lambda_1 \sim$  $\lambda_2 \sim$ 

gauge transformation δ  $=$  $+$ 

curvature $R(S) \sim$ 

Zero Massless D.O.F.

"Einstein tensor" $G(S) \sim \star\star$ 

Requirement : $G(S) \sim$  $\Rightarrow$ E.O.M. : $G(S) = 0$

two columns : $p + q = D - 1$

Example : $p = q = 1\,, D = 3\,,$ $S \sim$ 

"Boosting Up the Derivatives"

Second-Order Derivative Generalized FP

Curtright (1980)

$$\left(\Box - m^2\right) S = 0\,, \qquad S^{\rm tr} = 0\,, \quad \partial \cdot S = 0$$

$$\partial \cdot S = 0 \quad \Rightarrow \quad S = G(T)$$

$$\left(\Box - m^2\right) G(T) = 0\,, \qquad G(T)^{\rm tr} = 0$$

Higher-Derivative Gauge Theory

Example: p-forms

Condition : rank dual curvature = p $\rightarrow$

$$p = \frac{1}{2}(D - 1)$$

1-forms in 3D

$$R_{\mu\nu}(S) = 2\partial_{[\mu}S_{\nu]}\,, \qquad G_\mu(S) = \tfrac{1}{2}\epsilon_\mu{}^{\nu\rho}R_{\nu\rho}(S)$$

$$\mathcal{L} = \tfrac{1}{2}\epsilon^{\mu\nu\rho}\, S_\mu\, R_{\nu\rho}(S) \;:\quad \text{zero d.o.f.}$$

Proca : $\quad \left(\Box - m^2\right) S_\mu = 0\,, \qquad \partial^\mu S_\mu = 0$

- boosting up Proca: $S_\mu = G_\mu(T) \;\to\; \left(\Box - m^2\right) G_\mu(T) = 0$
- Integrating E.O.M. to action leads to ghosts
- This is a general feature of 3D odd spin

I will not discuss the parity-odd 3D TME and 3D TMG theories

These are based on a factorisation of the 3D Klein-Gordon operator

Now on to spin two !

Outline

3D Einstein-Hilbert Gravity

Deser, Jackiw, 't Hooft (1984)

There are no massless gravitons : "trivial" gravity

Adding higher-derivative terms leads to "massive gravitons"

Free Fierz-Pauli

- $\left(\Box - m^2\right) \tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \quad \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\mathrm{FP}} = \frac{1}{2}\tilde{h}^{\mu\nu} G^{\mathrm{lin}}_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no obvious non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Note : the numbers become 2 (4D) and 0 (3D) for $m = 0$

Higher-Derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \epsilon_\nu{}^{\gamma\delta} \partial_\alpha \partial_\gamma h_{\beta\delta} \equiv G_{\mu\nu}(h)$$

$$\left(\Box - m^2\right) \; G^{\text{lin}}_{\mu\nu}(h) = 0 \,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g} \left[-R - \frac{1}{2m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right) \right]$$

"New Massive Gravity" : unitary !

Mode Analysis

- Take NMG with metric $g_{\mu\nu}$, cosmological constant Λ and coefficient $\sigma = \pm 1$ in front of R

- lower number of derivatives from 4 to 2 by introducing an auxiliary symmetric tensor $f_{\mu\nu}$

- after linearization and diagonalization the two fields describe a massless spin 2 with coefficient $\bar{\sigma} = \sigma - \frac{\Lambda}{2m^2}$ and a massive spin 2 with mass $M^2 = -m^2\bar{\sigma}$

- special cases:
  - 3D NMG — Hohm, Townsend + E.B. (2009)
  - $D \geq 3$ "chiral/critical gravity" for special value of Λ

    Li, Song, Strominger (2008); Lü and Pope (2011)

Chiral/Critical Gravity

- a massive graviton disappears but a log mode re-appears
- In general one ends up with a non-unitary theory
- are there unitary truncations ?

Is NMG perturbative renormalizable?

D=4

- $\mathcal{L} \sim +R + R^2$: scalar field coupled to gravity

  unitarity: $\surd$ but renormalizability: X

  propagator $\sim \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_0 + \left(\frac{1}{p^2}\right)_2$

- $\mathcal{L} \sim R + \left(C_{\mu\nu}{}^{ab}\right)^2$: Weyl tensor squared

  propagator $\sim \left(\frac{1}{p^2}\right)_0 + \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_2$

  unitarity: X and renormalizability: X

D=3

How do the NMG propagators behave ?

$$\mathcal{L} \;=\; \sqrt{-g}\left[\sigma R + \frac{a}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right) + \frac{b}{m^2}R^2\right] \qquad \sigma = \pm 1$$

$$\text{propagator} \sim \left(\frac{1}{p^2} + \frac{b}{p^4}\right)_0 + \left(\frac{1}{p^2} + \frac{a}{p^4}\right)_2 \quad \Rightarrow \quad ab \neq 0$$

Nishino, Rajpoot (2006)

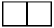
However, we also need $ab = 0$ $\Rightarrow$

NMG is (most likely) not perturbative renormalizable !

What did we learn?

- two theories can be equivalent at the linearized level (FP and boosted FP) but only one of them allows for a unique non-linear extension i.e. interactions !

- we need massive spin 2 whose massless limit describes 0 d.o.f.

Example :  in 3D

- what about 4D?

New Massive Gravity in 4D

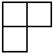
An alternative approach to 4D Massive Gravity ?

Generalized spin-2 FP

standard spin-2 :

describes
$$\begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 2 \quad \text{d.o.f.} \quad m = 0 \end{cases}$$

generalized spin-2 :



describes
$$\begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 0 \quad \text{d.o.f.} \quad m = 0 \end{cases}$$

Curtright (1980)

Connection-metric Duality

- Use first-order form with independent fields $e_\mu{}^a$ and $\omega_\mu{}^{ab}$

- linearize around Minkowski: $e_\mu{}^a = \delta_\mu{}^a + h_\mu{}^a$

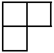
  and add a FP mass term $-m^2(h^{\mu\nu}h_{\nu\mu} - h^2) \quad \rightarrow$

$$\mathcal{L} \sim \text{“}\, h\,\partial\omega + \omega^2\,\text{”} - m^2(h^{\mu\nu}h_{\nu\mu} - h^2)$$

- solve for $\omega \;\rightarrow$ spin-2 FP in terms of h and auxiliary $h_{[\mu\nu]}$

- solve for $h_{\mu\nu}$ and write $\omega_\mu{}^{ab} = \frac{1}{2}\epsilon^{abcd}\,\tilde{h}_{\mu cd} \;\rightarrow$ generalized spin-2 FP in terms of $\tilde{h}$ after elimination of auxiliary $\tilde{h}_{[\mu cd]}$

Massive versus Massless Duality

Massive duality :  $\leftrightarrow$ 

$$\mathcal{L}_{\text{massive dual}} = \frac{1}{2}\tilde{h}^{\mu\nu,\rho}\, G_{\mu\nu,\rho}(\tilde{h}) - \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu,\rho}\tilde{h}_{\mu\nu,\rho} - 2\tilde{h}^{\mu}\tilde{h}_{\mu}\right)$$

- massless limit describes zero d.o.f. : “trivial” gravity

Massless duality :  $\leftrightarrow$ 

West (2001)

- Dual Einstein gravity describes two d.o.f.

Duality and taking massless limit do not commute !

Boosting up the Derivatives

- start with generalized spin-2 FP in terms of

  and subsidiary conditions

$$\tilde{h}_{\mu\nu,\rho}\,\eta^{\nu\rho} = 0\,, \qquad\qquad \partial^{\rho}\,\tilde{h}_{\rho\mu,\nu} = 0$$

- solve for $\partial^{\rho}\,\tilde{h}_{\rho\mu,\nu} = 0 \to \tilde{h}_{\mu\nu,\rho} = G_{\mu\nu,\rho}(h) \ \to$ "NMG in 4D" :

$$\mathcal{L}_{\rm NMG} \ \sim\ -\tfrac{1}{2}h^{\mu\nu,\rho}G_{\mu\nu,\rho}(h) \ +\ \frac{1}{2m^2}\ \underbrace{h^{\mu\nu,\rho}\,C_{\mu\nu,\rho}(h)}_{\text{"conformal invariance"}}$$

- mode analysis $\ \to$

$$\mathcal{L}_{\rm NMG} \ \sim\ \text{massless spin 2 plus massive spin 2}$$

Interactions ?

cp. to Bekaert, Boulanger, Cnockaert (2005)

- compare to Eddington-Schrödinger theory

$$\mathcal{L}'_{\mathrm{ES}} = \sqrt{-\det g}\,[g^{\mu\nu} R_{\mu\nu}(\Gamma) - 2\Lambda] \quad \Leftrightarrow \quad \mathcal{L}_{\mathrm{ES}} = \sqrt{|\det R_{(\mu\nu)}(\Gamma)|}$$

$$g_{\mu\nu} = \frac{(D-2)}{2\Lambda} R_{(\mu\nu)}(\Gamma)$$

- consider non-trivial background or couple to matter

$$h^{\mu\nu,\rho}\ \text{“}(\epsilon\partial T)\text{”}_{\mu\nu,\rho} \qquad \text{or} \qquad \text{“}(\epsilon\partial h)\text{”}^{\mu\nu}\ T_{\mu\nu}$$

Curtright and Freund (1980)

4D "Trivial" Gravity

avoids no-go theorem !

Example : $\square\square$ in 3D

- Chern-Simons formulation $\mathcal{L} \sim AdA + A^3$: $\left(e_\mu{}^a, \omega_\mu{}^a\right)$

  Achúcarro and Townsend (1986); Witten (1988)

first-order formulation of 4D "trivial" gravity :

- $\left(T_{\mu\nu}{}^a\,, \Omega_\mu{}^a\right)$ Zinoviev (2003); Alkalaev, Shaynkman and Vasiliev (2003)

  - interactions via CS formulation ?

Outline

A Short Review

- no symmetry principle
- fine-tuning is needed
- reference metric is needed "$g^{\mu\nu} g_{\mu\nu} = 1$"

Question : does massive gravity reduce to GR for $m \to 0$?

Problem : $5 \neq 2$!

FP : $5 \to 2 + \not{2} + 0$

- this is the vDVZ discontinuity (1970)

The Vainshtein Radius

Vainshtein : vDVZ discontinuity is artifact of linear approximation

- linear approximation of GR can be trusted for

$$r > r_{\mathrm{S}} \sim \frac{M}{M_{\mathrm{P}}^2} \qquad r_S \sim 1 \text{ km}$$

- in massive gravity extra attractive force is screened for

$$r < r_{\mathrm{V}} \sim \left(\frac{M}{m^4 M_{\mathrm{P}}^2} \right)^{1/5}$$

Other Issues

- instabilities: Boulware-Deser ghost (1972) $\phi(\phi\Box^2\phi)$
- the extent of the quantum regime : $r > r_{\mathrm{Q}}$

we want $r_{\mathrm{Q}} < r < r_{\mathrm{V}}$ to be large enough

There are several models in the market:

see talk by Deffayet

A Common Origin

Both 3D NMG and 4D Massive Gravity stem from a

general class of bi-gravity models!

Bañados and Theisen (2009); Hassan and Rosen (2011); Paulos and Tolley (2012)

- 4D Massive Gravity: promote fixed reference metric to dynamical metric
- 3D NMG: exchange higher derivatives for auxiliary symmetric tensor

Generalizations

Can the class of bi-gravity models be extended to poly-gravity or models bi-metric models of different symmetry type ?

Outline

Summary

- we discussed a general procedure for constructing Higher-Derivative Gravity Theories

- we investigated a new massive modification of 4D gravity

- Higher-Derivative gravity and Massive gravity have common origin

Open Issues

- Interactions ?

- Extension to Higher Spins ?