Some recent results on massive gravity

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1. (The) 3 sins of massive gravity.

2. Cures ?

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Part 1. The 3 sins of massive gravity

1.1. Introduction: why « massive gravity » ?

1.2. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

1.3. Non linear Pauli-Fierz theory and the Vainshtein Mechanism

1.4. The Goldstone picture (and « decoupling limit ») of non linear massive gravity, and what can one get from it ?

1.1. Introduction: Why « massive gravity » ?

➡ One way to modify gravity at « large distances »
… and get rid of dark energy (or dark matter) ?

$$H^2 = \frac{8\pi G}{3}\rho$$

Changing the dynamics
of gravity ?

Dark matter
dark energy ?

for this idea to work…

I.e. to « replace » the cosmological constant by a non vanishing graviton mass…

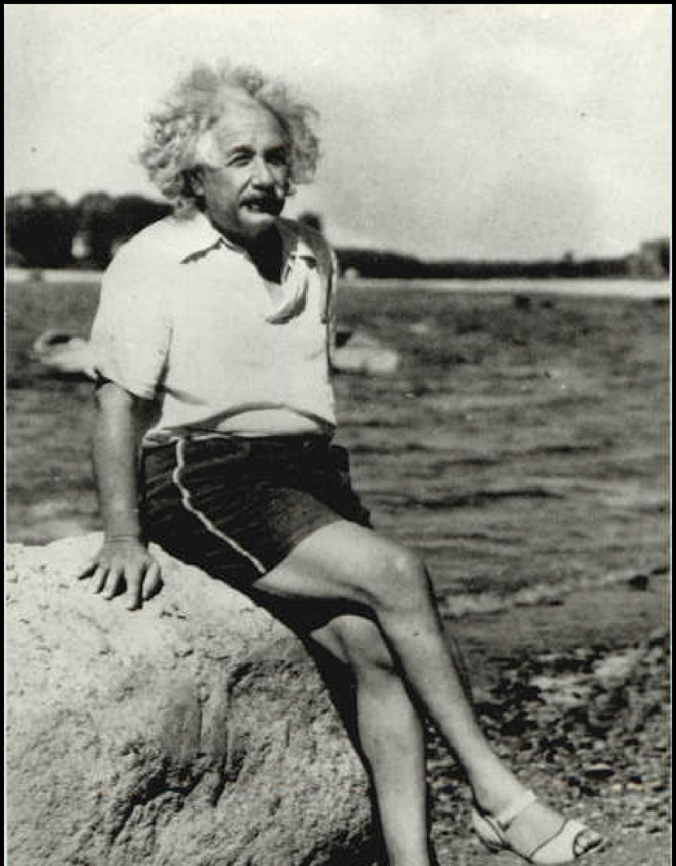
➡ One obviously needs a very light graviton (of Compton length of order of the size of the Universe)

for this idea to work…

I.e. to « replace » the cosmological constant by a non vanishing graviton mass…

NB: It seems one of the Einstein's motivations to introduce the cosmological constant was to try to « give a mass to the graviton »

(see « Einstein's mistake and the cosmological constant »
by A. Harvey and E. Schucking,
Am. J. of Phys. Vol. 68, Issue 8 (2000))



1.2. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

Pauli-Fierz action: second order action for a massive spin two $h_{\mu\nu}$

$$\int d^4x \underbrace{\sqrt{g} R_g} + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

second order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$

Only Ghost-free (quadratic) action for a massive spin two Pauli, Fierz 1939

(NB: breaks explicitly gauge invariance)

The propagators read

propagator for $m=0$ $\quad D_0^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2p^2} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{2p^2} + \mathcal{O}(p)$

propagator for $m\neq 0$ $\quad D_m^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2(p^2-m^2)} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{3(p^2-m^2)} + \mathcal{O}(p)$

Coupling the graviton with a conserved energy-momentum tensor

$$S_{int} = \int d^4x \, \sqrt{g} \, h_{\mu\nu} T^{\mu\nu}$$

$$h^{\mu\nu} = \int D^{\mu\nu\alpha\beta}(x - x') T_{\alpha\beta}(x') d^4 x'$$

The amplitude between two conserved sources T and S is given by

$$\mathcal{A} = \int d^4 x S^{\mu\nu}(x) h_{\mu\nu}(x)$$

For a massless graviton: $\mathcal{A}_0 = \left(\hat{T}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \hat{T} \right) \hat{S}^{\mu\nu}$

For a massive graviton: $\mathcal{A}_m = \left(\hat{T}_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} \hat{T} \right) \hat{S}^{\mu\nu}$

In Fourier space

e.g. amplitude between two non relativistic sources:

$$\left.\begin{aligned}\hat{T}^{\mu}_{\nu} &\propto \mathrm{diag}(\hat{m}_1, 0, 0, 0)\\ \hat{S}^{\mu}_{\nu} &\propto \mathrm{diag}(\hat{m}_2, 0, 0, 0)\end{aligned}\right\} \quad \mathcal{A} \sim \tfrac{2}{3}\hat{m}_1\hat{m}_2 \quad \text{Instead of} \quad \mathcal{A} \sim \tfrac{1}{2}\hat{m}_1\hat{m}_2$$

Rescaling of Newton constant $\quad G_{\mathrm{Newton}} = \frac{4}{3} G_{(4)}$

defined from Cavendish experiment

appearing in the action

but amplitude between an electromagnetic probe and a non-relativistic source is the same as in the massless case (the only difference between massive and massless case is in the trace part) ⇨ wrong light bending! (factor ¾)

N.B., the PF mass term reads

$$M_P^2 m^2 \int d^4x \, (h_{ij}h_{ij} - 2h_{0i}h_{0i} - h_{ii}h_{jj} + 2h_{ii}h_{00})$$

h_{00} enters linearly both in the kinetic part and the mass term, and is thus a Lagrange multiplier of the theory…

… which equation of motion enables to eliminate one of the a priori 6 dynamical d.o.f. h_{ij}

By contrast the h_{0i} are not Lagrange multipliers

5 propagating d.o.f. in the quadratic PF

$h_{\mu\nu}$ is transverse traceless in vacuum.

1.3. Non linear Pauli-Fierz theory and the « Vainshtein Mechanism »

Can be defined by an action of the form

Isham, Salam, Strathdee, 1971

$$S = \int d^4x\sqrt{-g} \ \left(\frac{M_P^2}{2}R_g + L_g\right) + S_{int}[f,g],$$

Einstein-Hilbert action
for the *g* metric

Matter action
(coupled to metric *g*)

Interaction term coupling
the metric *g* and the <u>non
dynamical</u> metric *f*

1.3. Non linear Pauli-Fierz theory and the « Vainshtein Mechanism »

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$$S = \int d^4x \sqrt{-g} \ \left(\frac{M_P^2}{2} R_g + L_g \right) + S_{int}[f,g],$$

The interaction term $S_{int}[f,g],$ is chosen such that

- It is invariant under diffeomorphisms
- It has flat space-time as a vacuum
- When expanded around a flat metric ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $f_{\mu\nu} = \eta_{\mu\nu}$)
  It gives the Pauli-Fierz mass term

Leads to the e.o.m. $M_P^2 G_{\mu\nu} = \left(T_{\mu\nu} + T^g_{\mu\nu}(f,g) \right)$

Matter energy-momentum tensor

Effective energy-momentum tensor (*f,g* dependent)

Some working examples

$$S_{int}^{(2)} = -\frac{1}{8}m^2 M_P^2 \int d^4x \sqrt{-f}\, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau}\right)$$

Boulware Deser, 1972, BD in the following

$$S_{int}^{(3)} = -\frac{1}{8}m^2 M_P^2 \int d^4x \sqrt{-g}\, H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau}\right)$$

Arkani-Hamed, Georgi, Schwarz, 2003
AGS in the following

with

$$H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$$

(infinite number of models with similar properties)

(in the « Pauli-Fierz universality class » [Damour, Kogan, 2003])

⇨ Look for static spherically symmetric solutions

With the ansatz (not the most general one)

$$\begin{cases} g_{AB}dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\ f_{AB}dx^A dx^B &= -dt^2 + dr^2 + r^2 d\Omega^2 \end{cases}$$

Gauge transformation

$$\begin{cases} g_{\mu\nu}dx^\mu dx^\nu &= -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \\ f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 \end{cases}$$

Which can easily be compared to Schwarzschild

Then look for an expansion in
G_N (or in $R_S \propto G_N M$) of the would-be solution

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$

$$\nu(R) = -\frac{R_S}{R}(1 \;\; + \ldots$$

$$\lambda(R) = +\frac{1}{2}\;\; \frac{R_S}{R}(1 \;\; + \ldots$$

Wrong light bending!

This coefficient equals +1
in Schwarzschild solution

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$

$$\nu(R) = -\frac{R_S}{R}(1 + \mathcal{O}(1)\epsilon + \ldots$$

$$\lambda(R) = +\frac{1}{2} \quad \frac{R_S}{R}(1 + \mathcal{O}(1)\epsilon + \ldots$$

with $\quad \epsilon = \frac{R_S}{m^4 R^5}$

Vainshtein 1972

In « some kind »

[Damour et al. 2003]

of non linear PF

Wrong light bending!

This coefficient equals +1
in Schwarzschild solution

Introduces a new length scale R_v in the problem
below which the perturbation theory diverges!

For the sun: bigger than solar system!

with $\quad R_v = (R_S m^{-4})^{1/5}$

So, what is going on at smaller distances?

Vainshtein 1972

There exists an other perturbative expansion at smaller distances, defined around (ordinary) Schwarzschild and reading:

$$\left.\begin{aligned} \nu(R) &= -\frac{R_S}{R}\left\{1+\mathcal{O}\left(R^{5/2}/R_v^{5/2}\right)\right\} \\ \lambda(R) &= +\frac{R_S}{R}\left\{1+\mathcal{O}\left(R^{5/2}/R_v^{5/2}\right)\right\} \end{aligned}\right\} \quad \text{with} \quad R_v^{-5/2} = m^2 R_S^{-1/2}$$

- This goes smoothly toward Schwarzschild as *m* goes to zero

- This leads to corrections to Schwarzschild which are non analytic in the Newton constant

To summarize: 2 regimes

$$\nu(R) = -\frac{R_S}{R}\left(1 + \mathcal{O}(1)\epsilon + \cdots\right) \quad \text{with} \quad \epsilon = \frac{R_S}{m^4 R^5}$$

Valid for $R \gg R_V$ with $R_V = \left(R_S m^{-4}\right)^{1/5}$

Standard perturbation theory around flat space

Crucial question: can one join the two regimes in a single existing non singular (asymptotically flat) solution? **(Boulware Deser 72)**

Expansion around Schwarzschild solution

$$\nu(R) = -\frac{R_S}{R}\left(1 + \mathcal{O}\left(R^{5/2}/R_V^{5/2}\right)\right)$$

Valid for $R \ll R_V$

This was investigated (by numerical integration) by
Damour, Kogan and Papazoglou (2003)

No non-singular solution found matching the two behaviours (always singularities appearing at finite radius)

(see also Jun, Kang 1986)

In the 2nd part of this talk:

A new look on this problem using in particular the « Goldstone picture » of massive gravity in the « Decoupling limit. »

(in collaboration with E. Babichev and R.Ziour 2009-2010)

1.4. The Goldstone picture (and « decoupling limit ») of non linear massive gravity, and what can one get from it ?

Originally proposed in the analysis of Arkani-Hamed, Georgi and Schwartz (2003) using « Stückelberg » fields …

and leads (For a generic theory in the PF universality class) to the cubic action in the scalar sector (helicity 0) of the model

$$\frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{M_P}\tilde{\phi}T + \frac{1}{\Lambda^5}\left\{(\nabla^2\tilde{\phi})^3 + \underbrace{\ldots}_{\text{Other cubic terms omitted}}\right\}$$

« Strong coupling scale »
(hidden cutoff of the model ?)

With $\Lambda = (m^4 M_P)^{1/5}$

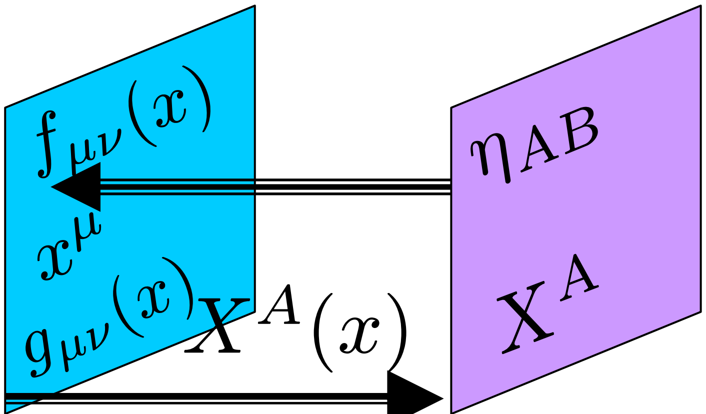
Basic idea

The theory considered has the usual diffeo invariance

$$\begin{cases} g_{\mu\nu}(x) &= \partial_\mu x'^\sigma(x)\partial_\nu x'^\tau(x) g'_{\sigma\tau}(x'(x)) \\ f_{\mu\nu}(x) &= \partial_\mu x'^\sigma(x)\partial_\nu x'^\tau(x) f'_{\sigma\tau}(x'(x)) \end{cases}$$

This can be used to go back and forth from a « unitary gauge » where $f_{AB} = \eta_{AB}$

To a « non unitary gauge » where some of the d.o.f. of the *g* metric are put into *f* thanks to a gauge transformation of the form



$$\begin{aligned} f_{\mu\nu}(x) &= \partial_\mu X^A(x)\partial_\nu X^B(x)\eta_{AB}(X(x)) \\ g_{\mu\nu}(x) &= \partial_\mu X^A(x)\partial_\nu X^B(x) g_{AB}(X(x)) \end{aligned}$$

X^A: 4 scalar fields

[cf. Chamseddine, Mukhanov 2010-2011]

Expand then the theory around the unitary gauge as

$$\begin{cases} X^A(x) = \delta^A_\mu x^\mu + \pi^A(x) \\ \\ \pi^A(x) = \delta^A_\mu \left(A^\mu(x) + \eta^{\mu\nu}\partial_\nu \phi\right) . \end{cases}$$

Unitary gauge coordinates

« pion » fields

The interaction term $S_{int}[f,g]$ expanded at quadratic order in the new fields A^μ and ϕ reads

$$\frac{M_P^2 m^2}{8}\int d^4x \qquad \left[h^2 - h_{\mu\nu}h^{\mu\nu} - F_{\mu\nu}F^{\mu\nu} \right.$$
$$\left. -4(h\partial A - h_{\mu\nu}\partial^\mu A^\nu) - \boxed{4(h\partial^\mu\partial_\mu\phi - h_{\mu\nu}\partial^\mu\partial^\nu\phi)}\right]$$

A^μ gets a kinetic term via the mass term

ϕ only gets one via a mixing term

One can demix ϕ from *h* by defining

$$h_{\mu\nu} = \hat{h}_{\mu\nu} - m^2 \eta_{\mu\nu} \phi$$

And the interaction term reads then at quadratic order

$$S = \frac{M_P^2 m^2}{8} \int d^4x \quad \left\{ \hat{h}^2 - \hat{h}_{\mu\nu} \hat{h}^{\mu\nu} - F_{\mu\nu} F^{\mu\nu} - 4(\hat{h}\partial A - \hat{h}_{\mu\nu} \partial^\mu A^\nu) \right.$$
$$\left. +6m^2 \left[\phi(\partial_\mu \partial^\mu + 2m^2)\phi - \hat{h}\phi + 2\phi\partial A \right] \right\}$$

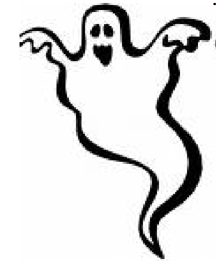
The canonically normalized ϕ is given by $\tilde{\phi} = M_P m^2 \phi$

Taking then the
« **Decoupling Limit** »

$$\left\{ \begin{array}{rcl} M_P & \to & \infty \\ m & \to & 0 \\ \Lambda = (m^4 M_P)^{1/5} & \sim & const \\ T_{\mu\nu}/M_P & \sim & const, \end{array} \right.$$

One is left with …

« Strong coupling scale »
(hidden cutoff of the model ?)

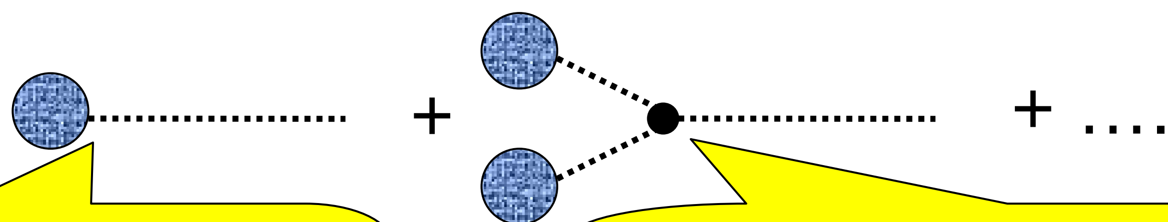


$$\frac{1}{2}\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu} - \frac{1}{M_P}\tilde{\phi}T - \frac{1}{\Lambda^5}\left\{\alpha\,(\Box\tilde{\phi})^3 + \beta\,(\Box\tilde{\phi}\;\tilde{\phi}_{,\mu\nu}\;\tilde{\phi}^{,\mu\nu})\right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$ and α and β model dependent coefficients

In the decoupling limit, the Vainshtein radius is kept fixed, and one can understand the Vainshtein mechanism as

E.g. around a heavy source: of mass M



Interaction M/M $_P$ of the external source with $\tilde{\phi}$

The cubic interaction above generates O(1) coorrection at $R = R_v \equiv (R_S m^{-4})^{1/5}$

An other non trivial property of non-linear Pauli-Fierz: at non linear level, it propagates 6 instead of 5 degrees of freedom, the energy of the sixth d.o.f. having no lower bound!

Using the usual ADM decomposition of the metric, the non-linear PF Lagrangian reads (for $\eta_{\mu\nu}$ flat)

$$M_P^2 \int d^4x \left\{ \left(\pi^{ij} \dot{g}_{ij} - N R^0 - N_i R^i \right) - m^2 \left(h_{ij} h_{ij} - 2 N_i N_i - h_{ii} h_{jj} + 2 h_{ii} \underbrace{\left(1 - N^2 + N_k g^{kl} N_l \right)}_{} \right) \right\}$$

With
$$\begin{cases} N \equiv \left(-g^{00}\right)^{-1/2} \\ N_i \equiv g_{0i} \end{cases}$$

Neither N_i , **nor *N*** are Lagrange multipliers

The e.o.m. of N_i **and *N*** determine those as functions of the other variables

Boulware, Deser '72

6 propagating d.o.f., corresponding to the g_{ij}

Moreover, the reduced Lagrangian for those propagating d.o.f. read

Boulware, Deser 1972

$$M_P^2 \quad \underbrace{\int d^4x \left\{ \pi^{ij}\dot{g}_{ij} - m^2\left(h_{ij}h_{ij} - h_{ii}h_{jj}\right) - \frac{1}{8m^2} R^l \left(\eta - h_{ii} g\right)^{-1}_{lm} R^m - \frac{1}{8m^2 h_{ii}} (R^0)^2 - 2m^2 h_{ii} \right\}}$$

$\Rightarrow$ Unbounded from below Hamiltonian

This can be understood in the « Goldstone » description

C.D., Rombouts 2005
(See also Creminelli, Nicolis, Papucci, Trincherini 2005)

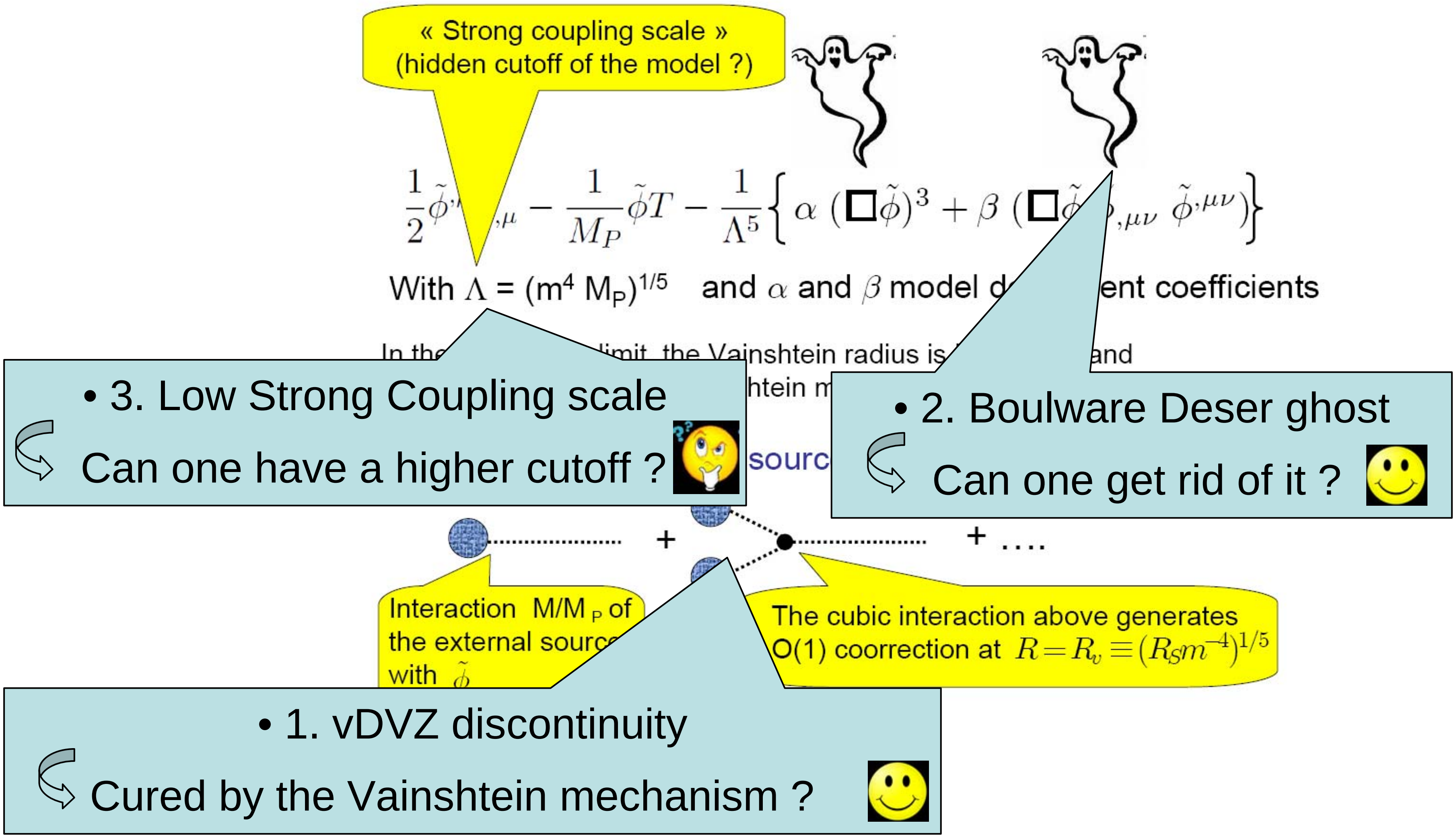
Indeed the action for the scalar polarization

$$\frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{M_P}\tilde{\phi}T + \frac{1}{\Lambda^5}\left\{(\nabla^2\tilde{\phi})^3 + \quad \ldots \quad \right\}$$

Leads to order 4 E.O.M. $\Rightarrow$, it describes two scalars fields, one being ghost-like

Summary of the first part: the 3 sins of massive gravity

They can all be seen at the Decoupling Limit level



The end of part 1

Part 2. Some cures and open issues.

2.1. The Vainshtein mechanism

2.2. Vainshtein does not work for Black Holes.

2.3. Getting rid of the Boulware-Deser ghost

2.4. Strong coupling and UV completion (back to DGP like models ?)

2.5. Some other approaches to non linear massive gravity

2.1. The Vainshtein mechanism

Babichev, C.D., Ziour, 2009, 2010

Framework: non linear Pauli-Fierz theory

$$S = \int d^4x\sqrt{-g}\,\left(\frac{M_P^2}{2}R_g + L_g\right) + S_{int}[f,g]$$

Leads to the e.o.m. $M_P^2 G_{\mu\nu} = \left(T_{\mu\nu} + T^g_{\mu\nu}(f,g)\right)$

Matter energy-momentum tensor

Effective energy-momentum tensor (f,g) dependent

Bianchi indentity $\Rightarrow\ \nabla^\mu T^g_{\mu\nu} = 0$

$$S^{(3)}_{int} = -\frac{1}{8}m^2M_P^2\int d^4x\,\sqrt{-g}\,H_{\mu\nu}H_{\sigma\tau}\left(g^{\mu\sigma}g^{\nu\tau} - g^{\mu\nu}g^{\sigma\tau}\right)$$

(Arkani-Hamed, Georgi, Schwartz)

Ansatz (« λ, μ, ν » gauge)

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \\
f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2
\end{aligned}
$$

With this ansatz the e.o.m (+ Bianchi) read

$$
\begin{aligned}
"G_{tt}" \Longrightarrow e^{\nu-\lambda}\left(\frac{\lambda'}{R} + \frac{1}{R^2}(e^\lambda - 1)\right) &= 8\pi G_N \left(T^g_{tt} + \rho e^\nu\right) \\
"G_{RR}" \Longrightarrow \frac{\nu'}{R} + \frac{1}{R^2}\left(1 - e^\lambda\right) &= 8\pi G_N \left(T^g_{RR} + P e^\lambda\right) \\
"Bianchi" \Longrightarrow \nabla^\mu T^g_{\mu R} &= 0
\end{aligned}
$$

$$T^g_{tt} = m^2 M_P^2 \, f_t, \quad T^g_{RR} = m^2 M_P^2 \, f_R, \quad \nabla^\mu T^g_{\mu R} = -m^2 M_P^2 f_g,$$

$$f_t = \frac{e^{-\lambda-2\mu}}{4}$$

$$\times \left[\left(3e^{\mu+\nu} + e^{\mu} - 2e^{\nu}\right) \left(1 - \frac{R\mu'}{2}\right)^2 + e^{\lambda} \left(2e^{\mu} - e^{\nu}\right) - 3e^{\lambda+\mu} \left(2e^{\mu+\nu} + e^{\mu} - 2e^{\nu}\right) \right]$$

$$f_R = \frac{e^{-\nu-2\mu}}{4}$$

$$\times \left[\left(3e^{\mu+\nu} - e^{\mu} - 2e^{\nu}\right) \left(1 - \frac{R\mu'}{2}\right)^2 + e^{\lambda} \left(2e^{\mu} + e^{\nu}\right) - 3e^{\lambda+\mu} \left(-2e^{\mu+\nu} + e^{\mu} + 2e^{\nu}\right) \right]$$

$$f_g = -\left(1 - \frac{R\mu'}{2}\right) \frac{e^{-\lambda-2\mu-\nu}}{8R}$$

$$\times \left[8\left(e^{\lambda} - 1\right)\left(3e^{\mu+\nu} - e^{\mu} - e^{\nu}\right) + 2R\left(\left(3e^{\mu+\nu} - 2e^{\nu}\right)\left(\lambda' + 4\mu' - \nu'\right) - e^{\mu}\left(\lambda' + 4\mu' + \nu'\right)\right)\right.$$

$$\left. -R^2 \left(\left(3e^{\mu+\nu} - 2e^{\nu}\right)\left(\lambda'\mu' - 2\mu'' - \mu'\nu' + (\mu')^2\right) - e^{\mu}\left(\lambda'\mu' - 2\mu'' + \mu'\nu' + (\mu')^2\right) - 2e^{\nu}(\mu')^2\right)\right]$$

To obtain our solutions, we used the Decoupling Limit, we first…

« shooted »

Then « relaxed »

We used a combination of shooting and relaxation methods
+ some analytic insight relying on (asymptotic) expansions,

with appropriate Boundary conditions
(asymptotic flatness, no singularity in R=0)

For setting boundary (or initial) conditions for the numerical integration, and better understand the result, we used crucially the Decoupling Limit.

To obtain the Decoupling Limit here, first do the rescaling

$$\begin{cases} \tilde{\nu} & \equiv & M_P \nu \\ \tilde{\lambda} & \equiv & M_P \lambda \\ \tilde{\mu} & \equiv & m^2 M_P \mu \end{cases}$$

And then let

$$\begin{cases} M_P & \rightarrow & \infty \\ m & \rightarrow & 0 \\ \Lambda = (m^4 M_P)^{1/5} & \sim & const \\ T_{\mu\nu}/M_P & \sim & const, \end{cases}$$

The full (non linear) system of e.o.m collapses to

$$\boxed{\begin{array}{rcl} \frac{\tilde{\lambda}'}{R} + \frac{\tilde{\lambda}}{R^2} & = & -\frac{1}{2}(3\tilde{\mu} + R\tilde{\mu}') + \tilde{\rho} \\ \frac{\tilde{\nu}'}{R} - \frac{\tilde{\lambda}}{R^2} & = & \tilde{\mu} \\ \frac{\tilde{\lambda}}{R^2} & = & \frac{\tilde{\nu}'}{2R} + \frac{Q(\tilde{\mu})}{\Lambda^5} \end{array}}$$

System of equations to be solved in the DL

$$\begin{aligned}
\frac{\tilde{\lambda}'}{R} + \frac{\tilde{\lambda}}{R^2} &= -\frac{1}{2}(3\tilde{\mu} + R\tilde{\mu}') + \tilde{\rho} \\
\frac{\tilde{\nu}'}{R} - \frac{\tilde{\lambda}}{R^2} &= \tilde{\mu} \\
\frac{\tilde{\lambda}}{R^2} &= \frac{\tilde{\nu}'}{2R} + \frac{Q(\tilde{\mu})}{\Lambda^5}
\end{aligned}$$

System of equations to be solved in the DL

$$\Downarrow$$

$$\frac{1}{\Lambda^5}\left[6Q(\tilde{\mu}) + 2RQ(\tilde{\mu})'\right] + \frac{9}{2}\tilde{\mu} + \frac{3}{2}R\,\tilde{\mu}' = \tilde{\rho}$$

Which can be integrated once to yield the first integral

$$\frac{2}{\Lambda^5}Q(\tilde{\mu}) + \frac{3}{2}\,\tilde{\mu} = -\frac{K}{R^3}$$

This first integral $-\frac{3}{2}\ \tilde{\mu} - \frac{2}{\Lambda^5} Q(\tilde{\mu}) = \frac{K}{R^3}$

$$f_{AB} dx^A dx^B \quad = \quad -dt^2 + dr^2 + r^2 d\Omega^2$$

Recall that μ is encoding the gauge transformation

$$f_{\mu\nu} dx^\mu dx^\nu \quad = \quad -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)} dR^2 + e^{-\mu(R)} R^2 d\Omega^2$$

upon the substitution

$$\tilde{\mu} = -\frac{2}{R}\tilde{\phi}'$$

Yields exactly one which is obtained using the Stückelberg field in the scalar sector $\tilde{\phi}$

$$3\ \frac{\tilde{\phi}'}{R} \quad + \quad \frac{2}{\Lambda^5}\left\{3\alpha\ \left(-4\frac{\tilde{\phi}'}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 2\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + \frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right) + \right.$$
$$\left. +\beta\left(-6\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 4\frac{\tilde{\phi}''^2}{R^2} + 4\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + 3\frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right)\right\} = \frac{K}{R^3}$$

To summarize, in the decoupling limit the full non linear system reduces to

$$
\boxed{
\begin{aligned}
\frac{\tilde{\lambda}'}{R} + \frac{\tilde{\lambda}}{R^2} &= -\frac{1}{2}(3\tilde{\mu} + R\tilde{\mu}') + \tilde{\rho} \\
\frac{\tilde{\nu}'}{R} - \frac{\tilde{\lambda}}{R^2} &= \tilde{\mu} \\
\frac{2}{\Lambda^5} Q(\tilde{\mu}) + \frac{3}{2}\,\tilde{\mu} &= -\frac{K}{R^3}
\end{aligned}
}
$$

Which can be shown to give the leading behaviour of the solution in the range $\underbrace{R_S \ll R \ll m^{-1}}$

The Vainshtein radius is in this range

Solving the DL (one only needs to solve the non linear ODE)

$$\frac{3}{2}\ \tilde{\mu} + \underbrace{\frac{2}{\Lambda^5} Q(\tilde{\mu})} = -\frac{K}{R^3}$$

Depends on the interaction term $S_{int}[f,g]$

E.g. in the Case of the two interaction terms (α+β=0)

$$S_{int}^{(2)} = -\frac{1}{8} m^2 M_P^2 \int d^4x\, \sqrt{-f}\, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau} \right)$$

(Boulware Deser)

$$S_{int}^{(3)} = -\frac{1}{8} m^2 M_P^2 \int d^4x\, \sqrt{-g}\, H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)$$

(Arkani-Hamed, Georgi, Schwarz)

This equation boils down to the simple form

$$3w - s\left(\dot{w}^2 + 2w\ddot{w} + 8\frac{w\dot{w}}{\xi} \right) = \frac{2c_0}{\xi^3}$$

With s = $\pm$ 1 and the dimensionless quantities

$$\left\{ \begin{array}{rcl} w &=& (R_v m)^{-2} \mu \\ \xi &=& R/R_V \\ c_0 &=& \dfrac{K}{R_V^2 \Lambda^5} \end{array} \right.$$

$$3w - s\left(\dot{w}^2 + 2w\ddot{w} + 8\frac{w\dot{w}}{\xi}\right) = \frac{2c_0}{\xi^3}$$

With s = $\pm$ 1 and the dimensionless quantities

$$\left\{\begin{array}{lcl} w & = & (R_v m)^{-2}\mu \\ \xi & = & R/R_V \\ c_0 & = & \dfrac{K}{R_V^2\Lambda^5} \end{array}\right.$$

How to read the Vainshtein mechanism and scalings ?

For $\xi \gg 1$ — Keep the linear part — $3w = \frac{2c_0}{\ldots}$ … ξ^{-3}

For ξ … — $s\left(\dot{w}^2 + 2w\ddot{w} + 8\frac{w\dot{w}}{\xi}\right) = \frac{2c_0}{\xi^3}$

Assume a power law scaling $\Rightarrow w \propto \xi^{-1/2}$

However, the situation here is in fact more complicated !

Indeed …

$$3w - s\left(\dot{w}^2 + 2w\ddot{w} + 8\frac{w\dot{w}}{\xi}\right) = \frac{2c_0}{\xi^3}$$

At large ξ (expect w $\propto$ 1/ ξ^3)

A power law expansion of the would-be solution to this problem can be found (here with $c_0 = 1$)

$$w(\xi) = \frac{2}{3\xi^3} + s\frac{4}{3\xi^8} + \frac{1024}{27\xi^{13}} + s\frac{712960}{243\ \xi^{18}} + \frac{104910848}{243\ \xi^{23}} + s\frac{225030664192}{2187\ \xi^{28}} + \ldots$$

Unique « solution » of perturbation theory

However… this series is divergent….

… but seems to give a good asymptotic expansion of the numerical solution at large ξ

- This can easily been checked numerically for
s= *-1* (Boulware Deser)
(where the Vainshtein solution does not exist at small ξ, becoming complex [Damour, Kogan, Papazoglou, 2003] !)

- For *s*=+*1* (Arkani-Hamed et al.) solution is numerically highly unstable, singularities are seemingly arising at finite ξ…

However by using a combination of relaxation method / Runge-Kutta/ Asymptotic expansion ,

one can see that solutions (infinitely many !) with Vainshtein asymptotics at large ξ do exist.

In our case, using « extended » Borel resummation (J. Ecalle)

Formal (divergent) serie $\sum_k a_k \xi^{-k} \xrightarrow{\text{Borel transform}} \sum_k \frac{a_k}{(k-1)!} \tilde{\xi}^{k-1}$

Laplace transform or rather « convolution average» extension

Solution of the ODE

(proof provided to us by J. Ecalle)

s = -1

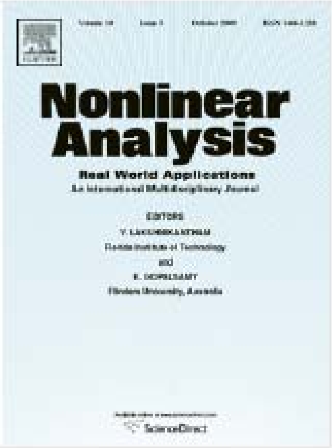
Unique solution with $w \propto 1/\xi^3$ decay at infinity

The difference between any two solutions is given (asymptotically) by $\xi^{3/2} \exp\left(-k\ 3/5\ \xi^{5/2}\right)$ (with integer *k* !)

s = 1

Infinitely many solutions with $w \propto 1/\xi^3$ decay at infinity

Accepted Manuscript



Existence of infinitely many solutions for second–order singular initial value problems with an application to nonlinear massive gravity

J. Ángel Cid, Óscar López Pouso, Rodrigo López Pouso

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DOI:	10.1016/j.nonrwa.2010.09.030
Reference:	NONRWA 1586

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Received date: 25 June 2010
Accepted date: 2 September 2010

So, in the $s=+1$, the perturbation theory does not uniquely fix the solution of the DL at infinity.

Back to the full non linear case

Flat space perturbation theory,
Starting with
($z=R\,m^{-1}$ and $\epsilon \propto G_N$)

$$\left\{\begin{array}{lcl} \nu_0 & = & -\frac{4\epsilon}{3z}e^{-z} \\ \lambda_0 & = & \frac{2\epsilon}{3}\left(1+\frac{1}{z}\right)e^{-z} \\ \mu_0 & = & \frac{2\epsilon}{3z}\left(1+\frac{1}{z}+\frac{1}{z^2}\right)e^{-z} \end{array}\right.$$

$$\left\{\begin{array}{lcl} \lambda & = & \lambda_0+\lambda_1+\ldots \\ \nu & = & \nu_0+\nu_1+\ldots \\ \mu & = & \mu_0+\mu_1+\ldots \end{array}\right.$$

where λ_i, ν_i, μ_i are assumed to be proportional to ϵ^{i+1}

One finds the
unique expansion
At large z (large R)

$$\left\{\begin{array}{lcl} \mu_n & = & \epsilon^{n+1}e^{-(n+1)z}\sum_{i=-\infty}^{i=0}\mu_{n,i}z^i \\ \lambda_n & = & \epsilon^{n+1}e^{-(n+1)z}\sum_{i=-\infty}^{i=0}\lambda_{n,i}z^i \\ \nu_n & = & \epsilon^{n+1}e^{-(n+1)z}\sum_{i=-\infty}^{i=0}\nu_{n,i}z^i \end{array}\right.$$

However, this misses a subdominant (non perturbative) correction of the form

$$\left\{\begin{array}{rcl} \delta\mu &=& F_\infty(z)\exp\left(-\frac{3}{\sqrt{\epsilon}}\,z\,e^{z/2}\right) \\ \delta\lambda &=& -F_\infty(z)\frac{z^2}{2}\exp\left(-\frac{3}{\sqrt{\epsilon}}\,z\,e^{z/2}\right) \\ \delta\nu &=& -F_\infty(z)\frac{\sqrt{\epsilon}}{3}\exp\left(-\frac{z}{2}-\frac{3}{\sqrt{\epsilon}}\,z\,e^{z/2}\right) \end{array}\right.$$

With $F_\infty(z) \sim \mathcal{O}\left(e^{z/4}z^{-3/2}\right)$

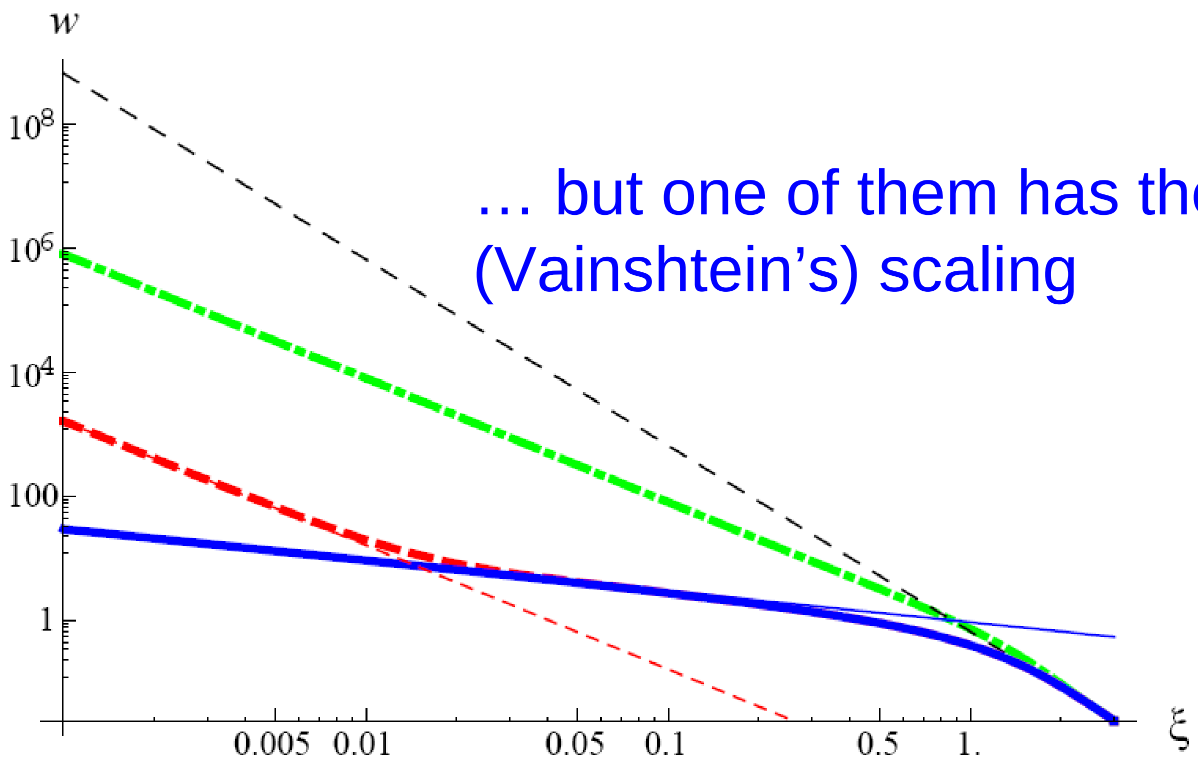
Hence, the solution at large z is not unique !

At small ξ (expect $w \propto 1/\xi^{1/2,}$ when the solution is real)

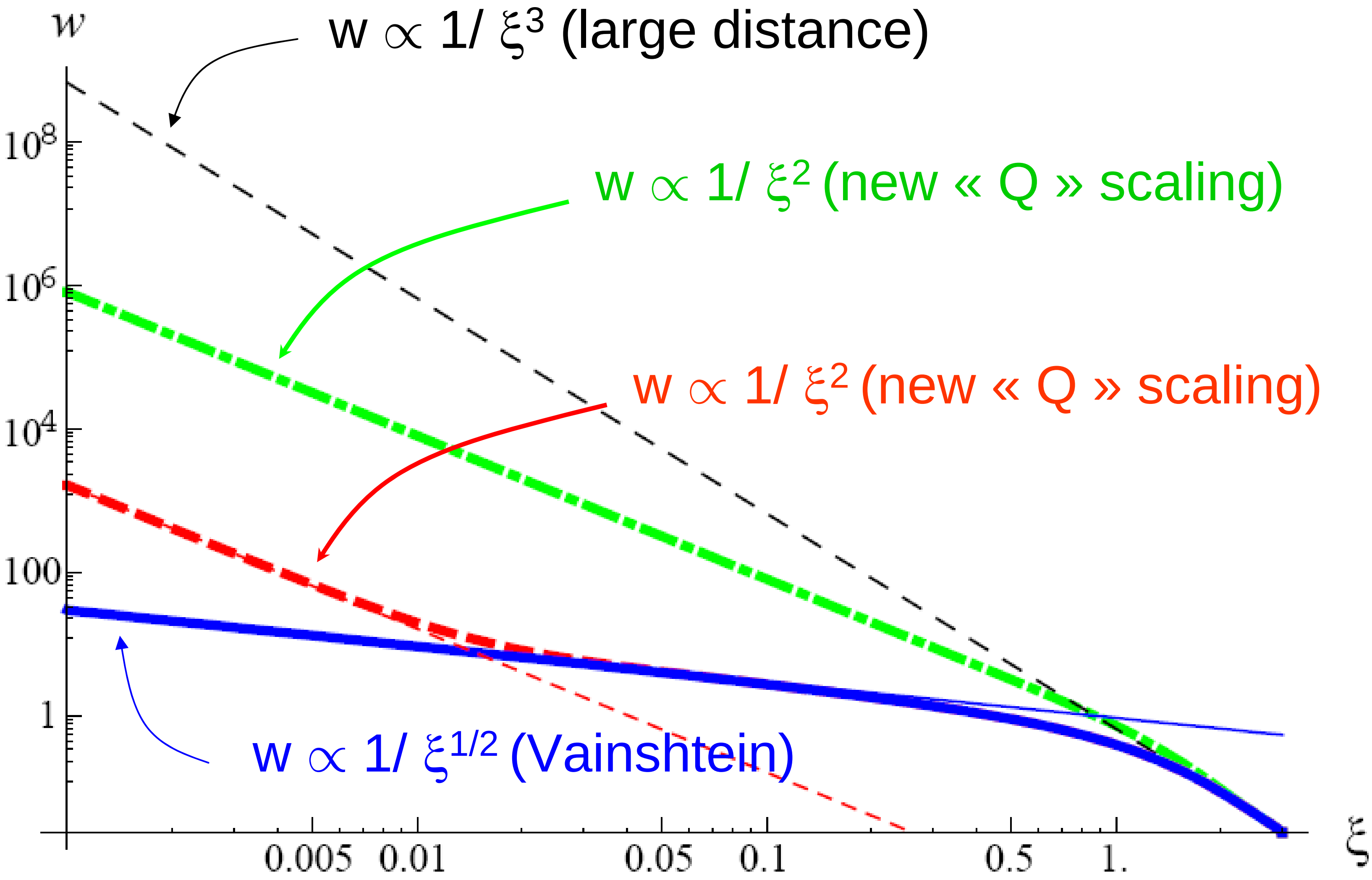
Here we discuss only the *s=+1* case (Arkani-Hamed et al.)

In this case the large distance behaviour $w(\xi) \sim \frac{2}{3\xi^3}$

Does not lead to a unique small distance ($\xi \ll 1$) behaviour (and solution)…



… but one of them has the expected (Vainshtein's) scaling

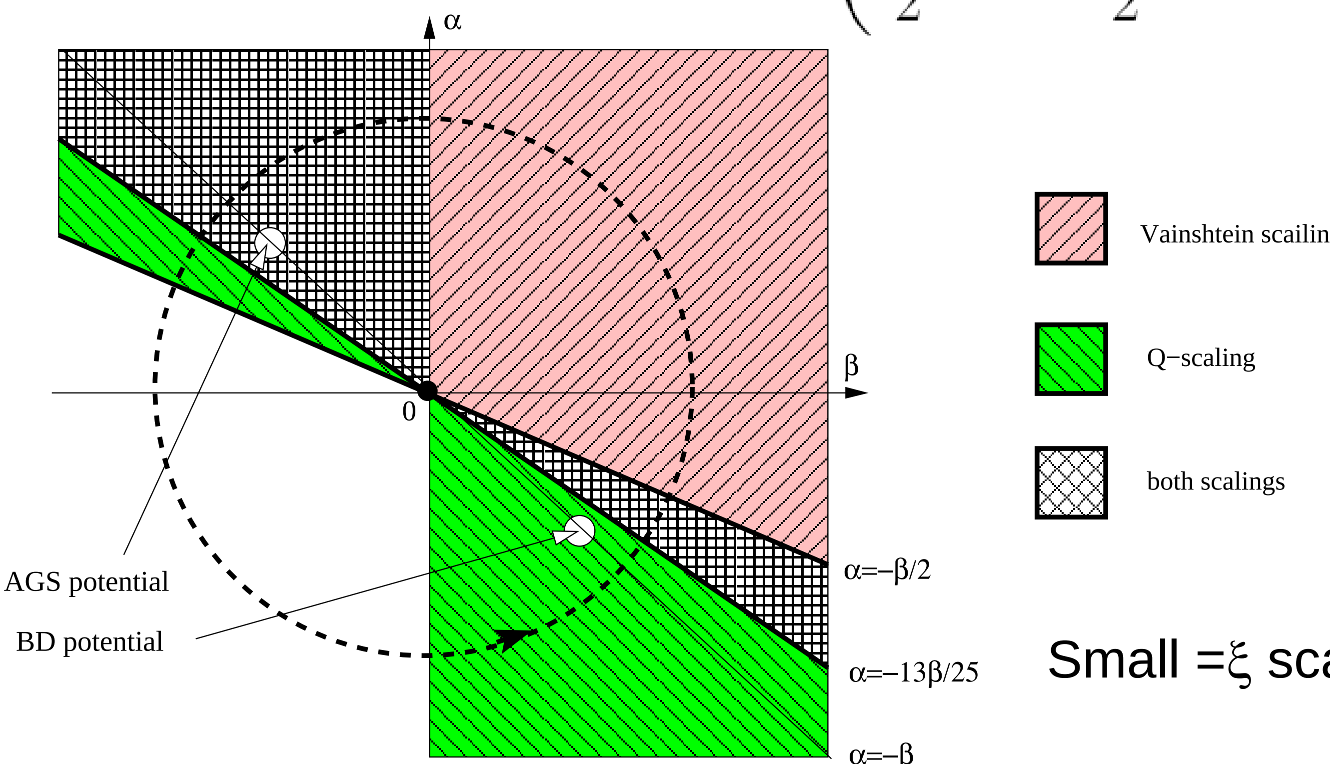


NB: in other cases (e.g. s=-1),
the Vainshtein scaling can be absent

Most general case (general α, β)

We have to solve

$$\begin{cases} 2\,Q(w) + \dfrac{3}{2}w = \dfrac{1}{\xi^3} \\ \text{with} \\ Q(w) = -\dfrac{1}{2}\left\{ 3\alpha\left(\dfrac{\xi}{2}\dot{w}\ddot{w} + \dfrac{3}{2}w\ddot{w} + 2\dot{w}^2 + \dfrac{6w\dot{w}}{\xi}\right) \right. \\ \qquad\qquad \left. + \beta\left(\dfrac{3\xi}{2}\dot{w}\ddot{w} + \dfrac{5}{2}w\ddot{w} + 5\dot{w}^2 + \dfrac{10w\dot{w}}{\xi}\right) \right\} \end{cases}$$



Small =ξ scalings

To summarize our DL findings

- One can find non singular solutions in the DL (but this can be hard because of numerical instabilities).

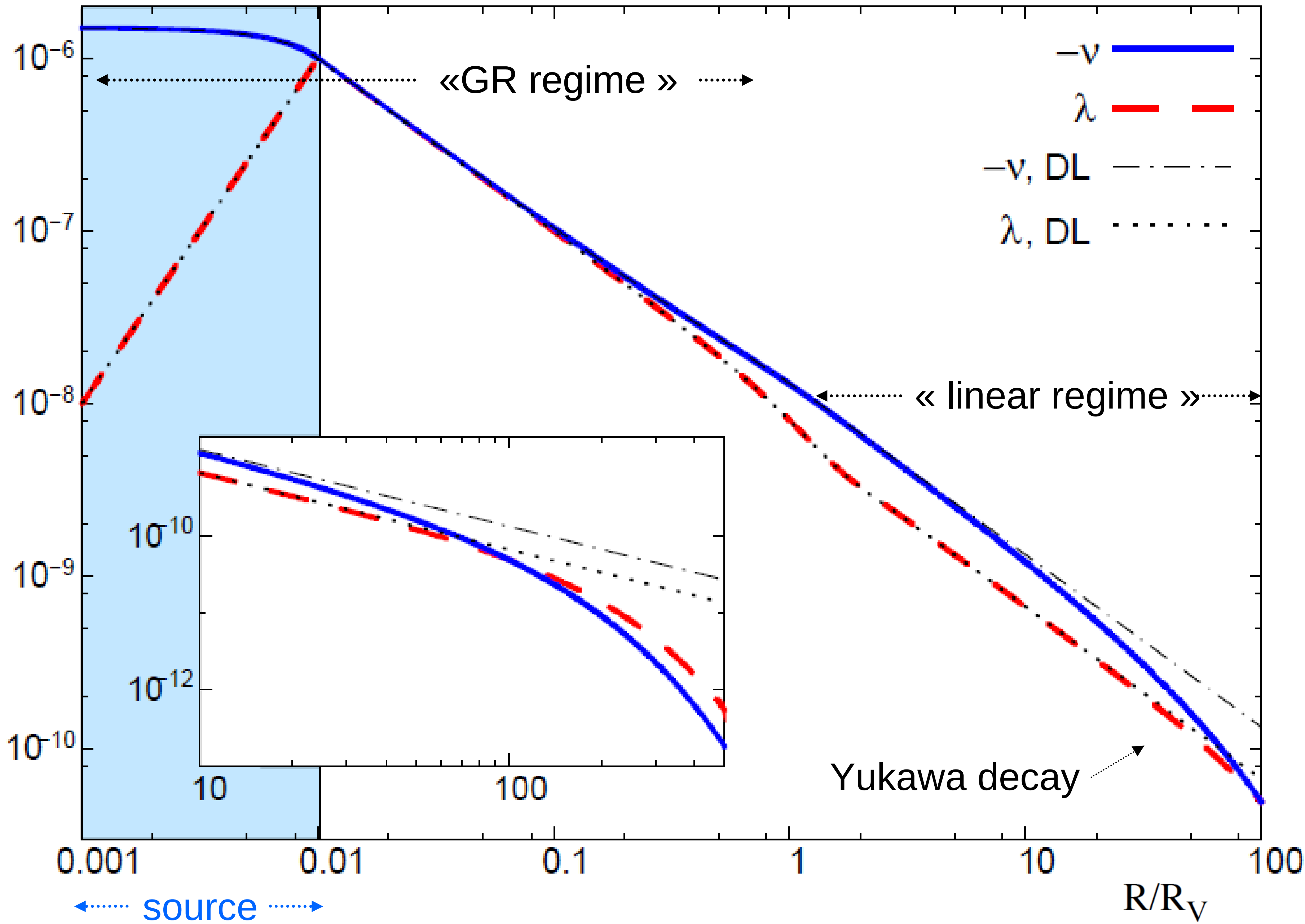
- The ghost does not prevent the existence of those solutions.

- The perturbative expansion (at large R) can be (depending on the potential) not enough to fix uniquely the solution.

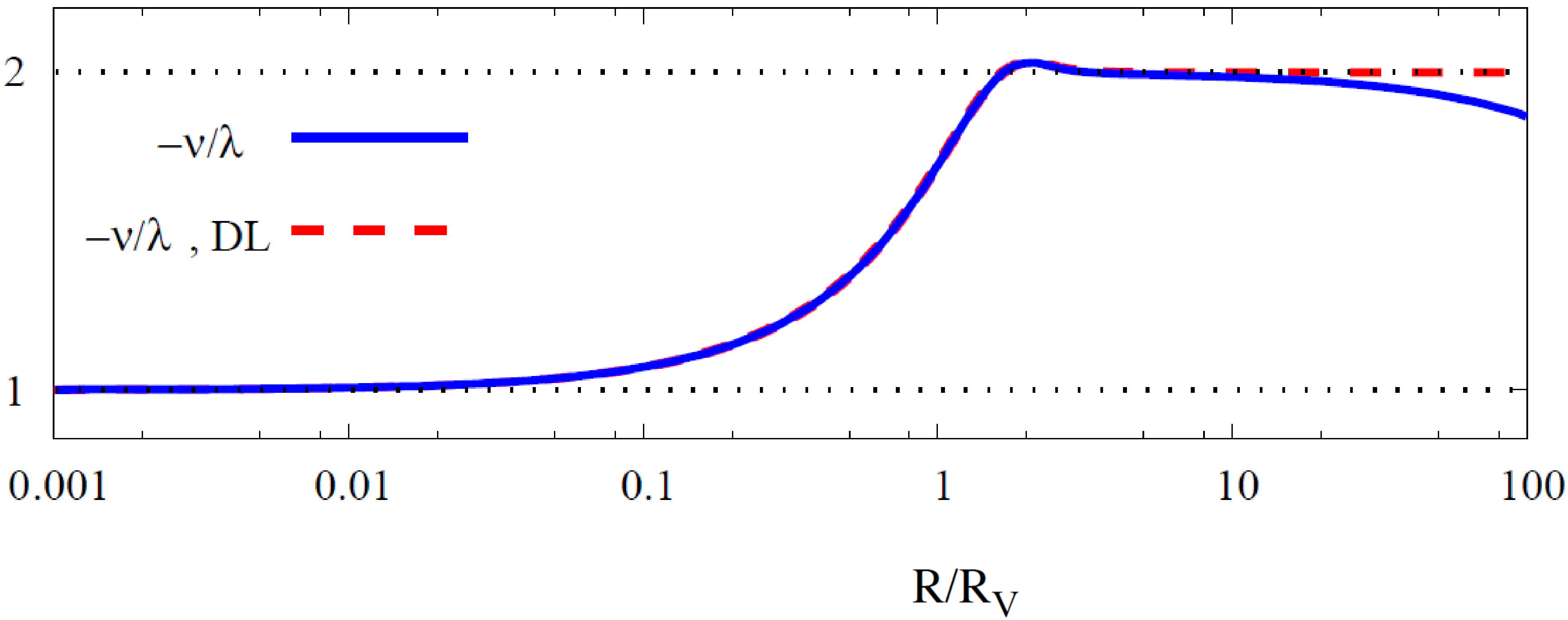
- There is a new possible scaling at small R

- Solution with the correct large R asymptotics cannot always be extended all the way to small R (depending on parameters α and β).

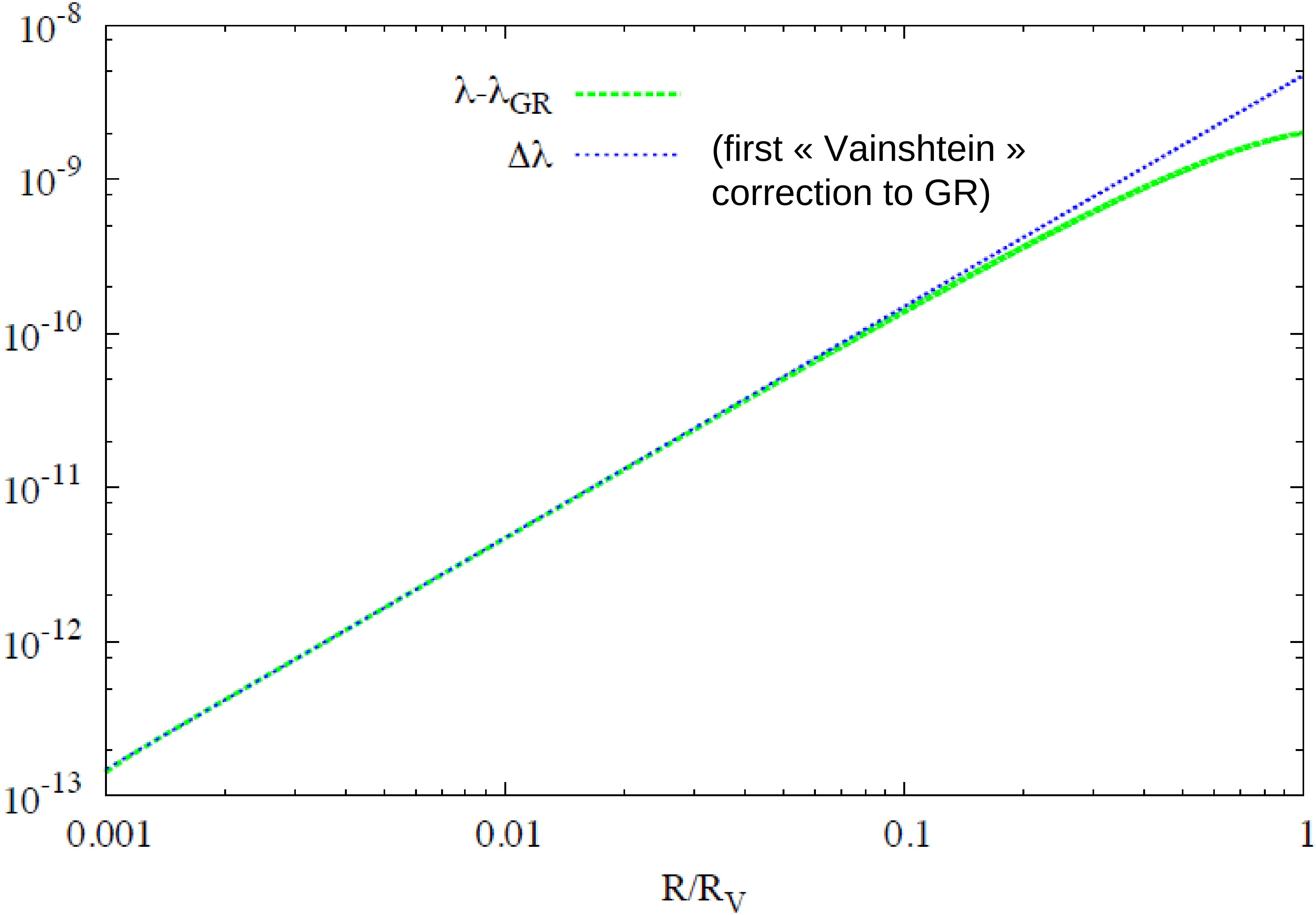
Numerical solutions of the full non linear system



The vDVZ discontinuity gets erased for distances smaller than R_V as expected



Corrections to GR in the $R \ll R_V$ regime



Solutions were obtained for very low density objects. We did (and still do) not know what is happening for dense objects (for BHs we now do know, see thereafter).

The « Q-scaling » does not lead to a physical solution (singularities in R=0)

Conclusion (Vainshtein mechanism in massive gravity)

- It works (numerical results also confirmed by Volkov).
- What is going on for dense object ?
- Black Holes (see next part) ?
- In other models ?
- Gravitational collapse ?

2.2. (Standard) Vainshtein mechanism does not work for black holes.

C.D.,T. Jacobson, CQG 2012
« On horizon structure of bimetric spacetimes »

Can be applied to many cases where one considers space-times hosting two « metrics » [rank-2 covariant tensors]

- Bimetric theories
  Belinfante-Swihart-Lightman-Lee (1957), Isham-Salam-Strathdee (1971), Rosen (1973), Ni (1973), Rastall (1975)…

- Theories with a prefered frame (with a unit vector u^a) where some mode can propagate in an effective metric
  $$g_{ab}^{(i)} = g_{ab} + (v_i^2 - 1)u_a u_b$$
  e.g. Einstein-Aether (Jacobson, Mattingly), Horava gravity, …

- Bimetric theory for MOND (Milgrom), Ghost-condensate related (Dubovsky, Sibiryakov), k-essence (Babichev, Mukhanov, Vikman) …

- (old and recent) « Massive gravity »
  (Isham, Salam, Strathdee; Gababadze, de Rham, Tolley)
  $$f_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \eta_{AB}$$

2.2.1. Generic properties of horizon structure (and some consequences)

C.D.,T.Jacobson, CQG 2012

Consider a theory with two metrics, $g_{\mu\nu}$ and $f_{\mu\nu}$

We want to investigate the consequence of one of the metrics (say *g*) to have a **Killing horizon** (in the static-spherically symmetric or stationary-axisymmetric cases)

Consider first the case where the two metrics are static and spherically symmetric

Proposition 1: Suppose the Killing vector ∂_t is null at $r = r_H$ with respect to $g_{\mu\nu}$. Then if both metrics are diagonal and describe smooth geometries at r_H, ∂_t must also be null with respect to $f_{\mu\nu}$ at $r = r_H$.

i.e. both metric must have the same horizon

First proof (1a)

When both metrics are static and spherically symmetric, they can be put in the form (in a common coordinate system)

$$\begin{aligned} f_{\mu\nu} dx^\mu dx^\nu &= -J(r)dt^2 + K(r)dr^2 + r^2 d\Omega^2 \\ g_{\mu\nu} dx^\mu dx^\nu &= -A(r)dt^2 + 2B(r)dtdr + C(r)dr^2 + D(r)d\Omega^2 \end{aligned}$$

Consider the scalar (assuming $B=0$ at the horizon)

$$g^{\mu\nu} f_{\mu\nu} = J/A + K/C + 2r^2/D$$

It must be regular at the horizon $r=r_H$ if both metrics are regular there

But $A(r_H)=0$, and J/A, K/C and r^2/D have the same sign, so cannot cancel

One must have $J(r_H) = 0$
(and hence the killing horizon of g is also one for f)

Second proof (1b)

(based on theorems by Racz and Wald 1992, 1996)

If a space-time has a Killing horizon, then, under rather general assumptions, it has a « virtual » bifurcation surface.

More precisely:

if a space-time is static (with « t » reflection symmetry) or stationary axisymmetric with « t-ϕ » reflection symmetry, and if the surface gravity of the horizon is non zero (and then constant)

then

There is an extension of a neighborhood of the horizon to one with a bifurcate Killing horizon

(i.e. a Killing horizon which contains a bifurcation surface)

(NB: this applies to any space-time without assuming anything concerning the field equations)

Moreover (Racz-Wald 1996)

Any <u>Killing invariant tensor field sharing</u> the t or the t-ϕ reflection symmetry of the metric

<u>can be extended globally to the enlarged space-time.</u>

Proof 1b: If both metrics $f_{\mu\nu}$ and $g_{\mu\nu}$ are diagonal then $g_{\mu\nu}$ shares the t reflection symmetry of $f_{\mu\nu}$. If the surface gravity of the g-horizon is nonzero, then the Racz-Wald theorem implies that both metrics can be extended to a regular bifurcation surface of the ∂_t Killing horizon for g. The scalar $f_{\mu\nu}\chi^\mu\chi^\nu = J(r)$ vanishes at the bifurcation surface where $\chi^\mu = 0$, and it cannot change along the Killing flow, so it vanishes everywhere at $r = r_H$.

(where χ is the killing vector)

NB: This extends to the stationary-axisymmetric case

This does not preclude the existence of two geometries one with a Killing horizon and one without….

But only implies that the non-horizon geometry cannot possess the t reflection symmetry

E.g.: the existence of a non zero *dt dr* component in the g metric can allow both geometries to be regular at the horizon.

$$\left(\begin{array}{rcl} f_{\mu\nu}dx^\mu dx^\nu & = & -J(r)dt^2 + K(r)dr^2 + r^2 d\Omega^2 \\ g_{\mu\nu}dx^\mu dx^\nu & = & -A(r)dt^2 + 2B(r)dtdr + C(r)dr^2 + D(r)d\Omega^2 \end{array}\right)$$

When this is the case (i.e. when the Killing horizon is not a Killing horizon for the other metric)

The bifurcation surface of the g spacetime cannot lie in the interior of the f space-time

Conversely, when the horizons coincide, they must have the same surface gravity

This can be put together as

If a Killing horizon of a metric g has a bifurcation surface that lies in the interior of the spacetime of another metric f with the same Killing vector, then it must also be a Killing horizon of f, and with the same surface gravity.

2.2.2. Some consequences for non-linear Pauli Fierz

$$S = \int d^4x \sqrt{-g} \; \left(\frac{M_P^2}{2} R_g + L_g \right) + S_{int}[f, g]$$

- (Standard) Vainshtein mechanism does not work for black holes

- Causal structure of static spherically symmetric solutions

• (Standard) Vainshtein mechanism does not work for black holes

Indeed, in the standard way of looking at Vainshtein mechanism of « massive gravity » one has two bi-diagonal metric

« Massive metric »

Flat metric

$$
\begin{aligned}
g_{AB}dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\
f_{AB}dx^A dx^B &= -dt^2 + dr^2 + r^2 d\Omega^2
\end{aligned}
$$

In any theory where the Vainshtein mechanism is working for recovering a solution close to the Schwarschild Black Hole, the *g* metric must have a (spherical) Killing horizon at $r=r_H$ … this must also be a killing horizon for *f*

Impossible:
Minkowski ST has no spherical Killing horizons (but only planar)

NB: this applies also to the new massive gravity of
de Rham, Gabadadze, Tolley (and in particular to solutions of Nieuwenhuizen; Gruzinov, Mirbabayi)

• Causal structure of « type I » static spherically symmetric solutions

$$
\begin{aligned}
f_{\mu\nu} dx^\mu dx^\nu &= -J(r)dt^2 + K(r)dr^2 + r^2 d\Omega^2 \\
g_{\mu\nu} dx^\mu dx^\nu &= -A(r)dt^2 + 2B(r)dtdr + C(r)dr^2 + D(r)d\Omega^2
\end{aligned}
$$

« Type I » solutions: those with $B \neq 0$

Salam, Strathdee 1977
Isham, Storey 1978

(as opposed to « type II » solutions, with B = 0, such as the ones discussed so far when addressing the Vainshtein mechanism - (cf. « λ, μ, ν ansatz ») previous part of this talk)

2.2.1. some Type I solutions are known analytically and simple

(Salam, Strathdee 1977, Isham, Storey, 1978, Damour, Kogan, Papazoglou 2003; see also Berezhiani, Comelli, Nesti, Pilo, 2008)

$$g_{\mu\nu}dx^\mu dx^\nu = (1-q)dt^2 - (1-q)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

$$f_{\mu\nu}dx^\mu dx^\nu = \tfrac{2}{3\beta}(1-p)dt^2 - 2Ddtdr - Adr^2 - 2/3r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

With
$$\begin{cases} A = \frac{2}{3\beta}(1-q)^{-2}\left(p+\beta - q - \beta q\right) \\ D^2 = \left(\frac{2}{3\beta}\right)^2 (1-q)^{-2}(p-q)(p+\beta-1-\beta q) \end{cases}$$

Integration constant

and
$$\begin{cases} p = \dfrac{2M_f}{r} + \dfrac{2\Lambda_f}{9}r^2 \\ q = \dfrac{2M_g}{r} + \dfrac{\Lambda_g}{3}r^2 \end{cases}$$

Both metric are of Schwarzschild-(A)dS form (no sign of vDVZ or massive gravity!)

Namely, the change of variable $d\tilde{t} = \frac{1}{\sqrt{\beta}}\left\{dt \mp \frac{\sqrt{(p-q)(p+\beta-1-\beta q)}}{(1-q)(1-p)}dr\right\}$

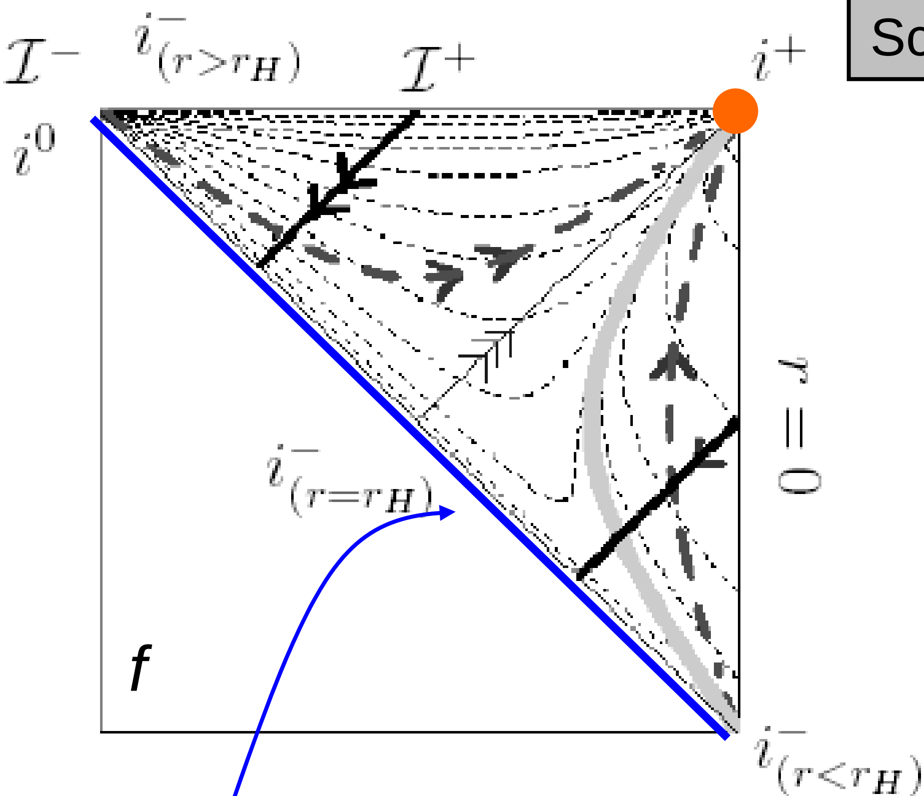
Put the metric $f_{\mu\nu}$ in the usual static form of S(A)dS:

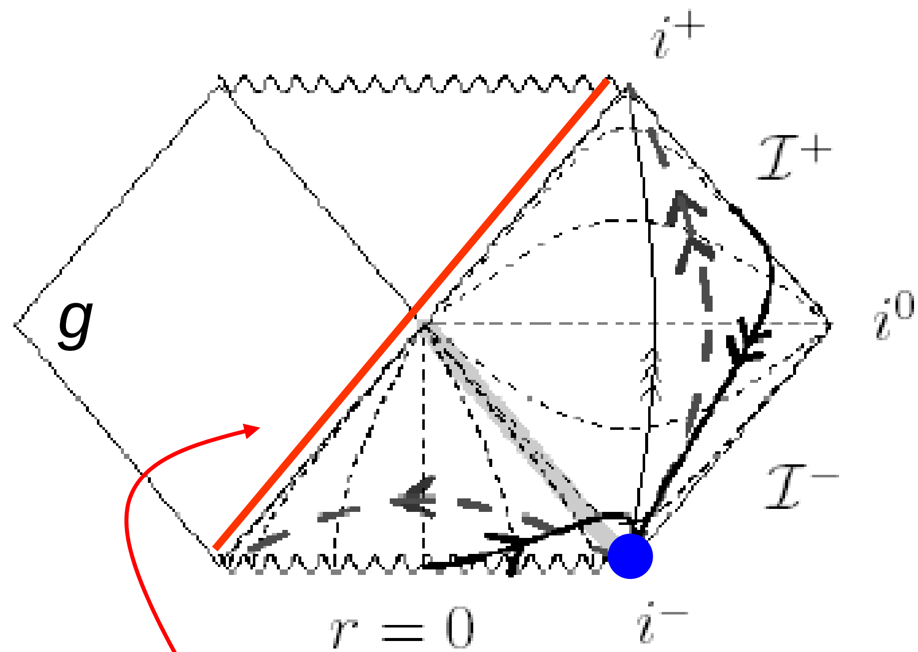
$$f_{\mu\nu}dx^\mu dx^\nu = \tfrac{2}{3}\left\{(1-p)d\tilde{t}^2 - (1-p)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right\}$$

Causal Structure

Blas, C.D., Garriga 2005

E.g. de Sitter (r_H) with Schwarzschild (r_s) with $r_s < r_H$





Part of the dS horizon mapped into the past timelike infinity of $r=r_H$ 2-sphere of Schwarzschild

Part of the Schwarzshild horizon mapped into the future timelike infinity of $r=r_s$ 2-sphere of de Sitter

Bifurcation sphere of one space-time does not lie in the interior of the other …

Conclusions

There exist interesting global constraints on putting together two metrics on a same manifold

- One simple consequence: failure of the usual Vainshtein mechanism to recover Black holes (but there exist non diagonal solutions crossing the horizon)

- Consequence for superluminal issues ?

- One simple question: What is the ending point of spherical collapse ?

2.3. Getting rid of the Boulware Deser ghost

de Rahm, Gabadadze; de Rham, Gababadze, Tolley 2010, 2011

Claim: the most general massive gravity (in the sense above) devoid of a Boulware Deser ghost is given by the 3 (4 counting Λ) parameters set of theories:

$$S = M_P^2 \int d^4x \sqrt{-g} \left\{ R + 2m^2 \sum_{n=1}^{3} \beta_n e_n(\mathbf{K}) \right\}$$

With

$$\begin{cases} K^{\mu}{}_{\nu} = \sqrt{g^{\mu\rho} f_{\rho\nu}} \\ e_1(\mathbf{K}) = tr\, \mathbf{K} \\ e_2(\mathbf{K}) = \frac{1}{2}\left((tr\, \mathbf{K})^2 - tr\, \mathbf{K}^2 \right) \\ e_3(\mathbf{K}) = \frac{1}{6}\left((tr\, \mathbf{K})^3 - 3\, tr\, \mathbf{K}\, tr\, \mathbf{K}^2 + 2\, tr\, \mathbf{K}^3 \right) \end{cases}$$

The absence of ghost is first seen in the decoupling limit

(using the observations of C.D., Rombouts 2005; Creminelli, Nicolis, Papucci, Trincherini 2005)

Which instead of the generic

$$\frac{1}{2}\tilde{\phi}\Box\tilde{\phi} + \frac{1}{\Lambda^5}\left\{\alpha\left(\Box\tilde{\phi}\right)^3 + \beta\left(\Box\tilde{\phi}\right)\tilde{\phi}_{,\mu\nu}\tilde{\phi}^{,\mu\nu}\right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$

Looks like (de Rham, Gabadadze, 2010)

$$\begin{aligned}\frac{1}{2}\tilde{\phi}\Box\tilde{\phi} \quad &+ \quad \frac{1}{\Lambda_3^3}\tilde{\alpha}\left(\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu}\right)\Box\tilde{\phi} \\ &+ \quad \frac{1}{\Lambda_3^6}\tilde{\beta}\left(\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu}\right)\left(\tilde{\phi}_{,\mu\nu}\tilde{\phi}^{,\mu\nu} - \left(\Box\tilde{\phi}\right)^2\right) \\ &+ \quad \cdots\end{aligned}$$

With $\Lambda_3 = (m^2 M_P)^{1/3}$

The absence of ghost in the full theory has been heavily debated (NB: nothing clear about positivity of the Hamiltonian)

Gabadadze, de Rham, Tolley;
Alberte, Chamseddine, Mukhanov;
Hassan, Rosen, Kluson…

One simple reason for which one gets an extra constraint:

(C.D., Mourad, Zahariade in preparation;
Hinterblicher, Rosen arXiv:1203.5783 [hep-th])

$$\begin{cases} g^{\mu\nu} &= \eta^{AB} e^{\mu}_{A} e^{\nu}_{B} \\ f_{\mu\nu} &= \eta_{AB} \omega^{A}_{\mu} \omega^{B}_{\nu} \end{cases}$$

with

$$e^{\mu}_{A} \omega_{B\mu} = e^{\mu}_{B} \omega_{A\mu}$$

The mass term

$$S = M_P^2 m^2 \int d^4x \sqrt{-g} \sum_{n=1}^{3} \beta_n e_n \left(\mathbf{K}\right)$$

Can be written as L.C. of

$$S_\kappa = M_P^2 m^2 \int d^4x \det\left(\kappa \omega^A_\nu - e^A_\nu\right)$$

2.4. Strong coupling and UV completion

A crucial question for the sake of massive gravity and also …. for the DGP model:

Find a proper UV completion of the model .

For DGP model, ⇨ Yes/ May be ?

Antoniadis, Minasian, Vanhove; Kohlprath, Vanhove; Kiritsis, Tetradis, Tomaras; Corley, Lowe, Ramgoolam.

String theory? ⇨ No/ May be not ?

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi.

2.5. Some other approaches to Non-Linear massive gravity

« Torsion massive gravity »

Nair, Randjbar-Daemi, Rubakov, 2009
Nikiforova, Randjbar-Daemi, Rubakov, 2009
C.D., Randjbar-Daemi, 2011

$$\begin{cases} S_m(u,u) = -\dfrac{\tilde{M}^2}{2}\displaystyle\int \sqrt{-g}(u_{ij}u^{ij} - u^2) \\ S_W(u,u) = \dfrac{s}{2}\displaystyle\int \sqrt{-g}W_{iklj}u^{ij}u^{kl} \end{cases}$$

« New Massive Gravity » in 3D and 4D

Bergshoeff, Hohm, Townsend, 2009
Bergshoeff, Fernandez-Melgarejo, Rosseel, Townsend, 2012

Conclusions

Massive gravity is a nice arena to explore large distance modifications of gravity.

A first, possibly consistent (?), non linear theory has recently been proposed (after about 10 years of progresses following the DGP model)…

… with many things still to be explored (in particular, stability issues).