Matrix Theories and Emergent Space

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This talk is based on arXiv:1207.0886, 1301.3722, 1301.3738, 1301.7062 and on ongoing research which will soon appear on the archive.

Many thank's to Jan Troost, for very useful discussions, and to my students Antonin Rovai and Micha Moskovic with whom this project is being continued.

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Black holes in high energy scattering, UV/IR relations, holographic properties, ...

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Weinberg and Witten 1980: rules out the simplest models (something that background independence and the lack of local observables clearly do)

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Our main example of a theory of emergent space along these ideas is of course the AdS/CFT correspondence. However, the correspondence has been mainly used to study properties of strongly coupled large N field theories from gravity. The other direction in the correspondence, studying gravity from field theory, is much less explored. This is not surprising: classical gravity is more tractable that strongly coupled field theories...

Moreover, typical field theory calculations yield expansions in the coupling constants, from which it is highly non-trivial to find hints about a geometrical interpretation.

Our aim in this talk will be to present a strategy to overcome these difficulties, and to briefly review a few simple applications, like for a model of D-particles in the presence of a large number of D4-branes in type IIA or D-instantons in the presence of a large number of D3-branes in type IIB.

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Step two: we have to understand how to sum up the usual multi-loop large N diagrams that are relevant to the observables mentioned in step one.

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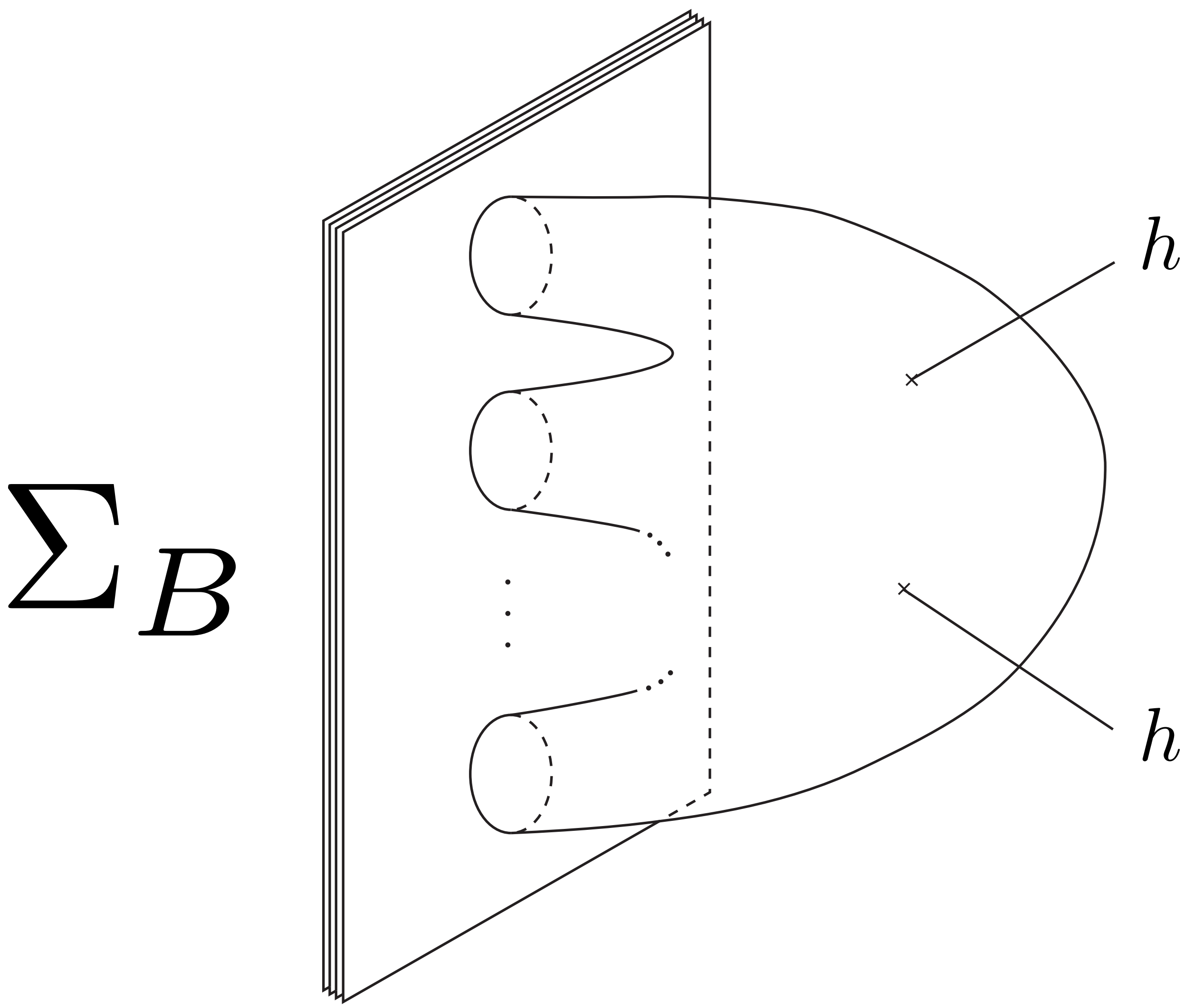
We shall explicitly find full supergravity backgrounds, *including non-trivial dilaton profile, Neveu-Schwarz B field and correctly quantized Ramond-Ramond forms,* without ever solving a supergravity equation of motion. To our knowledge, such detailed information on the backgrounds has never been obtained by any other method.

Maybe more importantly, the basic ideas we use can be applied to many other cases, even in the absence of conformal invariance and supersymmetry.

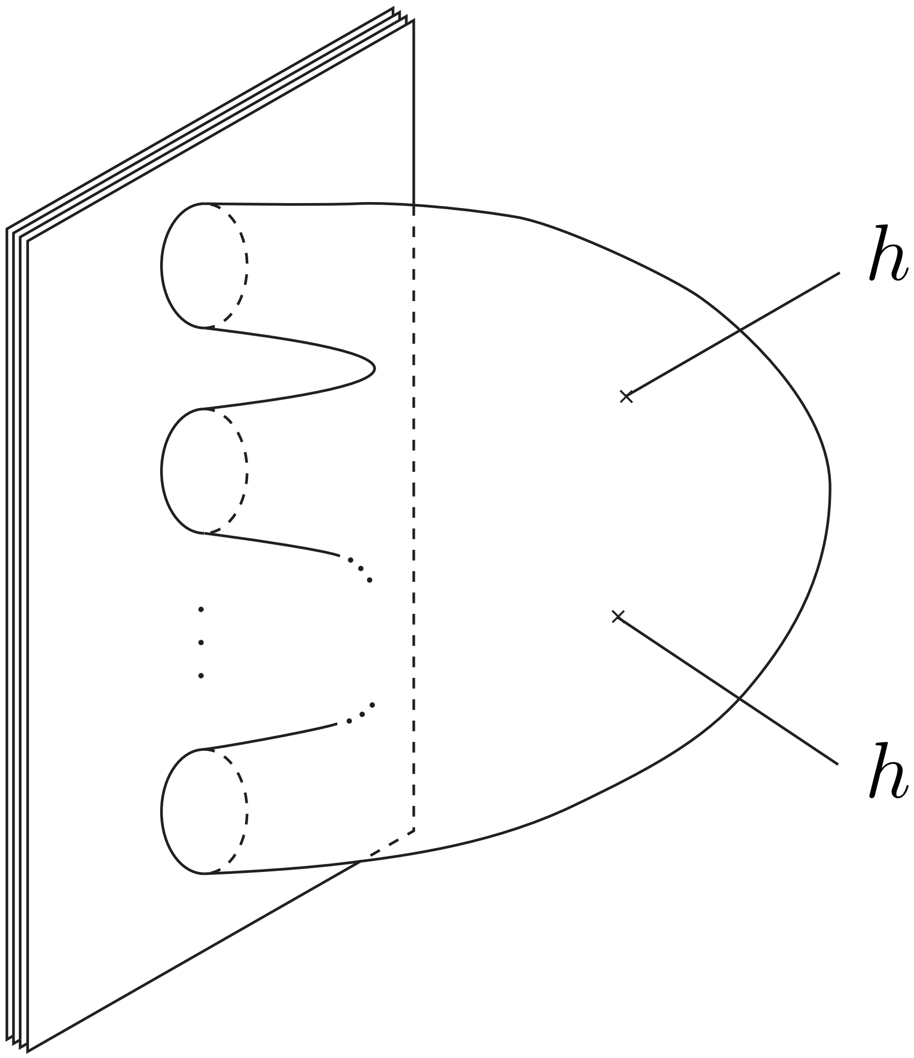


Σ_B
h
h

$N \to \infty$

$g_{\mathrm{s}} \sim 1/N$

Σ_B



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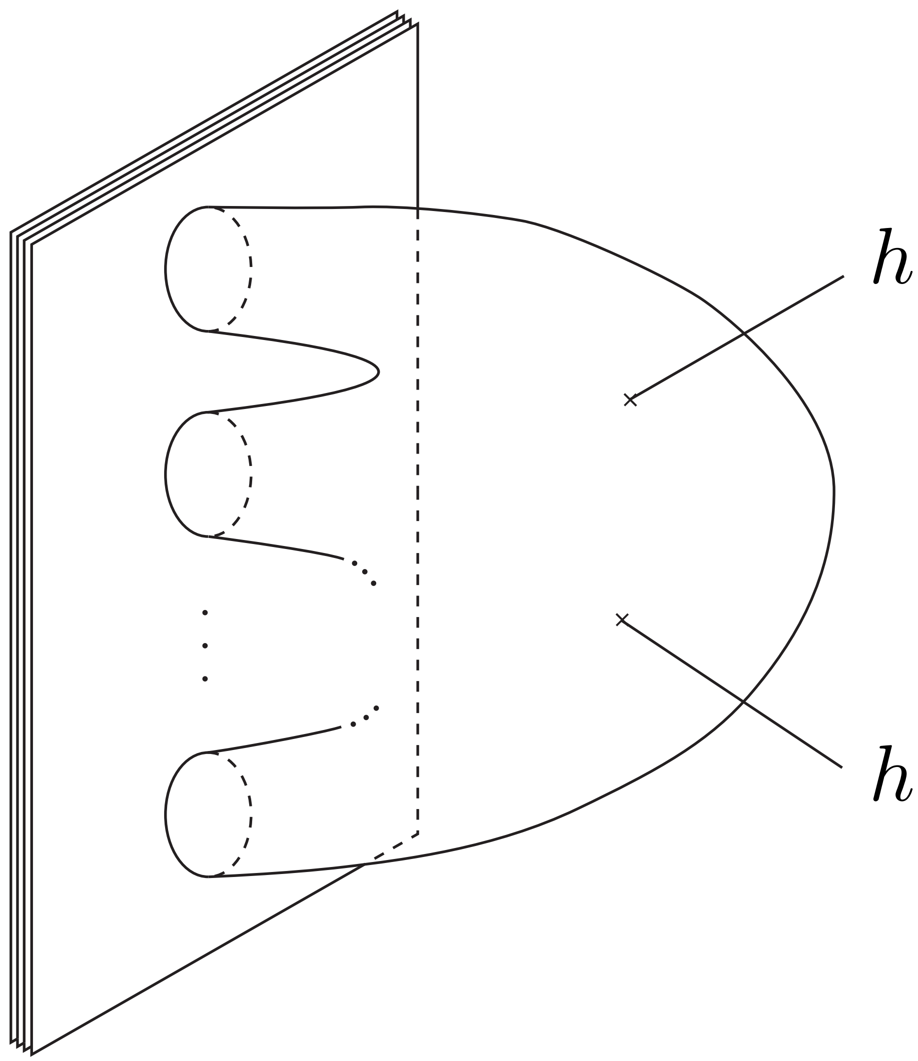

$$\sim \quad \langle \mathcal{O}\,\mathcal{O} \rangle_{\mathrm{Y}M}$$

$N \to \infty$

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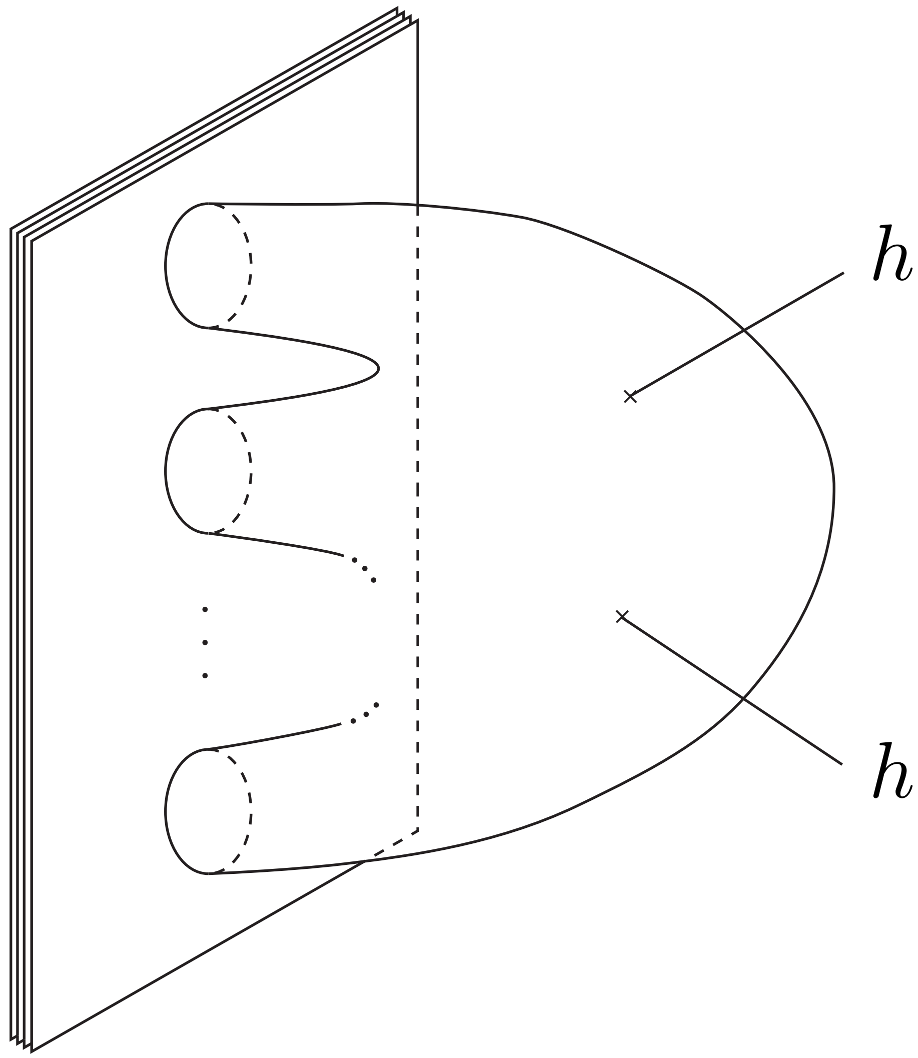
Near horizon limit $\alpha' \to 0$

Σ_B



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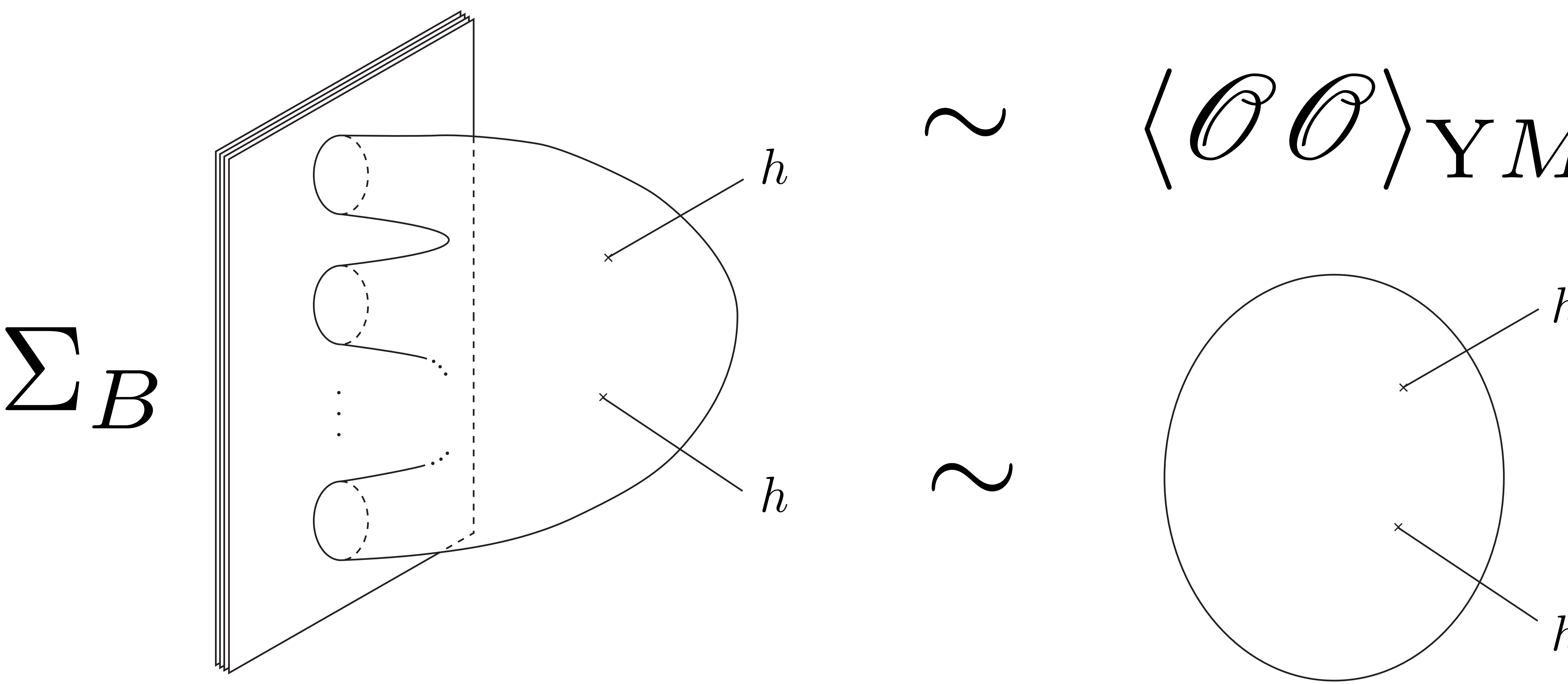

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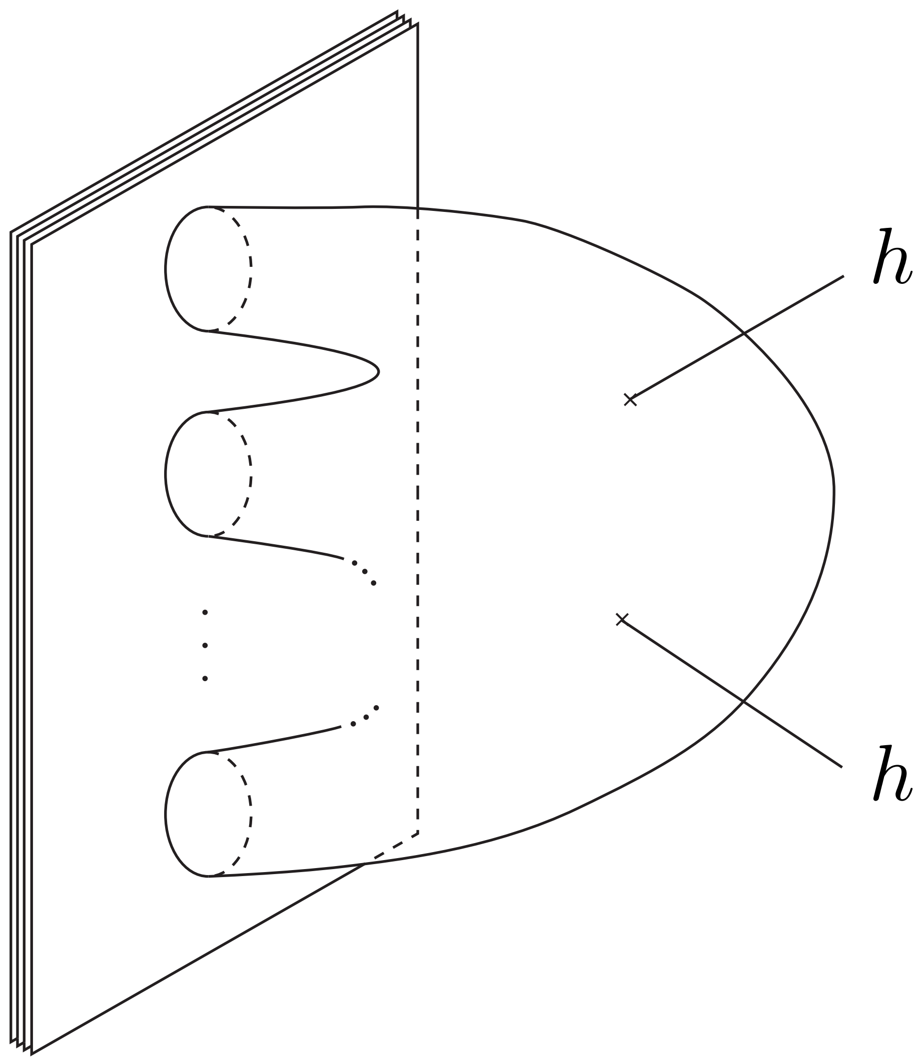

N branes
Gauge theory microscopic calculation

$$\sim \langle \mathcal{O}\,\mathcal{O} \rangle_{\mathrm{Y}M}$$

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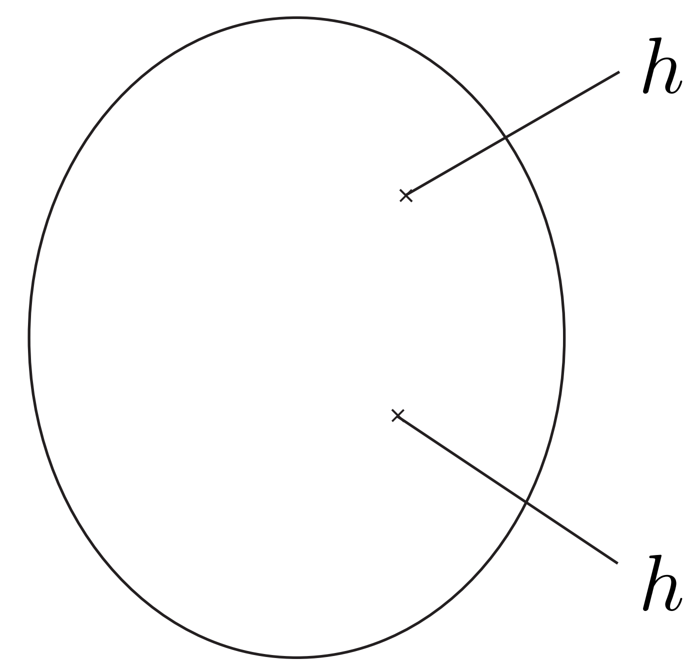

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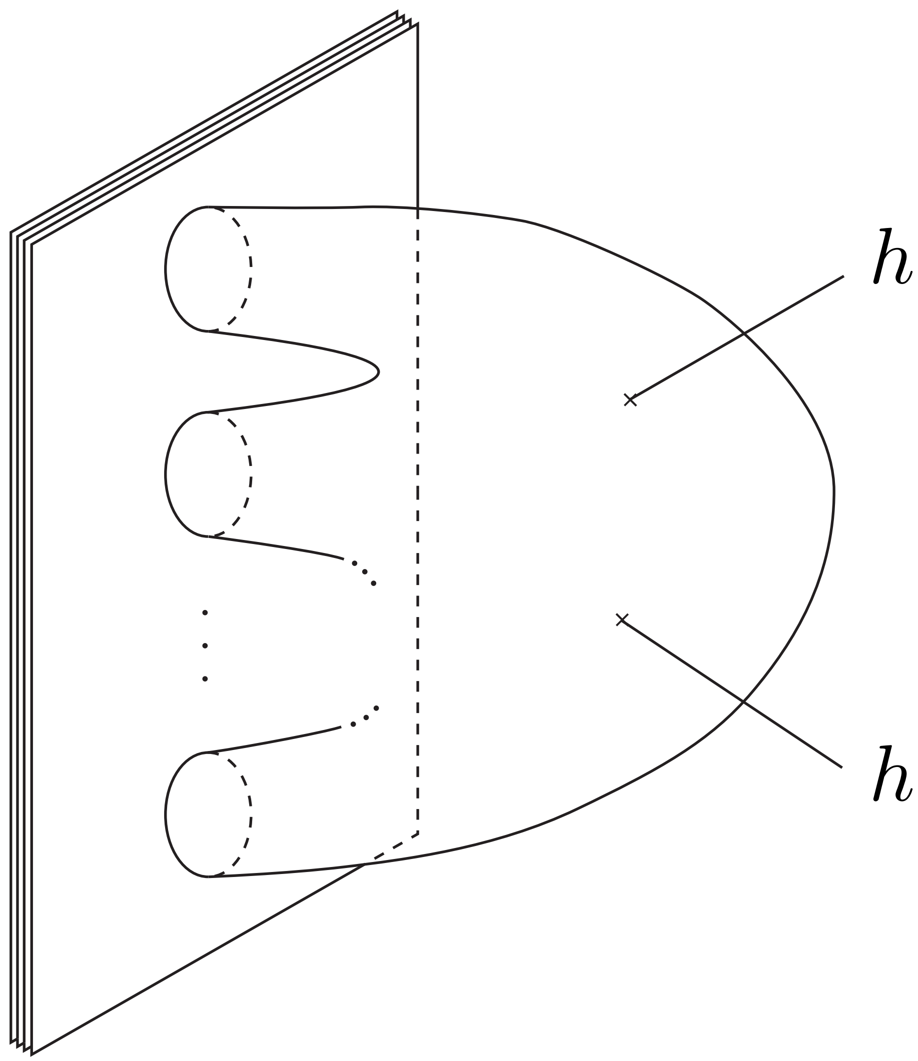
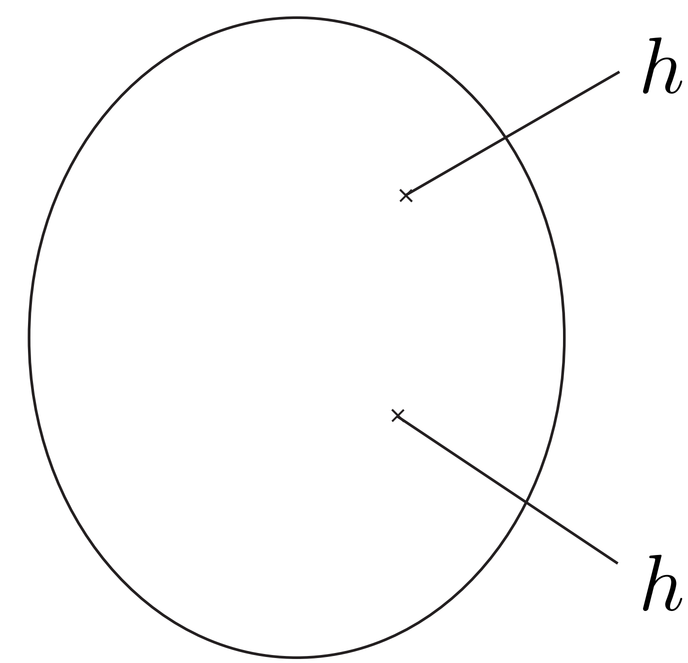


$\sim \langle \mathcal{O}\mathcal{O} \rangle_{\mathrm{Y}M}$

$\sim$



N branes
Gauge theory microscopic calculation

No brane
Non-trivial emergent gravitational background

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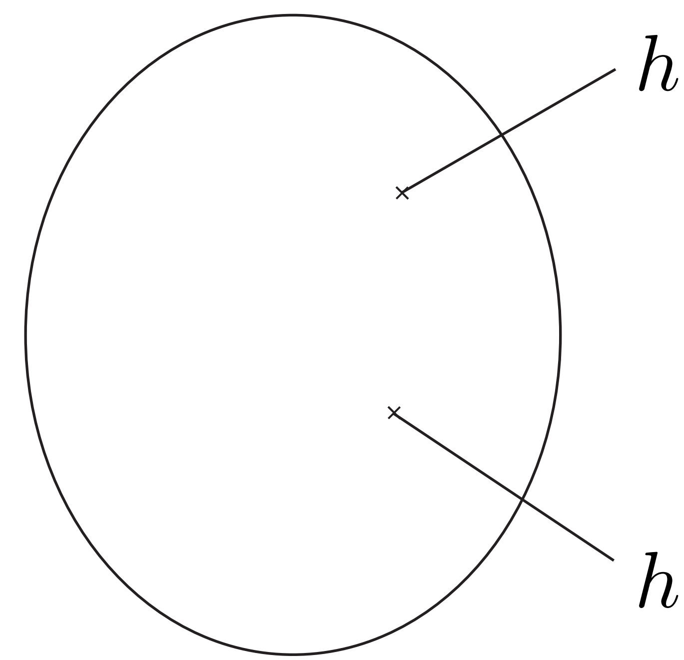
$g_s \sim 1/N$

Near horizon limit $\alpha' \to 0$

Σ_L  $\sim \langle \mathcal{O}\,\mathcal{O} \rangle_{\mathrm{Y}M}$

$\sim$ 

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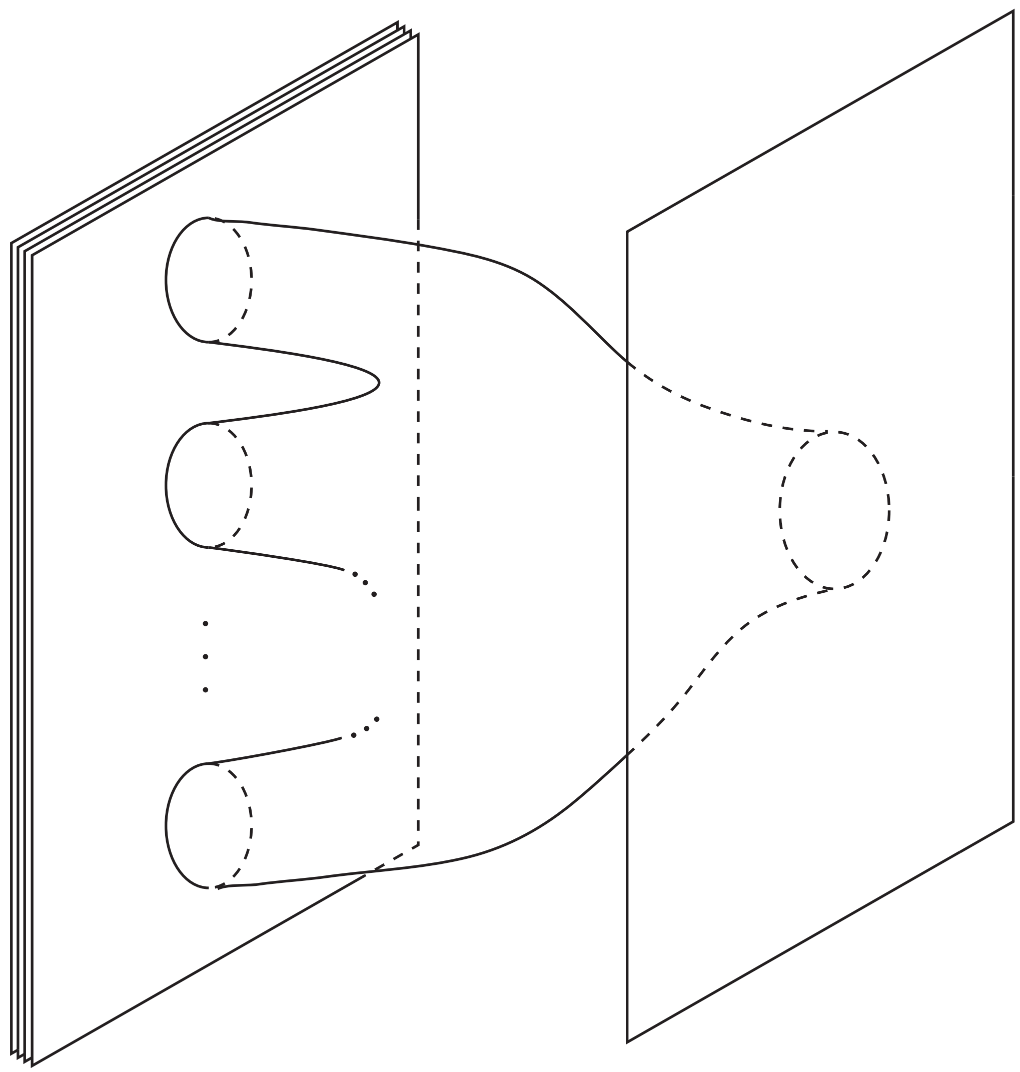
$$\sum_B$$



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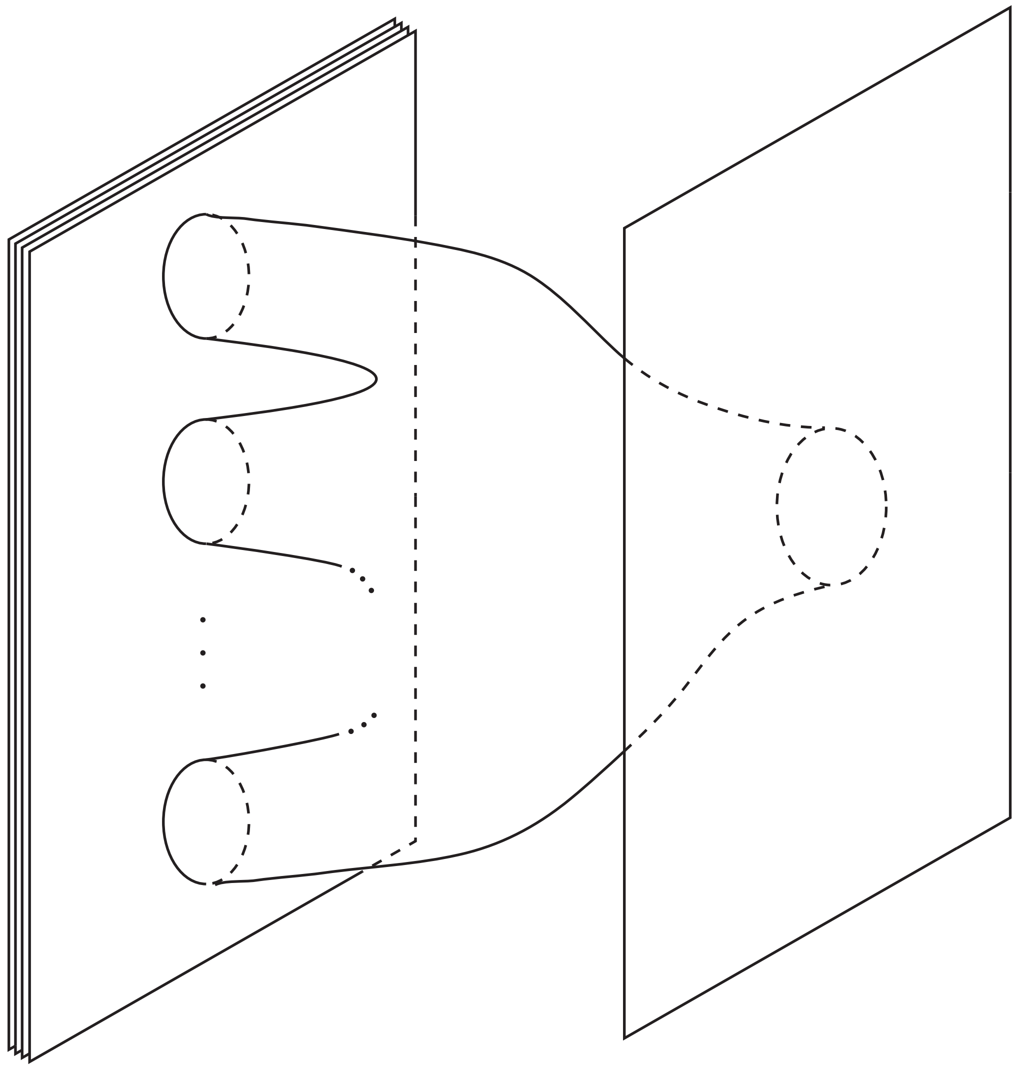

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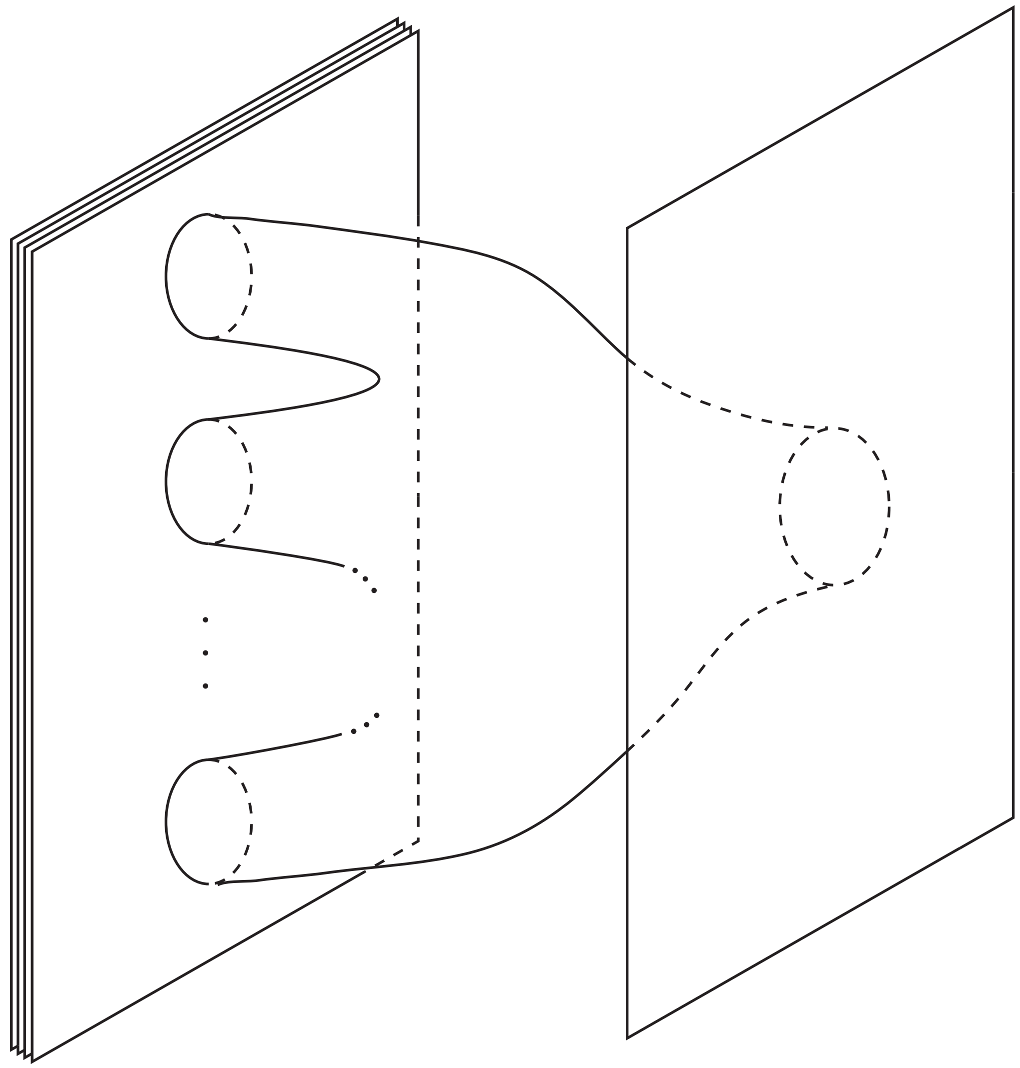

$\sim \quad NS_{\text{D-brane}}(Z)$

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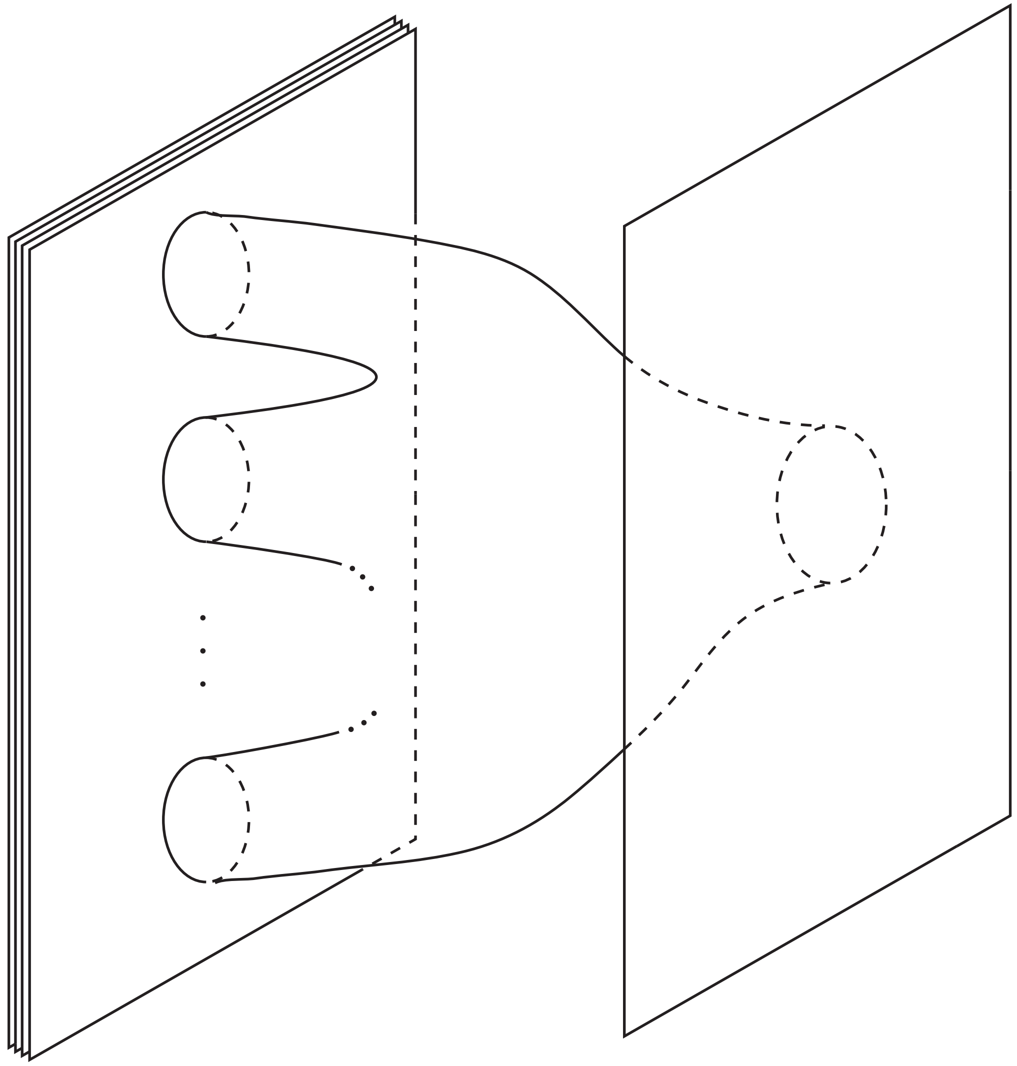

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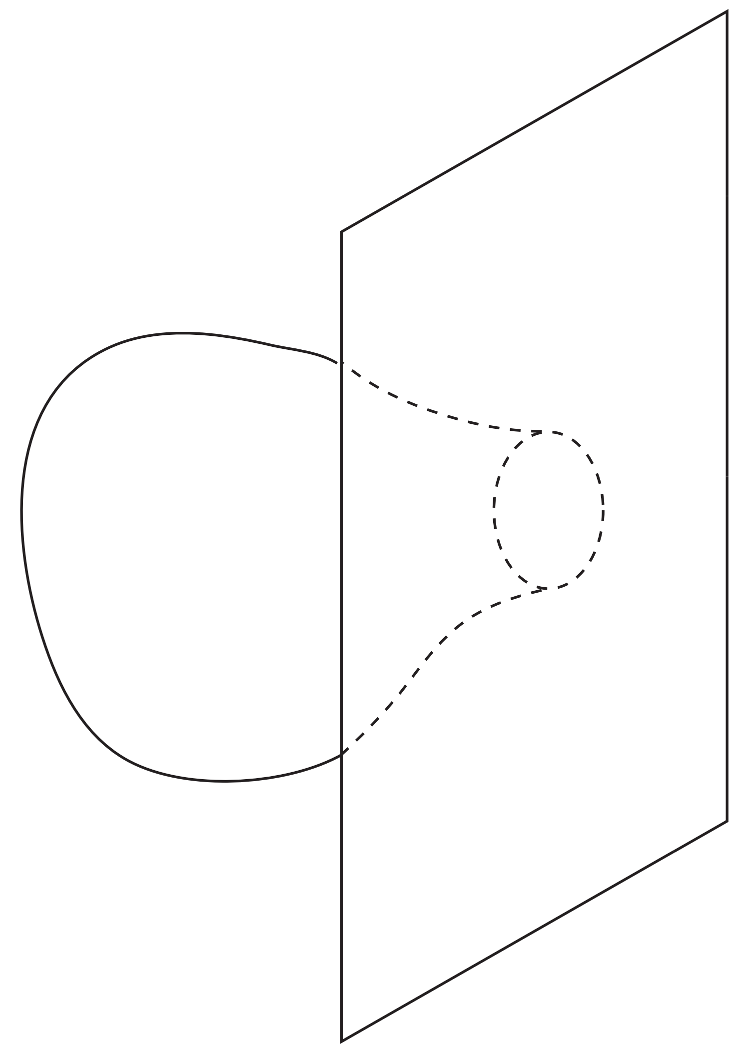

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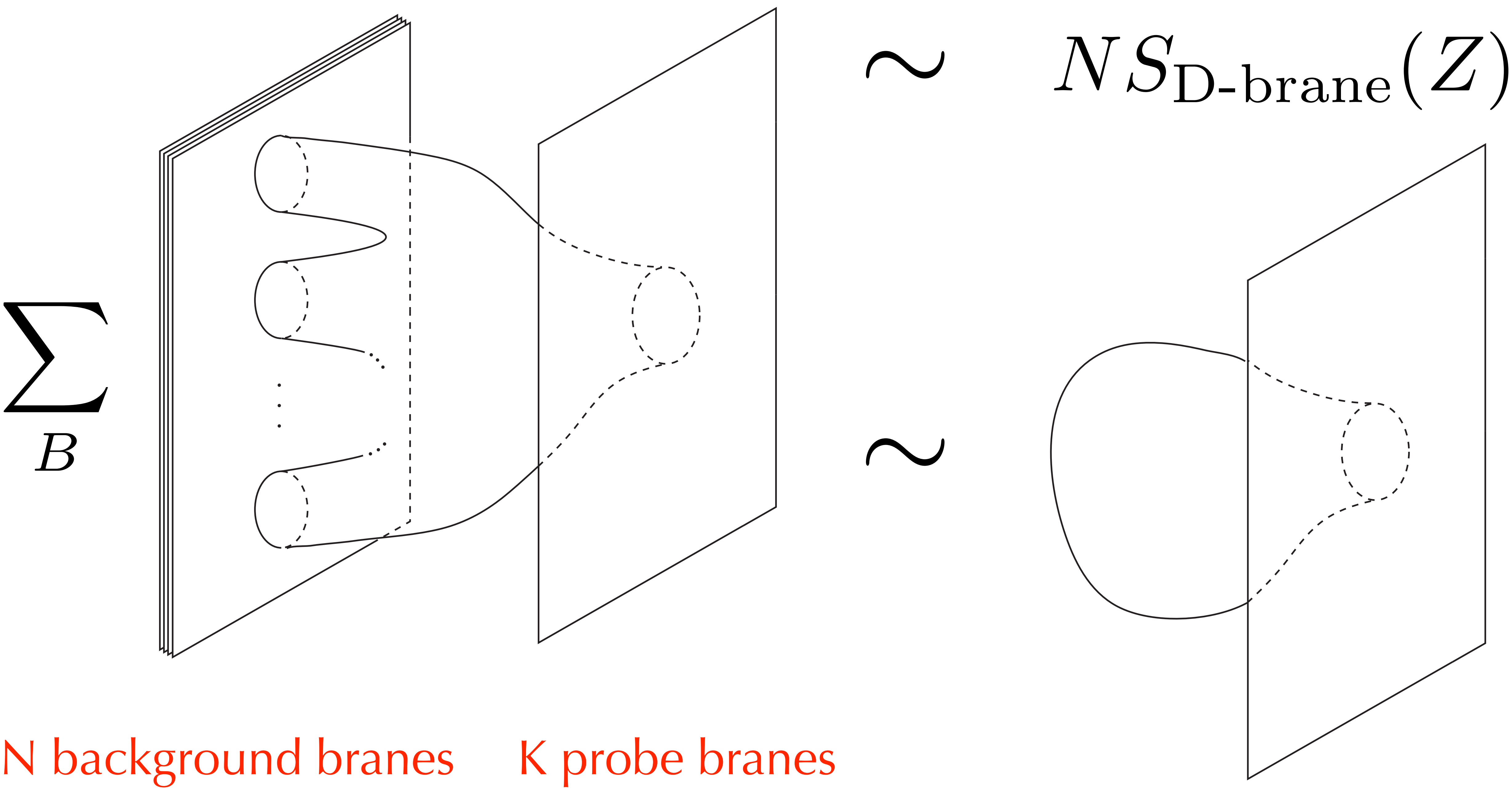

N background branes   K probe branes

Gauge theory microscopic
calculation

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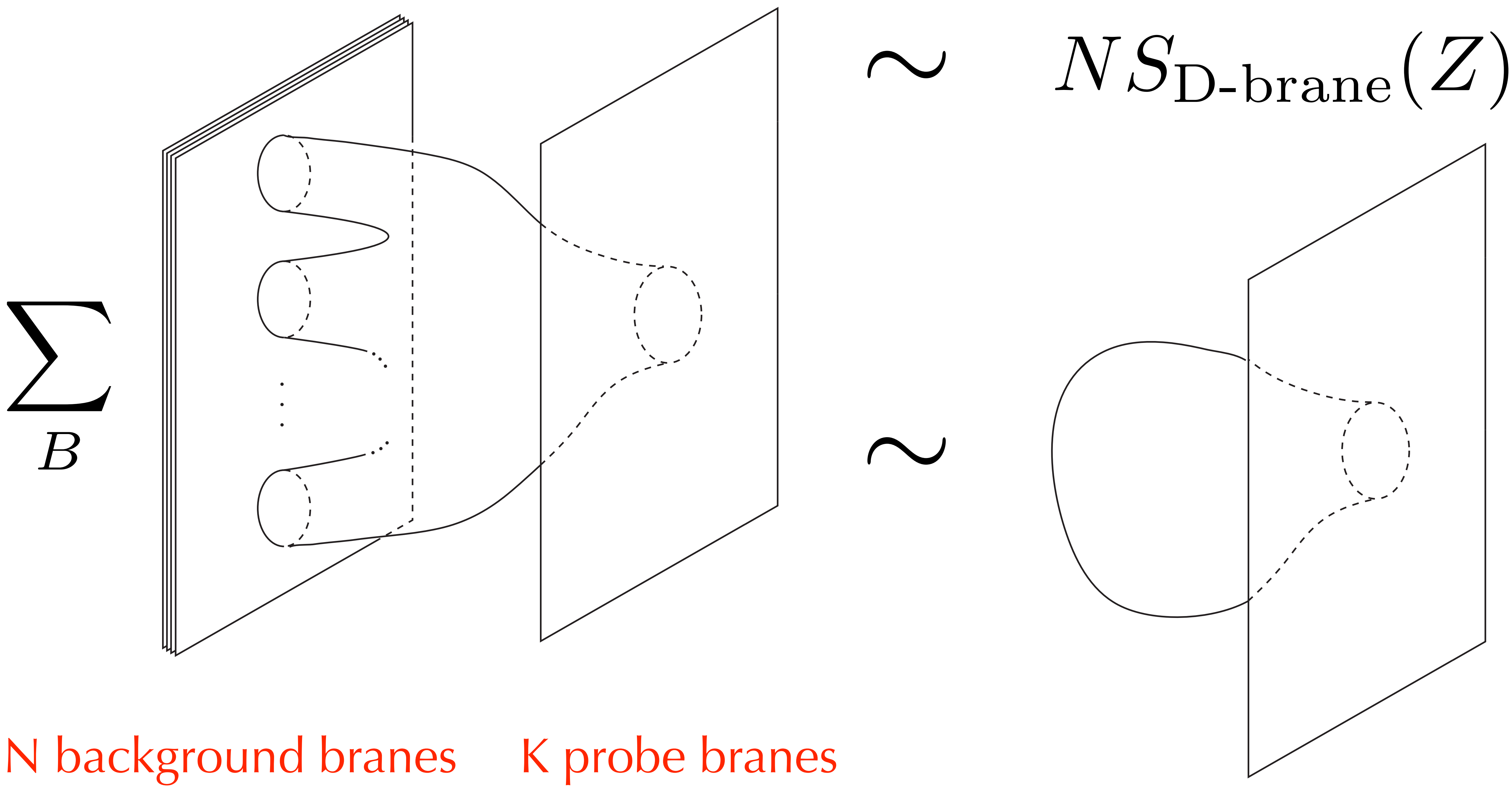

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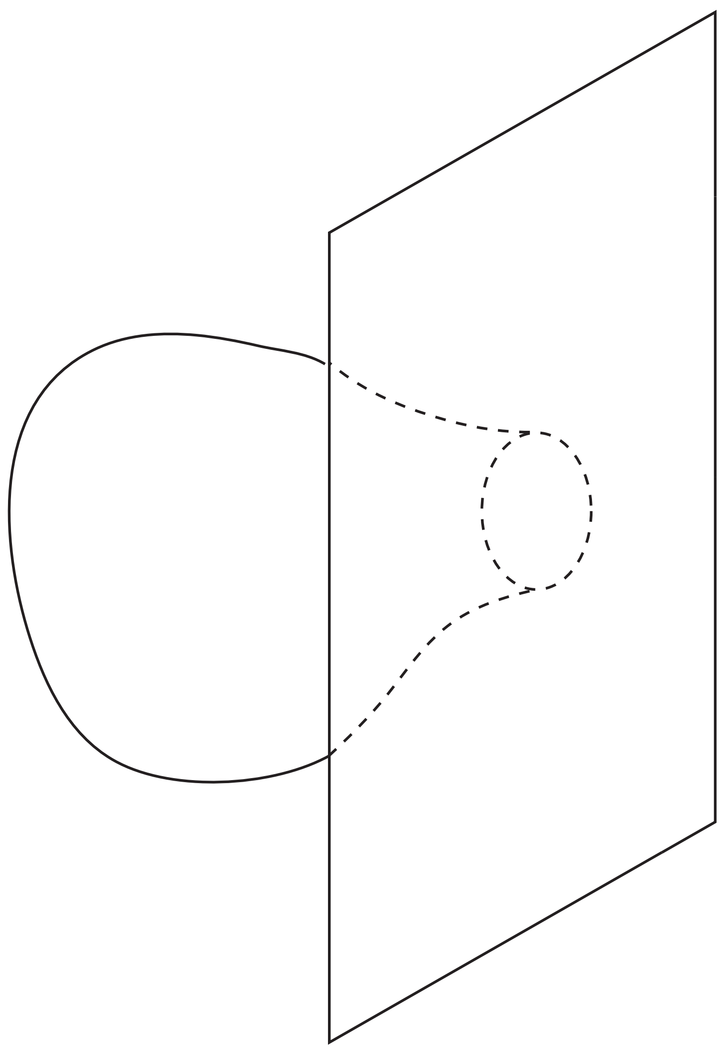
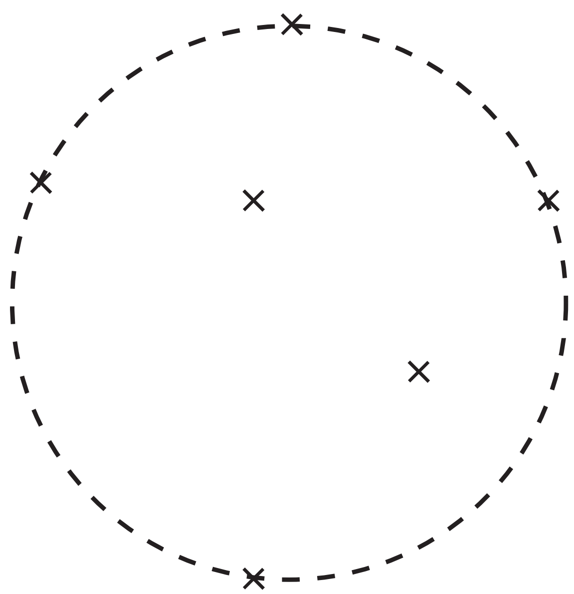
Non-trivial emergent gravitational background

Reading off the geometry





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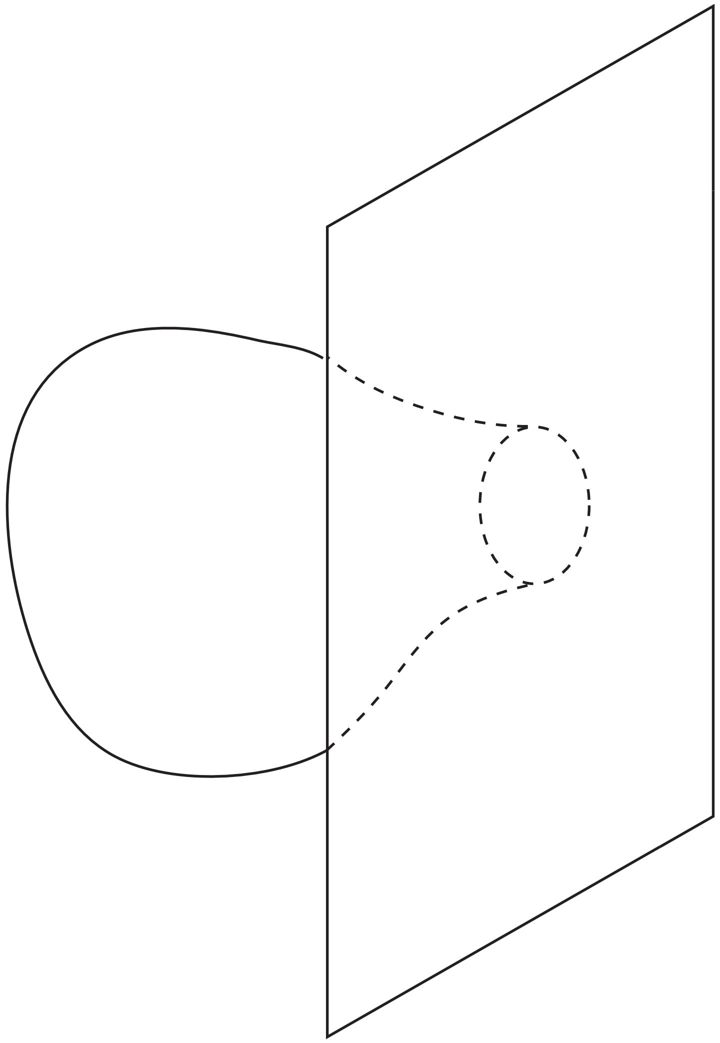

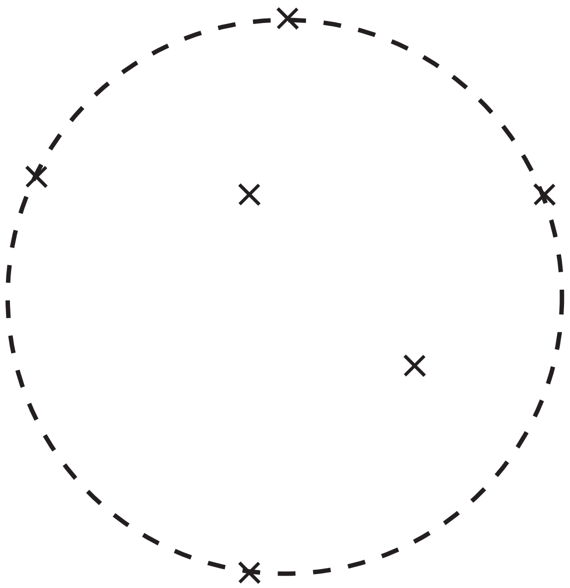

$$Z = z + l_s^2 \epsilon$$

$$S_{\text{D-brane}}(Z) = \sum_{n \geq 0} \frac{1}{n!} l_s^{2n} c_{i_1 \ldots i_n}(z) \, \mathrm{tr} \, \epsilon^{i_1} \cdots \epsilon^{i_n}$$

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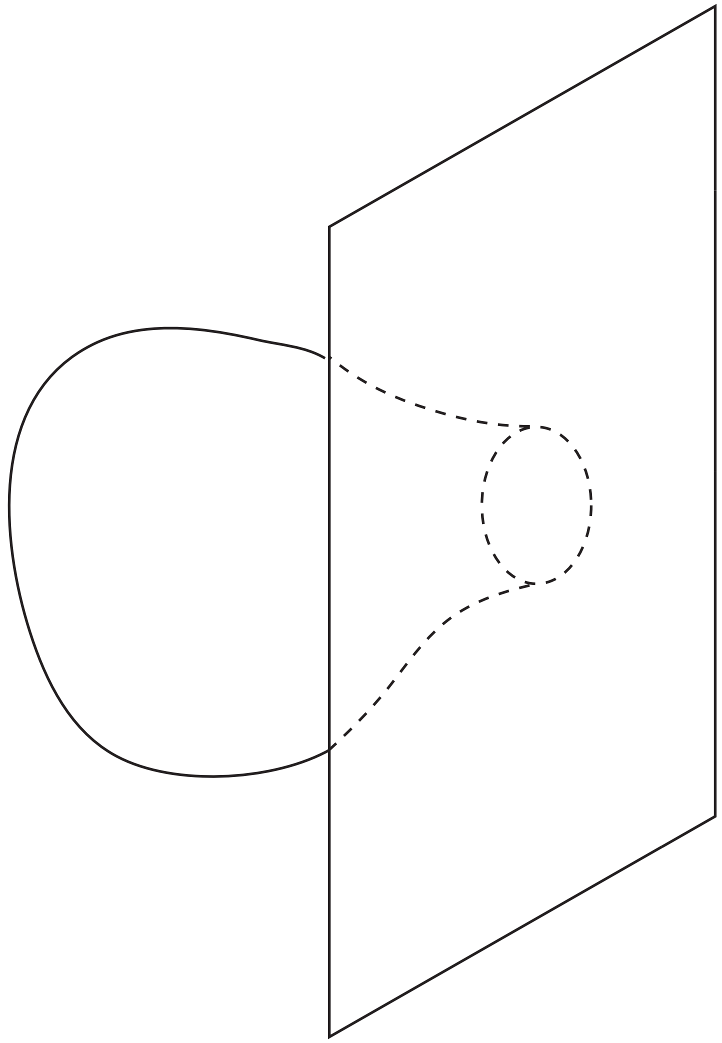

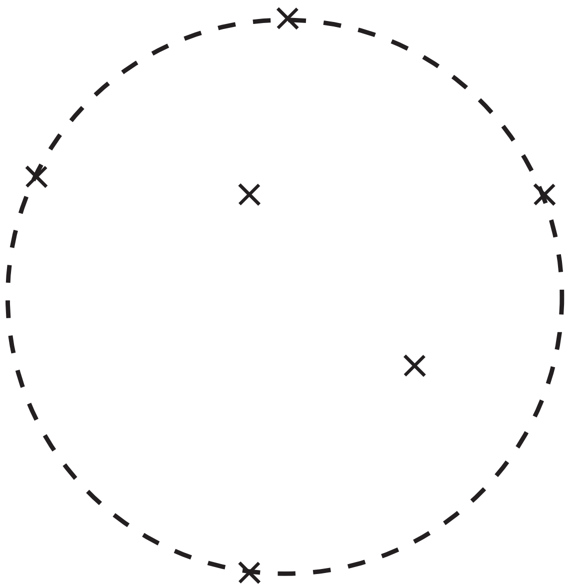

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The coefficients in the effective action depend on the closed string background and thus computing the effective action is a very effective tool to derive the background.

The coefficients $c_{i_1 \dots i_n}$ can be classified according to their symmetry type, and must also satisfy general consistency conditions, both algebraic and differential.

F. Ferrari, 1301.3722

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For example,

$$c_{(i_1 \cdots i_n)} = \partial_{i_1 \cdots i_n} c$$

$$\partial_{[i_1} c_{i_2 i_3 i_4]} = 0$$

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Myers 1999

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$$S_{\text{B. I}} = 2\pi \, \mathrm{Str} \, e^{-\phi} \sqrt{\det Q}$$

$$Q_{MN} = \delta_{MN} + i l_{\mathrm{s}} [Z_M, Z_N] E_{MN}$$

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$$S_{\text{C. S}} = 2i\pi \,\mathrm{Str}\, e^{i l_{\mathrm{s}}^2 i_Z i_Z} \sum_q C_q \wedge e^B$$

$$\left(i_Z\right)^p \omega = Z^{M_p} \cdots Z^{M_1} \omega_{M_1 \cdots M_p}$$

Reading off the geometry

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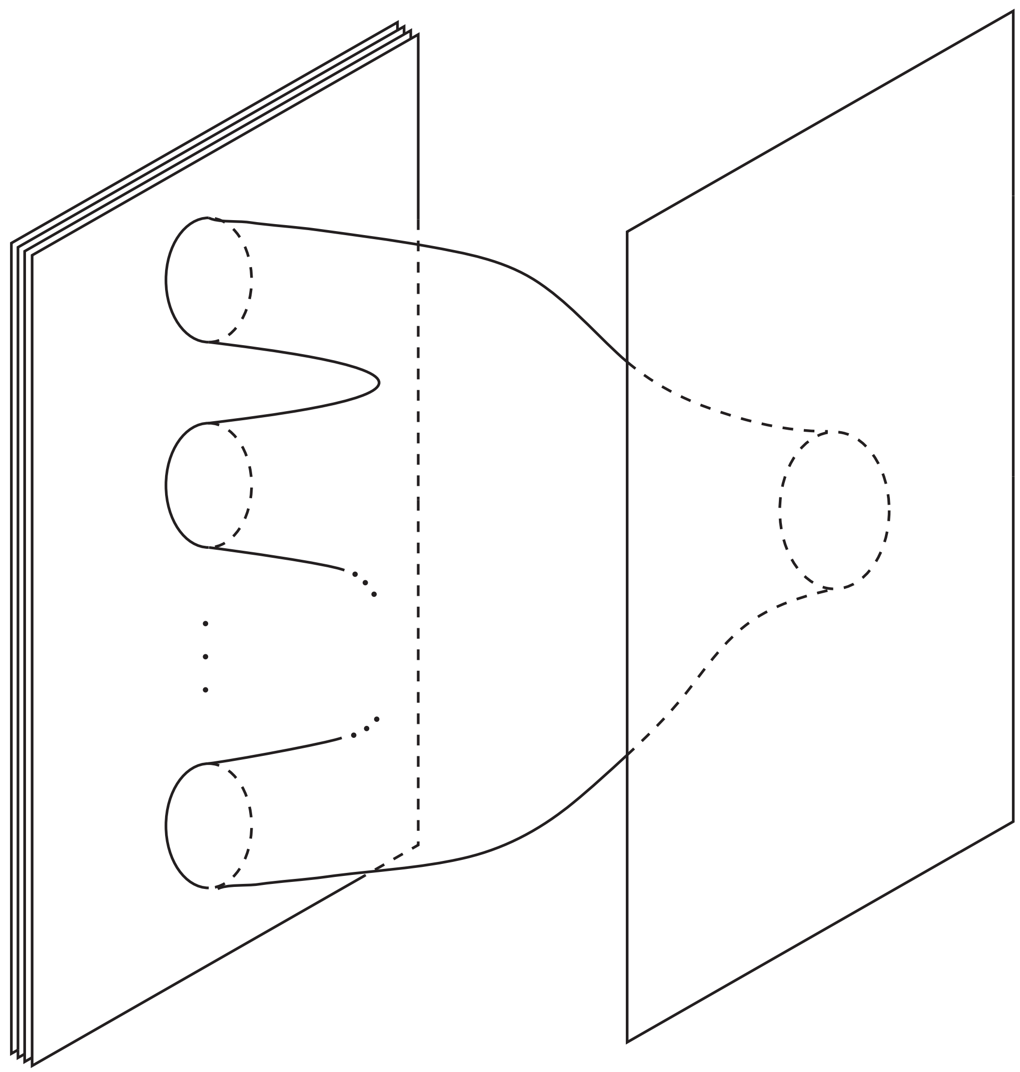
$$c_{[i_1 i_2 i_3 i_4 i_5]} = -\frac{120i\pi}{l_{\mathrm{s}}^4}\partial_{[i_1}\left(C_4 + C_2 \wedge B - \frac{1}{2}\tau B \wedge B\right)_{i_2 i_3 i_4 i_5]}$$

The microscopic calculation

The microscopic calculation

$$\sum_{B}$$



The microscopic calculation

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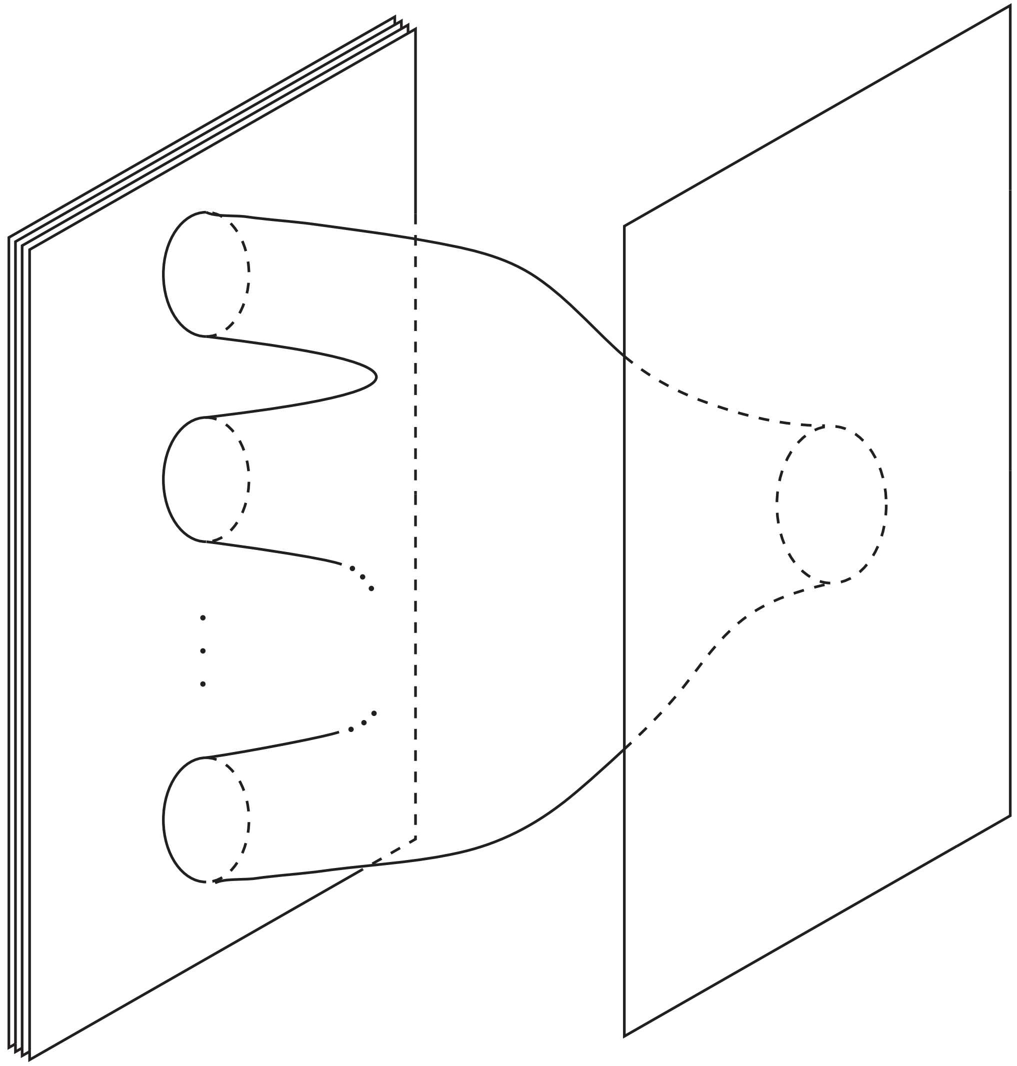

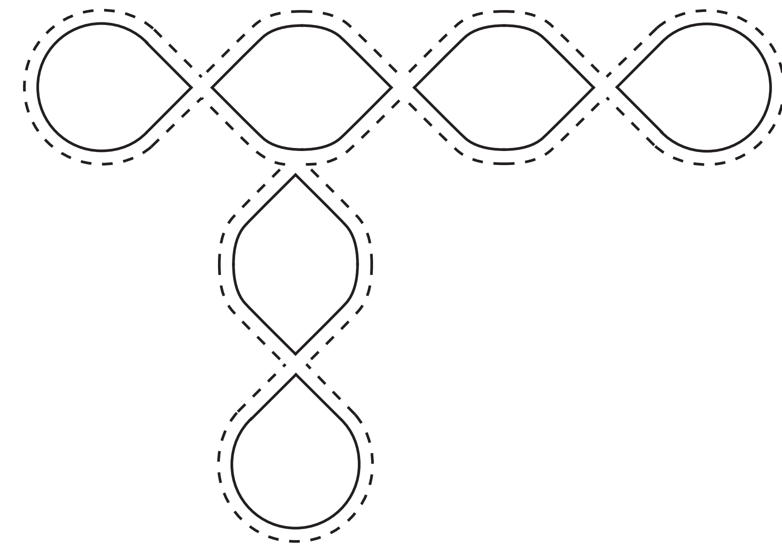

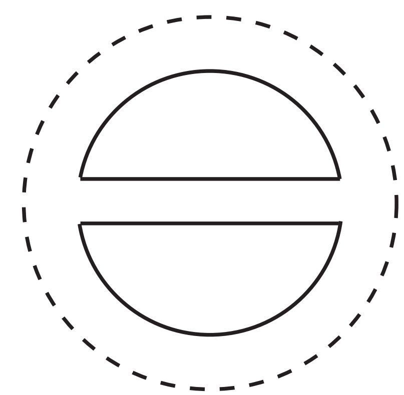

$$\int \mathrm{d}\mu_{\mathrm{D3}} \int \mathrm{d}\mu_{\mathrm{D(\text{-}1)}}\, e^{-S_{\mathrm{D3}}-S_{\mathrm{D(\text{-}1)}}} = \int \mathrm{d}Z\, e^{-N S_{\text{D-brane}}(Z)}$$

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The variables Z must be constructed in the microscopic, pre-geometric model of the left-hand side, in such a way that their quantum fluctuations are suppressed at large N. If the number of variables Z is independent of N, this property is automatically encoded in the right-hand side of the formula.

$$\int \mathrm{d}\mu_{\mathrm{D3}} \int \mathrm{d}\mu_{\mathrm{D(-1)}}\, e^{-S_{\mathrm{D3}}-S_{\mathrm{D(-1)}}} = \int \mathrm{d}Z\, e^{-NS_{\text{D-brane}}(Z)}$$

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These variables (or some of them, in the D(-1)/D3 case the (10-4)K^2 = 6K^2 variables associated with the 6 dimensions that are not present in the SYM theory) will be composite from the point of view of the microscopic model.

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We can then read off the full background from $S_{\text{D-brane}}(Z)$, by expanding around Z=z.

$$\int \mathrm{d}\mu_{\mathrm{D3}} \int \mathrm{d}\mu_{\mathrm{D(\text{-}1)}}\, e^{-S_{\mathrm{D3}}-S_{\mathrm{D(\text{-}1)}}} = \int \mathrm{d}Z\, e^{-N S_{\text{D-brane}}(Z)}$$

The a priori hard part is of course to construct Z and to show that the left hand side can be computed from the right hand side for some suitable and computable action $S_{\text{D-brane}}(Z)$.

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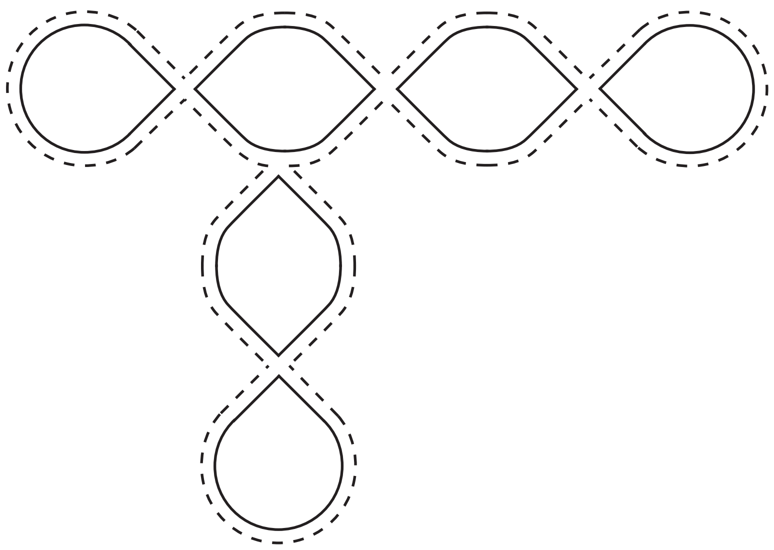

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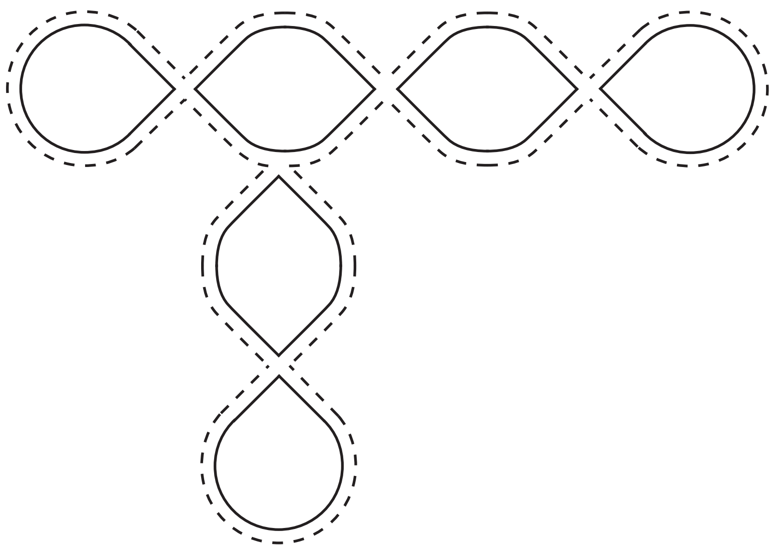

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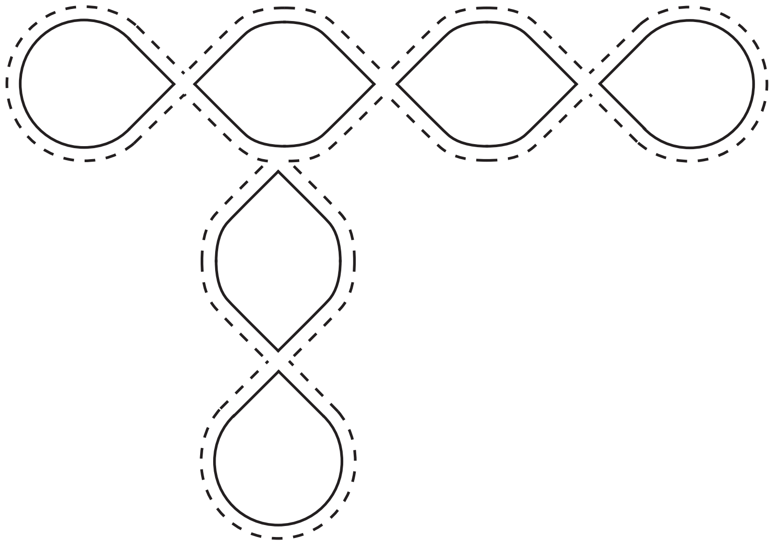

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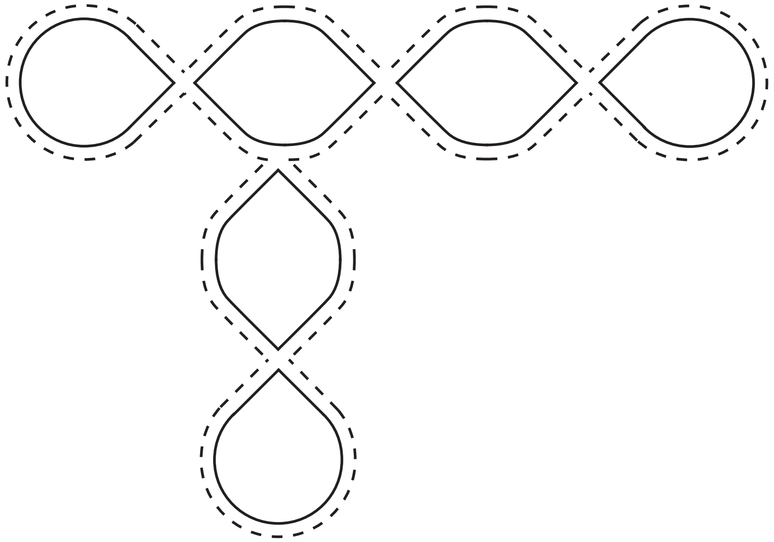

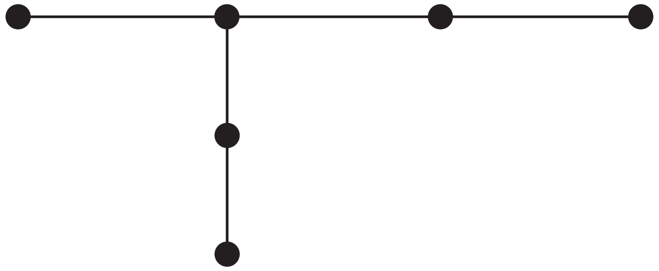

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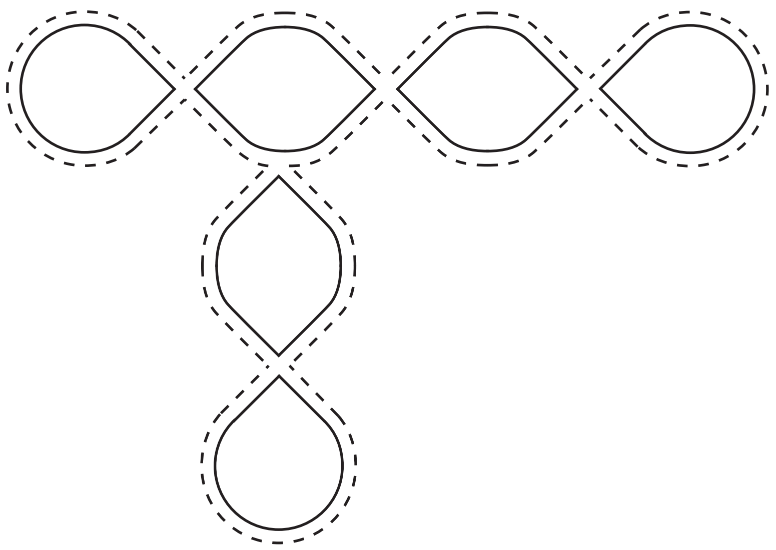

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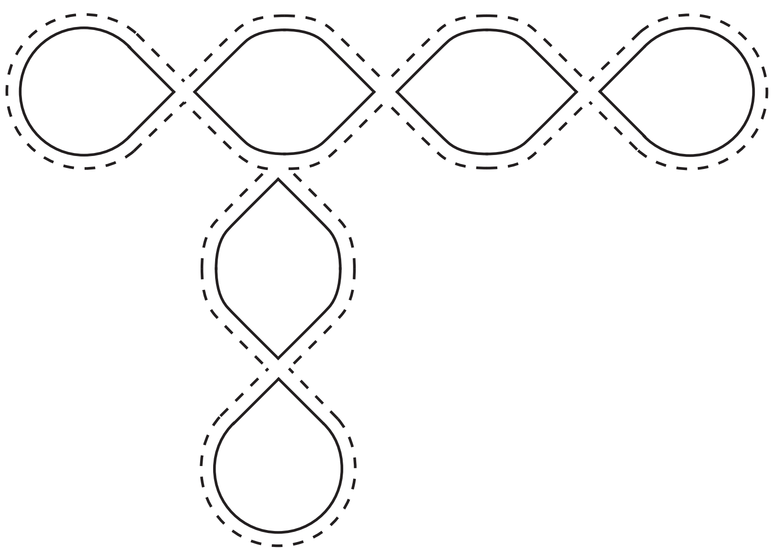

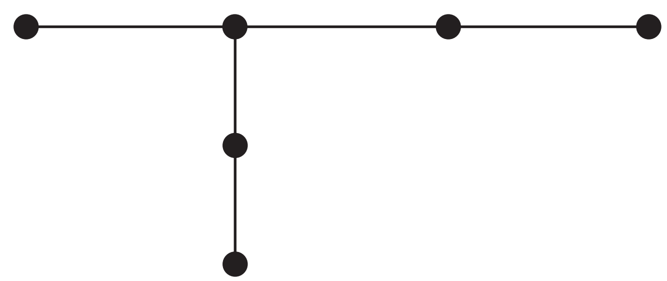

These "bubble diagrams" can be easily summed up: they are vector models diagrams!

The simplest vector model is the O(N)-invariant theory of N scalar fields $\vec{\phi}$ with a potential term

$$\frac{1}{2}m\vec{\phi}^2 + g\vec{\phi}^4$$

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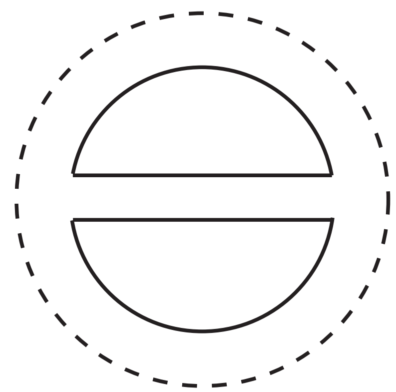
Analogues of the field σ, associated with the sum of the bubble diagrams, are thus natural candidates for the emergent coordinates of space.

The bubble diagrams are associated with the interactions between the strings stretched between the background branes and the probe branes, for example the D3-D(-1) strings.

But we also have to deal with the diagrams of the second class, for example



which are associated with the interactions between the D3-D(-1) and the D3-D3 strings or, in other words, with the couplings between the D-instanton moduli and the D3-branes fields.

These diagrams are the typical "matrix model" diagrams that are so hard to deal with. For the simple case of the D3 brane background, we shall see that a simple argument implies that their contribution to the integral

$$\int \mathrm{d}\mu_{\mathrm{D3}} \int \mathrm{d}\mu_{\mathrm{D(\text{-}1)}}\, e^{-S_{\mathrm{D3}}-S_{\mathrm{D(\text{-}1)}}} = \int \mathrm{d}Z\, e^{-NS_{\text{D-brane}}(Z)}$$

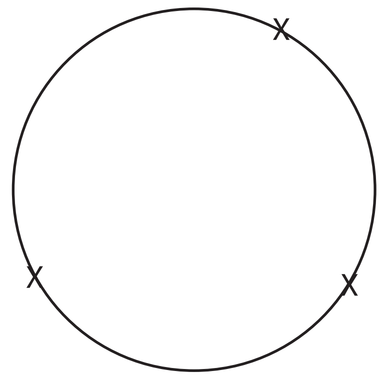
is trivial.

More generally, for example in non-supersymmetric set-ups, these diagrams will play a rôle. However, let us note that the sum over the undecorated bubble diagrams is already a sum over an infinite number of loops that yields a highly non-trivial dependence on the 't Hooft coupling. Evaluating, in such circumstances, to what extent the "decoration" of the bubbles modifies the result can be investigated in simple models. Ferrari, to appear

Bosonic symmetries of the D3/D(-1) system: $\mathrm{SO}(4) \times \mathrm{SO}(6)$

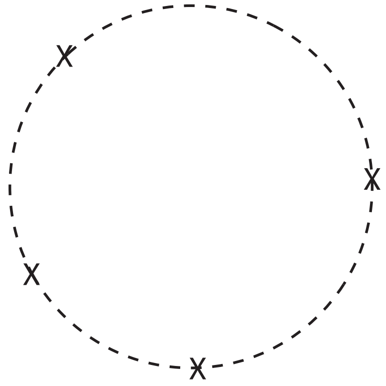
$$\alpha\,,\ \dot{\alpha}\,,\ \mu \quad a\,,\ A$$

S_{D3} is the N=4 super Yang-Mills action

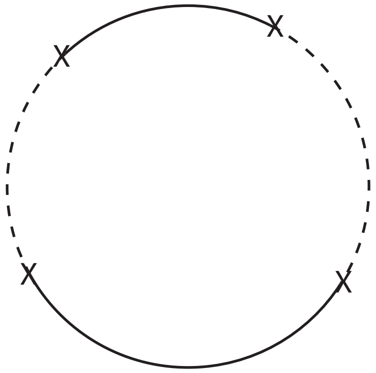


$$A_\mu\,,\ \varphi_A\,,\ \lambda_{\alpha a}\,,\ \bar{\lambda}^{\dot{\alpha} a}$$

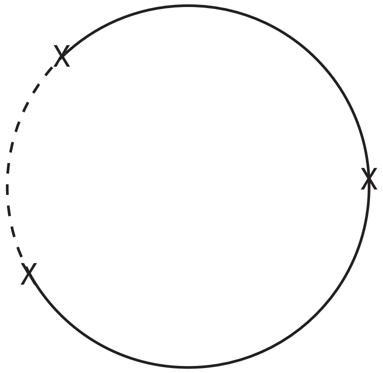
To get $S_{\mathrm{D}(-1)}$ we have to consider three types of string diagrams



$$X_\mu\,,\ \bar{\psi}^{\dot{\alpha} a}$$

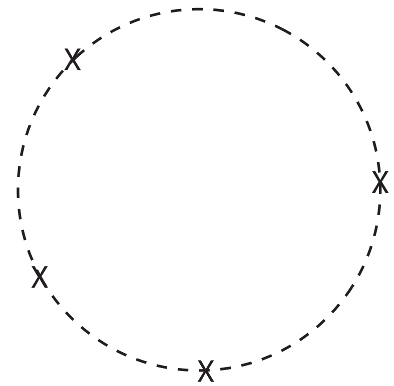


$$\tilde{q}^{\alpha I i}\,,\ q_{\alpha I i}\,,\ \tilde{\chi}^{a I i}\,,\ \chi^{a}_{I i}$$

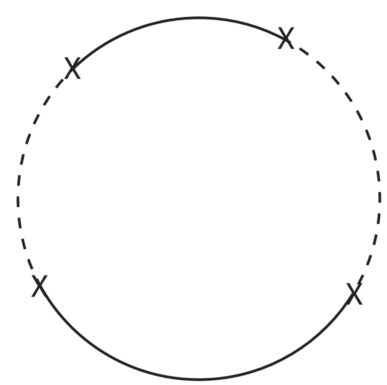


$$\tilde{\chi}\varphi\chi$$

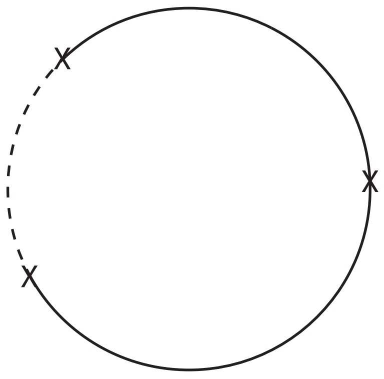
Green and Gutperle 2000, Billó, Frau, Pesando, Fucito, Lerda, Liccardo 2002



$$\frac{4\pi^2 N}{\lambda}\,\mathrm{tr}\Big\{-[Y_A, X_\mu][Y_A, X_\mu] - \bar{\psi}^a_{\dot\alpha}\Sigma_{Aab}[Y_A, \bar{\phi}^{\dot\alpha b}] - 2\Lambda^\alpha_a \sigma_{\mu\alpha\dot\alpha}[X_\mu, \bar{\psi}^{\dot\alpha a}] + 2iD_{\mu\nu}[X_\mu, X_\nu]\Big\}$$



$$\frac{1}{2}\tilde{q}Y_AY_Aq \quad - \quad \frac{1}{2}\tilde{\chi}\Sigma_AY_A\chi \quad + \quad \frac{1}{\sqrt{2}}\tilde{q}\Lambda\chi \quad + \quad \frac{1}{\sqrt{2}}\tilde{\chi}\Lambda q \quad + \quad \frac{i}{2}\tilde{q}D_{\mu\nu}\sigma_{\mu\nu}q$$



$$Y_A \to Y_A - \hat{\varphi}_A(\hat{X})$$

$$\Lambda \to \Lambda - \hat{\lambda}(\hat{X})$$

$$D \to D - F^+(\hat{X})$$

$$\hat{X} = \frac{1}{K}\,\mathrm{tr}\,X + \mathcal{O}(\bar{\psi})$$

The coupling to the D3 brane fields is through local operators evaluated at one point, the position of the instanton.

$$\int \mathrm{d}\mu_{\mathrm{D3}} \mathrm{d}X \mathrm{d}\bar{\psi} \mathrm{d}Y \mathrm{d}\Lambda \mathrm{d}D \, \mathrm{d}\tilde{q} \mathrm{d}q \mathrm{d}\tilde{\chi} \mathrm{d}\chi \, e^{-S_{\mathrm{D3}} - S_{\mathrm{D}(-1)}}$$

$$= \int \mathrm{d}X \mathrm{d}\bar{\psi} \mathrm{d}Y \mathrm{d}\Lambda \mathrm{d}D \, \langle e^{-S_{eff}(X, Y, \bar{\psi}, \Lambda; \mathscr{O}(x_{inst}))} \rangle$$

$$= \int \mathrm{d}X \mathrm{d}\bar{\psi} \mathrm{d}Y \mathrm{d}\Lambda \mathrm{d}D \, e^{-S_{\text{D-brane}}(X, Y, \bar{\psi}, \Lambda)}$$

$$X^\mu = x^\mu + l_{\mathrm{s}}^2 \epsilon^\mu$$

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To get the full type IIB background, one must integrate out D exactly from this action and then expand in ϵ and η up to terms of order six. This is some rather tedious algebra, but the calculation is completely straightforward.

$$i = (\mu, A)$$

$$c = -2i\pi\big(C_0 + ie^{-\phi}\big) = -2i\pi\tau$$

$$c_{[i_1 i_2 i_3]} = -\frac{12\pi}{l_{\mathrm{s}}^2}\partial_{[i_1}\big(\tau B_{i_2 i_3]} - C_{2\, i_2 i_3]}\big)$$

$$c_{[i_1 i_2][i_3 i_4]} = -\frac{18\pi}{l_{\mathrm{s}}^4}e^{-\phi}\big(G_{ik}G_{jl} - G_{il}G_{jk}\big)$$

$$c_{[i_1 i_2 i_3 i_4 i_5]} = -\frac{120i\pi}{l_{\mathrm{s}}^4}\partial_{[i_1}\Big(C_4 + C_2 \wedge B - \frac{1}{2}\tau B \wedge B\Big)_{i_2 i_3 i_4 i_5]}$$

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$$\mathrm{d}s^2 = \left(\frac{r}{R}\right)^2 \vec{d}x^2 + \left(\frac{R}{r}\right)^2 \mathrm{d}r^2 + R^2 \mathrm{d}\Omega_5^2$$

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$$c_{[i_1 i_2 i_3 i_4 i_5]} = -\frac{24 i\pi}{l_{\mathrm{s}}^4} F_{5\, i_1 i_2 i_3 i_4 i_5}$$

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$$R^4 = \frac{\lambda}{4\pi^2} l_{\mathrm{s}}^4$$

$$F_{5\,\mu\nu\rho\sigma A} = -\frac{64 i \pi^3 N}{l_{\mathrm{s}}^4 \lambda^2} \vec{y}^2 y^A \epsilon_{\mu\nu\rho\sigma}$$

$$F_{5\,ABCDE} = \frac{4 N l_{\mathrm{s}}^4}{\pi} y^F \epsilon_{ABCDEF}$$

Calculations along the same lines for D-particles and D-strings allow to find the D4-brane and D5-brane backgrounds in type IIA and IIB respectively.

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Ferrari, Moskovic, Rovai, 1301.3738, 1301.7062, to appear

General conclusions

A conceptually important (and certainly well-known!) comment can be made by way of conclusion.
The fluctuations of space and geometry are traditionally associated with the quantum corrections to a purely classical picture of gravity and thus, strictly speaking, to the genuine quantum gravity effects.
<u>This interpretation is misleading in the present context.</u> Indeed, the microscopic, pre-geometric model we start with will always be treated quantum mechanically, and the emergence of space and gravity are possible only as a consequence of strong quantum mechanical effects in this model. In other words, the notions of space and gravity are fundamentally quantum mechanical, including in the regime where they superficially look classical.

This property is a generic feature of any model of emergent space and gravity. It contradicts sharply the standard lore about the difficulties in quantum gravity, which is still advocated by a large fraction of the modern literature which presents gravity and quantum mechanics as incompatible or at best hard to reconcile. If space emerges, as in the model we have discussed, there is really nothing to reconcile. Quite the contrary, we can find space and gravity only as a consequence of quantum mechanics.

This tantalizing paradigm for gravity, which underlies most of our modern thinking about string theory, would certainly be universally accepted if only more effort were devoted to the construction of tractable models, which we have modestly tried to do in the simplest and most symmetric case.

Thank you for your attention!