Deep-Space Navigation: a Tool to Investigate the Laws of Gravity

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Outline

• Laws of gravity in the solar system: observables, space probe dynamics, anomalies
• Cassini, Pioneer and the Pioneer anomaly
• Juno: Lense-Thirring at Jupiter
• Planned tests at Mercury with BepiColombo
How well do we know gravity at various scales?

- Poorly
- Reasonably well
- Well
- No precise data
- Poorly
- Poorly

Theories that predict deviations from General Relativity:

- Large Extra dim.
- Scalar-Tensor Extra dimensions
- Chameleon dark energy
- MOND TeVeS, STVG
- Dark energy, IR-modified gravity, f(R) gravity, branes, strings and extra dim.

Experimental Approach:

- Controlled experiments
  - Laboratory experiments
  - Space-based experiments
- Astronomical observations
  - Astronomy
  - Astrophysics
  - Cosmology

Techniques available to explore gravity:

- Clocks, interferometers, pendula
- LLR, GPS
- Ongoing space exploration missions
- Precision spectroscopy
- Galaxy surveys, pulsars
- Cosmology missions
  - CMB surveys
- Gravitational waves
At what level is General Relativity violated?

• In spite of the experimental success, there are strong theoretical arguments for violations of GR at some level.

• Unfortunately no reliable predictive, alternative theory has been proposed yet.

• The theoretical uncertainties are so large that every experiment able to improve over previous tests is significant.

• Violations of GR from a single experiment will be accepted with great caution (if not skepticism). Confirmation with different techniques is essential.
Which tools are available?

- Geodesic motion of test masses (deep space probes, solar system bodies)
- Propagation of photons in a gravity field
- Measurements of angles, distances and velocities
### Observables used in deep space navigation

**Range (light travel time)**

- Phase comparison of modulation tones or codes in coherent radio links
- Current accuracies:
  - 1 - 3 m (incl. station bias)
  - 0.2 m (BepiColombo Ka-band /multilink radio systems with wideband code modulation and delay calibration)

**VLBI (angles)**

- Time delay at two widely separated ground antennas
- Current accuracies:
  - ≈2-4 nrad (ΔDOR)
  - (up to ×100 better with phase referencing – but absolute accuracy limited by quasar position error)

**Range rate**

- Phase comparison (carrier) in coherent radio links
- Current accuracies:
  - $3 \times 10^{-6}$ m/s @1000 s integr. times (Ka-band /multilink radio systems)
Angle measurements: Delta Differential One-way Ranging (ΔDOR)

\[ \Delta \rho = \tau_g c = B \sin \theta \]
Fighting Noise

- Uncertainties in the dynamical model (solar system ephemerides, asteroid masses)
- Non-gravitational accelerations (onboard accelerometer)
- Propagation noise (solar corona, interplanetary plasma, troposphere)
- Spacecraft and ground instrumentation

Dynamical noise and non-gravs must be reduced to a level compatible with the accuracy of radio-metric measurements:

\[ \sigma_a = \frac{1}{\tau} \sigma_v = 3 \times 10^{-8} \text{ cm s}^{-2} \quad \text{at} \quad \tau = 10^4 \text{ s} \]  
\( \text{(range rate)} \)

\[ \sigma_a = \frac{1}{\tau^2} \sigma_\rho = 1 \times 10^{-13} \text{ cm s}^{-2} \quad \text{at} \quad \tau = 10^7 \text{ s} \]  
\( \text{(range)} \)
Power spectrum of frequency residuals

Cassini 2002 SCE

Errors in solid tides models (1-2 cm)

Power spectrum of GWE1 normalized CL residuals

Cassini 2001 solar opposition
Tests based on propagation of photons

**Deflection of light**

\[ \theta_{gr} = 2(1 + \gamma) \frac{M_{\text{sun}}}{b} = 4 \times 10^{-6} (1 + \gamma) \frac{R_{\text{sun}}}{b} \text{ rad} \]

**Main advantage:**
short time scale!
[7-10 days]

**Solar Gravity**

**Time delay**

\[ \Delta t = (1 + \gamma) M_{\text{sun}} \ln \frac{l_0 + l_1 + l_{01}}{l_0 + l_1 - l_{01}} \approx 70 \text{ km for a grazing beam} \]

**Frequency shift**

\[ \frac{\Delta \nu}{\nu} = \frac{d}{dt} \Delta t \approx 4(1 + \gamma) \frac{M_{\text{sun}}}{b} \frac{db}{dt} \approx 8 \times 10^{-10} \text{ for a grazing beam} \]
From:

Clifford M. Will,
"The Confrontation between General Relativity and Experiment",
http://www.livingreviews.org/lrr-2006-3
The Cassini Solar Conjunction Experiment
SCE1 30 days coverage from DSN
RMS range rate residuals:

\[ 2 \times 10^{-6} \text{ m/s @ 300 s} \]

\[ \gamma = 1 + (2.1 \pm 2.3) \times 10^{-5} \]

\[ \gamma_{\text{Viking}} = 1 \times 10^{-3} \]
The trajectory of Cassini in the sky during SCE1

LASCO images - SOHO
Plasma noise in the X/X, X/Ka, Ka/Ka links and the calibrated Doppler observable (daily Allan dev. @1000s, Cassini SCE1)
Minimum impact parameter: $1.6 \, R_s$ (DOY 172)
Power spectrum of relative frequency shift residuals
Noise Signatures in 2-way Doppler Link
ACF of Doppler residuals

Two-way light time minus earth-sun two-way light time

Two-way light time

ACF of Doppler residuals

"149kk.cor"

"149nd.cor"
Saturn-centered B-plane plot of the Cassini orbital solutions

Pioneer anomaly - Facts

1) Pointing toward the sun
2) Almost constant

\[ a_p = (8.74 \pm 1.33) \cdot 10^{-8} \text{ cm/s}^2 \]
The other 12 things that do not make sense: missing mass, varying constants, cold fusion, life, death, sex, free will ...
Pioneer anomaly: non-conventional hypotheses

- Dark matter
- Interplanetary dust
- Modified gravity
- ...

Yukawa-like force

\[ F_Y(r) = -k \frac{e^{-r/\lambda}}{r^2} \]

Phase referencing of Cassini:

\[ a_p < 10^{-12} \text{ cm/s}^2 \quad \text{(Folkner et al., 2009)} \]

PA would cause inconsistency in planetary ephemerides

\[ \omega = \tilde{\omega} \sqrt{1 + \frac{A_p r^2}{\mu}} \]

Corrections to planetary mean motion

For the Earth:

\[ \Delta r = 200 \text{ km} \]

in one year!
Pioneer’s RTG
(Radioisotope Thermoelectric Generators)

$^{238}$Pu → Half life = 88 y

RTG thermal power = 2500 W

Accelration is nearly constant!

In 1991 RTG power was 20% lower (2000 W)

This power is just 2.5% of the total RTG power at launch (2500 W)

63 W, anisotropically radiated, would produce an acceleration equal to the “Pioneer anomaly”
Cassini’s RTG

The 13 kW thermal emission is strongly anisotropic due to thermal shields

• RTG anisotropic emission is by far the largest non-gravitational acceleration experienced by the spacecraft during cruise and tour

\[ a_{CAS} \approx 4.5 \cdot 10^{-7} \text{ cm/s}^2 \]

• 30% of total dissipated power must be radiated anisotropically

\[ a_{CAS} = \frac{P_{\text{anisotropic}}}{Mc} \]

Is \( a_{CAS} \) hiding a “Pioneer anomaly”

\[ a_{CAS} \approx 5a_p \]
Disentangling RTG and “Pioneer” acceleration

• Induce controlled orbital polarizations by orienting the spacecraft in different directions – Requires a undisturbed operations – Possible only in a the Post-Extended Mission

A 180 deg turn produces a $2 a_p$ variation of the total acceleration

• Exploit the large (2500 kg) mass decrease after SOI

$$a_c = a_{RTG} + a_{PA}$$
$$a_t = \frac{m_2}{m_1} a_{RTG} + a_{PA}$$

$$\sigma_{PA} = \left( \frac{m_2}{m_1} \sigma_t + \sigma_c \right) \left( \frac{m_1}{m_2} - 1 \right)^{-1}$$

$$\sigma_c / a_c \approx 4\% \quad \text{well determined!}$$
SOI: 1 July 2004

Spacecraft mass decreased from 4.6 tons to 2.8 tons after SOI/PRM/Huygens release.
Non-gravitational accelerations – cruise

Leading sources:
- RTG
- Solar radiation pressure \( \left( \frac{A}{M} \approx 0.0023 \text{ m}^2 \text{ kg}^{-1} \right) \)

At the epoch of the first radio science experiment (6.65 AU, Nov. 2001):

\[
\frac{a_{RTG}}{a_{SP}} \approx \frac{3 \times 10^{-12} \text{ km s}^{-2}}{5 \times 10^{-13} \text{ km s}^{-2}} = 6
\]

The two accelerations are nearly aligned (within 3°) and highly correlated. Disentangling the two effects was complicated by variations of HGA thermo-optical coefficients.

HGA thermo-optical properties have been inferred by temperature readings of two sensors mounted on the HGA back side.
Thermal Equilibrium
Infinite thermal conductivity
$\alpha$ spec value=0.15

$\varepsilon = \frac{\alpha \Phi}{\sigma T^4}$

Thermal emission properties are mostly unaffected by radiation and outgassing

$\alpha = \frac{\sigma T^4}{\Phi}$

Specular reflectivity neglected
Lambertian diffuse reflectivity

$F_{SP} \propto A \frac{(5 - 2\alpha)}{3}$

Source: S.C. Clark (JPL)
4.3 kW of net thermal emission required (30% of total RTG power - 13 kW)

\[ P = aMc \]
The non-gravitational acceleration experienced by Cassini in the radial direction can in principle hide a Pioneer-like effect \((a_{\text{Cas}} \approx 3 \cdot a_{\text{Pioneer}})\). This can be assessed by comparing the non-gravitational accelerations after a large mass decrease.

\[
a_{\text{rad}} = a_{\text{RTG}} \left( \frac{m_0}{m_i} \right) + a_{\text{Pioneer}} + a_{\text{SP}} \left( \frac{m_0}{m_i} \right) \left( \frac{r_0}{r_i} \right)^2
\]

<table>
<thead>
<tr>
<th>Residual</th>
<th>Accounted for</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_c a_c = F_{\text{RTG}})</td>
<td>If the radial force experienced by Cassini is due only to RTG anisotropic thermal emission, the acceleration must be inversely proportional to the mass.</td>
</tr>
<tr>
<td>(m_t a_t = F_{\text{RTG}})</td>
<td></td>
</tr>
</tbody>
</table>
MA and NAV estimates

RTG - Radial Acceleration

ESTIMATED VALUES ARE SCALED TO A MASS OF 2781 kg
Weighted mean value of NAV estimates up to T49 (Dec. 2008)
61 independent solutions (data arcs spanning intervals of at least 1.5 revs)

\((-3.12 \pm 0.12) \times 10^{-12} \text{ km s}^{-2}\)
Upper limit to anomalous acceleration

RTG and Pioneer accelerations

\[ a_{rad} = a_{RTG} \left( \frac{m_0}{m_i} \right) \pm a_{Pioneer} \]

\((8.74 \pm 1.33) \times 10^{-13} \text{ km s}^{-2}\)

(Di Benedetto and Iess, 20° International Symposium on Space Flight Dynamics, 2009)
Flyby anomaly

Appears only during Earth flybys of deep space probes. No anomaly during planetary and satellite flybys. Effects: impossibility to fit simultaneously inbound and outbound arcs. Solving for an impulsive burn at pericenter allows a global fit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GLL-I</th>
<th>GLL-II</th>
<th>NEAR</th>
<th>Cassini</th>
<th>Rosetta</th>
<th>M*GER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>12/8/90</td>
<td>12/8/92</td>
<td>1/23/98</td>
<td>8/18/99</td>
<td>3/4/05</td>
<td>8/2/05</td>
</tr>
<tr>
<td>$H$ (km)</td>
<td>960</td>
<td>303</td>
<td>539</td>
<td>1175</td>
<td>1956</td>
<td>2347</td>
</tr>
<tr>
<td>$\phi$ (deg)</td>
<td>25.2</td>
<td>-33.8</td>
<td>33.0</td>
<td>-23.5</td>
<td>20.20</td>
<td>46.95</td>
</tr>
<tr>
<td>$\lambda$ (deg)</td>
<td>296.5</td>
<td>354.4</td>
<td>47.2</td>
<td>231.4</td>
<td>246.8</td>
<td>107.5</td>
</tr>
<tr>
<td>$V_\infty$ (km/s)</td>
<td>8.949</td>
<td>8.877</td>
<td>6.851</td>
<td>16.010</td>
<td>3.863</td>
<td>4.056</td>
</tr>
<tr>
<td>DA (deg)</td>
<td>47.7</td>
<td>51.1</td>
<td>66.9</td>
<td>19.7</td>
<td>99.3</td>
<td>94.7</td>
</tr>
<tr>
<td>$I$ (deg)</td>
<td>142.9</td>
<td>138.7</td>
<td>108.0</td>
<td>25.4</td>
<td>144.9</td>
<td>133.1</td>
</tr>
<tr>
<td>$\alpha_1$ (deg)</td>
<td>266.76</td>
<td>219.35</td>
<td>261.17</td>
<td>334.31</td>
<td>346.12</td>
<td>292.61</td>
</tr>
<tr>
<td>$\delta_1$ (deg)</td>
<td>-12.52</td>
<td>-34.26</td>
<td>-20.76</td>
<td>-12.92</td>
<td>-2.81</td>
<td>31.44</td>
</tr>
<tr>
<td>$\alpha_\phi$ (deg)</td>
<td>219.97</td>
<td>174.35</td>
<td>183.49</td>
<td>352.54</td>
<td>246.51</td>
<td>227.17</td>
</tr>
<tr>
<td>$\delta_\phi$ (deg)</td>
<td>-34.15</td>
<td>-4.87</td>
<td>-71.96</td>
<td>-4.99</td>
<td>-34.29</td>
<td>-31.92</td>
</tr>
<tr>
<td>$M_{SC}$ (kg)</td>
<td>2497</td>
<td>2497</td>
<td>730</td>
<td>4612</td>
<td>2895</td>
<td>1086</td>
</tr>
<tr>
<td>$\Delta V_\infty$ (mm/s)</td>
<td>3.92</td>
<td>-4.6</td>
<td>13.46</td>
<td>-2</td>
<td>1.80</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma V_\infty$ (mm/s)</td>
<td>0.3</td>
<td>1.0</td>
<td>0.01</td>
<td>1</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Equation (1) (mm/s)</td>
<td>4.12</td>
<td>-4.67</td>
<td>13.28</td>
<td>-1.07</td>
<td>2.07</td>
<td>0.06</td>
</tr>
</tbody>
</table>

From Anderson et al., 2008
Flyby anomaly

Post-perigee data zero-weighted

Solving for prograde delta-V

From Morley and Budnik, 2006

New physics?
Errors in the model used in the OD codes? (It is the same in all SW used in deep space navigation!)
## NASA Juno Mission

- **New Frontiers** mission
- **Investigation of atmosphere, magnetosphere and interior of Jupiter**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch</td>
<td>August 2011</td>
</tr>
<tr>
<td>Jupiter Orbit Insertion (JOI)</td>
<td>August 2016</td>
</tr>
<tr>
<td>Mission duration</td>
<td>1 year (32 orbits)</td>
</tr>
<tr>
<td>Orbit inclination</td>
<td>Polar (90°)</td>
</tr>
<tr>
<td>Orbit eccentricity</td>
<td>0.9466</td>
</tr>
<tr>
<td>Orbit period</td>
<td>11 days</td>
</tr>
<tr>
<td>Pericenter altitude</td>
<td>5000 km</td>
</tr>
<tr>
<td>Spacecraft mass @ Jupiter</td>
<td>1300 kg</td>
</tr>
<tr>
<td>Power</td>
<td>Solar arrays (54 m²)</td>
</tr>
<tr>
<td>Attitude control</td>
<td>Spin stabilized</td>
</tr>
</tbody>
</table>

At pericenter, $v = 70$ km/s
At pericenter, $v = 70 \text{ km/s}$

Juno: orbit geometry and tracking
At arrival orbit is nearly face on, then Earth view angle increases up to nearly 30 deg.
Lense-Thirring Precession of Juno

\[ \mathbf{\dot{r}} = \frac{(1+\gamma) \mu_J}{c^2 r^3} \mathbf{J} \left[ \frac{3}{r^2} (\mathbf{r} \times \mathbf{v})(\mathbf{r} \cdot \hat{P}) + (\mathbf{v} \times \hat{P}) \right] \]

\[ \frac{dS}{dt} = \mathbf{\Omega} \times \mathbf{S} \quad \mathbf{\Omega} = \frac{(1+\gamma) G}{2c^2 r^3} \left[ -\mathbf{J} + \frac{3(J \cdot \mathbf{r}) \mathbf{r}}{r^2} \right] \]

\( \mathbf{J} \) = Jupiter specific angular momentum
\( m_J \)

\( \mathbf{r} \) = Jupiter-S/C position vector

\( \mathbf{v} \) = S/C velocity relative to Jupiter

\( \hat{P} \) = direction of the Jupiter spin axis

\( \mu_J \) = Jupiter gravitational constant

(proposed by Iorio, 2008)

L-T effect is large! SNR ≈ 100

\[ \frac{C}{MR^2} = 0.26 \]
BepiColombo: ESA’s mission to Mercury

Launch: Ariane 5 (2014)
Solar Electric Propulsion
Chemical Propulsion
Arrival at Mercury: 2020

Magnetospheric Orbiter
Planetary Orbiter
SEPM - CPM
MPO: 400x1500 km
2000 simulations of 1y experiment
No preferred frame – $\eta$ free

Cruise SCE

Correlation ellipses

$\eta$ free

$\eta = 4\beta - \gamma - 3$
The effect of SEP violations on the Earth-Mercury distance

Orbit projected displacements due to $\eta=1 \times 10^{-5}$

Displacement (cm)

Time (days from MJD56226)
Current accuracies of selected PN parameters and values expected from the BepiColombo MORE experiment. Metric theories of gravity with no preferred frame effects are assumed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Present accuracy</th>
<th>MORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$2 \times 10^{-5}$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1 \times 10^{-4}$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$5 \times 10^{-4}$</td>
<td>$8 \times 10^{-6}$</td>
</tr>
<tr>
<td>$J_2^\odot$</td>
<td>$4 \times 10^{-8}$</td>
<td>$2 \times 10^{-9}$</td>
</tr>
<tr>
<td>$\dot{G}/G$</td>
<td>$9 \times 10^{-13} \text{ yr}^{-1}$</td>
<td>$3 \times 10^{-13} \text{ yr}^{-1}$</td>
</tr>
</tbody>
</table>

Prospects for future missions

• Free-flying spacecraft
  • Subject to stray accelerations and uncertainties in the masses of solar system bodies
  • Onboard accelerometers of limited use (unless LISA class or better): must be bias-free and work to very low frequencies

• Planetary orbiters
  • Tied to central body, nearly immune to stray accelerations
  • Subject to uncertainties in the masses of solar system bodies

• Planetary landers
  • Immune to stray accelerations, but subject to the effects of rotational dynamics (and again to unmodelled accelerations from asteroids)
  • Planetary rotation is of paramount interest to geophysics; opportunity for synergies
Final remarks

• Advances in solar system tests of gravity have been painfully slow.
• So far, progress has relied upon piggy-back experiments (Viking, Cassini, BepiColombo, GAIA)
• Progress has been made in ruling out claims of violations of GR at solar system scales.
• Lacking a predictive theoretical framework for violations of GR, space agencies are not willing to invest on dedicated missions.
• In addition, any experiment claiming a violation will not be immediately accepted! Concurrence of different measurements is crucial.
• However, cosmological evidence for a new physics should boost the experimental efforts also at solar system level. Indeed, violations at cosmological scales will almost surely affect laws of gravity at short scales, maybe with detectable effects in classical tests.
Additional material
Jupiter’s metric with Newtonian quadrupole correction:

\[
d s^2 = \left(1 - 2 \frac{GM_J}{rc^2} + J_2 \frac{GM}{rc^2} \left( \frac{R_J}{r} \right)^2 (3 \cos^2 \theta - 1) \right) dt^2 \\
- \left(1 + 2 \frac{GM_J}{rc^2} - J_2 \frac{GM}{rc^2} \left( \frac{R_J}{r} \right)^2 (3 \cos^2 \theta - 1) \right) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)
\]

At Juno’s pericenter \( r \approx R_J \), the correction due to the quadrupole is simply of order \( J_2 \). Both the monopole time delay and relativistic Doppler shift must therefore contain a correction of the same order.

Only order of magnitude estimates beyond this point. N. Ashby has carried out more precise calculations.
Two-way monopole time delay and relativistic Doppler shift:

\[ \Delta t = 4 \frac{R_g}{c} \ln \frac{l_0 + l_1 + l_{12}}{l_0 + l_1 - l_{12}} = 4.6 \times 10^{-8} \text{ s} = 13.8 \text{ m} \]

\[ \left( \frac{\Delta f}{f} \right)_0 \approx 8 \frac{R_g}{cb} \frac{db}{dt} \approx 8 \frac{R_g}{b} \frac{v}{c} = 3.4 \times 10^{-11} \]

\[ \left( \frac{\Delta f}{f} \right) = \frac{d}{dt} \Delta t \]

\( b = \) impact parameter = 74000 km
\( v = \) velocity at pericenter = 60 km/s
\( Rg = \) gravitational radius of Jupiter = 1.5 m

Thi Doppler shift is only a factor of 6 smaller than the one experienced by Cassini during SCE1!
Two-way quadrupole relativistic Doppler shift:

\[
\left( \frac{\Delta f}{f} \right)_2 \approx J_2 \left( \frac{\Delta f}{f} \right)_0 = 6.8 \times 10^{-13}
\]

about a factor of 70 larger than the measurement error. The effect is asymmetric across pericenter and mimics a Newtonian J3. Note that the correction to the light time is below the accuracy of current ranging systems.

The effect is large and must be accounted for in the OD software (currently it is not).
The 34m beam waveguide tracking station DSS 25, NASA’s Deep Space Network, Goldstone, California

The Advanced Media Calibration System for tropospheric dry and wet path delay corrections.
Precession of Jupiter’s spin axis

Provides also the angular momentum of Jupiter!

- The quadrupole and the inertia tensors share the same eigenvectors.

\[
Q = \frac{\sqrt{5}}{3} MR^2 \begin{bmatrix}
-\left( C_{20} - \sqrt{3} C_{22} \right) & \sqrt{3} S_{22} & \sqrt{3} C_{21} \\
\sqrt{3} S_{22} & -\left( C_{20} + \sqrt{3} C_{22} \right) & \sqrt{3} S_{21} \\
\sqrt{3} C_{21} & \sqrt{3} S_{21} & 2C_{20}
\end{bmatrix}
\]

\[
Q = \frac{1}{3} I \cdot Tr(S) - S
\]

- By diagonalizing the quadrupole tensor one computes the principal axes of inertia and their associated uncertainties.
Lense-Thirring Precession of Juno

- Option 1: assume GR is true and estimate Jupiter’s angular momentum from L-T
- Option 2: assume GR is true and combine estimates of LT and pole precession in an improved solution for Jupiter’s angular momentum
- Option 3: combine estimates of LT and pole precession solving simultaneously for the L-T parameter and Jupiter’s angular momentum

Caveat: how separable is L-T from other effects, e.g. accelerations due to zonal harmonics?

Currently L-T at Jupiter is not modelled in any OD software.