Effective field theory methods in the post-Newtonian framework for the 2-body problem in General Relativity

Riccardo Sturani

Università di Urbino/INFN (Italy)

IHES, November 8^{th}, 2012

Outline

EFT principles: known fundamental theory

- Fundamental theory known:

effects of short distance physics r_s (heavy d.o.f. Λ)
on large distance physics $r \gg r_s$ (light modes $\omega \ll \Lambda$)

$$\begin{aligned} \exp\left(iS_{eff}[\phi]\right) &= \int \mathcal{D}\Phi(x) e^{iS[\phi,\Phi]} \\ S_{eff} &= \sum_i c_i \int d^d x \mathcal{O}_i(x) \end{aligned}$$

Wilson Coefficients
$c_i(\mu = \Lambda) \sim \Lambda^{\Delta - d}$

local operators of $\phi(x)$
mass dim. Δ: Decoupling

Renormalize existing coefficients and generates new ones
Dependence of large scale theory on small scale r given by simple power counting rule

EFT principles: unknown fundamental theory

- Fundamental theory unknown:
large scale effective Lagrangean can be expanded in terms of local operators

$\downarrow$

write down the most general set of operators consistent with long scale system symmetries with unknown coefficients.

$\downarrow$

Example: finite size effects in gravitational coupling of isolated bodies

EFT for isolated compact object

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

Effective

- List generic operators coupled to particle world-line
- Diffeomorphism invariance

$$S_{pp-fun} = -\sum_i m_i \int d\tau$$

Integrating out

$$S_{pp-eff} = -m \int d\tau + c_R \int d\tau R + c_V \int d\tau R_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + c_2 \int d\tau \left(R_{\mu\nu\rho\sigma}\right)^2 + \dots$$

(for a spherical body)

EFT for isolated compact object

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

Effective

- List generic operators coupled to particle world-line
- Diffeomorphism invariance

$$S_{pp-fun} = -\sum_i m_i \int d\tau$$

Integrating out

$$S_{pp-eff} = -m\int d\tau + \cancel{c_R \int d\tau R} + c_V \cancel{\int d\tau R_{\mu\nu}} \dot{x}^\mu \dot{x}^\nu + c_2 \int d\tau \left(R_{\mu\nu\rho\sigma}\right)^2 + \ldots$$

(for a spherical body)

EFT applications

- Cosmology
  Generic gravity Lagrangean invariant under spatial rotations (time-dependent space diffeomorphisms)
  Short vs. Large
  inflaton fluctuation vs. Hubble scale of the background

  See P. Cheung et al. 2007

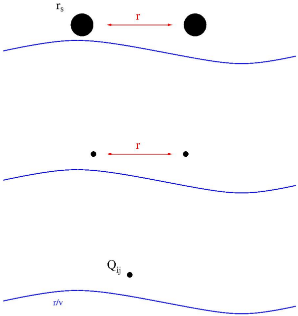
- Hydrodynamics
  Derivative expansions:
  Short vs. Large
  Field time derivative vs. mean free time
  Field space derivatives vs. mean free length

  See Dubovsky et al. 2011

Outline

Different scales in EFT

- Very short distance $\lesssim r_s$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $k_\mu \sim (v/r, 1/r)$
- Long distance: GW's $k_\mu \sim (v/r, v/r)$ coupled to point particles with moments



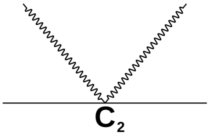
Goldberger and Rothstein PRD '04

Matching example

Cross section for graviton scattering by a single black hole:

$$\sigma_{fund-BH} = r_s^2 f(r_s\omega) \sim \ldots r_s^6\omega^4 + \ldots r_s^{10}\omega^8 \ldots$$

Effective contribution to the amplitude:



$$G_N c_2 \omega^4$$

$$\sigma_{EFT-BH} \sim \ldots + r_s G_N c_2 \omega^4 + \ldots + G_N^2 c_2^2 \omega^8 \implies c_2 \propto \frac{r_s^5}{G_N}$$

Goldberger and Rothstein PRD '04

Physical vs. gauge degrees of freedom

$h_{\mu\nu}$ includes

1. 4 gauge degrees of freedom
2. 2 physical, radiative degrees of freedom
3. 4 physical, non-radiative degrees of freedom

1&3 propagate with "the speed of thought" (Eddington '22)

After fixing the diffeomorphism invariance:

$$h_{\mu\nu} = \begin{pmatrix} -2\Phi & \Xi_i \\ \Xi_i & h_{ij}^{TT} + \theta\delta_{ij} \end{pmatrix}$$

$\partial_i \Xi^i = h_{ij}^{TT}\delta^{ij} = \partial^i h_{ij} = 0$: 6 degrees of freedom left, 4 eaten by gauge fixing

Einstein eq's:

$$\begin{aligned} \nabla^2\Phi = \nabla^2\Xi_i = \nabla^2\Theta &= 0 \\ \Box h_{ij}^{TT} &= 0 \end{aligned}$$

Conservative dynamics

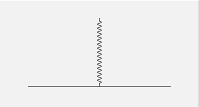
$$\exp\left[iS_{eff}(x_a)\right] = \int \mathcal{D}h(x)\exp\left[iS_{EH}(h) + iS_{pp}(h, x_a)\right]$$

$$S_{pp} = -\sqrt{G_N}m\int dt\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + \sqrt{G_N}h_{00}^2\ldots\right)$$

$$S_{EH} = \int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N}h(\partial h)^2 + \ldots\right]$$

Power counting to integrate out potential gravitons

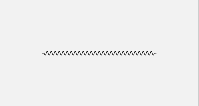
h-M Vertex: $\sim dt\, d^3k\sqrt{G_N}m$



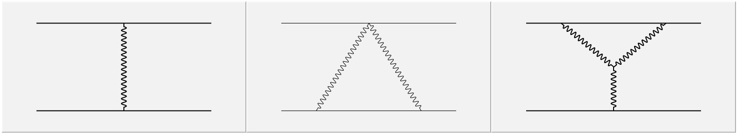
Propagator: $\delta(t)\delta^{(3)}(k)\frac{1}{k^2}(1 + \frac{k_0^2}{k^2} + \ldots)$

In $\int dt\, d^dk\, e^{ik(x_1(t)-x_2(t))}/k^2$

$k \to 1/r$, $k_0 \to \partial_t \sim kv \sim v/r$

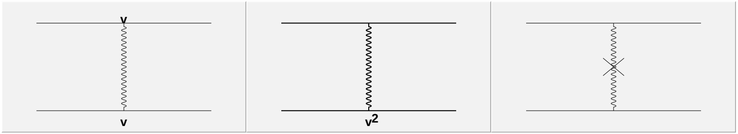


The 1PN potential



Scaling: L $\qquad\qquad\qquad$ Lv^2

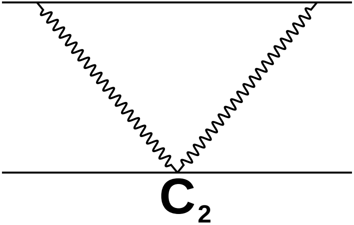
Using virial theorem $v^2 \sim G_N M/r$



$$V = -\frac{Gm_1m_2}{2r}\left[1-\frac{G_Nm_1}{2r}+\frac{3}{2}(v_1^2)-\frac{7}{2}v_1v_2-\frac{1}{2}v_1\hat{r}v_2\hat{r}\right]+$$

$$1 \leftrightarrow 2$$

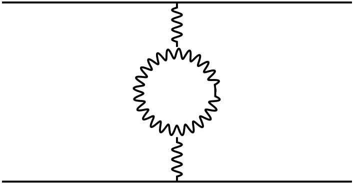
Finite size effects enters at 5PN



Lv^{10}

Re-derivation of the "Effacement principle" (Damour '92)

Quantum corrections are irrelevant



$$S_{eff} \sim \int dt\, \hbar\, G_N^2 \frac{m_1 m_2}{r^3} \simeq \hbar v^3$$

vs. leading order:

$$S_{eff} \int dt\, G_N \frac{m_1 m_2}{r} \simeq L$$

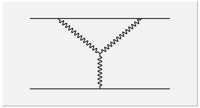
See Donoghue PRD 1994

What's new to EFT in gravity?

- Systematic use of Feynman diagram with manifest power counting rule, enabling the construction of automatized algorithms
- Effective 2-body action is produced without the need to solve for the metric (however as in traditional ADM calculations)
- recast old problems in a field theory language: integrals in momentum space "look" easier to compute

The 3PN computation automatized

Topologies



Graphs — v and time derivative-insertions

Amplitudes — $$A = G_N m_i v_i \int d^d k \, d^d k_1 \frac{1}{k^2 (k-k_1)^2} \ldots$$

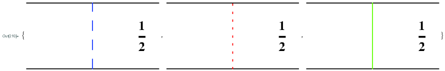
Evaluation — Analytic integral in a database

S. Foffa & RS PRD 2011

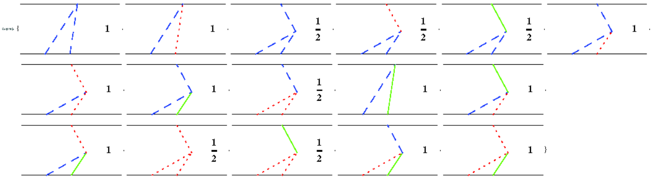
original result Damour, Jaranowski and Schäfer PRD 2001

Feynman diagrams at 3PN order

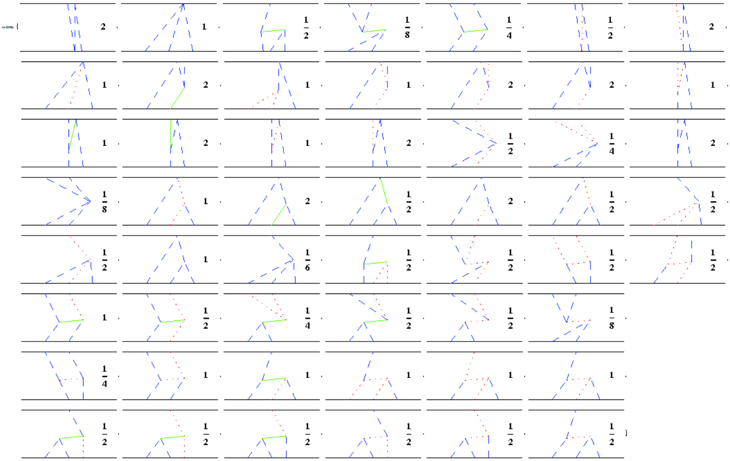
$G_N v^6$



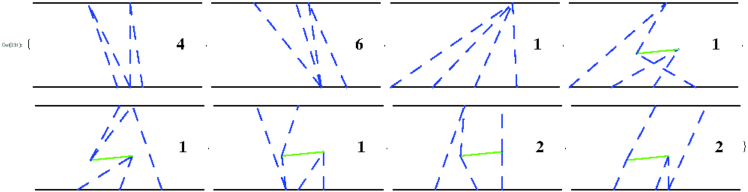
$G_N^2 v^4$



Feynman diagrams at 3PN order: $G_N^3 v^2$



Feynman diagrams at 3PN order: G_N^4



Final result matches previous derivation of 3PN Hamiltonian see eq. (174) of Blanchet's Living Review on Relativity

The 4 PN status

- 3 graphs @ $G_N v^8$ order
- 23 @ $G_N^2 v^6$
- 202 @ $G_N^3 v^4$
- 307 @ $G_N^4 v^2$
- 50 @ G_N^5

See also Jaranowski and Schäfer PRD12

Outline

- General EFT approach: Decompose binary motion into central wordline + moments describing internal dynamics

$$S = -m\int d\tau \quad + \quad \sum_n \int d\tau c_n^{(I)} I^{abi_1\ldots i_n}\nabla_{i_1}\ldots\nabla_{i_n}E_{ab}$$
$$+ \quad \sum_n \int d\tau c_n^{(J)} J^{abi_1\ldots i_n}\nabla_{i_1}\ldots\nabla_{i_n}B_{ab}$$

$$E_{\mu\nu} \equiv C_{\mu\alpha\nu\beta}\dot{x}^\alpha\dot{x}^\beta \qquad B_{\mu\nu} = \epsilon_{\mu\rho\sigma\alpha}C^{\rho\sigma}{}_{\nu\beta}\dot{x}^\alpha\dot{x}^\beta$$

- in GR

$$S \supset \frac{\sqrt{G_N}}{2}\int d^4x\, h_{\mu\nu}T^{\mu\nu}$$

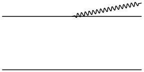
Take the multipole expansion:

$$h_{\mu\nu}(t,x) = \sum_{n=0}^{\infty}\frac{1}{n!}x^{i_1}\cdots x^{i_n}\partial_{i_1}\ldots\partial_{i_n}h_{\mu\nu}(t,0)$$

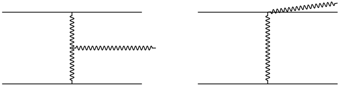
$\Longrightarrow$ compute moment of the Energy Momentum Tensor, decomposed into $SO(3)$ irreducible representations

The dissipative sector

Coupling gravitational waves in all possible ways to the composite systems e.g.



Leading order



$\subset$ NLO

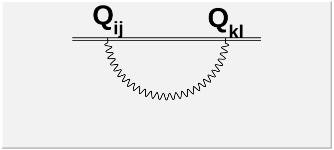
$$S_{EFT-diss} = -\frac{m}{m_{Pl}}\int h_{00} - \left[\frac{1}{2}\sum_a m_a v_a^2 - \frac{G_N m_1 m_2}{r}\right]\frac{h_{00}}{2m_{Pl}} \\ -\frac{1}{2m_{Pl}}\epsilon_{ijk}L_k\partial_j h_{0i} - \frac{1}{2m_{Pl}}\sum_a m_a x_i x_j R^0{}_{i0j}$$

Leading radiative coupling: $T^{ij}h_{ij} \sim \partial_t^2\left(T_{00}x^i x^j\right)h_{ij}$

Radiation reaction

Integrating out the radiation field only the sources are left:

Radiation emitted and absorbed



Using Feynman propagators Effective action modified

$$\Delta S_{real} = -\frac{G_N}{10}\int dt\, Q_{ij}(t)\,\frac{d^5 Q_{ij}(t)}{dt^5}$$

Real part → should modifies e.o.m. giving Burke-Thorne potential:

$$\Delta^{(RR)}\ddot{x}_{ai}(t) = \frac{2G_N}{5} x_{aj}(t) Q^{(5)}_{ij}(t)$$

In the standard Lagrangian formalism

$$e^{iW[J+Q]} = \int \mathcal{D}h_{GW} e^{i\int \partial h^2_{GW} + \partial^2 h^3_{GW} + h_{GW}(J+\ddot{Q})} \propto e^{i\int (J+Q)\Delta_F(J+Q)} + \ldots$$

$$h_{GW}(x) = \frac{\delta W}{\delta J(x)}\bigg|_{J=0} \stackrel{?}{=} \int d^4x'\, D_F(x-x')J(x')$$

$$\begin{aligned} \Delta_F(x) &= \theta(t)\Delta_+(x) + \theta(-t)\Delta_-(x) \\ \Delta_{R,A}(x) &= \theta(\pm t)\left[\Delta_+(x) + \Delta_-(x)\right] \end{aligned}$$

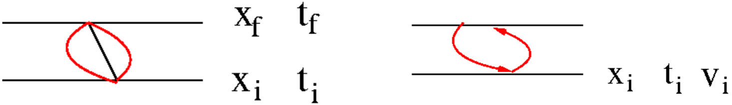
$$\Delta_F = -\tfrac{i}{2}(\Delta_R + \Delta_A) - \tfrac{1}{2}\Delta_+ + \Delta_- \implies h_{GW} \propto \int (\Delta_R + \Delta_A) J$$

A-causal!

Lagrangean formalism *looks* not suitable to describe dissipative systems

(Galley a Tiglio PRD '09)

Solution: Doubling of the degrees of freedom



$$e^{iW[J_i+Q_i]} = \int \mathcal{D}h_1 \mathcal{D}h_2 e^{i\left[S(h_1)-S(h_2)+\int (J_1+Q_1)h_1-(J_2+Q_2)h_2\right]}$$

Specific causal structure:

$$\begin{aligned} W[J_1,J_2] &= \frac{i}{2}\int (J_1,J_2)\begin{pmatrix} G_F & -G_- \\ -G_+ & G_D \end{pmatrix}\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} \\ &= \frac{i}{2}\int (J_+,J_-)\begin{pmatrix} 0 & -iG_R \\ -iG_A & G_H \end{pmatrix}\begin{pmatrix} J_+ \\ J_- \end{pmatrix} \end{aligned}$$

with $J_\pm \propto J_1 \pm J_2$ and e.o.m.'s: $h_{GW} = \left.\frac{\partial W}{\partial J_-}\right|_{\substack{J_-=Q_-=0 \\ Q_+=Q}} \propto \int G_R Q$

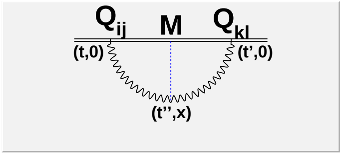
Keldish '65, Chou et al. '80

The radiation-reaction diagram

- Radiation emitted and absorbed

$$S_{eff} \propto \int dt Q_{-ij} Q^{(5)}_{+ij}$$

- Radiation emitted, scattered and absorbed



$$iS_{eff} \quad \propto \quad G_N^2 M \int dt\, Q^{(2)}_{-ij}(t) \int dt' Q^{(2)}_{+ij}(t') \times$$
$$\int dt'' d^3x\, \partial_t^2 G_R(t-t'',x) G_R(t''-t',x) \frac{1}{r}$$

Conservative part of the self force

- At leading order the rad-reaction affects e.o.m. at 2.5PN order

Burke-Thorne potential

$$\Delta^{(SF)}\ddot{x}_{ai}(t) = \frac{2G_N}{5}x_{aj}(t)Q^{(5)}_{ij}(t) - \frac{8}{5}G_N^2 M x_{aj}\int_{-\infty}^{t} dt' Q^{(7)}_{ij}(t')\log\left[\frac{(t-t')}{T}\right]$$

$+$ relative 1.5PN tail correction

- Conservative part associated with tail integral

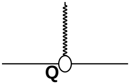
$$\Delta^{(SF)}\ddot{x}_{ai}(t) = \frac{8G_N^2 M}{5}x_{aj}(t)Q^{(6)}_{ij}(t)\log\left(\frac{r}{\lambda}\right)$$

Gravitational radiation emitted, scattered, and absorbed.

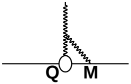
L. Blanchet and T. Damour PRD '88
L. Blanchet, S.L. Detweiler, A. Le Tiec, B. F. Whiting PRD '10

Observables associated with GW emission

$iA_h(k) =$



$+$



$\dots$

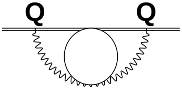
Emission rate

$$d\Gamma_h(k) = \frac{1}{T}\frac{d^3k}{(2\pi)^3 2k}|A_h(k)|^2$$

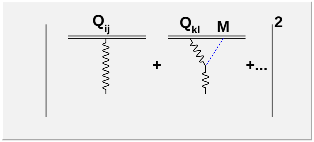
$$F = \int |k| d\Gamma_h(k) = \frac{G_N}{5}\langle\left(\frac{d^3Q^{ij}(t)}{dt^3}\right)^2\rangle + \frac{16G_N}{45}\langle\left(\frac{d^3J^{ij}(t)}{dt^3}\right)^2\rangle + \dots$$

The optical theorem:

Im



$=$



with Feynman propagators → time averaged flux

Logs and renormalization

$$A_{h-Quad} = \frac{i\sqrt{G_N}k^2}{4}\epsilon^*_{ij}(k)I_{ij}(|k|)$$

$$\left|\frac{A}{A_{h-Quad}}(k)\right|^2 = 1 + \ldots + (G_N M|k|)^2\left[-\frac{214}{105}\left(\frac{1}{d-4} + \ln\frac{k^2}{\mu^2} + \ldots\right)\right]$$

True ultraviolet divergence and log appearance → renormalization

$$I_{ij}(|k|) = Z(|k|,\mu)I^R{}_{ij}(|k|,\mu) \qquad Z(|k|,\mu) = 1 + \frac{107}{105}(G_N M|k|)^2 \times \frac{1}{d-4}$$

leading to a classical RG equation

$$\mu\frac{dI^R_{ij}}{d\mu} = -\frac{214}{105}(G_N M|k|)^2\, I^R_{ij}(|k|,\mu) \qquad I^R_{ij}(|k|,\mu) = \left(\frac{\mu}{\mu_0}\right)^{-\frac{107}{105}(Gmk)^2} I^R_{ij}(|k|,\mu_0)$$

Prediction of leading logs at higher order, e.g. $\#(G_N M|k|)^4 \log^2\left(\frac{k^2}{\mu^2}\right)$

Goldberger and Ross PRD '10

Conclusions

- EFT is a powerful and flexible tool, applicable to problems exhibiting clear scale separation
- PN computations within EFT are equivalent to computations performed with tradition method: predictions for the same physical observables must give same values
- PN computations within EFT methods provide a healthy competition with traditional methods
- Merit of EFT methods in gravity: ricycle huge knowledge accumulated in theoretical particle physics

Spare slides
Binary inspiral phenomenology

GW detection

Inspiral

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} E(v) &= -\tfrac{1}{2}\nu M v^2 \left(1 + \#(\nu) v^2 + \#(\nu) v^4 + \ldots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \tfrac{32}{5G_N} v^{10} \left(1 + \#(\nu) v^2 + \#(\nu) v^3 + \ldots\right) \end{aligned}$$

E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T \omega(t) dt &= -\int^{v(T)} \frac{\omega(v) dE/dv}{P(v)} dv \\ &\sim \int \left(1 + \#(\nu) v^2 + \ldots + \#(\nu) v^6 + \ldots\right) \frac{dv}{v^6} \end{aligned}$$

GW detection

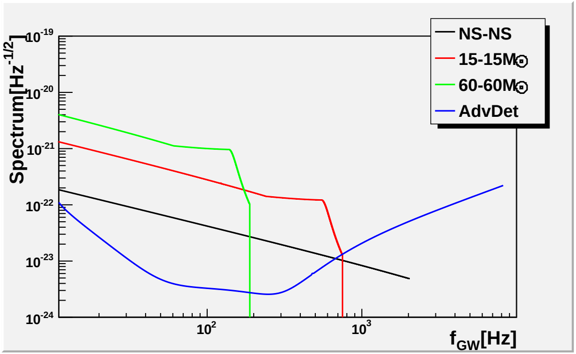
$$N_{cycles} \simeq 1.6 \cdot 10^4 \left(\frac{10\text{Hz}}{f_{min}}\right)^{5/3} \left(\frac{1.2 M_\odot}{M_c}\right)^{5/3}$$

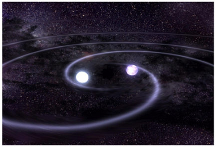
Sensitivity $\propto M_c^{5/3} \sqrt{N_{cycles}} \propto M_c^{5/6}$

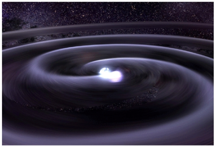
$f_{Max} \propto M^{-1}$, $M_c \equiv (m_1 m_2)^{3/5} (m_1 + m_2)^{2/5}$

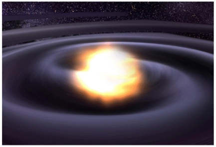
Important to know the phase at $O(1)$ when taking correlation of detector's output and model waveform

Detector sensitivity









Observational rate estimates

LIGO/Virgo Advanced Observatories will detect

	NS-NS	10 $M_\odot$ BH-BH
Distance (Mpc)	$300\mathrm{Mpc}$	$1\mathrm{GPc}$
Rates $\mathrm{MWEG}^{-1}\mathrm{Myear}^{-1}$	$1 \div 10^3$	$4 \cdot 10^{-2} \div 100$

$$N = 0.011 \times \frac{4}{3}\pi \left(\frac{D_H}{2.26Mpc}\right)^3 \mathrm{MWEG}$$

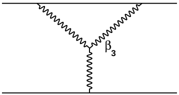
Best case:

$$r_{NS-NS} \sim 400\mathrm{yr}^{-1} \qquad r_{BH-BH} \sim 10^3\mathrm{yr}^{-1}$$

I. Mandell et al. PRD 2010

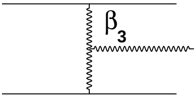
Fundamental gravity tests: Graviton self-interactions

- Conservative dynamics



$$V \supset \beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2}$$

- Emission



$$L_{pp} \supset h_{ij} \beta_3 (\nu M x_i \ddot{x}_j)$$

Example of tagging of fundamental physics effects

Bound on self-interaction triple vertex

At present the binary pulsars give best constraint on non-conservative effect from β_3

$$\dot{P}_{\beta_3} = \dot{P}_{GR}(1 + c\beta_3) \qquad c \simeq 3.21$$

Given that $\frac{\dot{P}_{obs}}{\dot{P}_{GR}} - 1 \simeq 0.1\% \implies \beta_3 = (4.0 \pm 6.4) \cdot 10^{-4}$
Conservative effect of β_3 already constrained by Lunar Laser Ranging, as @ 1PN

$$\beta_3 = \beta_{PPN} < 2 \cdot 10^{-4}$$

Cannella et al. '09

Bayesian analysis of GR vs. modGR

Searching for waveforms whose phase is modified at any PPN waveforms

$$\begin{aligned}\phi(t) &= \phi_N(t)\left[1+\phi_1(t)(1+\delta_1)+\phi_{1.5}(1+\delta\phi_{1.5})\right.\\ &\left.+\phi_2(1+\delta\phi_2)\right]\end{aligned}$$

and injecting fake signals with

$$\phi_{inj}(t) = \phi_{GR}(t) + \phi_N(t)\delta\phi_A(t)$$

Li et al. 2011

Baysian analysis of GR vs. modGR

$$
\begin{aligned}
O_i &= P(H_i|d) \\
&= P(H_i)\frac{P(d|H_i)}{P(d)}
\end{aligned}
$$

$$
\begin{aligned}
O_{GR}^{mGR} &= \frac{O_{mGR}}{O_{GR}} \\
&\propto \frac{P(d|H_{mGR})}{P(d|H_{GR})}
\end{aligned}
$$

(1 catalog = 15 sources)