Fundamental constants, gravitation and cosmology

Jean-Philippe UZAN

Constants

Physical theories involve constants

These parameters cannot be determined by the theory that introduces them.

These arbitrary parameters have to be assumed constant:

- *experimental validation*
- *no evolution equation*

By testing their constancy, we thus test the laws of physics in which they appear.

A physical measurement is always a comparison of two quantities, one can be thought as a unit

- *it only gives access to dimensionless numbers*
- *we consider variation of dimensionless combinations of constants*

JPU, Rev. Mod. Phys. **75**, 403 (2003); Liv. Rev. Relat. (to appear, 2010)
JPU, [astro-ph/0409424, arXiv:0907.3081]
R. Lehoucq, JPU, *Les constantes fondamentales* (Belin, 2005)
G.F.R. Ellis and JPU, Am. J. Phys. **73** (2005) 240
JPU, B. Leclercq, De *l'importance d'être une constante* (Dunod, 2005)
translated as "*The natural laws of the universe*" (Praxis, 2008).

Reference theoretical framework

The number of physical constants depends on the level of description of the laws of nature.

In our present understanding [*General Relativity* + *SU(3)xSU(2)xU(1)*]:

- G : Newton constant (**1**)

- **6** Yukawa coupling for quarks
- **3** Yukawa coupling for leptons

- mass and VEV of the Higgs boson: **2**

- CKM matrix: **4** parameters
- coupling constants: **3**
- Λ_{uv}: **1**

- c, ħ : **2**

- cosmological constant

22 constants
19 parameters

Number of constant may change

This number is « time-dependent ».

Neutrino masses

Add **3** Yukawa couplings + **4** CKM parameters = **7** more

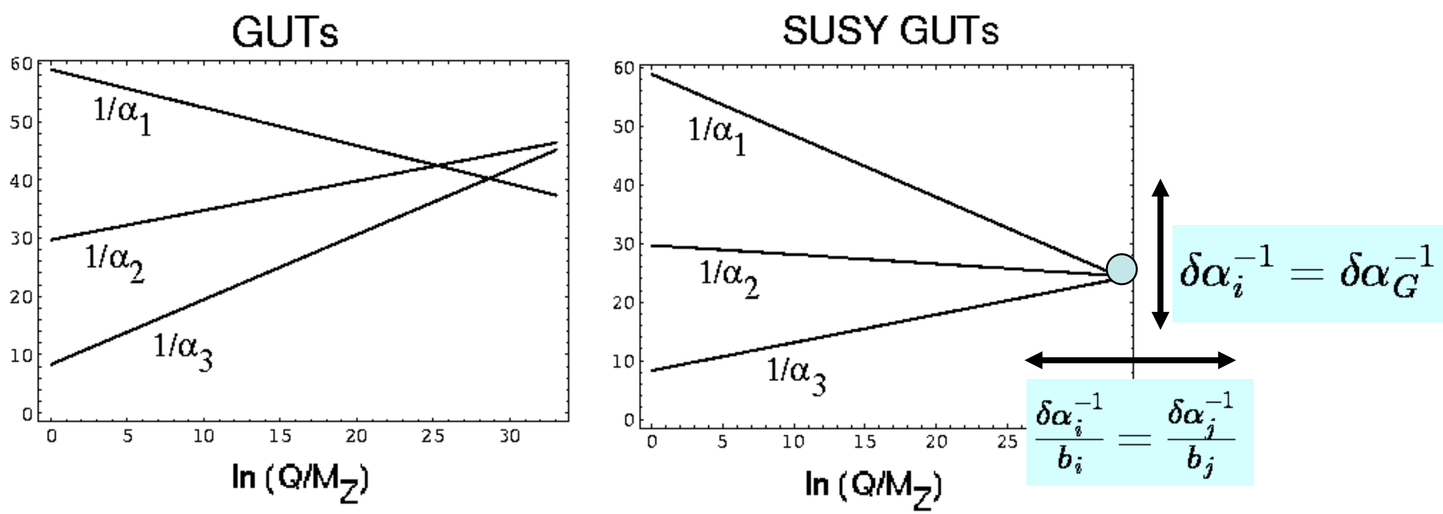
Unification

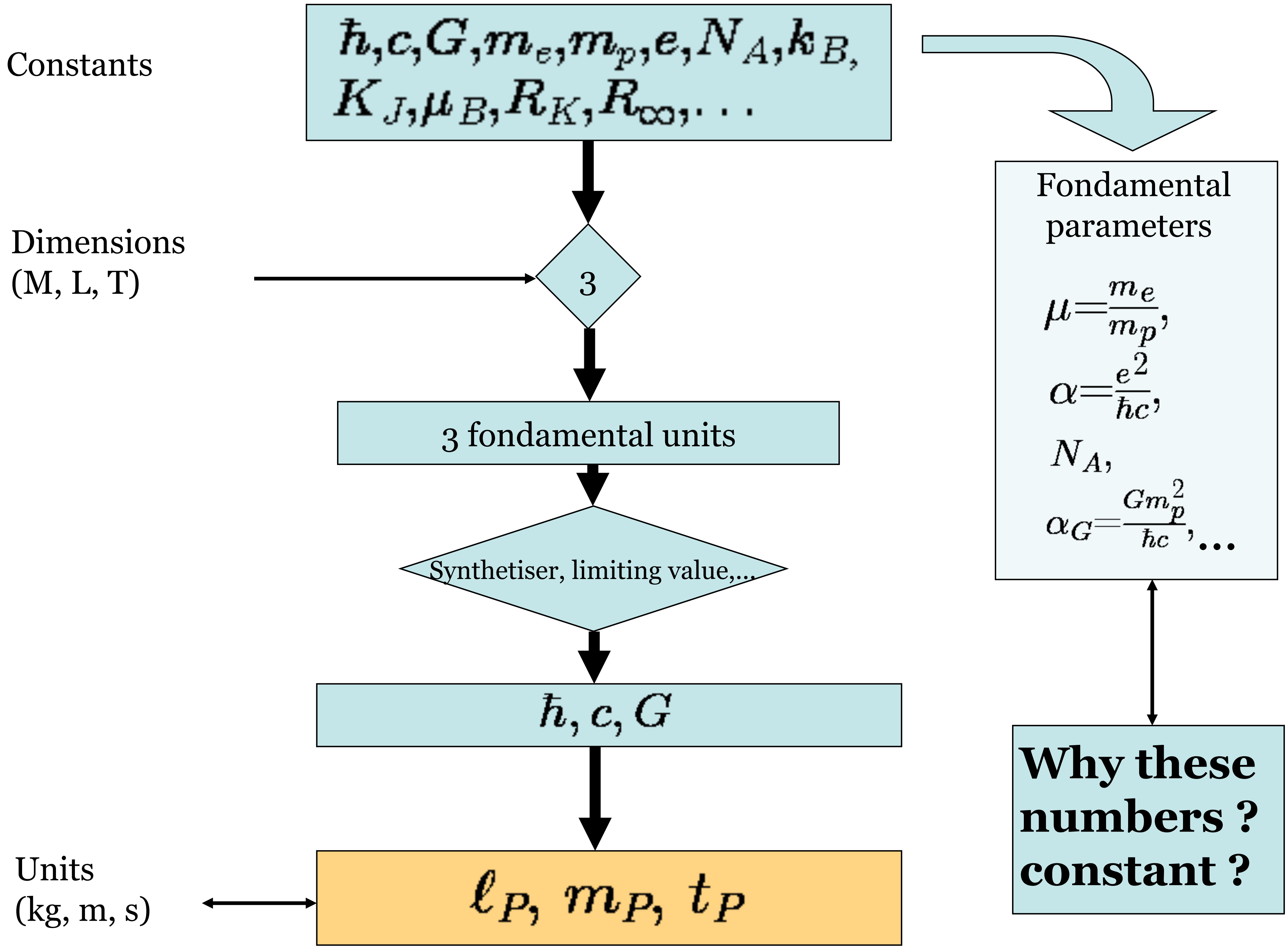
$$\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi}\ln\frac{M_{GUT}}{E}$$

SM : $b_i = (41/10, -19/6, -7)$

MSSM : $b_i = (33/5, 1, -3)$

$$\alpha^{-1} = \tfrac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$$




Constants
$\hbar, c, G, m_e, m_p, e, N_A, k_B, K_J, \mu_B, R_K, R_\infty, \ldots$
Dimensions
(M, L, T)
3
3 fondamental units
Synthetiser, limiting value,...
$\hbar, c, G$
Units
(kg, m, s)
ℓ_P, m_P, t_P
Fondamental parameters
$\mu = \frac{m_e}{m_p}$,
$\alpha = \frac{e^2}{\hbar c}$,
N_A,
$\alpha_G = \frac{G m_p^2}{\hbar c}$,...
Why these numbers ? constant ?

Constants

Are they constant?

Test of physcis and GR,
Variations are predicted by most extensions of general relativity.
Important question from a cosmological point of view

Why do they have the value we measure?

Why is the universe just so?
Cosmology allows a way to attack this question

I- Links to general relativity and example of theories with varying constants

II- Setting constraints on variation of fundamental constants
Phyisical systems – general approach

III- 2 detailed examples: BBN – 3alpha

IV- links to cosmology & conclusions

Part I: constants and gravity

GR in a nutshell

Underlying hypothesis

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

$$S_{matter}(\psi, g_{\mu\nu})$$

Dynamics

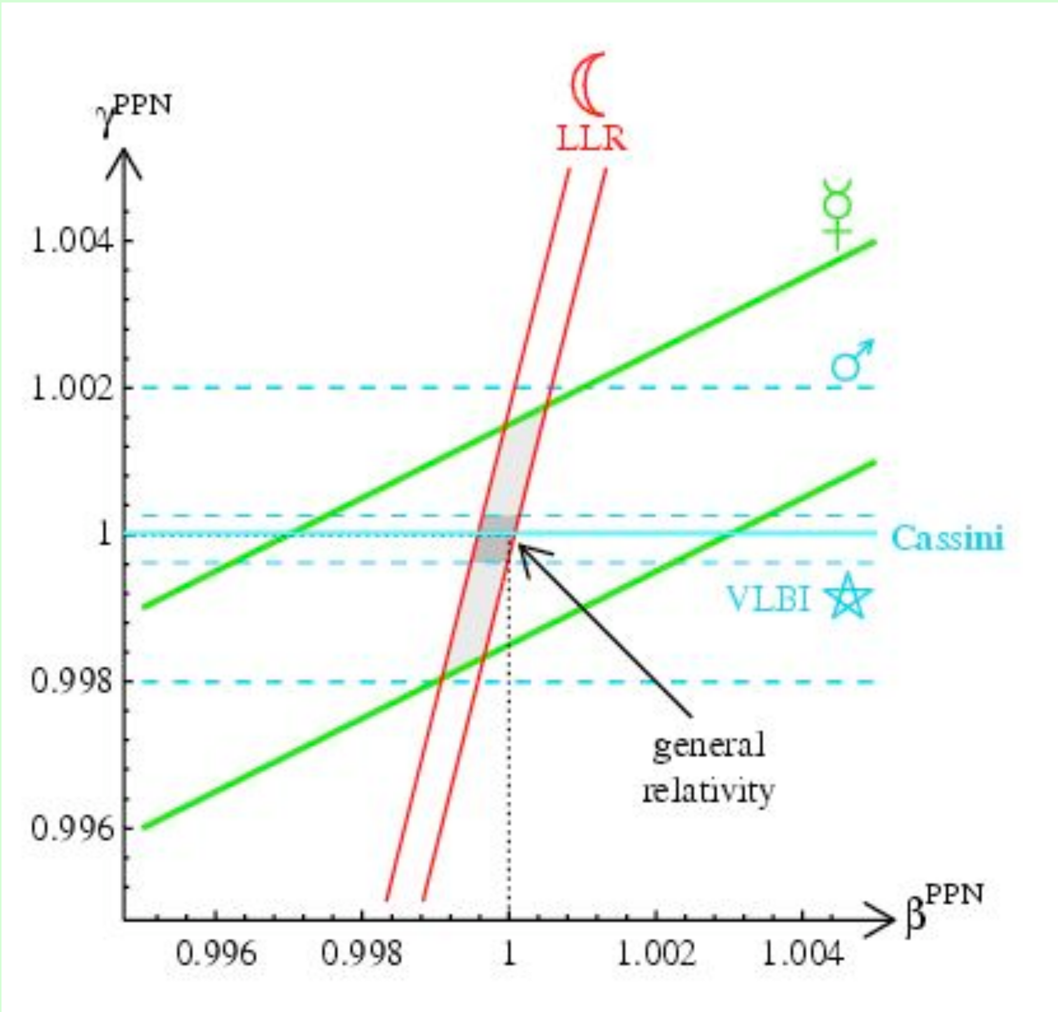
$$S_{grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}\,R_*\,d^4x$$

Relativity

$$g_{\mu\nu} = g^*_{\mu\nu}$$

General relativity: experimental validity

Courtesy of G. Esposito-Farèse

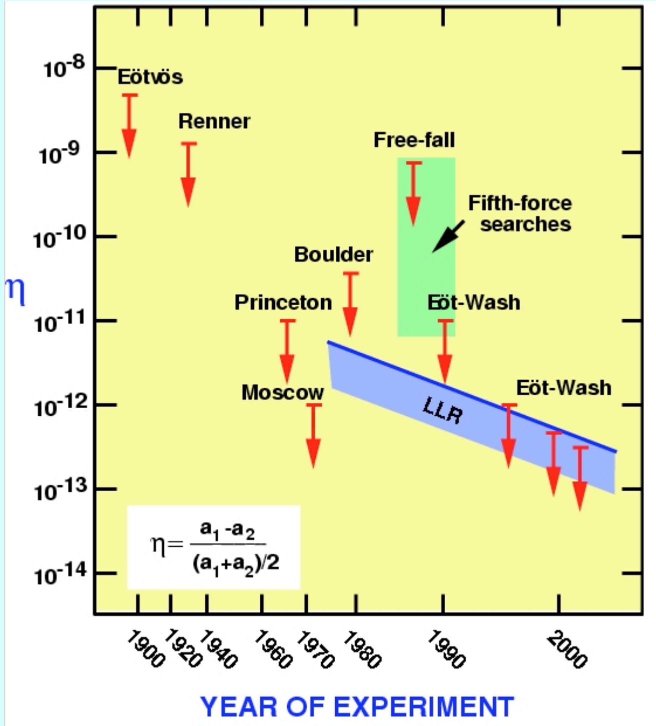


$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

Universality of free fall

$$\eta = 2\frac{|a_1 - a_2|}{|a_1 + a_2|}$$



C. Will, gr-qc/0510072

« Constancy » of constants

JPU, RMP (2003)

Equivalence principle and constants

Action of a test mass:

$$S = -\int mc\sqrt{-g_{\mu\nu}v^\mu v^\nu}\,dt \qquad \text{with} \qquad \begin{array}{l} v^\mu = dx^\mu/dt \\ u^\mu = dx^\mu/d\tau \end{array}$$

$$\delta S = 0$$

$$a^\mu \equiv u^\nu \nabla_\nu u^\mu = 0 \qquad \text{(geodesic)}$$

$$g_{00} = -1 + 2\Phi_N/c^2 \qquad \text{(Newtonian limit)}$$

$$\dot{\mathbf{v}} = \mathbf{a} = -\nabla\Phi_N = \mathbf{g}_N$$

Equivalence principle and constants

Action of a test mass:

$$S = -\int m_A[\alpha_i] c \sqrt{-g_{\mu\nu} v^\mu v^\nu} dt$$

with $v^\mu = dx^\mu / dt$, $u^\mu = dx^\mu / d\tau$

Dependence on some constants

$$\delta S = 0$$

$$f_{A,i}$$

$$a_A^\mu = -\sum_i \left(\frac{\partial \ln m_A}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial x^\beta} \right) \left(g^{\beta\mu} + u^\beta u^\mu \right)$$

(NOT a geodesic)

$$g_{00} = -1 + 2\Phi_N / c^2$$

(Newtonian limit)

$$\mathbf{a} = \mathbf{g}_N + \delta \mathbf{a}_A$$

$$\delta \mathbf{a}_A = -c^2 \sum_i f_{A,i} \left(\nabla \alpha_i + \dot{\alpha}_i \frac{\mathbf{v}}{c^2} \right)$$

Anomalous force
Composition dependent

The same in purely Newtonian

If some constants vary then the mass of any nuclei becomes spacetime dependent

In Newtonian terms, a free motion implies

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = \vec{0}$$

If a constant varies then

$$\frac{d\vec{p}}{dt} = \vec{0} = m\vec{a} + \underbrace{\frac{dm}{d\alpha}\dot{\alpha}\vec{v}}_{m\vec{a}_{\text{anomalous}}}$$

Universality of free-fall is violated

Field theory

If a constant is varying, this implies that it has to be replaced by a dynamical field

This has 2 consequences:

1- the equations derived with this parameter constant will be modified

one cannot just make it vary in the equations

2- the theory will provide an equation of evolution for this new parameter

The field responsible for the time variation of the « constant » is also responsible for a long-range (composition-dependent) interaction

i.e. at the origin of the deviation from General Relativity.

Example: ST theory

Most general theories of gravity that include a scalar field beside the metric

Mathematically **consistent**

Motivated by **superstring**

dilaton in the graviton supermultiplet,

modulii after dimensional reduction

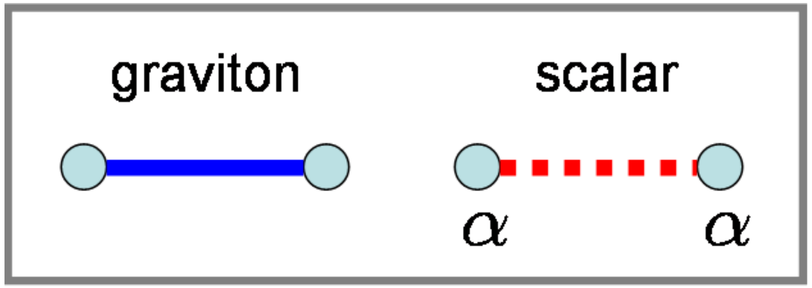
Consistent field theory to satisfy WEP

Useful extension of GR (simple but general enough)

spin 2

spin 0

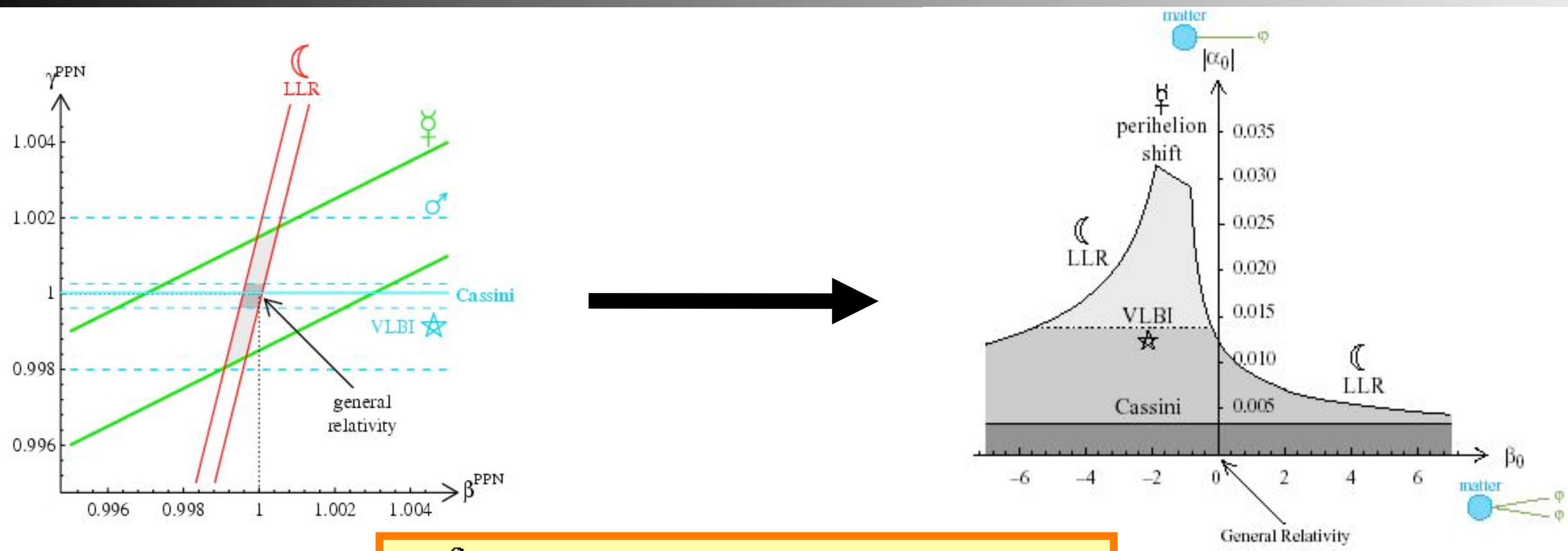
$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\{R - 2(\partial_\mu\phi)^2 - V(\phi)\} + S_m\{\mathrm{matter}, \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}\}$$



$$\alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$
$$\beta = \mathrm{d}\alpha/\mathrm{d}\phi$$

ST theory: déviation from GR and variation

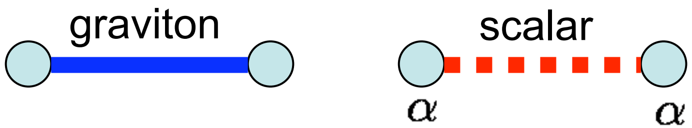
Courtesy of Esposito-Farèse



$$\alpha_0^2 < 10^{-5}, \qquad -4.5 < \beta_0$$

Time variation of G

$$G_{\mathrm{cav}} = G(1+\alpha^2)$$



$$\left.\frac{\dot{G}}{G}\right|_0 \equiv \sigma_0 H_0$$

$$\frac{\dot{G}}{G} < 10^{-12}\,\mathrm{yr}^{-1}$$

$$\sigma_0 < 10^{-2}$$

Constraints valid for a (almost) massless field.

Example of varying fine structure constant

It is a priori « easy » to design a theory with varying fundamental constants

Consider

$$S = \int \{ \frac{1}{16\pi G} R - 2(\partial_\mu \phi)^2 - V(\phi) - \frac{1}{4} B(\phi) F_{\mu\nu}^2 \} \sqrt{-g}\, d^4x$$

But that may have dramatic implications.

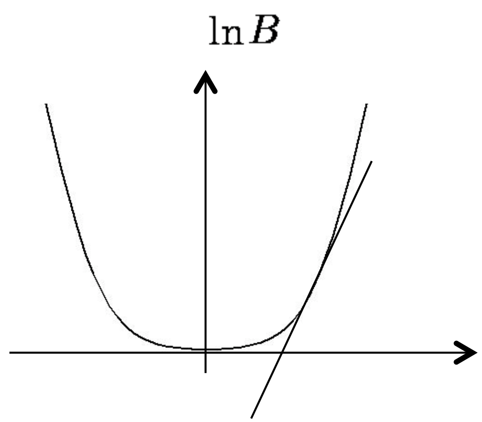
$$m_A(\phi) \supset 98.25\alpha \frac{Z(Z-1)}{A^{1/3}} \text{MeV} \longrightarrow f_i = \partial_\phi \ln m_i \sim 10^{-2} \frac{Z(Z-1)}{A^{4/3}} \alpha'(\phi)$$

Violation of UFF is quantified by

$$\eta_{12} = 2\frac{|\vec{a}_1 - \vec{a}_2|}{|\vec{a}_1 + \vec{a}_2|} = \frac{f_{\rm ext}|f_1 - f_2|}{1 + f_{\rm ext}(f_1 + f_2)/2}$$

It is of the order of

$$\eta_{12} \sim 10^{-9} \underbrace{\mathrm{X}_{1,2,\mathrm{ext}}(A, Z)}_{\mathcal{O}(0.1-10)} \times (\partial_\phi \ln B)_0^2$$



Requires to be close to the minimum

Extra-dimensions

Such terms arise when compatifying a higher-dimensional theories

Example:

$$S = \frac{1}{12\pi^2 G_5}\int \bar{R}\sqrt{|\bar{g}|}\,\mathrm{d}^5 x$$

5D theory

$$\bar{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \frac{1}{M^2}\phi^2 A_\mu A_\nu & \frac{1}{M}\phi^2 A_\mu \\ \frac{1}{M}\phi^2 A_\nu & \phi^2 \end{pmatrix}$$

Compactification

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4 x\sqrt{-g}\,\phi\left(R - \frac{\phi^2}{4M^2}F_{\alpha\beta}F^{\alpha\beta}\right)$$

4D effective theory

$$G = \frac{3\pi \bar{G}_5}{4V_{(5)}} \quad V_{(5)} = \int \mathrm{d}y$$

Varying fine structure constant

Varying G

String (inspired)

$$S = \int d^4x \sqrt{-g}\,(B_g R - B_\phi (\partial\phi)^2 - \tfrac{1}{4} B_{F_i} F_i^2 \; - B_\psi \psi D \bar{\psi} - V)$$

Little is known about these functions

$$B_i = e^{-\phi} + c_0^{(i)} + c_1^{(1)} e^{\phi} + \ldots$$

For the attracttion mechanism to exist:

they must have a minimum at a common value

Damour, Polyakov (1994)

In Jordan frame

$$S = \int \sqrt{-g}\, d^4x \tfrac{1}{16\pi G}(R - 2(\partial\phi)^2 - U) - \tfrac{1}{4} B_F F^2 - \int m_i(\phi) ds$$

Then all constants vary (correlated)

$$S = \int \sqrt{-g}\, d^4x \frac{1}{16\pi G}(R - 2(\partial\phi)^2 - U) - \frac{1}{4}B_F F^2 - \int m_i(\phi) ds$$

Masses are now field dependent

$$m_i = Z(m_p + m_e) + N m_n + E_3 + E_1$$

$$m_{p,n} = \Lambda_{QCD}\left(1 + f(m_q/\Lambda_{QCD})\right)$$

$$\Lambda_{QCD} \propto B_g^{-1/2} e^{-8\pi B_F/\alpha_3} \Lambda_S$$

$$m_i(\phi) = \Lambda_{QCD}(\phi)\left(1 + a^q \frac{m_q}{\Lambda_{QCD}} + a^e \alpha\right)$$

Composition idependent

$|\gamma - 1|, \beta - 1, \dot{G}$

Composition dependent

$\eta, \dot{\alpha}, \dot{\mu}$

$B \simeq -\frac{1}{2}\kappa(\phi - \phi_m)^2$ all deviations are proportional to $(\phi_0 - \phi_m)^2$

Avoiding UFF problem TODAY

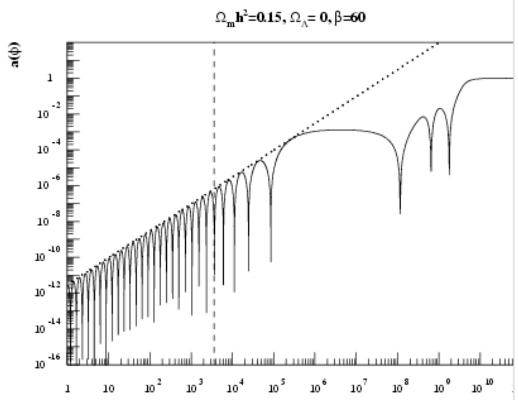
Klein-Gordon equation (ST theory)

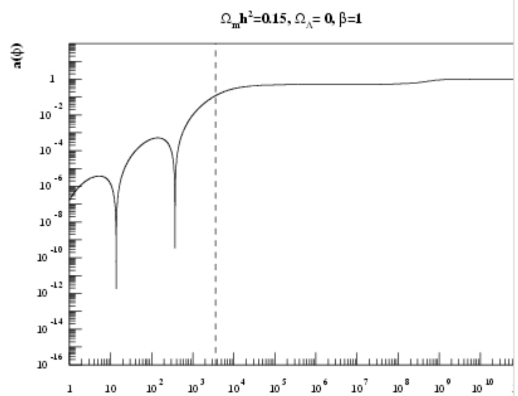
Damour, Nordtvedt (1993)

$$\frac{2}{3-\varphi_*'^2}\varphi_*'' + \left(1-\frac{P_T}{\rho_T}\right)\varphi_*' = -\alpha(\varphi_*)\frac{\rho_* - 3P_*}{\rho_T} - \alpha_V\frac{\rho_V - 3P_V}{\rho_T}$$

$$\alpha_V = \mathrm{d}\ln V^{1/4}/\mathrm{d}\varphi_*.$$

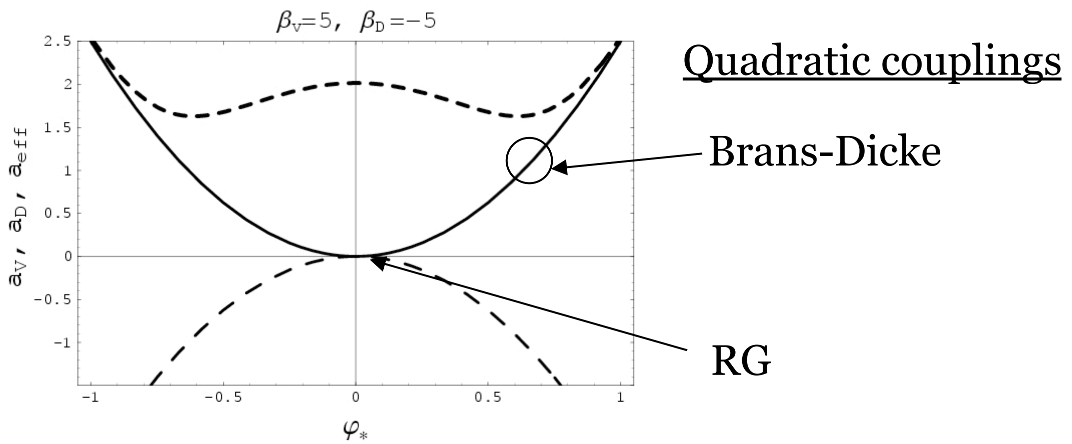
If *ln(A)* has a minimum, the field is driven toward the minimum and the ST theory attracted toward GR





Distinct minima: $+S_V[A_V^2(\varphi_*)g^*_{\mu\nu};\psi_V] + S_D[A_D^2(\varphi_*)g^*_{\mu\nu};\psi_D]$

$$A_{\rm eff}(\varphi_*) = A_V(\varphi_*) + \tilde{\Xi}_0 A_D(\varphi_*)$$



Coc, Olive, JPU, Vangioni, 2007

Summary

The constancy of fundamental constants is a test of the equivalence principle.

The magnitude of the variation of the constants, violation of the universality of free fall and other deviations from GR are of the same order.

« Dynamical constants » are generic in most extenstions of GR (extra-dimensions, string inspired model.

If one constant is varying then many other constants will also be varying (a consequence of unification).

They open a window on these theories or challenge them to explain why the constants vary so little (stabilisation mechanism).

In order to satisfy the constraints from the UFF today, there are 2 possibilities:

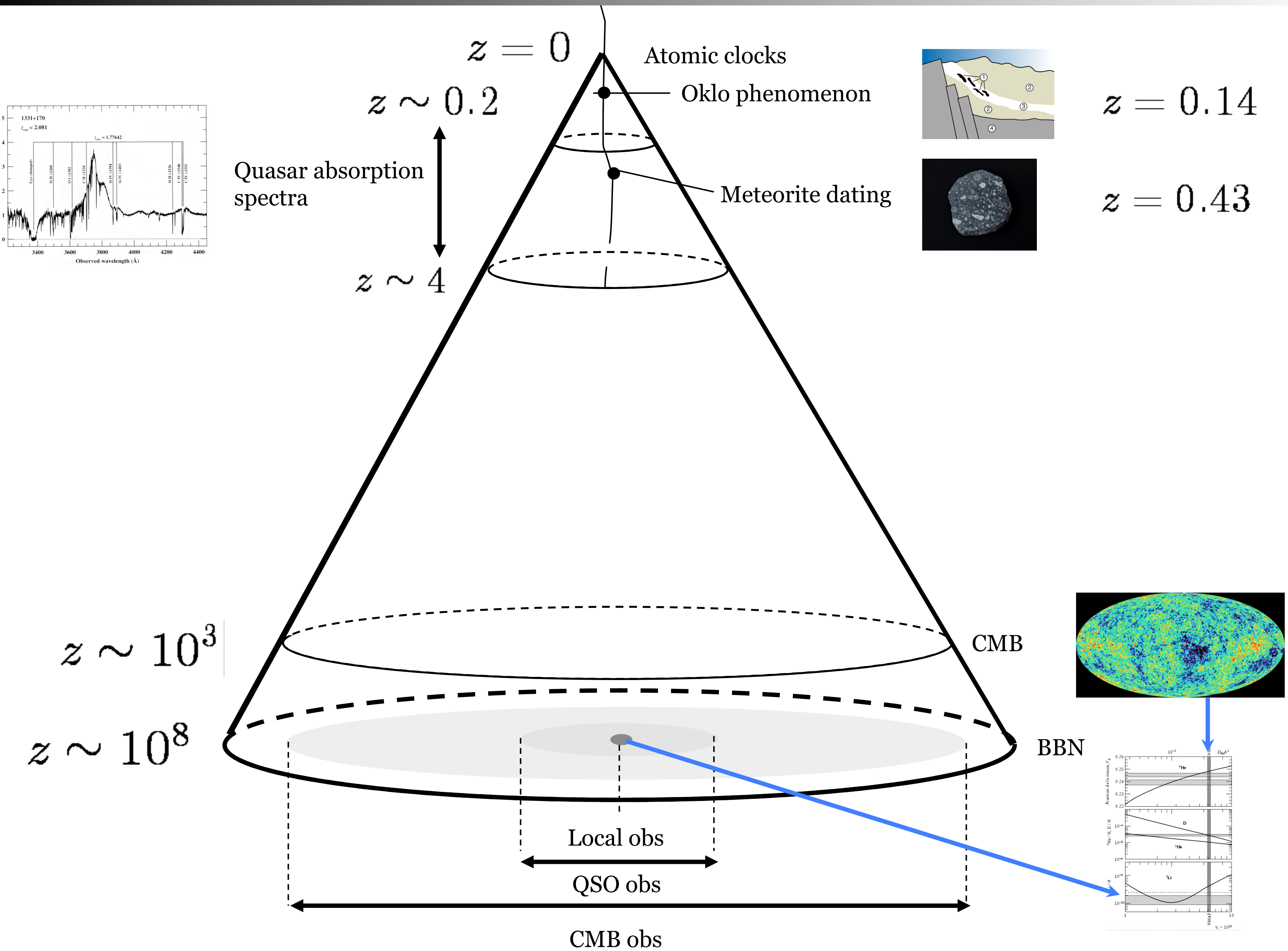
- Least coupling principle
- Chameleon mechanism

In both cases, the variations in the past are expected to be larger than on Solar system scales.

Part II: Testing for constancy

JPU, Rev. Mod. Phys. **75**, 403 (2003)
JPU, [astro-ph/0409424]
R. Lehoucq, JPU, *Les constantes fondamentales* (Belin, 2005)
G.F.R. Ellis and JPU, Am. J. Phys. **73** (2005) 240
JPU, B. Leclercq, De *l'importance d'être une constante* (Dunod, 2005)

Physical systems



Observables and primary constraints

A given physical system gives us an observable quantity

$$O(G_k, X)$$

→ External parameters: *temperature,...:*

→ Primary physical parameters

From a physical model of our system we can deduce the sensitivities
to the primary physical parameters

$$\kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k}$$

The primary physical parameters are usually not fundamental constants.

$$\Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i$$

Physical systems

System	Observable	Primary constraint	Other hypothesis
Atomic clocks	Clock rates	α, μ, g_i	-
Quasar spectra	Atomic spectra	α, μ, g_p	Cloud physical properties
Oklo	Isotopic ratio	E_r	Geophysical model
Meteorite dating	Isotopic ratio	λ	
CMB	Temperature anisotropies	α, μ	Cosmological model
BBN	Light element abundances	Q, τ_n, m_e, m_N, α, B_d	Cosmological model

Atomic clocks

Based the comparison of atomic clocks using different transitions and atoms:

e.g. hfs Cs vs fs Mg : **$g_p\mu$** ;

hfs Cs vs hfs H: **$(g_p/g_I)\alpha$**

$$\frac{\nu_{Cs}}{\nu_{Rb}} \propto \frac{g_{Cs}}{g_{Rb}}\alpha^{0.49} \qquad \frac{\nu_{Cs}}{\nu_H} \propto g_{Cs}\mu\alpha^{2.83}$$

Clock 1	Clock 2	Constraint (yr^{-1})	Constants dependence	Reference
	$\frac{d}{dt}\ln\left(\frac{\nu_{clock_1}}{\nu_{clock_2}}\right)$			
^{87}Rb	^{133}Cs	$(0.2 \pm 7.0) \times 10^{-16}$	$\frac{g_{Cs}}{g_{Rb}}\alpha_{EM}^{0.49}$	Marion (2003)
^{87}Rb	^{133}Cs	$(-0.5 \pm 5.3) \times 10^{-16}$		Bize (2003)
^{1}H	^{133}Cs	$(-32 \pm 63) \times 10^{-16}$	$g_{Cs}\mu\alpha_{EM}^{2.83}$	Fischer (2004)
^{199}Hg^{+}	^{133}Cs	$(0.2 \pm 7) \times 10^{-15}$	$g_{Cs}\mu\alpha_{EM}^{6.05}$	Bize (2005)
^{199}Hg^{+}	^{133}Cs	$(3.7 \pm 3.9) \times 10^{-16}$		Fortier (2007)
^{171}Yb^{+}	^{133}Cs	$(-1.2 \pm 4.4) \times 10^{-15}$	$g_{Cs}\mu\alpha_{EM}^{1.93}$	Peik (2004)
^{171}Yb^{+}	^{133}Cs	$(-0.78 \pm 1.40) \times 10^{-15}$		Peik (2006)
^{87}Sr	^{133}Cs	$(-1.0 \pm 1.8) \times 10^{-15}$	$g_{Cs}\mu\alpha_{EM}^{2.77}$	Blatt (2008)
^{87}Dy	^{87}Dy			Cingöz (2008)
^{27}Al^{+}	^{199}Hg^{+}	$(-5.3 \pm 7.9) \times 10^{-17}$	$\alpha_{EM}^{-3.208}$	Blatt (2008)

Oklo- a natural nuclear reactor

It operated 2 billion years ago, during 200 000 years !!

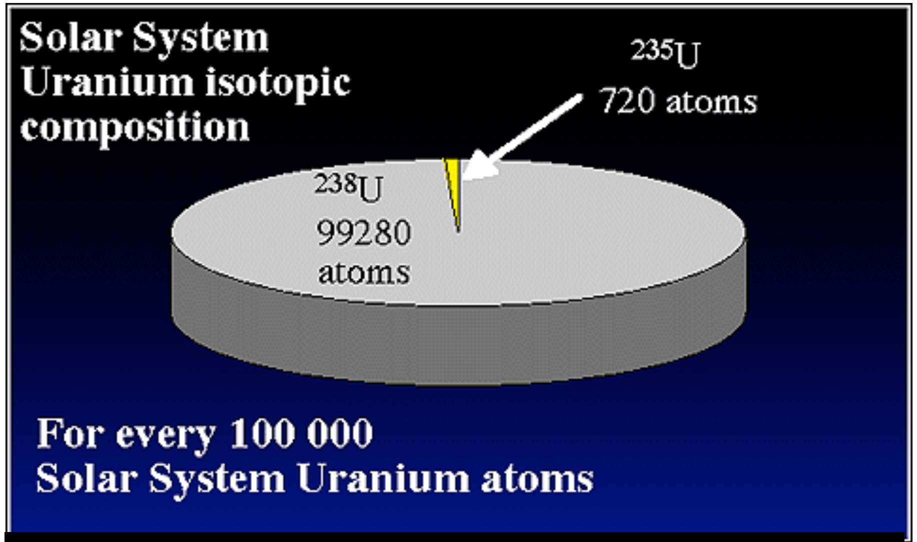
4 conditions :

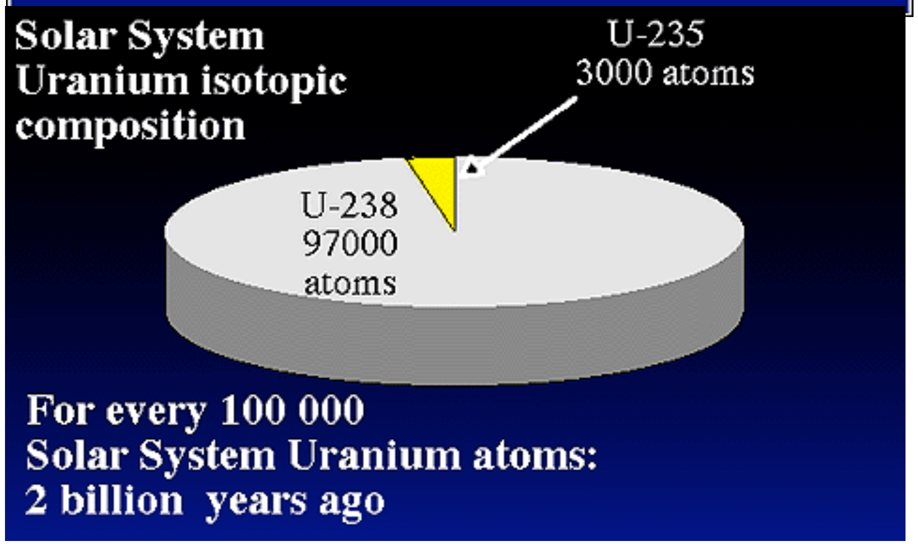
1- Naturally high in U^{235},

2- moderator : water,

3- low abundance of neutron absorber,

4- size of the room.





Oklo-constraints

Shlyakhter, Nature **264** (1976) 340
Damour, Dyson, NPB **480** (1996) 37
Fujii et al., NPB **573** (2000) 377
Lamoreaux, torgerson, nucl-th/0309048
Flambaum, shuryak, PRD**67** (2002) 083507

Natural nuclear reactor in Gabon,
operating 1.8 Gyr ago (**z~0.14**)

Abundance of Samarium isotopes

$$^{149}\mathrm{Sm} + n \rightarrow ^{150}\mathrm{Sm} + \gamma \qquad E_r = 0.0973\,\mathrm{eV}$$

From isotopic abundances of Sm, U and Gd, one can
measure the cross section averaged on the thermal neutron flux

$$\hat{\sigma}_{149}(T, E_r) = 91 \pm 6\,\mathrm{kb}$$

From a model of Sm nuclei, one can infer $s = \Delta E_r / \Delta \ln \alpha$

s~1Mev so that

$$\Delta\alpha/\alpha \sim 1\mathrm{Mev}/0.1\mathrm{eV} \sim 10^{-7}$$

$\Delta\alpha/\alpha = (0.5 \pm 1.05) \times 10^{-7}$ Damour, Dyson, NPB **480** (1996) 37

Fujii et al., NPB **573** (2000) 377 2 branches.

Meteorite dating

Bounds on the variation of couplings can be obtained by
Constraints on the lifetime of long-lives nuclei (α and β decayers)

For β decayers, $$\lambda \sim \Lambda(\Delta E)^p \propto G_F^2 \alpha^s$$

Rhenium: $${}^{187}_{75}\mathrm{Re} \longrightarrow {}^{187}_{76}\mathrm{Os} + \bar{\nu}_e + e^-$$ Peebles, Dicke, PR **128** (1962) 2006

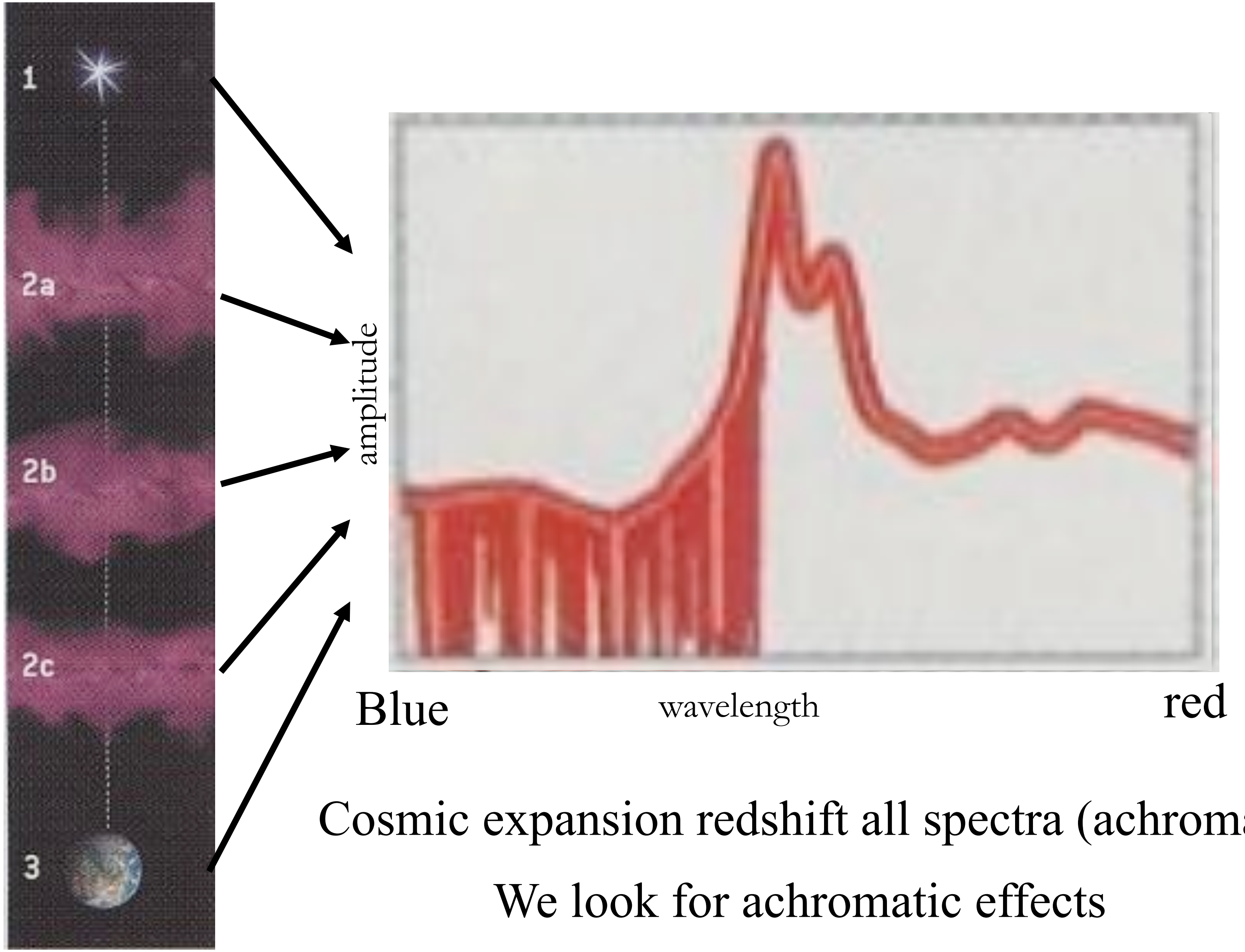
$$\Delta E \sim 2.5\,\mathrm{keV}, \qquad s \sim -18000$$

Use of laboratory data +meteorites data

$$-24 \times 10^{-7} < \Delta\alpha/\alpha < 8 \times 10^{-7}$$ Olive et al., PRD **69** (2004) 027701

Caveats: meteorites datation / averaged value

Absorption spectra



Cosmic expansion redshift all spectra (achromatic)

We look for achromatic effects

QSO

3 main methods:

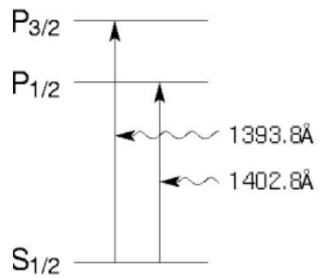
Alkali doublet (AD) Savedoff 1956

Fine structure doublet, $\Delta\lambda/\lambda \propto \alpha^2$

Single atom

Rather weak limit

Si IV alkali doublet



VLT/UVES: Si IV in 15 systems, *1.6<z<3*

$$\frac{\Delta\alpha}{\alpha} = (0.15 \pm 0.43) \times 10^{-5}$$

Chand et al. 2004

HIRES/Keck: Si IV in 21 systems, *2<z<3*

$$\frac{\Delta\alpha}{\alpha} = (-0.5 \pm 1.3) \times 10^{-5}$$

Murphy et al. 2001

Many multiplet (MM) Webb et al. 1999

Compares transitions from multiplet and/or atoms
s-p vs d-p transitions in heavy elements
Better sensitivity

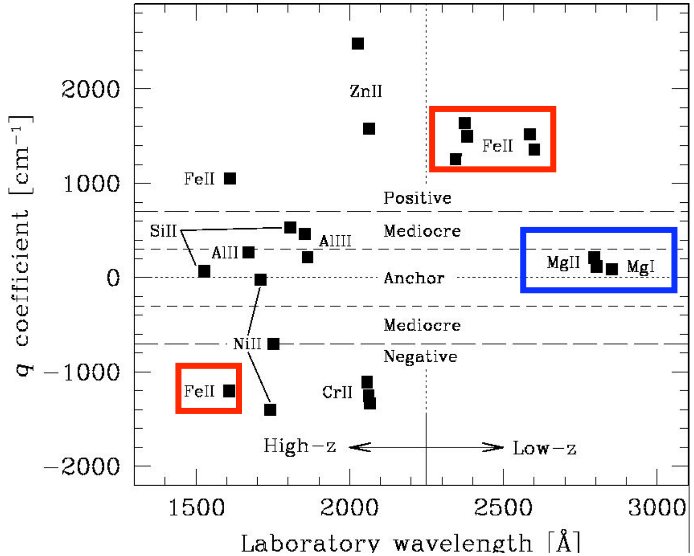
Single Ion Differential α Measurement (SIDAM) Levshakov et al. 1999

Analog to MM but with a single atom / FeII

QSO: many multiplets

The many-multiplet method is based on the corrrelation of the shifts of different lines of different atoms.

Dzuba et al. 1999-2005



Relativistic N-body with varying α:

$$\omega = \omega_0 + 2\,q\,\frac{\Delta\alpha}{\alpha}$$

First implemented on 30 systems with MgII and FeII

Webb et al. 1999

HIRES-Keck, 153 systems, *0.2<z<4.2*

$$\frac{\Delta\alpha}{\alpha} = (-\,0.57 \pm 0.11) \times 10^{-5}$$

Murphy et al. 2004

5σ detection !

QSO: VLT/UVES analysis

Selection of the absorption spectra:

- lines with similar ionization potentials

  most likely to originate from similar regions in the cloud
- avoid lines contaminated by atmospheric lines
- at least one anchor line is not saturated

  redshift measurement is robust
- reject strongly saturated systems

Only 23 systems

lower statistics / better controlles systematics

VLT/UVES

$$\frac{\Delta\alpha}{\alpha} = (-\,0.06 \pm 0.06) \times 10^{-5}$$

Chand et al. 2004

DOES NOT CONFIRM HIRES/Keck DETECTION

Controversy

VLT/UVES:

selection a priori of the systems

data publicly available on the WEB

HIRES/Keck:

signal comes from only some systems

data not public

Reanalysis of the VLT/UVES data by Murphy et al.

χ^2 no smooth for some systems

argue

$$\frac{\Delta\alpha}{\alpha} = (-\,0.64 \pm 0.36) \times 10^{-5}$$

Murphy et al. 2006

χ^2 not smooth for some systems

2 problematic systems that dominate the analysis

If removed

$$\frac{\Delta\alpha}{\alpha} = (-\,0.01 \pm 0.15) \times 10^{-5}$$

Srianand et al. 2007

CMB

It changes the recombination history

1- modifies the optical depth $\dot{\tau} = x_e n_e c \sigma_T$

2- induces a change in the hydrogen and helium abundances (x_e)

Effect on the position of the Doppler peak

on polarization (reionisation)

Degeneracies:

cosmological parameters

electron mass

origin of primordial fluctuations

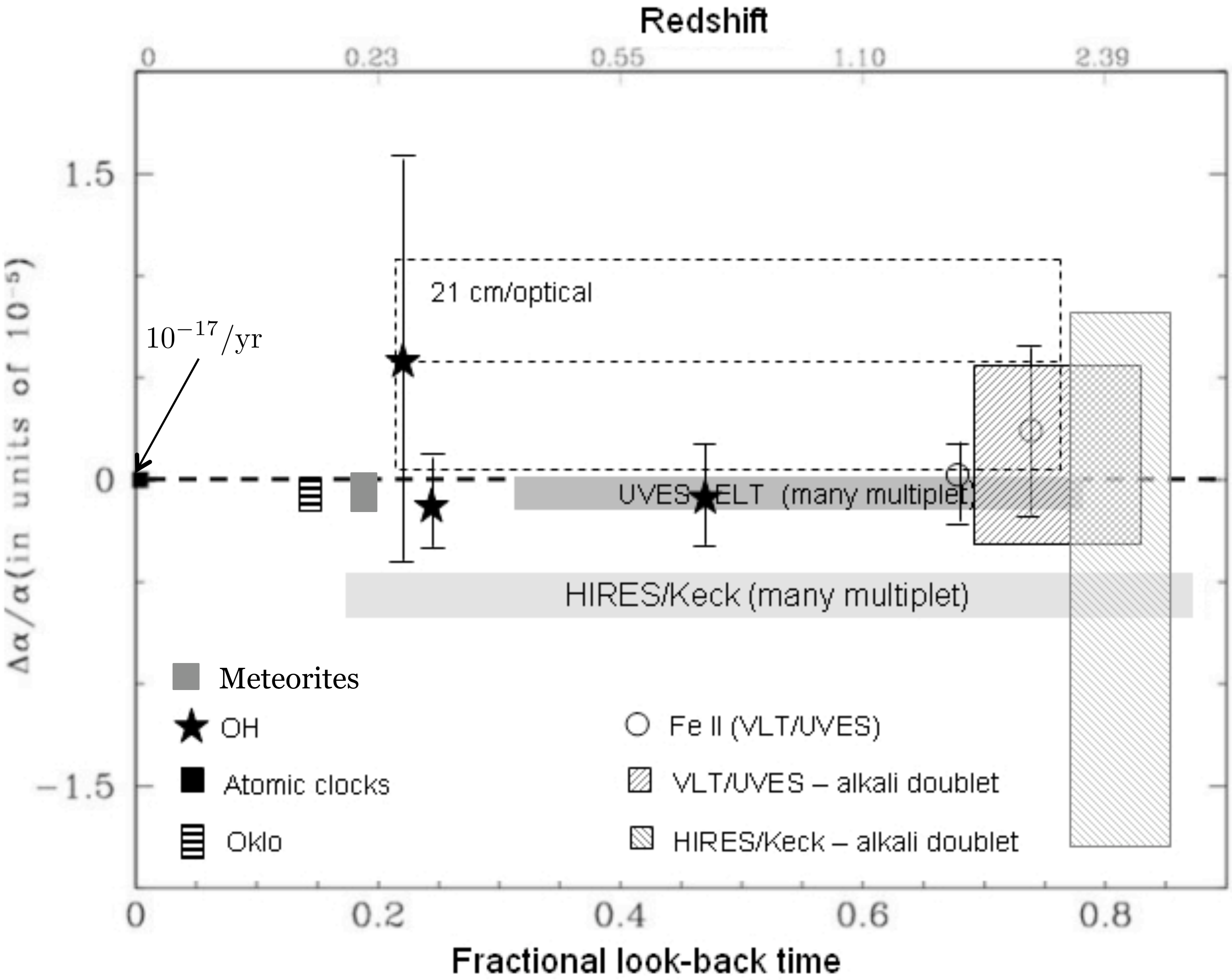
$$\sigma_T \propto \alpha^2 / m_e$$

Analysis of WMAP data

$$\Delta\alpha/\alpha = (-1.5 \pm 3.5) \times 10^{-2} \qquad z \sim 10^3$$

Martins et al. PLB **585** (2004) 29; G. Rocha et al, N. Astron. Rev. **47** (2003) 863

Summary of the constraints on α



Part III: Coupled variation

Example of BBN & 3α

BBN: generality

BBN predicts the primordial abundances of D, He-3, He-4, Li-7

Mainly based on the balance between

1- expansion rate of the universe

2- weak interaction rate which controls n/p at the onset of BBN

Example: helium production

$$Y = \frac{2(n/p)_N}{1+(n/p)_N}$$

$$(n/p)_f \sim \mathrm{e}^{-Q/k_B T_f}$$

$$(n/p)_N \sim (n/p)_f \, \mathrm{e}^{-t_N/\tau_\mathrm{n}}$$

(B_D, η) → t_N

freeze-out temperature is roughly given by $G_F^2 (k_B T_f)^5 = \sqrt{GN}\,(k_B T_f)^2$

Coulomb barrier: $\sigma = \frac{S(E)}{E}\,\mathrm{e}^{-2\pi\alpha Z_1 Z_2 \sqrt{\mu/2E}}$

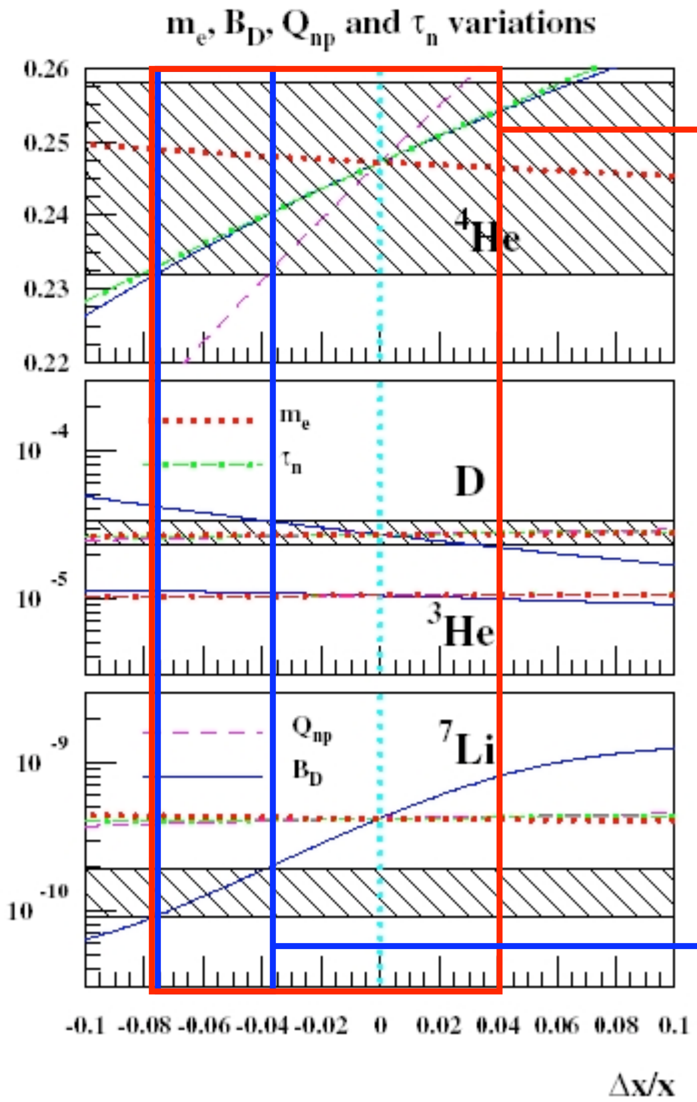
Predictions depend on

$$G_k = (G, \alpha, \tau_n, m_e, Q, B_D, \sigma_i)$$
$$X = (\eta, h, N_\nu, \ldots)$$

Coc,Nunes,Olive,JPU,Vangioni 2006

BBN: effective BBN parameters

Independent variations of the BBN parameters



$$- 7.5 \times 10^{-2} < \frac{\Delta B_D}{B_D} < 6.5 \times 10^{-2}$$

$$- 8.2 \times 10^{-2} < \frac{\Delta \tau_n}{\tau_n} < 6 \times 10^{-2}$$

$$- 4 \times 10^{-2} < \frac{\Delta Q}{Q} < 2.7 \times 10^{-2}$$

Abundances are very sensitive to B_D.
Equilibrium abundance of D and the reaction rate p(n,γ)D depend exponentially on B_D.

These parameters are not independent.

Difficulty: QCD and its role in low energy nuclear reactions.

$$- 7.5 \times 10^{-2} < \frac{\Delta B_D}{B_D} < - 4 \times 10^{-2}$$

Coc,Nunes,Olive,JPU,Vangioni 2006

BBN: fundamental parameters (1)

Neutron-proton mass difference:

$$Q = m_n - m_p = a\alpha\Lambda + (h_d - h_u)v$$

$$\frac{\Delta Q}{Q} = -0.6\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right) + 1.6\left(\frac{\Delta(h_d - h_u)}{h_d - h_u} + \frac{\Delta v}{v}\right)$$

Neutron lifetime:

$$\tau_n^{-1} = G_F^2 m_e^5 f(Q/m_e) \qquad m_e = h_e v$$

$$G_F = 1/\sqrt{2}\,v^2$$

$$\frac{\Delta\tau_n}{\tau_n} = -4.8\frac{\Delta v}{v} + 1.5\frac{\Delta h_e}{h_e} - 10.4\frac{\Delta(h_d - h_u)}{h_d - h_u} + 3.8\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right)$$

BBN: fundamental parameters (2)

D binding energy:

Use a potential model $$V_{nuc} = \frac{1}{4\pi r}\left(- g_s^2 e^{-r m_\sigma} + g_v^2 e^{-r m_\omega}\right)$$

$$\frac{\Delta B_D}{B_D} = -48\frac{\Delta m_\sigma}{m_\sigma} + 50\frac{\Delta m_\omega}{m_\omega} + 6\frac{\Delta m_N}{m_N}$$

Flambaum,Shuryak 2003

Most important parameter beside Λ is the strange quark mass.
One needs to trace the dependence in m_s.

$$\frac{\Delta m_\sigma}{m_\sigma} \sim 0.54\frac{\Delta m_s}{m_s}$$

$$\frac{\Delta m_\omega}{m_\omega} \sim 0.15\frac{\Delta m_s}{m_s}$$

$$\frac{\Delta m_N}{m_N} \sim 0.12\frac{\Delta m_s}{m_s}$$

$$\frac{\Delta B_D}{B_D} = 18\frac{\Delta\Lambda}{\Lambda} - 17\left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s}\right)$$

This allows to determine all the primary parameters in terms of (h_i, v, Λ,α)

BBN: assuming GUT

GUT:

The low-energy expression for the QCD scale

$$\Lambda = \mu \left(\frac{m_c m_b m_t}{\mu^3}\right)^{2/27} \exp\left(-\frac{2\pi}{9\alpha_3(\mu)}\right)$$

We deduce

$$\frac{\Delta\Lambda}{\Lambda} = R\frac{\Delta\alpha}{\alpha} + \frac{2}{27}\left(3\frac{\Delta v}{v} + \sum_{i=c,b,t}\frac{\Delta h_i}{h_i}\right)$$

The value of *R* depends on the particular GUT theory and particle content
Which control the value of M_{GUT} and of $\alpha(M_{GUT})$.
Typically R=36.

Assume (for simplicity) h_i=h

$$\frac{\Delta B_D}{B_D} = -13\left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right) + 18R\frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta Q}{Q} = 1.5\left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right) - 0.6(1+R)\frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta\tau_n}{\tau_n} = -4\frac{\Delta v}{v} - 8\frac{\Delta h}{h} + 3.8(1+R)\frac{\Delta\alpha}{\alpha}$$

$$(\alpha, v, h)$$

Stellar carbon production

- Helium burning
  - Triple alpha reaction $3\alpha\rightarrow{}^{12}C$
  - Competing with $^{12}C(\alpha,\gamma)^{16}O$
- Hydrogen burning (at Z = 0)
  - Slow pp chain
  - CNO with C from $3\alpha\rightarrow{}^{12}C$
- Three steps :
  - $\alpha\alpha\leftrightarrow{}^{8}Be$ (lifetime ~ 10^{-16} s) leads to an equilibrium
  - $^{8}Be+\alpha\rightarrow{}^{12}C^{*}$ (288 keV, l=0 resonance, the "Hoyle state")
  - $^{12}C^{*}\rightarrow{}^{12}C + 2\gamma$
- Resonant reaction unlike e.g. $^{12}C(\alpha,\gamma)^{16}O$
  - Sensitive to the position of the "Hoyle state"
  - Sensitive to the variation of "constants"

^{12}C production and variation of the strong interaction *[Rozental 1988]*

C/O in Red Giant stars *[Oberhummer et al. 2000; 2001]*

1.3, 5 and 20 $M_\odot$ stars, $Z=Z_\odot$ / Limits on effective N-N interaction

C/O in low, intermediate and high mass stars *[Schlattl et al. 2004]*

1.3, 5, 15 and 25 $M_\odot$ stars, $Z=Z_\odot$ / Limits on resonance energy shift

Stellar carbon production

Triple α coincidence (Hoyle)

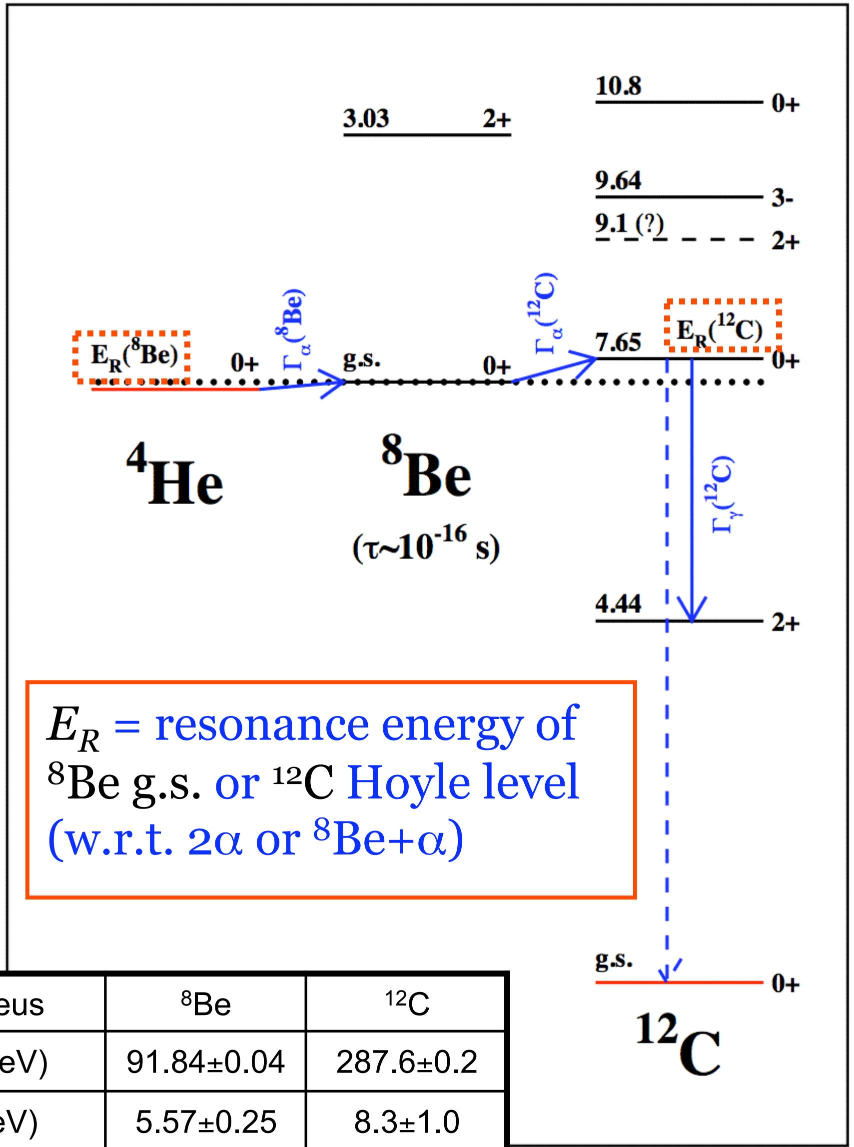
1. Equillibrium between ^{4}He and the short lived (~10^{-16} s) ^{8}Be : $\alpha\alpha \leftrightarrow {}^8$Be
2. Resonant capture to the (l=0, J^π=0$^+$) Hoyle state: ^{8}Be+α→^{12}C*(→^{12}C+γ)

Simple formula used in previous studies

1. Saha equation (thermal equilibrium)
2. Sharp resonance analytic expression:

$$N_A^2\langle\sigma v\rangle^{\alpha\alpha\alpha} = 3^{3/2} 6 N_A^2 \left(\frac{2\pi}{M_\alpha k_B T}\right)^3 \hbar^5 \gamma \exp\left(\frac{-Q_{\alpha\alpha\alpha}}{k_B T}\right)$$

with $Q_{\alpha\alpha\alpha} = E_R(^8\text{Be}) + E_R(^{12}\text{C})$ and $\gamma \approx \Gamma_\gamma$

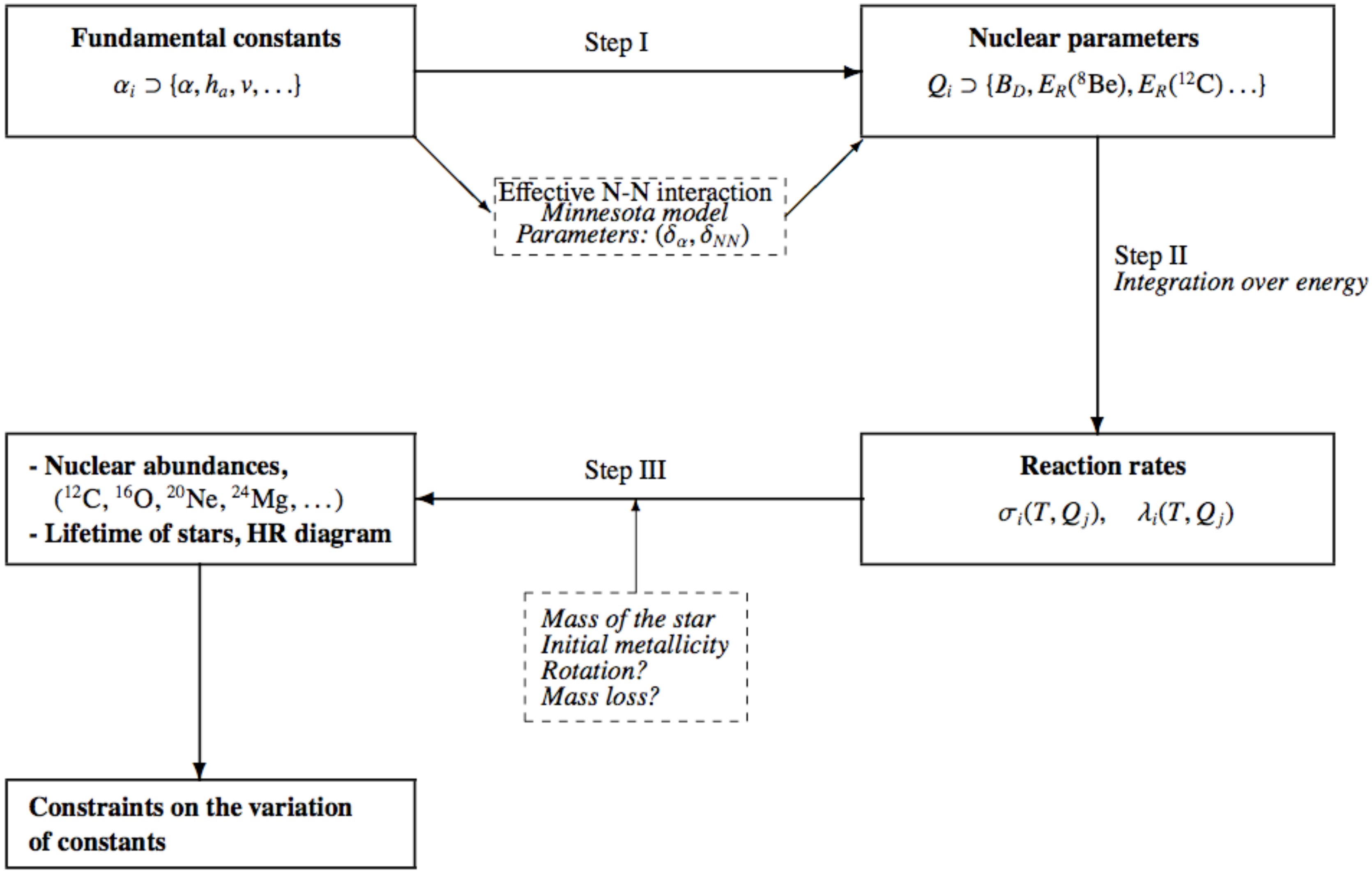


Nucleus	^{8}Be	^{12}C
E_R (keV)	91.84±0.04	287.6±0.2
Γ_α (eV)	5.57±0.25	8.3±1.0
Γ_γ (meV)	-	3.7±0.5

Approximations

1. Thermal equilibrium
2. Sharp resonance
3. ^{8}Be decay faster than α capture

Modelisation



Ekström, Coc, Descouvemont, Meynet, Olive, JPU, Vangioni, 2009

Microscopic calculation

❑ Hamiltonian:

$$H = \sum_{i=1}^{A} T(r_i) + \sum_{i<j=1}^{A} \left(V_{Coul.}(r_{ij}) + V_{Nucl.}(r_{ij})\right)$$

Where $V_{Nucl.}(r_{ij})$ is an effective Nucleon-Nucleon interaction

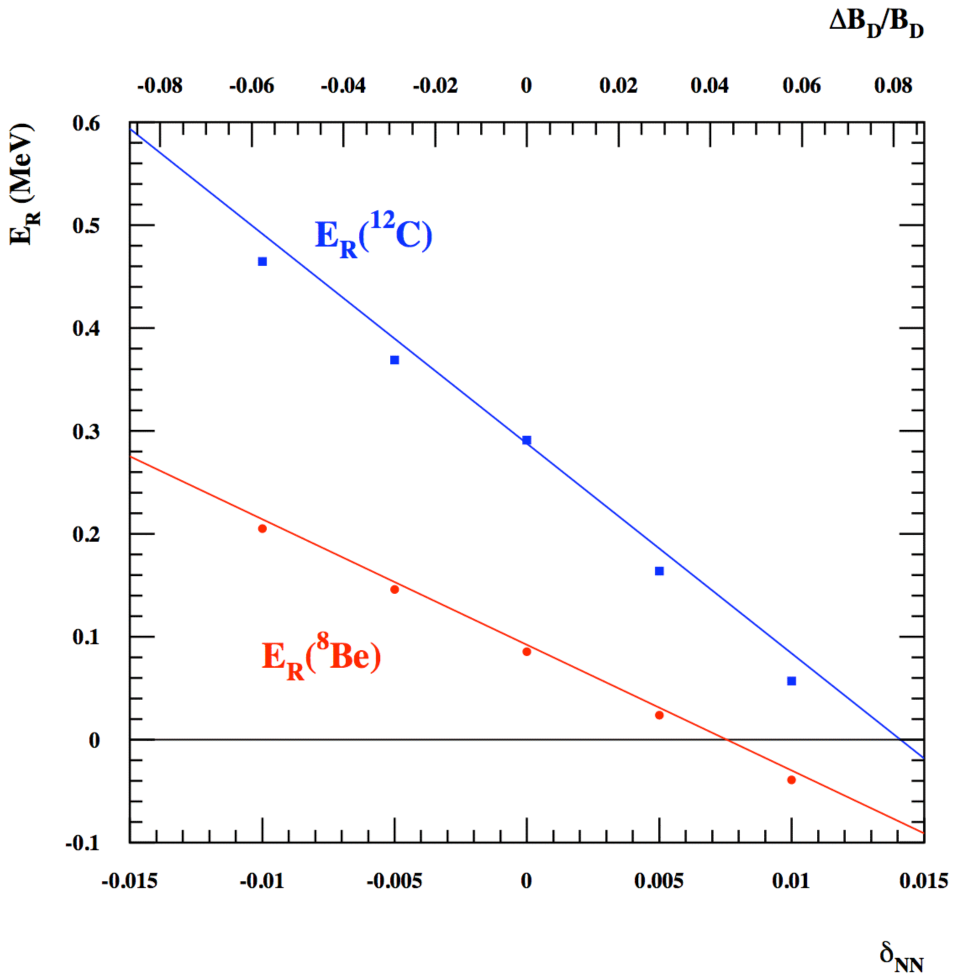
❑ Minnesota N-N force *[Thompson et al. 1977]* optimized to reproduce low energy N-N scattering data and B_D (deuterium binding energy)

❑ α-cluster approximation for ^{8}Be$^{g.s.}$ (2α) and the Hoyle state (3α) *[Kamimura 1981]*

❑ Scaling of the N-N interaction

$$V_{Nucl.}(r_{ij}) \rightarrow (1+\delta_{NN}) \times V_{Nucl.}(r_{ij})$$

to obtain B_D, $E_R(^8\text{Be})$, $E_R(^{12}\text{C})$ as a function of δ_{NN} :

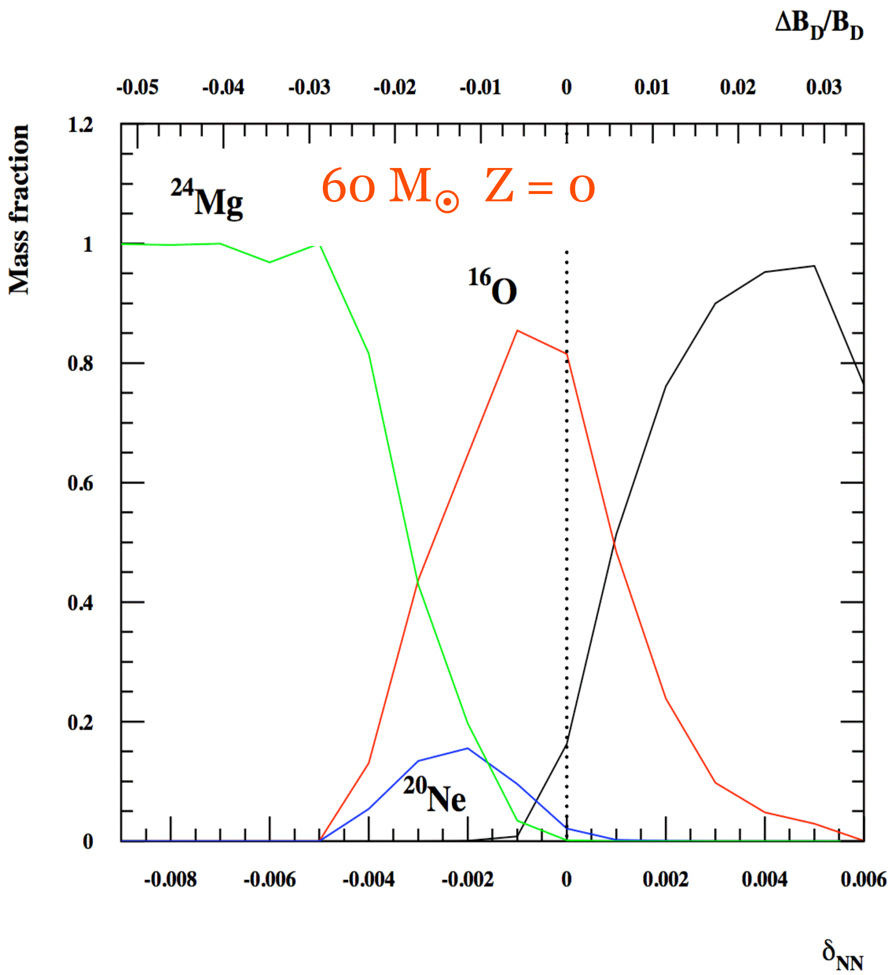


❑ Link to fundamental couplings through B_D or δ_{NN}

Composition at the end ofcore He burning

Stellar evolution of massive Pop. III stars

*We choose **typical** masses of 15 and 60 $M_\odot$ stars/ Z=0 ⇒Very specific stellar evolution*



- **The standard region:** Both ^{12}C and ^{16}O are produced.
- **The ^{16}O region:** The 3α is slower than $^{12}C(\alpha,\gamma)^{16}O$ resulting in a higher T_C and a conversion of most ^{12}C into ^{16}O
- **The ^{24}Mg region:** With an even weaker 3α, a higher T_C is achieved and $^{12}C(\alpha,\gamma)^{16}O(\alpha,\gamma)^{20}Ne(\alpha,\gamma)^{24}Mg$ transforms ^{12}C into ^{24}Mg
- **The ^{12}C region:** The 3α is faster than $^{12}C(\alpha,\gamma)^{16}O$ and ^{12}C is not transformed into ^{16}O

Constraints

From stellar evolution of zero metallicity 15 and 60 $M_\odot$ at redshift z = 10 - 15

- Excluding a core dominated by ^{24}Mg $\Rightarrow \delta_{NN} > -0.005$

  or $\Delta B_D/B_D > -0.029$

- Excluding a core dominated by ^{12}C $\Rightarrow \delta_{NN} < 0.003$

  or $\Delta B_D/B_D < 0.017$

- Requiring ^{12}C/^{16}O close to unity $\Rightarrow -0.0005 < \delta_{NN} < 0.0015$

  or $-0.003 < \Delta B_D/B_D < 0.009$

$$\Delta B_D/B_D \approx 5.77 \times \delta_{NN}$$

Conservative constraint on Nucleosynthesis

^{12}C/^{16}O ~1 $\Rightarrow -0.0005 < \delta_{NN} < 0.0015$

or $-0.003 < \Delta B_D/B_D < 0.009$

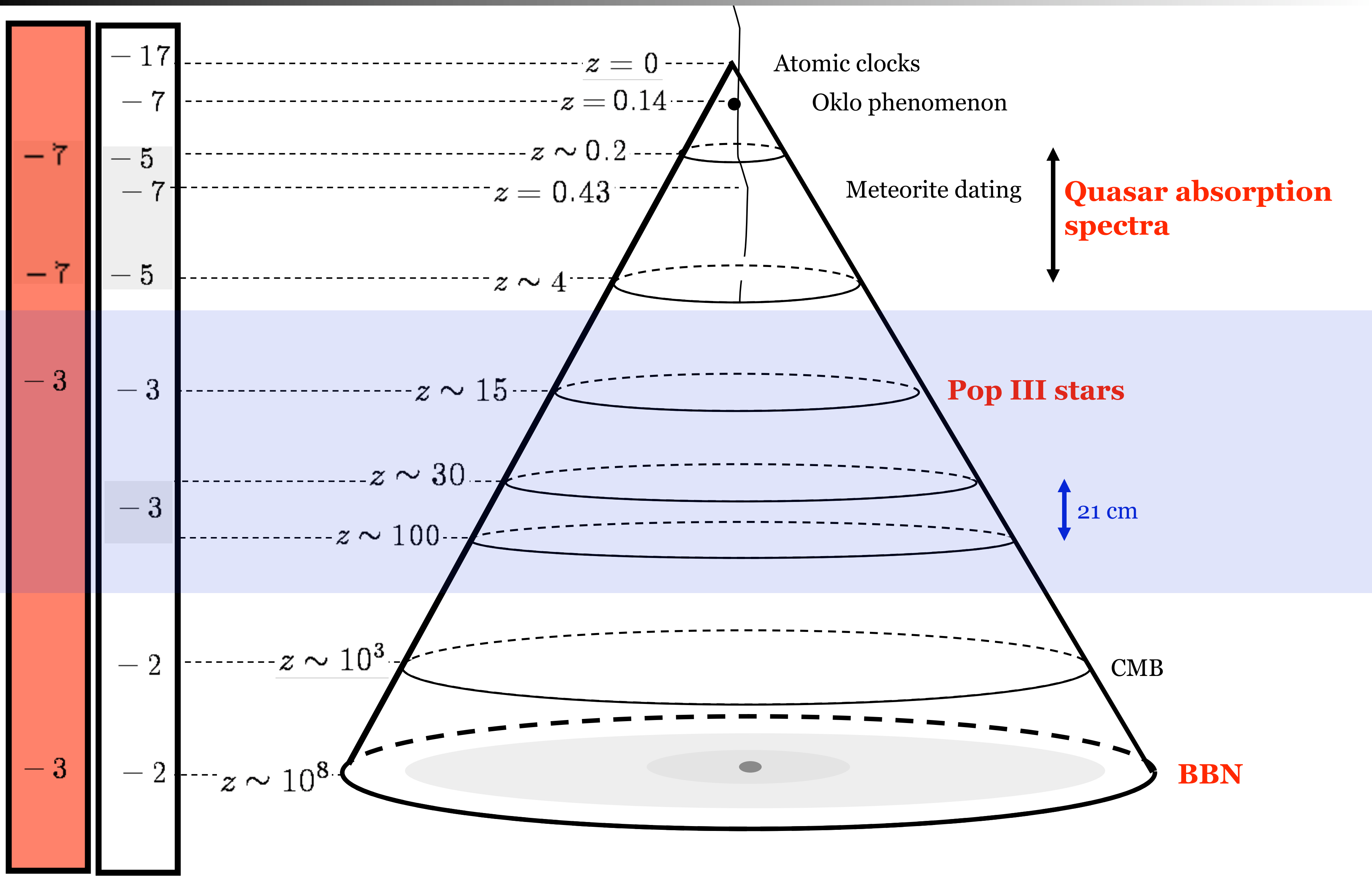
Conclusions

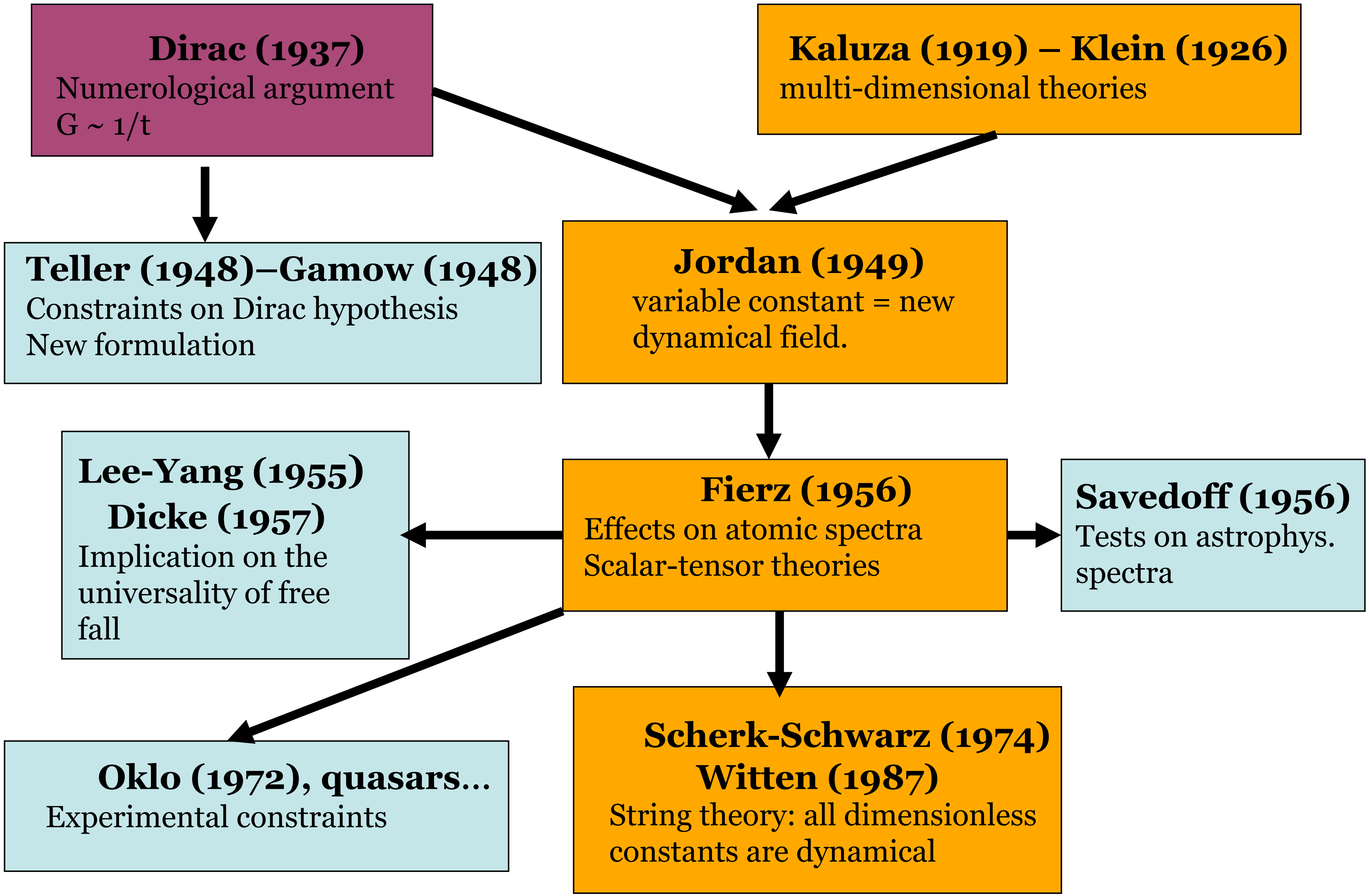
Constants are a transversal way to look at the history of physics and at the structure of its theory.

Observational developments allow to set strong constraints on their possible variation

They allow to test general relativity and may open a window on more fundamental theories of gravity

Future evolution




Dirac (1937)
Numerological argument
G ~ 1/t
Kaluza (1919) – Klein (1926)
multi-dimensional theories
Teller (1948)–Gamow (1948)
Constraints on Dirac hypothesis
New formulation
Jordan (1949)
variable constant = new dynamical field.
Lee-Yang (1955)
Dicke (1957)
Implication on the universality of free fall
Fierz (1956)
Effects on atomic spectra
Scalar-tensor theories
Savedoff (1956)
Tests on astrophys. spectra
Oklo (1972), quasars…
Experimental constraints
Scherk-Schwarz (1974)
Witten (1987)
String theory: all dimensionless constants are dynamical