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The transplanckian S-matrix: recent progress and open problems

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Introduction

There are many beautiful (analytical as well as numerical) GR results on determining whether some given initial data should lead to gravitational collapse or to a completely dispersed final state

The two phases would be typically separated by a critical hypersurface $S_{cr}^{(Cl)}$ in the parameter space $P^{(Cl)}$ of the initial states

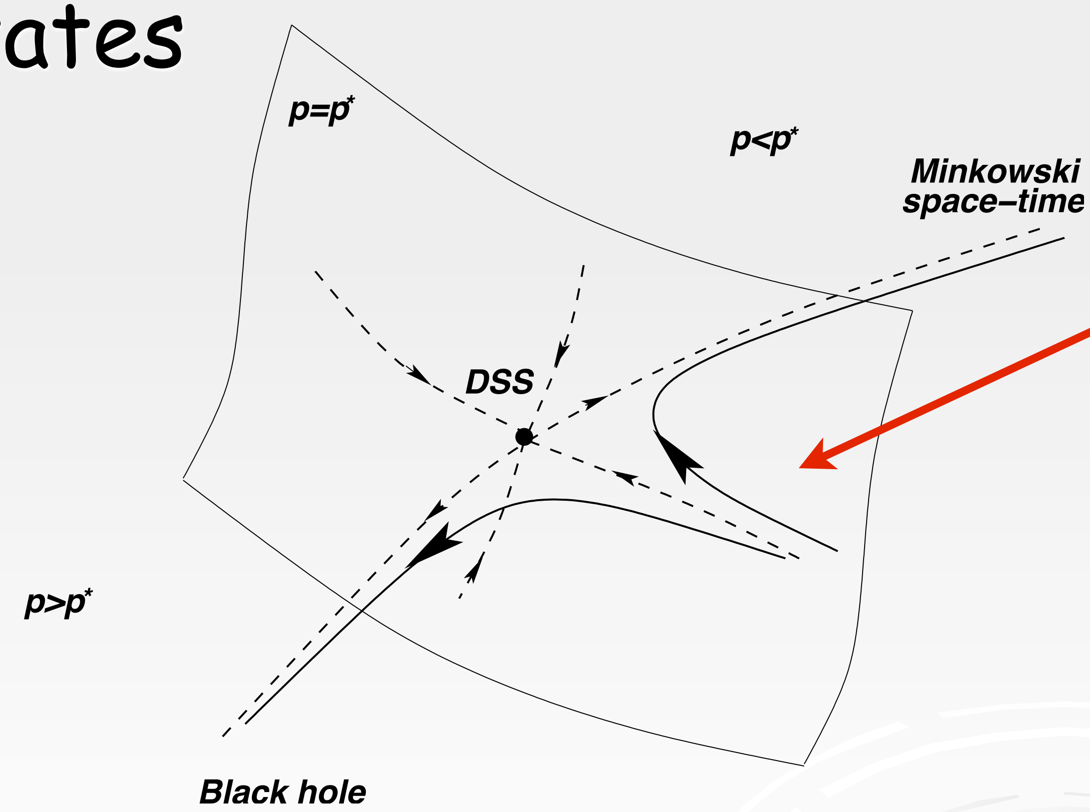


Figure 1: Phase space picture of the critical gravitational collapse.

from C. Gundlach's review ('02)

The approach to criticality resembles that of phase transitions (order, crit. exp. ...)

For pure gravity Christodoulou & Klainerman ('93) have found a region on the dispersion side of the critical surface;

Regions on the collapse side have been found for spherical symmetry by Christodoulou ('91, ...) and, numerically, by Choptuik and collaborators ('93, ...);

Last year, Christodoulou identified another such region in which a lower bound on (incoming energy)/(unit adv. time) holds uniformly over the full solid angle;

A few months ago Choptuik and Pretorius (0908.1780) have obtained new numerical results for a highly-relativistic axisymmetric situation (see below).

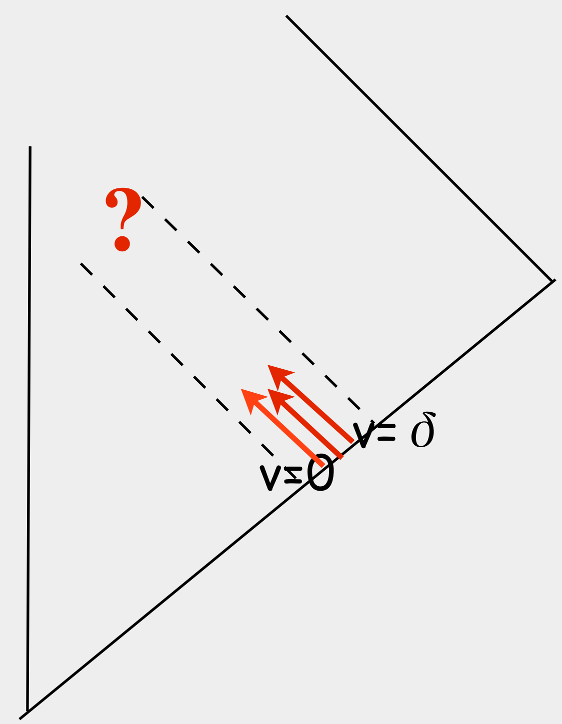
A useful (but only sufficiency) criterion for collapse is the indentification of a Closed Trapped Surface (CTS) at a certain moment in the system's evolution

0805.3880 [gr-qc] 26 May 2008, 594 pages

Christodoulou, Demetrios (2009).
The formation of black holes in general relativity.
Zurich: European Mathematical Society Publishing House.

Essence of new DC's criterion

Penrose diagram



incoming energy (G=1) per unit advanced time & solid angle

$$\text{If} \quad M(\theta,\phi,\delta) \equiv \int_0^\delta dv \frac{d\mathcal{M}(v,\theta,\phi)}{dv\ d\cos\theta\ d\phi} \geq \frac{k}{8\pi} \quad \text{for all } \theta,\phi$$

then, for $\delta/k \ll 1$, a CTS (hence a BH) forms with

$$R \geq k - O(\delta)$$

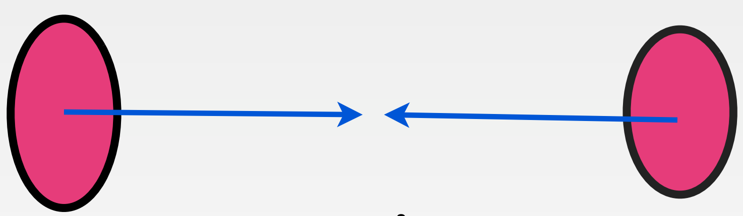
DC's result is not useful for two-body collisions, the energy being concentrated in two narrow cones but one can still get useful criteria through CTS constructions

- Point-particle collisions:

1. b=0: Penrose ('74) : $M_{BH} > E/\sqrt{2} \sim 0.71E$
2. b≠0: Eardley & Giddings ('02), one example:

$$\left(\frac{R}{b}\right)_{cr} \leq 1.25 \qquad (R = 2G\sqrt{s} = 4GE_1 = 4GE_2)$$

- Extended sources:



- Kohlprath & GV ('02), one example: central collision of 2 homogeneous null discs of radius L

$$\left(\frac{R}{L}\right)_{cr} \leq 1$$

Using infinite-L sln. & causality one can prove that a curvature singularity forms as well (GV, unpl.)

What about the quantum problem?

Quantum mechanically we can prepare pure initial states that correspond, roughly, to the classical data. They define a parameter space $P^{(Q)}$. Questions:

- Does a unitary S-matrix (evolution operator) describe the evolution of the system everywhere in $P^{(Q)}$?
- If yes, does such an S-matrix develop singularities as one approaches a critical surface $S_{cr}^{(Q)}$ in $P^{(Q)}$?
- If yes, what happens in the vicinity of this critical surface? Does the nature of the final state change as one goes through it? Is there a connection between $S_{cr}^{(Cl)}$ and $S_{cr}^{(Q)}$?
- What happens to the final state deep inside the BH region? Does it resemble at all Hawking's thermal spectrum for each initial pure state?
- Qs related to information paradox/puzzle (Hawking '75)

TPE collisions as a gedanken experiment (Amati, Ciafaloni & GV 1987-'08)

Trans-Planckian-Energy (TPE => $E \gg M_P c^2$, or $Gs/c^5 h \gg 1$) string collisions represent a perfect theoretical laboratory for studying these questions within a framework that claims to be a fully consistent quantum theory of gravity.

The need for TPEs comes from our wish to understand the physics of semiclassical -rather than Planck size- black holes. It will also simplify the theoretical analysis.

A different issue is what happens to black holes in string theory when their Schwarzschild radius R is smaller than the characteristic length scale l_s of string theory. Indications are that no such BHs exist.

A phenomenological motivation for studying TPE collisions?

Finding signatures of string/quantum gravity @ LHC:

- In KK models with large extra dimensions;
- In brane-world scenarios; in general:
- If the true Quantum Gravity scale is O(few TeV)

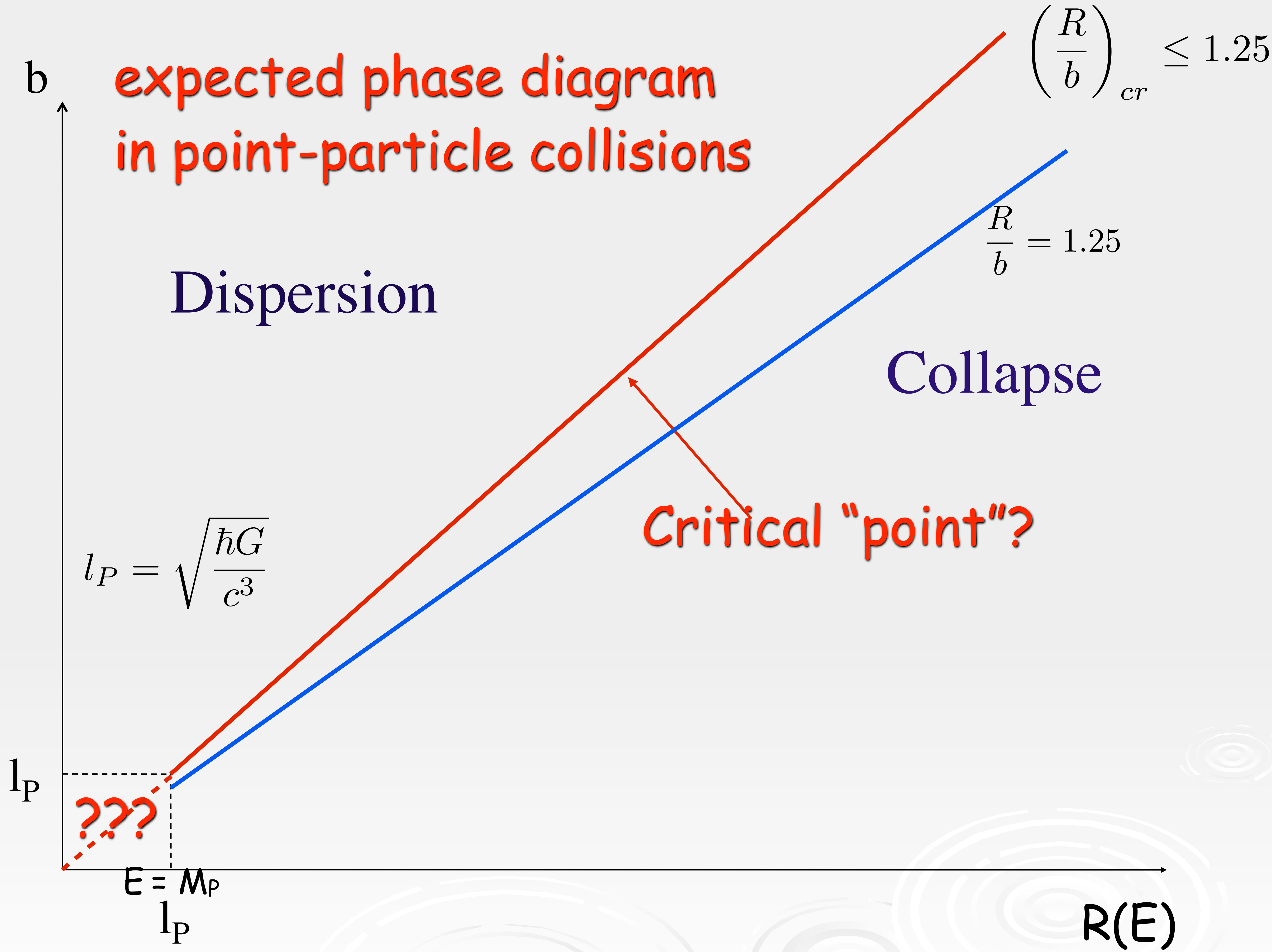
NB: In the most optimistic situation the LHC will be very marginal for producing BH, let alone semiclassical ones

Q: Can there be some precursors of BH behaviour even below the expected BH-production threshold?

The rest of the talk

- Scales & regimes in TPE collisions
- The small angle regime
- String corrections
- Classical corrections
- Towards a quantum description of gravitational collapse

expected phase diagram in point-particle collisions



If we collide strings, instead of point particles, there is another length scale, the characteristic size l_s of strings

$$l_s = \sqrt{\frac{\hbar c}{T}} \qquad \text{Cf.} \qquad l_P = \sqrt{\frac{\hbar G}{c^3}}$$

(T is the classical string tension)

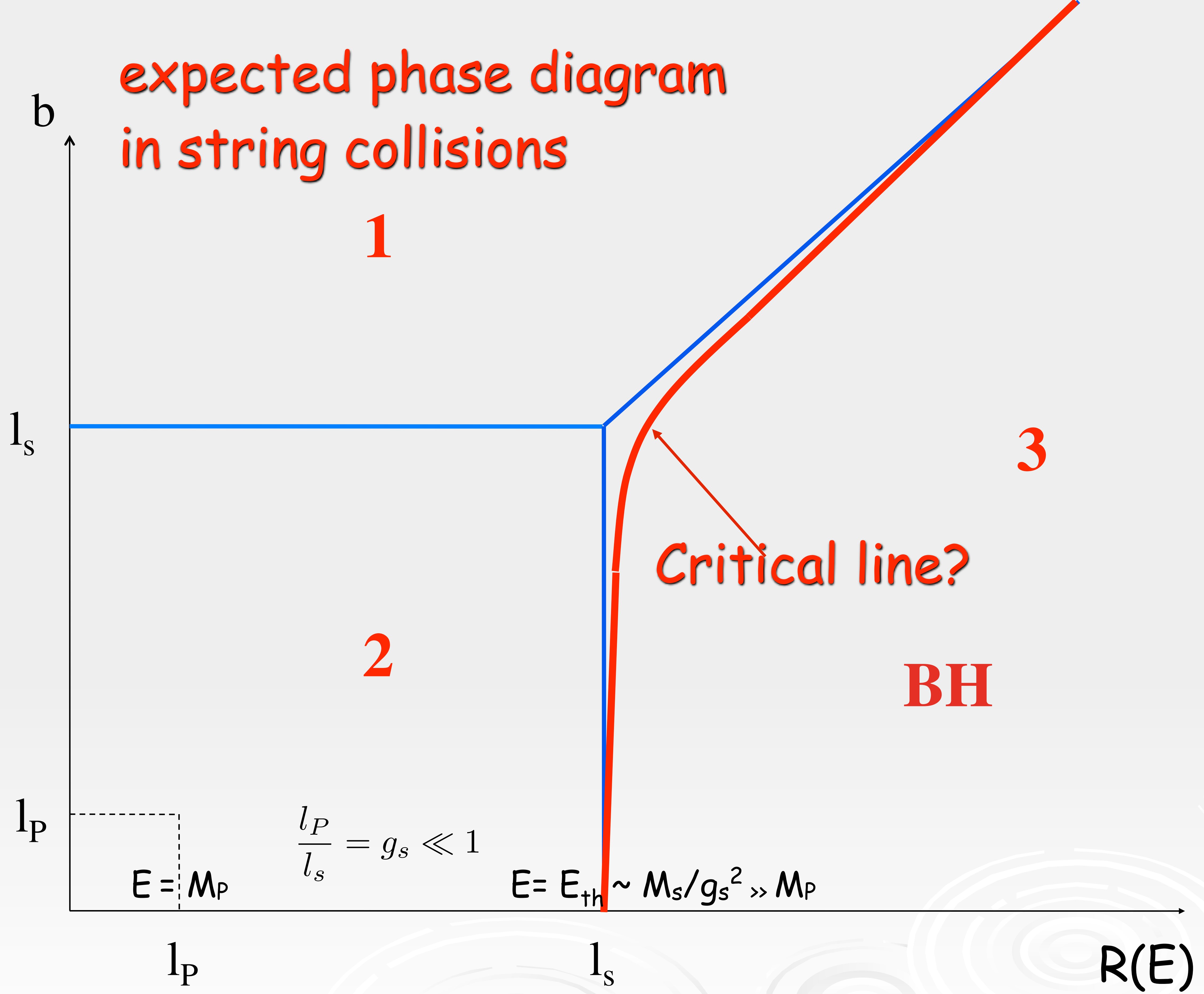
l_s plays the role of the beam size!

3 length scales: b, R and l_s =>

3 broad-band regimes in transplanckian string collisions

1) Small angle scattering ($b \gg R, l_s$)
2) Stringy ($l_s > R, b$)
3) Large angle scattering ($b \sim R > l_s$), collapse ($b, l_s < R$)

expected phase diagram in string collisions

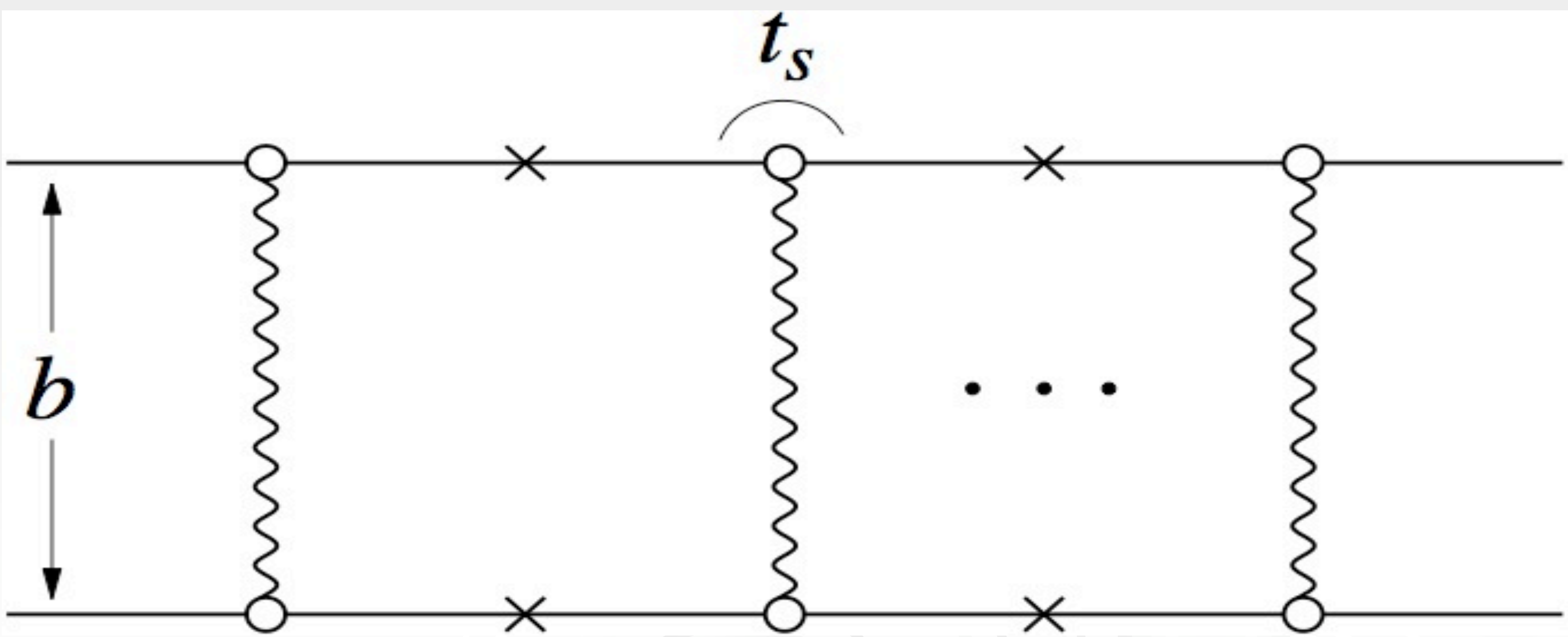


A semiclassical S-matrix @ TPE

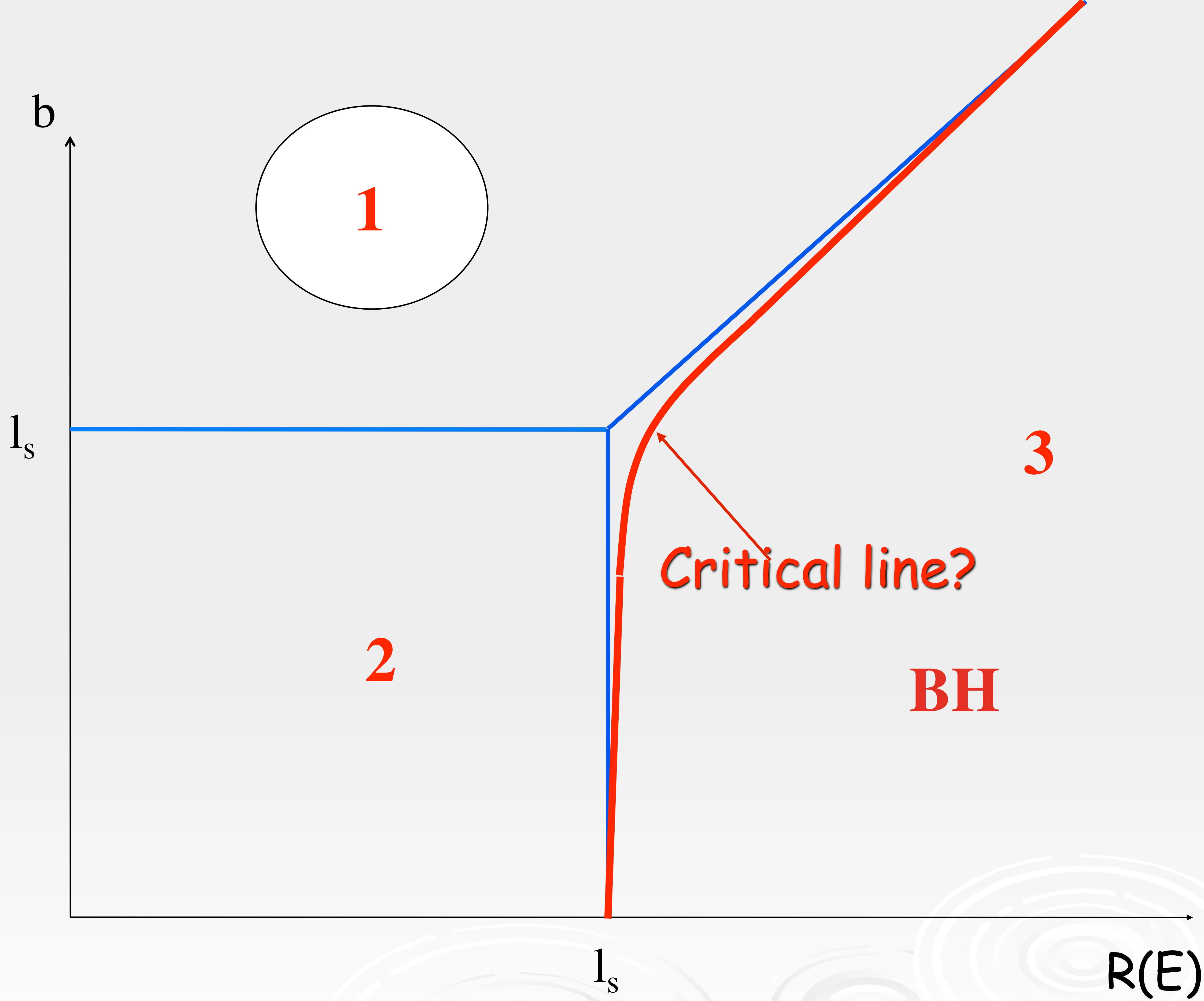
General arguments as well as explicit calculations suggest the following form for the elastic S-matrix:

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + O(l_s^2/b^2) + O(l_P^2/b^2) + \ldots)\right)$$

Leading eikonal diagrams (crossed ladders included)



NB: For Im A some terms may be more than just corrections...


b
1
l_s
3
Critical line?
2
BH
l_s
R(E)

Recovering CGR expectations @ large distance

$$S(E,b) \sim exp\left(-i\frac{Gs}{\hbar}logb^2\right) \;\; ; \;\; S(E,q) = \int d^2b \; e^{-iqb} S(E,b) \;\; ; \;\; s = 4E^2 \; , \; q \sim \theta E$$

The integral is dominated by a saddle point at:

$$b_s = \frac{4G\sqrt{s}}{\theta} \; , \; \theta = \frac{4G\sqrt{s}}{b} = 2\frac{R}{b} \; , \;\; R \equiv 2G\sqrt{s}$$

the generalization of Einstein's deflection formula for ultra-relativistic collisions. It corresponds precisely (and for any D) to the relation between impact parameter and deflection angle in the (Aichelburg-Sexl) metric generated by a relativistic point-particle of energy E. This effective metric is not put in: it's "emergent"

$$b_s = \frac{4G\sqrt{s}}{\theta}\ ,\ \theta = \frac{4G\sqrt{s}}{b} = 2\frac{R}{b}\ ,\ \ R \equiv 2G\sqrt{s}$$

- Note that, at fixed θ, larger E probe **larger** b
- The reason is quite simple: because of eikonal exponentiation, $A_{cl} \sim Gs/h$ also gives the average loop-number. The total momentum transfer **$q = \theta\, E$** is thus **shared** among $O(s \sim E^2)$ exchanged gravitons to give:

$$q_{ind} \sim \frac{\hbar q}{Gs} \sim \frac{\hbar\theta}{R} \sim \frac{\hbar}{b_s}$$

meaning that the process is soft at large s...

String corrections in region 1

(relevant because of imaginary part)

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + \cancel{O(R^2/b^2)} + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2)} + \ldots)\right)$$

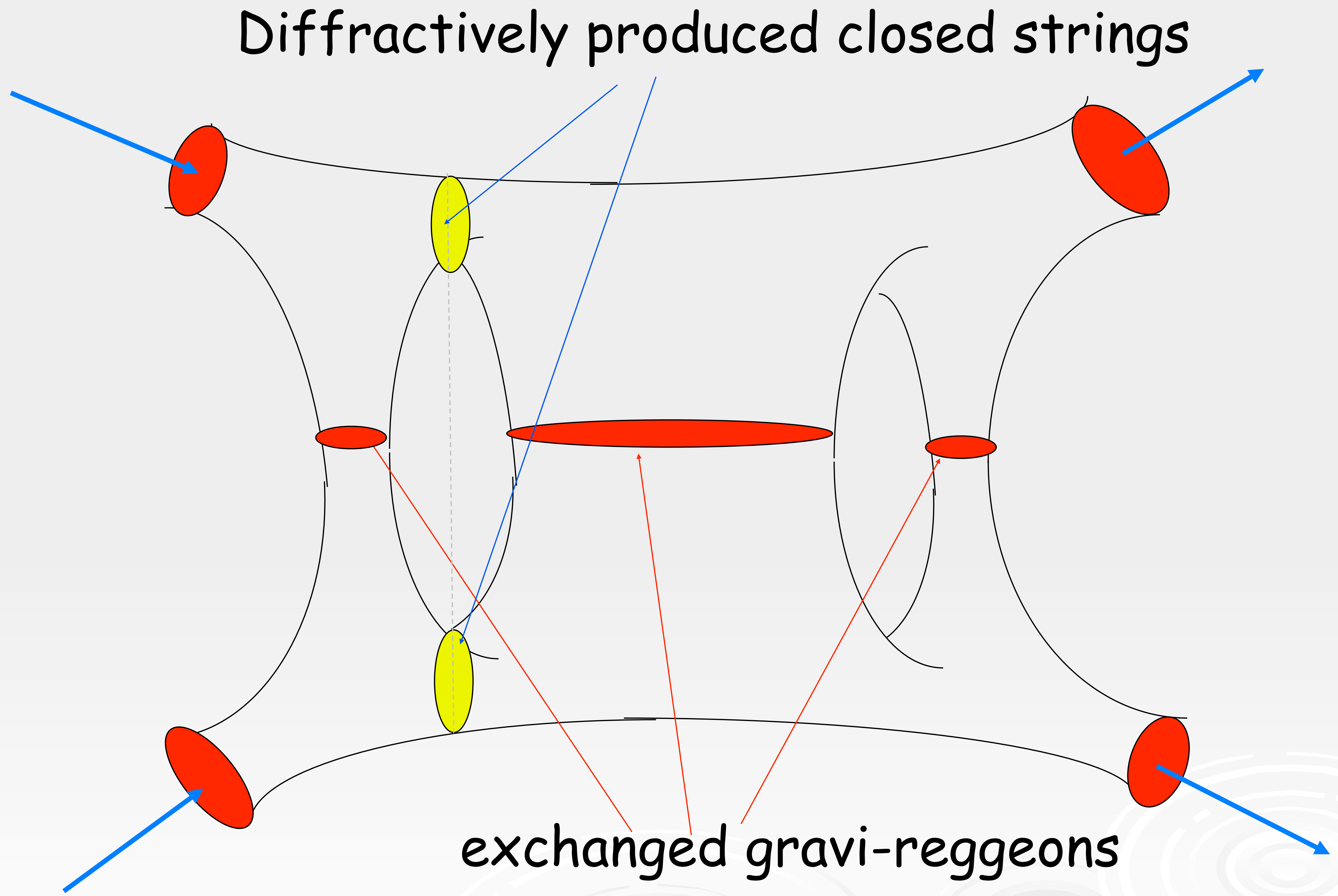
Graviton exchanges can excite one or both strings (figure). Reason (SG): a string moving in a non-trivial metric feels tidal forces as a result of its finite size. A simple argument (GV) gives, for any D, the critical impact parameter b_D below which the phenomenon kicks-in

$$\theta_1 \sim G_D\ E_2\ b^{3-D} \ \Rightarrow \Delta\theta_1 \sim G_D\ E_2\ l_s\ b^{2-D}$$

This angular spread provides an invariant mass:

$$M_1 \sim E_1\Delta\theta_1 \sim G_D\ s\ l_s\ b^{2-D} = M_2$$ Strings get excited if

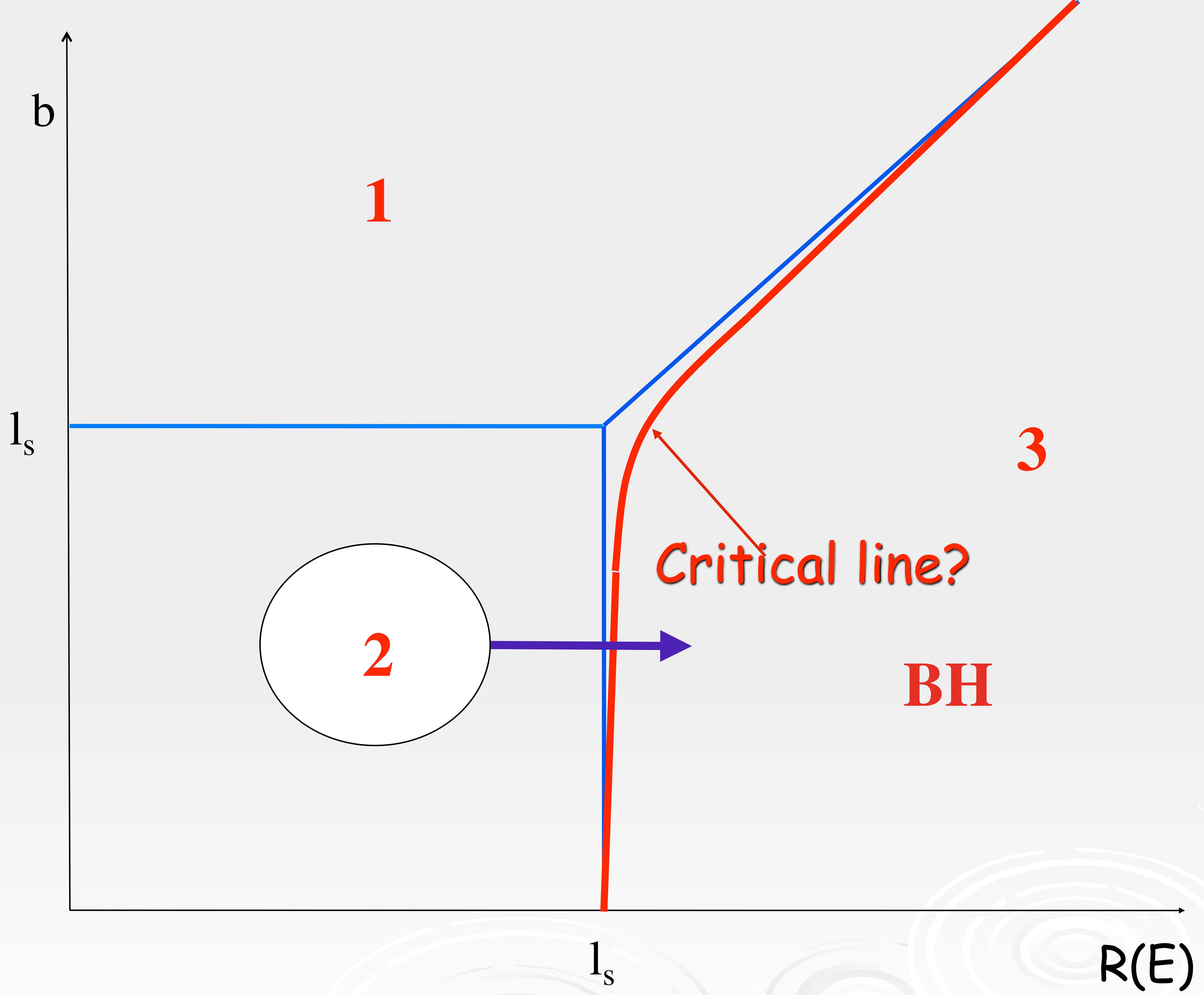
$$M_{1,2} \sim M_s = \hbar l_s^{-1} \Rightarrow b = b_D \sim \left(\frac{Gsl_s^2}{\hbar}\right)^{\frac{1}{D-2}}$$ as found by ACV


Diffractively produced closed strings
exchanged gravi-reggeons

Similar to diffractive excitation in hadron-hadron collisions through "soft-Pomeron" exchange

Q: Is this similarity between diff. diss. and tidal excitation more than superficial?

A: Perhaps yes if there is some gauge/gravity duality at work like in AdS/CFT...



b
1
l_s
3
Critical line?
2
BH
l_s
R(E)

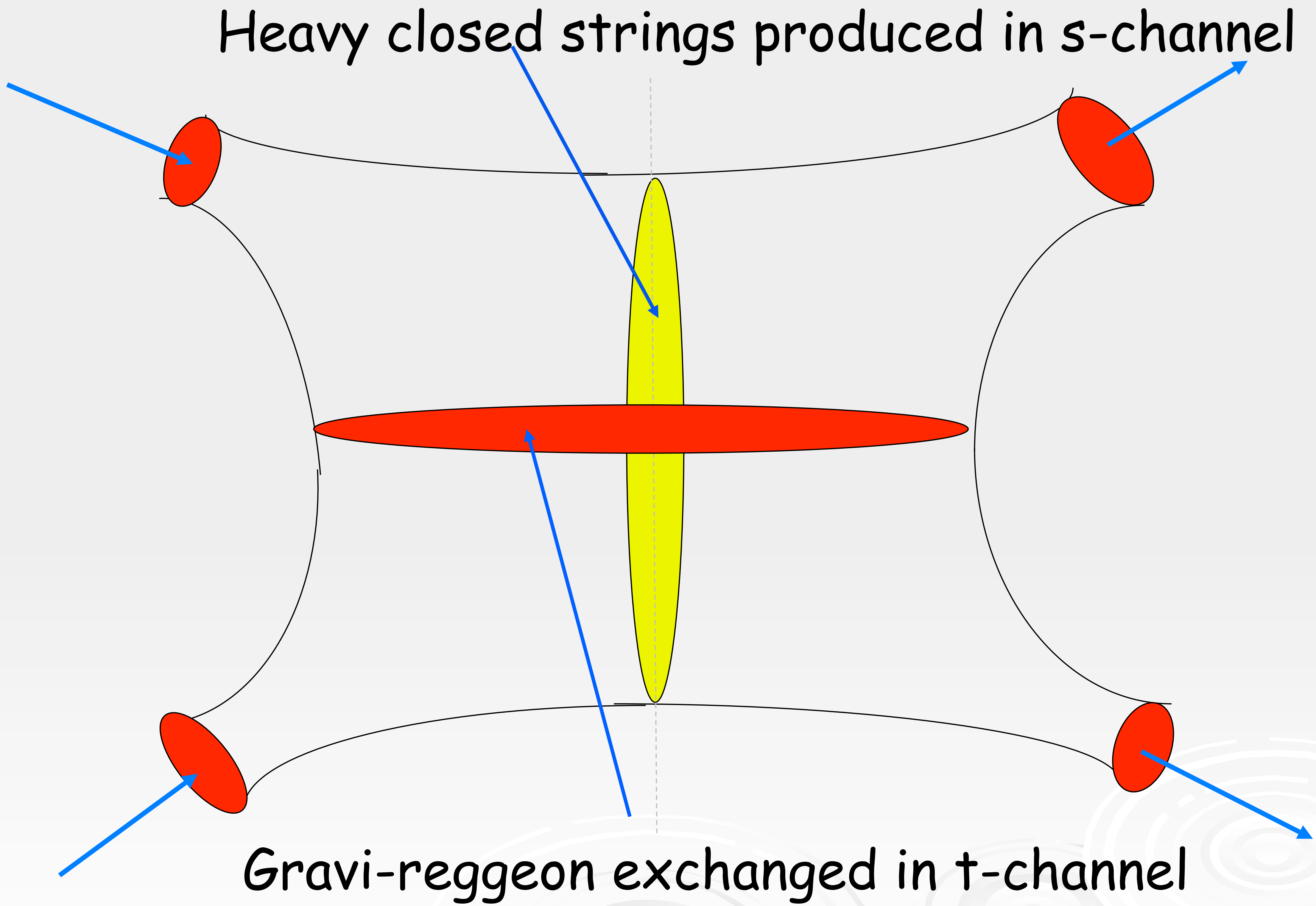
String-size corrections in region 2

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + \cancel{O(R^2/b^2)} + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2)} + \dots)\right)$$

Because of (DHS) duality even single graviton exchange does not give a real scattering amplitude. The imaginary part is due to formation of closed-strings in the s-channel.

It is exponentially damped at large impact parameter (=> irrelevant in region 1, important in region 2)

Im A is due to closed strings in s-channel (DHS duality)



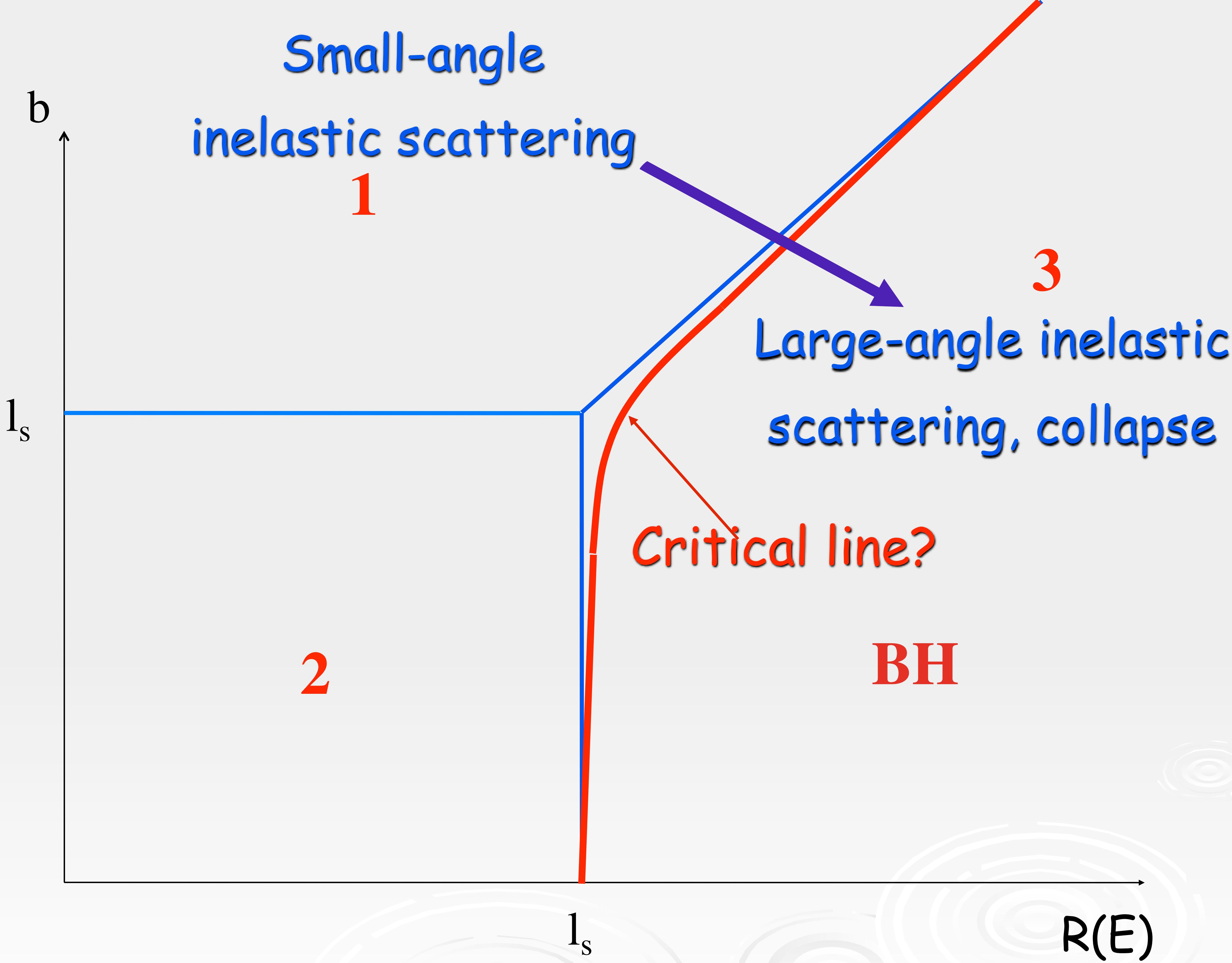
$$\mathrm{Im} A_{cl}(E,b) \sim \frac{Gs}{\hbar} \exp\left(-\frac{b^2}{l_s^2 \, \log s}\right)$$

As one goes to impact parameters below the string scale one starts producing more and more strings. The average number of produced strings grows (once more!) like $Gs \sim E^2$ so that, above M_{Pl}, the average energy of each final string starts decreasing as the incoming energy is increased

$$\langle E_{final} \rangle \sim \frac{M_{Pl}^2}{\sqrt{s}} \sim T_{BH}$$

Similar to what we expect in BH physics!

An interesting signature even below the actual threshold of BH production!


Small-angle
inelastic scattering
1
b
l_s
3
Large-angle inelastic
scattering, collapse
Critical line?
2
BH
l_s
R(E)

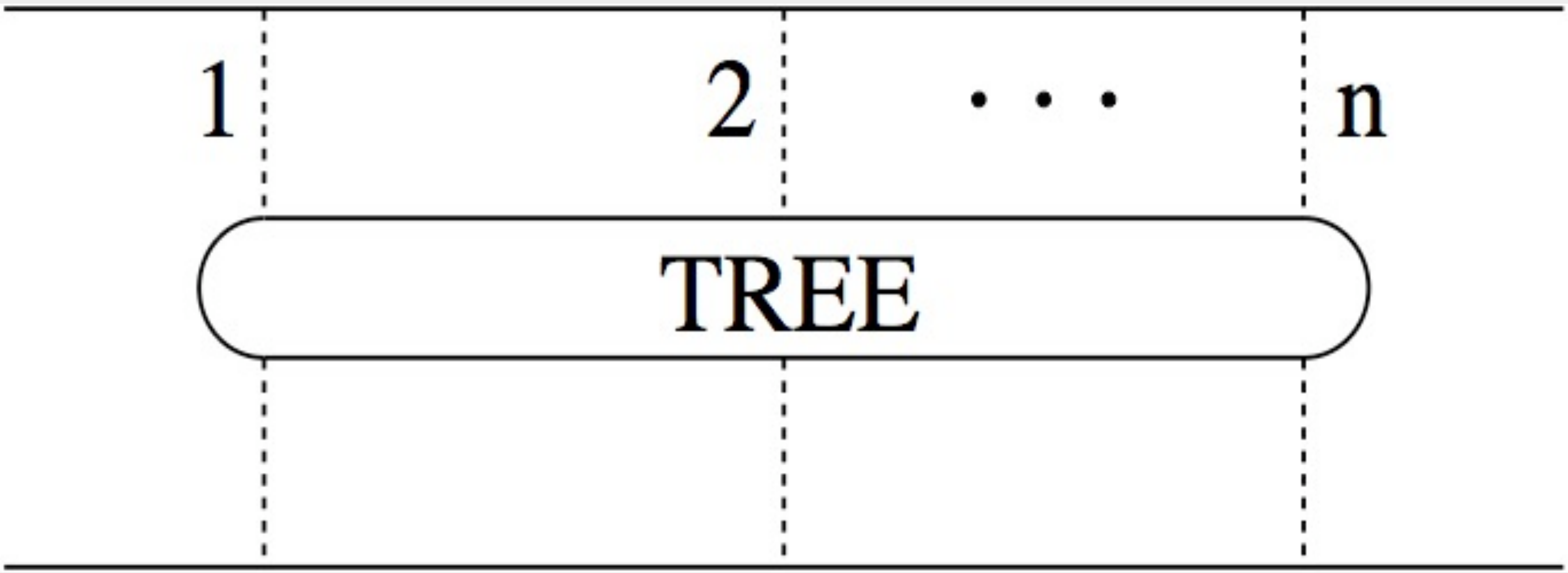
Classical corrections

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + O(l_s^2/b^2) + O(l_P^2/b^2) + \dots)\right)$$

From small to large-angle inelastic scattering ... and to gravitational collapse?

(ACV, hep/th-0712.1209, MO, VW, CC...'08)

Classical corrections are related to "tree diagrams"



Power counting for connected trees:

$$A_{cl}(E,b) \sim G^{2n-1} s^n \sim Gs\ R^{2(n-1)} \rightarrow Gs\ (R/b)^{2(n-1)}$$

Summing tree diagrams => solving a classical field theory.

Q: Which is the effective field theory for TP-scattering?

Reduced effective action & field equations

There is a D=4 effective action generating the leading diagrams (Lipatov, ACV '93). After (approximately) factoring out the longitudinal dynamics it becomes a D=2 effective action containing 4 fields:

a_1 and a_2, representing the longitudinal (++ and --) components of the metric, sourced by the EMT of the two fast particles;

ϕ, a complex field representing the TT components of the graviton field (i.e. the physical gravitons).

One polarization suffers from (well understood but bothersome) IR divergences. Limiting ourselves to the IR-safe polarization ϕ becomes real. In that case:

The action (neglecting rescattering for b not too small)

$$\frac{\mathcal{A}}{2\pi G s} = \int d^2x \left[a(x)\bar{s}(x) + \bar{a}(x)s(x) - \frac{1}{2}\nabla_i \bar{a} \nabla_i a \right] + \frac{(\pi R)^2}{2} \int d^2x \left(-(\nabla^2 \phi)^2 + 2\phi \nabla^2 \mathcal{H} \right) ,$$

$$-\nabla^2 \mathcal{H} \equiv \nabla^2 a \, \nabla^2 \bar{a} - \nabla_i \nabla_j a \, \nabla_i \nabla_j \bar{a} \, ,$$

(for point-particles s(x) is a δ -function)

The corresponding eom read:

$$\nabla^2 a + 2s(x) = 2(\pi R)^2 (\nabla^2 a \, \nabla^2 \phi - \nabla_i \nabla_j a \, \nabla_i \nabla_j \phi), \quad \nabla^2 \bar{a} + 2\bar{s}(x) = \ldots$$

$$\nabla^4 \phi = -(\nabla^2 a \, \nabla^2 \bar{a} - \nabla_i \nabla_j a \, \nabla_i \nabla_j \bar{a})$$

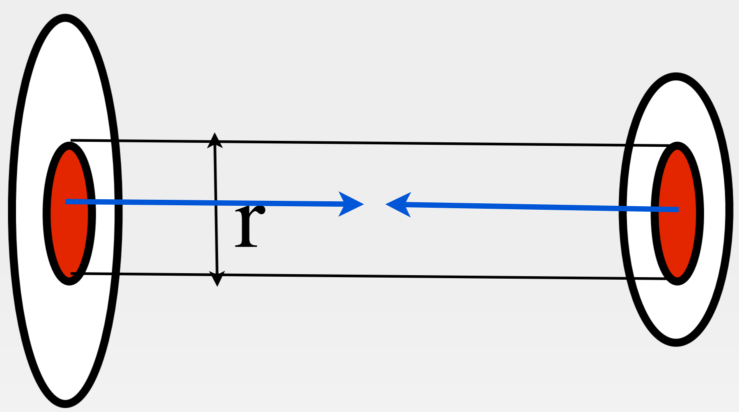
The semiclassical approximation amounts to solving the eom and computing the classical action on the solution. This is why we took Gs/h >> 1!

Still too hard for analytic study, for numerics: see below

Axisymmetric beam-beam collisions

(ACV '07, J.Wosiek & GV '08)

$R_1(r)=4GE_1(r)$



$R_2\,(r)=4GE_2(r)$

A simpler, yet rich, problem:

1. The sources contain several parameters & we can look for critical surfaces in their multi-dim.^{al} space

2. The CTS criterion is simple (see below)

3. Numerical results are coming in (see CP, 2009)

4. One polarization not produced

5. And, last but not least, PDEs become ODEs

Equations, results

Introducing the auxiliary field ($t = r^2$) $\rho = t\left(1 - (2\pi R)^2 \dot{\phi}\right)$

eom become:

$$\dot{a}_i = -\frac{1}{2\pi\rho}\frac{R_i(r)}{R}$$

$$\ddot{\rho} = \frac{1}{2}(2\pi R)^2 \dot{a}_1 \dot{a}_2 = \frac{1}{2}\frac{R_1(r)R_2(r)}{\rho^2}$$

subject to boundary conditions

$$\rho(0) = 0 \quad , \quad \dot{\rho}(\infty) = 1$$

ACV vs. CTS

KV's criterion for existence of CTS: if there exists an r_c s.t.

$$R_1(r_c)R_2(r_c) = r_c^2$$

we can construct a CTS and therefore a BH forms.

Theorem (VW08): whenever the KV criterion holds*) the ACV field equations do not admit regular real solutions.
Thus:

KV criterion ==> ACV criterion

but not necessarily the other way around!

*) actually the r.h.s. can be replaced by $\frac{2}{3\sqrt{3}} r_c^2$

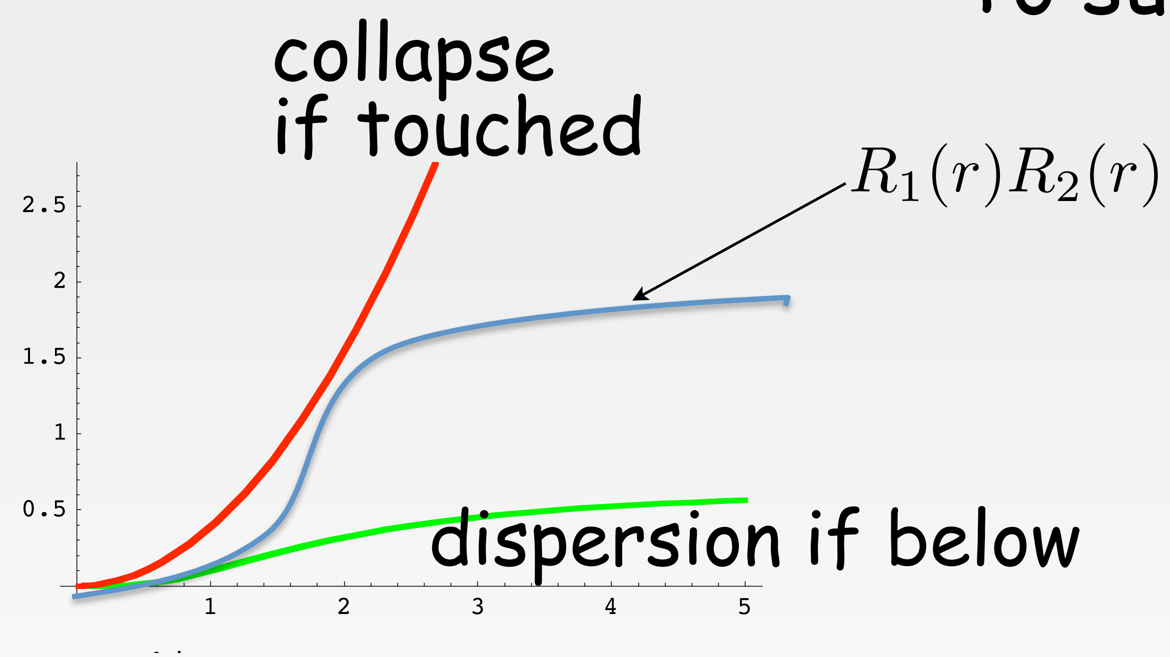
A sufficient criterion for dispersion

(P.-L. Lions, private comm.)

If
$$R_1(r)R_2(r) \leq \frac{8}{27}\frac{r^4}{(1+r^2)^2}\left[1+\frac{1}{2}\left(1-\frac{log(1+r^2)}{r^2}\right)\right]^2$$

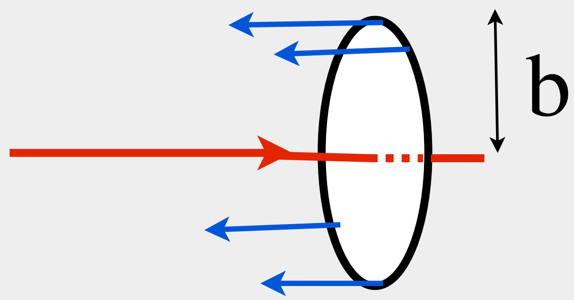
the ACV eqns do admit regular, real solutions.

To summarize



clearly, there is room for improvement...

Example 1: particle-scattering off a ring



Can be dealt with analytically:

$$\ddot{\rho} = \frac{R^2}{2\rho^2}\Theta(r^2 - b^2)$$

$$\begin{aligned} \rho &= \rho(0) + r^2\dot{\rho}(0) \quad , \quad (r < b) \\ \dot{\rho} &= \sqrt{1 - R^2/\rho} \quad , \quad (r > b) \end{aligned}$$

Since $\rho\,(0) = 0$:

$$\rho(b^2) = b^2\dot{\rho}(b^2) = b^2\sqrt{1 - R^2/\rho(b^2)}$$

This (cubic) equation has real solutions iff

$$b^2 > \frac{3\sqrt{3}}{2}R^2 \equiv b_c^2$$

$(b/R)_c \sim 1.61$

CTS: $(b/R)_c > 1$

Example 2: Two hom. beams of radius L.

The equation for ρ becomes

$$\ddot{\rho}(r^2) = \frac{R^2}{2\rho^2}\Theta(r-L) + \frac{R^2 r^4}{2L^4\rho^2}\Theta(L-r)$$

We can compute the critical value numerically:

$$\left(\frac{R}{L}\right)_{cr} \sim 0.47$$

It is compatible with (and a factor 2.13 below) the CTS upper bound of KV:

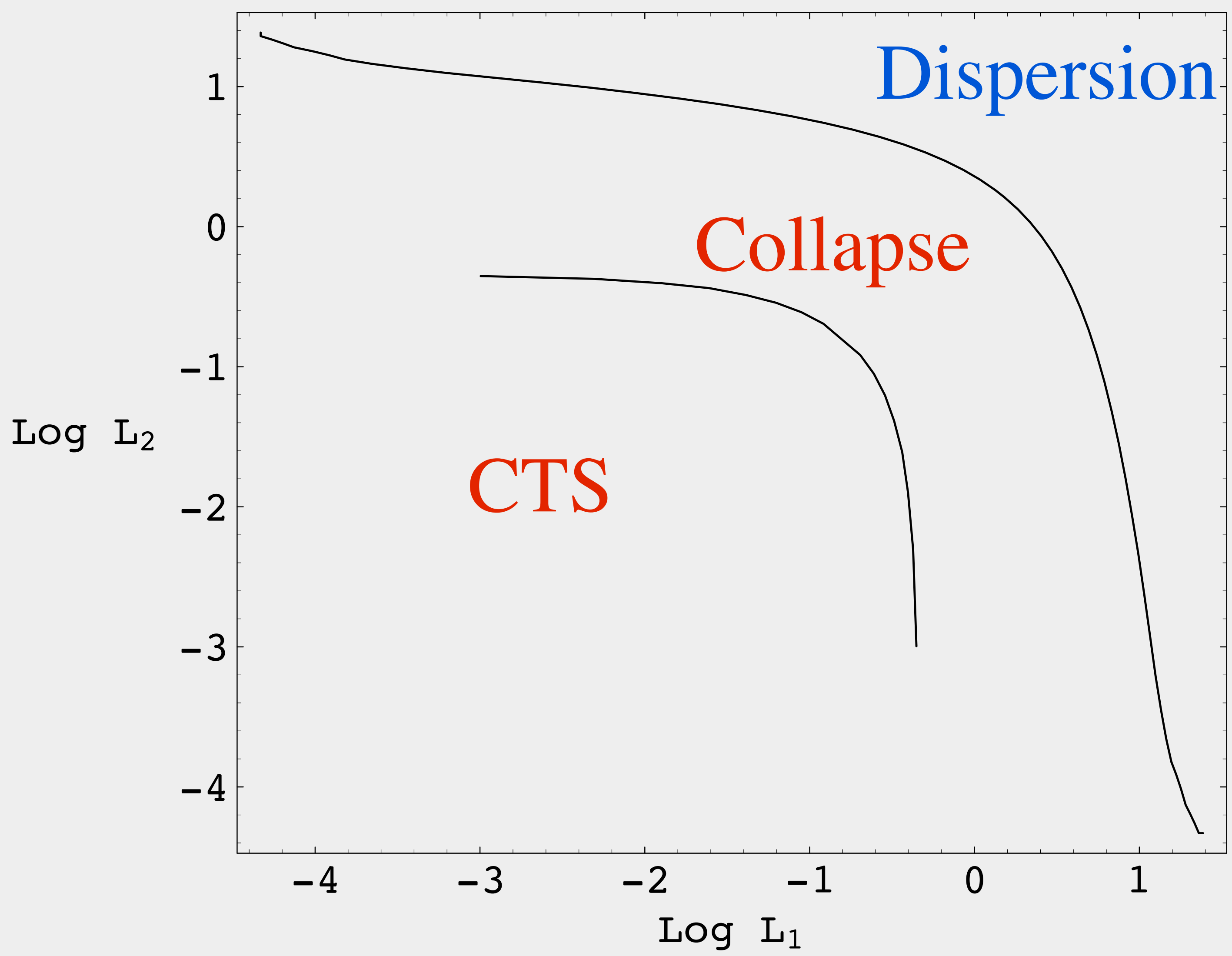
$$\left(\frac{R}{L}\right)_{cr} < 1.0$$

Example 3:
Two different Gaussian Beams
(GV&J.Wosiek '08)

Consider two extended sources (beams) with the same fixed total energy and Gaussian profiles centered at r=0 but with arbitrary widths L_1 and L_2

$$s_i(t) = \frac{1}{2\pi L_i^2} exp\left(-\frac{t}{2L_i^2}\right) \quad , \quad \frac{R_i(t)}{R} = 1 - exp\left(-\frac{t}{2L_i^2}\right)$$

Determine numerically critical line in (L_1 , L_2) plane and compare it with the one coming from the CTS criterion.



For $L_1 = L_2$, L_c is a factor 2.70 above CTS's lower bound

In 0908.1780 Choptuik & Pretorius analyzed a "similar" situation numerically. BH formation occurs about a factor 3 above the naive CTS value: a coincidence?

Particle-particle collisions at finite b

Numerical solutions

(G. Marchesini & E. Onofri, 0803.0250)

Solve directly PDEs by FFT methods in Matlab

Result: real solutions only exist for

$$b > b_c \sim 2.28R$$

Compare with EG's CTS lower bound on b_c

$$b_c > 0.80R$$

b_c is a factor 2.85 above CTS's lower bound

What happens below b_c?

(ACV '07, M. Ciafaloni & D. Colferai '08, '09)

If we insist on regularity at r=0, for $b < b_c$ we end up with complex classical solutions => Im $A_{cl} \neq 0$.

Im A_{cl} induces in the S-matrix a new absorption on top of the one due to graviton emission.The meaning of this new absorption is still unclear. One possibility is to associate it with the opening on new channels related to black-hole production.

Progress on this issue has been obtained by M. Ciafaloni & D. Colferai (0807.2117, 0909.4523) by adding a class of quantum corrections to the semiclassical approximation and by giving a tunnelling interpretation to complex solutions.

Some questions still remain open

My feeling is that the apparent loss of unitarity is due to our oversimplified treatment of the longitudinal dynamics

Particle Spectra: an "energy crisis"

(ACV07, VW08/2, M. Ciafaloni & GV in progress)

Within our approximations the spectrum of the produced gravitons gives the following result for GW emission:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\ \frac{Gs}{\hbar}\frac{R^2}{b^2} >> 1$$

Accordingly, the fraction of energy emitted in GW turns out to be O(1) already for b=b*>>R with Gs/h $(R/b^*)^2$ =O(1). This is puzzling from a GR perspective...and is related to a crucial

Q: What is the frequency cutoff on the GWs emitted in an ultrarelativistic small angle (b>>R) 2-body scattering?

Possible answers: 1/b, 1/R , γ /b (Galtsov et al. for b>>1/m, R).

My guess (1/R) would rather give:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp(-|k||b| - \omega R) \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim \frac{R^2}{b^2}$$

In both cases, while for b >> R gravitons are produced at small angles, as b -> b_c ~ R their distribution becomes more and more spherical w/ <n> ~ Gs and (again!) characteristic energy O(1/R ~T_H)

The classical answer to this problem seems to be unavailable...

THE GENERATION OF GRAVITATIONAL WAVES.
IV. BREMSSTRAHLUNG*†‡

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ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$(\text{angle of gravitational deflection of stars' orbits}) \ll (1 - v^2/c^2)^{1/2}\ .$$

For for $\theta < 1/\gamma$ (b > γ R) it agrees with GKST.

What's the answer for $\theta > 1/\gamma$? Hopefully it is in another paper...

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High-speed black-hole encounters and gravitational radiation

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(Received 15 March 1977)

Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or close encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.

If my guess turns out to be correct classically we have to find why ACV do not reproduce it (again the oversimplified treatment of long. dynamics? Neglect of rescattering?)

Summary

• Gedanken experiments have played an important role in the early developments of Quantum Mechanics.

• TPE collisions may well play a similar role for understanding whether & how QM & GR are mutually compatible

• Superstring theory in flat space-time offers a concrete framework where the quantum scattering problem is well-posed.

• The problem simplifies by considering $Gs/h \gg 1$ since a suitable semiclassical approximation can be justified. Within that constraint we have considered various regimes, roughly classified as follows:

- A large impact parameter regime, where an eikonal approximation holds and GR expectations are recovered (emerging AS metric, tidal excitation..)

- A stringy regime, where one finds an approximate S-matrix with some characteristics of BH-physics as the expected BH threshold is approached from below

- A strong-gravity (large R ~ GE) regime where an effective action approach can be (partly) justified and tested

•Critical points (lines) have emerged matching well CTS-based GR criteria (within an intriguing factor 2-3)

- As the critical line is approached, the final state starts resembling a Hawking-like spectrum: a fast growth (~ E^2) of multiplicity w/ a related softening of the final state.

- Progress was made towards constructing a unitary S-matrix and understanding the physics of the process as the critical surface is reached and possibly crossed

- Much more work remains to be done, but an understanding of the quantum analog/replacement of GR's gravitational collapse does no-longer look completely out of reach...

THANK YOU