Hairy black holes and self-accelerating cosmologies in the bigravity theory

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M.S.V., arXiv:1202.6682;

M.S.V., JHEP 1201 (2012) 035;

A.Chamseddine, M.S.V., Phys.Lett. B704 (2011) 652.

IHES, 29 March 2012

Massive Gravity

- A deformation of GR that allows to explain the observed universe acceleration $\Rightarrow m \sim 1/$(cosm. horizon size).
- Has problems: does not reduce to GR in the weak field when $m \to 0$ (VdVZ discontinuity), has a ghost, no uniqueness.
- Remedies seem to exist for some of these problems (Vainstein mechanism). Very recently a class of models has been proposed that seem to be free of the ghost.
- We wish to study black holes and cosmologies in these models.

Plan

- Massive gravity in D=4
- Checking Vainstein scenario
- Ghost-free theories
- Black holes and self-accelerating cosmologies
- More exotic cosmologies
- Hairy black holes
- Lumps of pure gravity
- Asymptotically flat stars and Vainstein mechanism

I. Massive gravity in D=4

Bimetric theory

4D manifold with two metrics

$$g_{\mu\nu}(x) \qquad \text{and} \qquad f_{\mu\nu}(x) = \eta_{AB}\partial_\mu X^A(x)\partial_\nu X^B(x)$$

and the action

$$S = -\frac{1}{8\pi G}\int \left(\frac{1}{2}R + m^2\mathcal{L}_{\rm int}\right)\sqrt{-g}\,d^4x + S_{\rm (mat)}$$

where $\mathcal{L}_{\rm int}$ is a scalar function of $H^\alpha_{\ \beta} = g^{\alpha\sigma}f_{\sigma\beta} - \delta^\alpha_\beta$

$$\mathcal{L}_{\rm int} = \frac{1}{8}((H^\alpha_{\ \alpha})^2 - H^\alpha_{\ \beta}H^\beta_{\ \alpha}) + O((H^\alpha_{\ \beta})^3)$$

Theory is not unique, but has a unique weak field limit.

EOM for $g_{\mu\nu}, X^A$

$$G_{\mu\nu} = m^2 T_{\mu\nu} + 8\pi G T_{\mu\nu}^{(\mathrm{mat})}$$

with

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}_{\mathrm{int}}}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}_{\mathrm{int}},$$

varying with respect to X^A gives

$$\nabla^\mu T_{\mu\nu} = 0.$$

The matter equations imply

$$\nabla^\mu T_{\mu\nu}^{(\mathrm{mat})} = 0.$$

In the unitary gauge, $X^\alpha = x^\alpha$ and $f_{\mu\nu} = \eta_{\mu\nu}$, in the weak field limit $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ one recovers the

Pauli-Fierz equations

$$\frac{1}{2}\{-\Box h_{\mu\nu} + \ldots\} \;=\; \frac{1}{2}m^2(h_{\mu\nu} - h\eta_{\mu\nu}) + 8\pi G T^{(\mathrm{mat})}_{\mu\nu}$$

which imply 4 constraints

$$\partial^\mu h_{\mu\nu} - \partial_\nu h \;=\; 0.$$

Taking the trace gives the fifth constraint

$$3m^2 h = 16\pi G T^{(\mathrm{mat})}$$

$\Rightarrow$ there remain 5 degrees of freeedom of massive graviton.

For generic $g_{\mu\nu}$ there are 5 degrees + 1 extra state with negative norm – Boulevard-Deser ghost.

VdVZ discontinuity

Choosing the two metrics as

$$
\begin{aligned}
ds^2 &= e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2 \\
df^2 &= dt^2 - U'^2(r)dr^2 - U^2(r)d\Omega^2 \qquad (U^2 \equiv r^2 e^{\mu})
\end{aligned}
$$

gives in the linear approximation at large r

$$
\nu = -\frac{C}{r}\,e^{-mr},\ \lambda = \frac{C(1+mr)}{2r}\,e^{-mr},\ \mu = \frac{C(1+mr+m^2r^2)}{2m^2r^3}\,e^{-mr}
$$

$\Rightarrow$ for r large but $rm \ll 1$ one has

$$
\nu = -\frac{C}{r}, \qquad \lambda = \frac{C}{2r} \qquad \Rightarrow \qquad \nu + \lambda = -\frac{C}{2r} \neq 0
$$

$\Rightarrow$ GR is not recovered for $m \to 0$, no correct Newton's law.

Vainstein solution

Non-linear corrections to the VdVZ

$$\nu = -\frac{r_g}{r}\left(1 + c_1\,\frac{A}{m^4 r^5} + \ldots\right), \quad \lambda \;=\; \frac{r_g}{2r}\left(1 + c_2\,\frac{A}{m^4 r^5} + \ldots\right)$$

are $\sim 10^{32}$ at the edge of solar system if $m \sim (10^{25} cm)^{-1}$. They become small only for (assuming that $C \sim r_g$)

$$r \gg r_{\rm V} = \left(r_g/m^4\right)^{1/5} \sim 100\,Kps$$

$\Rightarrow$ the VdVZ problem arises only for $r \gg r_V$. For $r \ll r_V$

$$\nu = -\frac{r_g}{r} + \ldots, \qquad \lambda = \frac{r_g}{r} + \ldots, \qquad \frac{U}{r} = 1 + a\sqrt{\frac{r_g}{r}} + \ldots,$$

$\Rightarrow$ GR is recovered at the non-linear level.

II. Checking Vainstein scenario

Damour, Kogan, Papazoglou’ 02
Babuchev, Deffayet, Ziour ’09
M.S.V.

The (AGS) model

$$\mathcal{L}_{\rm int} = \frac{m^2}{8}((H^{\mu}{}_{\mu})^2 - H^{\mu}{}_{\nu}H^{\nu}{}_{\mu}) \quad \text{with} \quad H^{\mu}{}_{\nu} = g^{\mu\sigma}f_{\sigma\nu} - \delta^{\mu}_{\nu}$$

Static, spherically symmetric case $(U^2 \equiv r^2 e^{\mu})$

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2 d\Omega^2, \quad df^2 = dt^2 - U'^2 dr^2 - U^2 d\Omega^2$$

$$\Rightarrow \quad H^{\alpha}_{\beta} = {\rm diag}(1 - e^{-\nu}, 1 - e^{-\lambda}U'^2, 1 - e^{\mu}, 1 - e^{\mu})$$

gravitons: $T^{\mu}_{\nu} = \delta^{\mu}_{\nu}\left(\frac{1}{8}((1 - H^{\mu}_{\mu})(H^{\mu}_{\mu} - \sum_{\gamma} H^{\gamma}_{\gamma}) + \sum_{\gamma}(H^{\gamma}_{\gamma})^2\right)$

matter: $8\pi G T^{({\rm mat})\mu}_{\nu} = {\rm diag}(\rho, -P, -P, -P)$

Field equations

$$e^{-\lambda}\left(\frac{\lambda'}{r}-\frac{1}{r^2}\right) \quad + \quad \frac{1}{r^2}=m^2T^0_0+\rho$$

$$e^{-\lambda}\left(\frac{\nu'}{r}-\frac{1}{r^2}\right) \quad + \quad \frac{1}{r^2}=m^2T^r_r-P$$

$$(T^r_r)' \quad = \quad \frac{\nu'}{2}(T^0_0-T^r_r)+\frac{2}{r}(T^\vartheta_\vartheta-T^r_r) \quad \Leftarrow \nabla_\mu T^\mu_\nu=0,\ \mu''$$

$$P' \quad = \quad \frac{\nu'}{2}(P+\rho) \quad \Leftarrow \nabla_\mu T^{(\mathrm{mat})\mu}_{\ \nu}=0$$

with $\rho=\rho_\star\Theta(r_\star-r)$ – star of radius $r_\star$ and density $\rho_\star$.

$$y_k(r)=\{\beta,\lambda,\mu,\mu',P\} \quad \Rightarrow \quad \frac{dy_k}{dr}=F_k(r,y_m)$$

Boundary conditions

Origin $r = 0$: curvature is bounded $\Rightarrow$

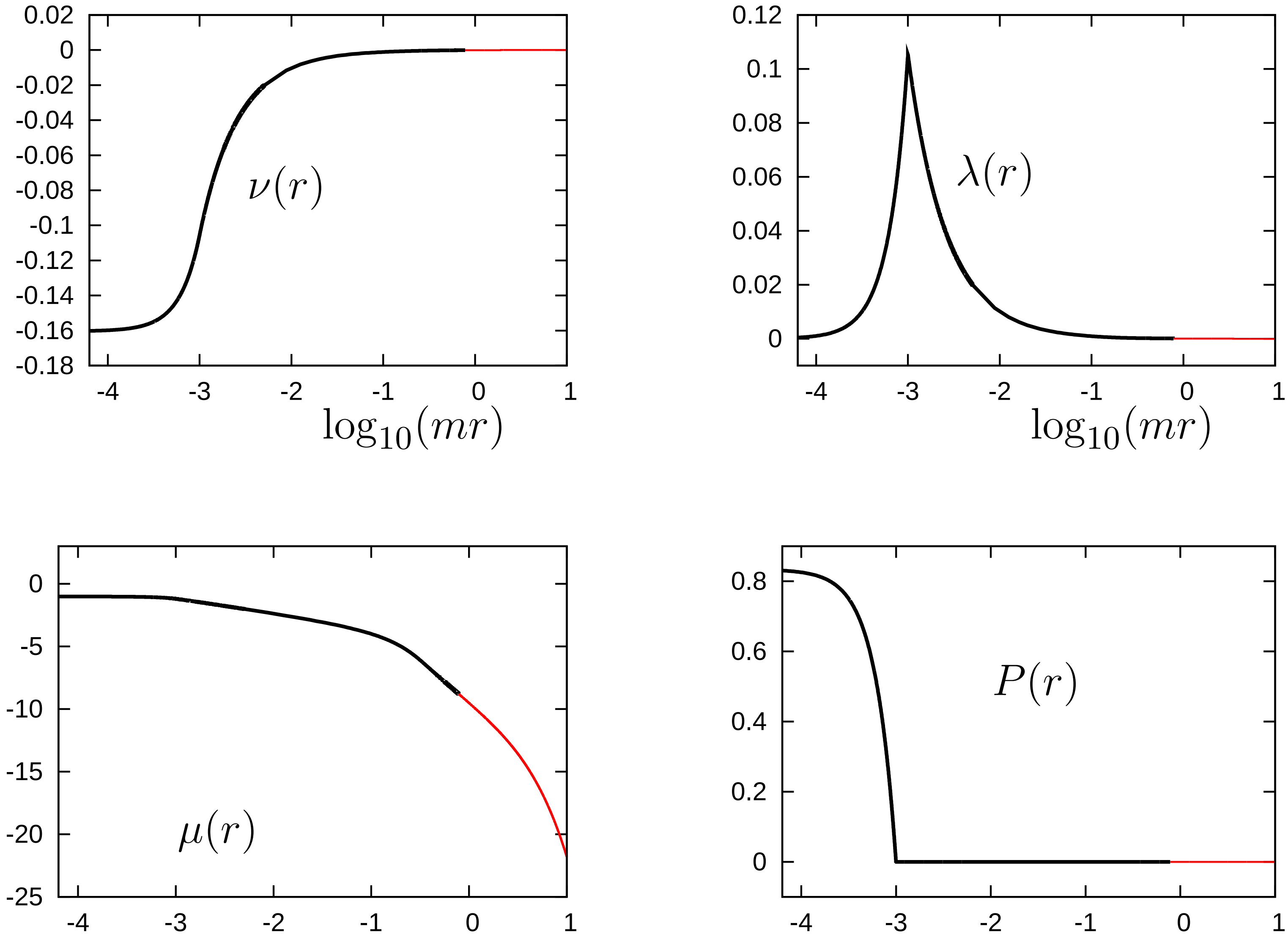
$$\nu = \nu_0 + \mathcal{O}(r^2), \quad \lambda = \mathcal{O}(r^2), \quad \mu = \mu_0 + \mathcal{O}(r^2), \quad P = P_0 + \mathcal{O}(r^2)$$

Star surface $r = r_\star$: ν, λ, μ, P are continues, $P = 0$ for $r \geq r_\star$.
Infinity $r = \infty$: VdVZ+ghost ($x = mr$)

$$\begin{aligned}
\nu &= -\frac{C}{x}\, e^{-x} + A\frac{\sqrt{C}}{2} e^{-\frac{x}{2} - \mathcal{M}x} + \ldots, \\
\lambda &= \frac{C}{2x}\,(1+x)\, e^{-x} + A\frac{x^2}{2} e^{-\mathcal{M}x} + \ldots, \\
\mu &= C\,\frac{1+x+x^2}{2x^3}\; e^{-x} + Ae^{-\mathcal{M}x} + \ldots
\end{aligned}
\quad \text{with} \quad \boxed{\mathcal{M} = \frac{2}{\sqrt{C}}\, e^{x/2}}$$

Here ν_0, μ_0, P_0, C, A are 5 integration constants.

Solution $m = 0.01, r_\star = 0.1, \rho_\star = 30$



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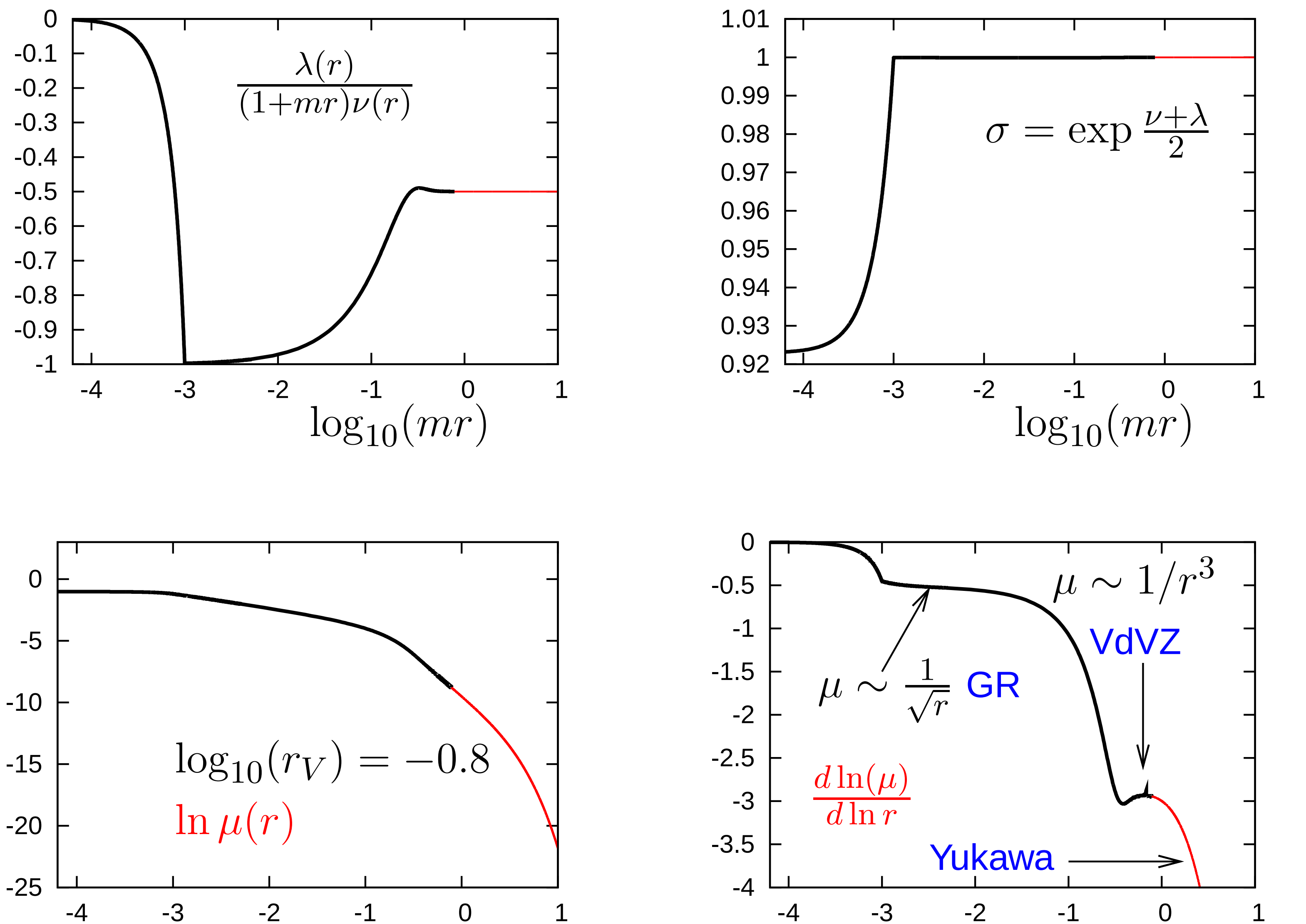
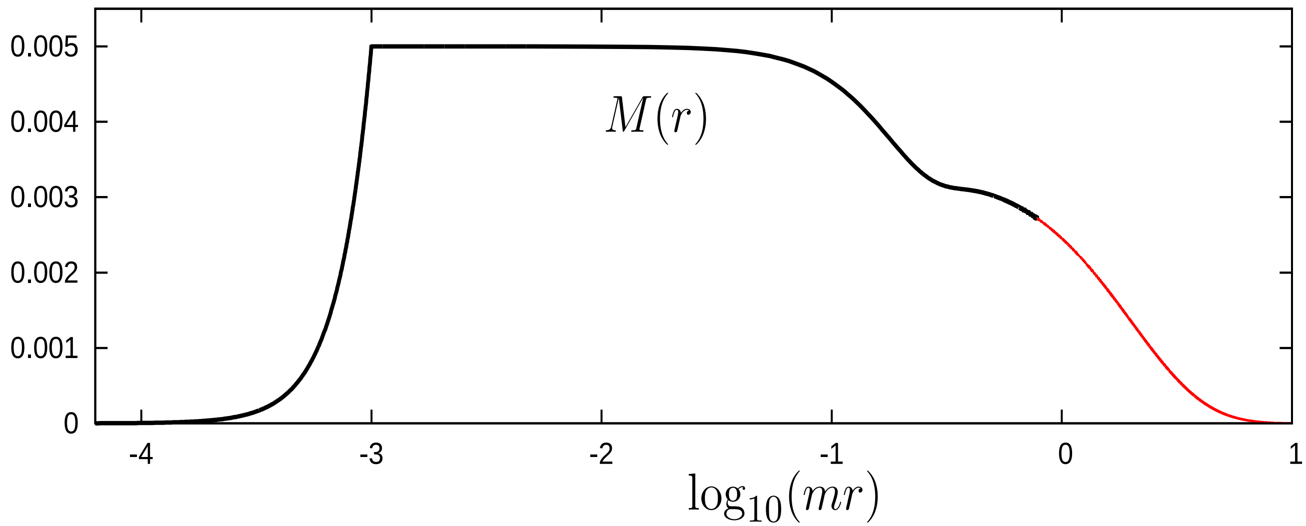


Mass function



$$(g_{rr})^{-1} \equiv 1 - \frac{2M(r)}{r}, \quad M' = \frac{r^2}{2}\{m^2 T^0_0 + \rho\} \equiv \frac{r^2}{2}\mathbf{T}^0_0$$

In GR $\mathbf{T}^0_0 \geq 0 \Rightarrow M(r)$ always grows till $M(\infty) = M_{\rm ADM}$.
In massive gravity $M(\infty) = 0 \Rightarrow \mathbf{T}^0_0$ must be non-positive and unbounded from below – negative energies.

III. Ghost free theories

de Rham, Gabadadze, Tolley ’10
Hassan, Rosen ’11

The RGT bimetric model

$$\mathcal{L}_{\rm int} = \frac{m^2}{2}(K^2 - K^\nu_\mu K^\mu_\nu) \quad \text{with} \quad \boxed{K^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\sigma} f_{\sigma\nu}}}$$

is claimed to be ghost-free and unique up to 2-parameter deformations /de Rham, Gabadadze, Tolley ’10/.

- Cosmologies /Chamseddine, M.S.V./
- Black holes /Nieuwenhuizen/,/Koyama, Niz, Tasinato/. No asymptotically flat black holes.
- If the metric $f_{\alpha\beta}$ is promoted to be dynamical, the theory rests ghost-free /Hassan, Rosen ’11/

The ghost-free bigravity

$$S[g_{\mu\nu}, f_{\mu\nu}] = -\frac{1}{8\pi G}\int\left(\frac{1}{2}R + m^2\cos^2\eta\,\mathcal{L}_{\rm int}\right)\sqrt{-g}\,d^4x - \frac{1}{16\pi\tan^2\eta\,G}\int\mathcal{R}\sqrt{-f}\,d^4x + S_{\rm (mat)}\,,$$

$$\mathcal{L}_{\rm int} = \frac{1}{2}(K^2 - K^\nu_\mu K^\mu_\nu) + \frac{c_3}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}K^\mu_\alpha K^\nu_\beta K^\rho_\gamma + \frac{c_4}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}K^\mu_\alpha K^\nu_\beta K^\rho_\gamma K^\sigma_\delta\,,$$

where m, η, c_3, c_4 are parameters and

$$K^\mu_\nu = \delta^\mu_\nu - \gamma^\mu{}_\nu\,, \qquad \boxed{\gamma^\mu{}_\sigma\gamma^\sigma{}_\nu = g^{\mu\sigma}f_{\sigma\nu}}$$

Varying the constraint

$$\delta\gamma^{\mu}{}_{\sigma}\gamma^{\sigma}{}_{\nu} + \gamma^{\mu}{}_{\sigma}\delta\gamma^{\sigma}{}_{\nu} = \delta g^{\mu\sigma} f_{\sigma\nu} + g^{\mu\sigma}\delta f_{\sigma\nu}$$

Let us introduce two tetrads e^{ν}_{B} and ω^{A}_{μ} such that

$$g^{\mu\nu} = \eta^{AB} e^{\mu}_{A} e^{\nu}_{B}\,, \qquad f_{\mu\nu} = \eta_{AB}\omega^{A}_{\mu}\omega^{B}_{\nu}\,,$$

and

$$\boxed{e^{\mu}_{A}\omega_{B\mu} = e^{\mu}_{B}\omega_{A\mu}} \qquad (\bullet)$$

Then

$$\gamma^{\mu}{}_{\nu} \equiv \sqrt{g^{\mu\sigma} f_{\sigma\nu}} = e^{\mu}_{A}\omega^{A}_{\nu}$$

so that one can directly vary with respect to e^{ν}_{B} and ω^{A}_{μ} imposing the condition $(\bullet)$ by a Lagrange multiplier.

Field equations

$$G^{\rho}_{\lambda} = m^2 \cos^2\eta\, T^{\rho}_{\lambda} + 8\pi G T^{(\text{mat})\,\rho}_{\lambda}\,, \qquad \mathcal{G}^{\rho}_{\lambda} = m^2 \sin^2\eta\, \mathcal{T}^{\rho}_{\lambda}\,,$$

with
$$T^{\rho}_{\lambda} = \tau^{\rho}_{\lambda} - \delta^{\rho}_{\lambda}\mathcal{L}_{\text{int}}\,, \qquad \mathcal{T}^{\rho}_{\lambda} = -\frac{\sqrt{-g}}{\sqrt{-f}}\,\tau^{\rho}_{\lambda}\,,$$

$$\begin{aligned}\tau^{\rho}_{\lambda} &= (\gamma^{\sigma}_{\sigma} - 3)\gamma^{\rho}_{\lambda} - \gamma^{\rho}_{\sigma}\gamma^{\sigma}_{\lambda} - \frac{c_3}{2}\,\epsilon_{\lambda\mu\nu\sigma}\epsilon^{\alpha\beta\gamma\sigma}\gamma^{\rho}_{\alpha}K^{\mu}_{\beta}K^{\nu}_{\gamma} \\ &- \frac{c_4}{6}\,\epsilon_{\lambda\mu\nu\sigma}\epsilon^{\alpha\beta\gamma\delta}\gamma^{\rho}_{\alpha}K^{\mu}_{\beta}K^{\nu}_{\gamma}K^{\sigma}_{\delta}\,.\end{aligned}$$

- Reduces to the bimetric RGT theory for $\eta \to 0$ if $f_{\mu\nu}$ becomes flat.
- $g_{\mu\nu} = f_{\mu\nu} \Rightarrow T^{\mu}_{\nu} = \mathcal{T}^{\mu}_{\nu} = 0 \Rightarrow G^{\mu}_{\nu} = 0 \Rightarrow$ theory contains vacuum GR

Conservation conditions

Diff.-invariance of the $S_{(\mathrm{mat})}$ and Bianchi identities imply

$$\overset{(g)}{\nabla}_\rho\, T^\rho_\lambda = 0\,, \qquad\qquad \overset{(g)}{\nabla}_\rho\, T^{(\mathrm{mat})\rho}_{\ \ \lambda} = 0\,.$$

Similarly, the Bianchi identities imply that

$$\overset{(f)}{\nabla}_\rho\, \mathcal{T}^\rho_\lambda = 0$$

but these are not independent, since under a diffeomorphism generated by $\xi^\mu(x)$ one has

$$0 \equiv \delta S_{\mathrm{int}} = -\int \xi^\mu \underbrace{\overset{(g)}{\nabla}_\sigma\, T^\sigma_\mu}_{0} \sqrt{-g}\, d^4x - \int \xi^\mu\, \overset{(f)}{\nabla}_\sigma\, \mathcal{T}^\sigma_\mu \sqrt{-f}\, d^4x.$$

Spherical symmetry

Most general case

$$e^0 = Q\,dt, \quad e^1 = \frac{1}{N}\,dr, \quad e^2 = R\,d\vartheta, \;\; e^3 = R\sin\vartheta\,d\varphi$$

$$\omega^0 = a\,dt + c\,dr, \;\; \omega^1 = -c\,QN\,dt + b\,dr, \;\; \omega^2 = U\,d\vartheta, \;\; \omega^3 = U\sin\vartheta d\varphi$$

where a, b, c, Q, N, U, R functions of t, r. One has

$$g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu \,, \qquad f_{\mu\nu} = \eta_{AB} \omega^A_\mu \omega^B_\nu \,, \qquad e^\mu_A \omega_{B\mu} = e^\mu_B \omega_{A\mu}$$

Two different cases:

- $c = f_{0r} \neq 0 \Rightarrow$ metrics are not simultaneously diagonal
- $c = f_{0r} = 0 \Rightarrow$ metrics are simultaneously diagonal

IV. Black holes and self-accelerating cosmologies

M.S.V. JHEP 1201 (2012) 035
M.S.V. arXiv:1202.6682

Non-diagonal $f_{\mu\nu}$

$$ds^2 = Q^2 dt^2 - \frac{dr^2}{N^2} - R^2 d\Omega^2$$

$$df^2 = \left(a^2 - c^2 Q^2 N^2\right) dt^2 + 2c(a + bQN)\, dt dr - \left(b^2 - c^2\right) dr^2 - U^2 d\Omega^2$$

If $g_{\mu\nu}$ is either static or FRW $\Rightarrow G^0_r = T^0_r = 0 \Leftrightarrow \boxed{U = CR}$

$$C = \frac{1}{c_3 + c_4}\left(2c_3 + c_4 - 1 \pm \sqrt{1 - c_3 + c_4 + c_3^2}\right)$$

$\Rightarrow T^0_0 = T^r_r = const. \Rightarrow \overset{(g)}{\nabla}_\mu T^\mu_\nu \sim T^r_r - T^\theta_\theta$ where

$$\overset{(g)}{\nabla}_\mu T^\mu_\nu \sim \overbrace{(c_3 C - C - c_3 + 2)}^{A} \overbrace{(C^2 Q - CQNb - aC + c^2 N^2 Q + abN)}^{B}$$

Equations

If $\overset{(g)}{\nabla}_\mu T^\mu_\nu = 0 \Rightarrow$ cosmological term + matter

$$
\begin{aligned}
(A) \qquad G^\mu_\nu &= m^2 \cos^2\eta\, \lambda \delta^\mu_\nu + 8\pi G T^{(\mathrm{mat})\mu}{}_\nu \\
(B) \qquad \mathcal{G}^\mu_\nu &= m^2 \sin^2\eta\, \tilde{\lambda} \delta^\mu_\nu
\end{aligned}
$$

$$\lambda = (C-1)(c_3 C - C - c_3 + 3), \qquad \tilde{\lambda} = \frac{1-C}{C^2}(c_3 C - c_3 + 2).$$

Here $T^{(\mathrm{mat})\mu}{}_\nu = 0$ in the static case, while for cosmologies

$$8\pi G T^{(\mathrm{mat})\mu}{}_\nu = \mathrm{diag}(\rho(t), -P(t), -P(t), -P(t))$$

Equations (A) decouple from (B), but there are constraints between $g_{\mu\nu}$ and $f_{\mu\nu}$. Let us first solve (A).

Solutions for $g_{\mu\nu}$

Cosmological term $\Lambda = \lambda\, m^2 \cos^2 \eta > 0 \Rightarrow$

Black holes: Schwarzschild-dS,

$$N^2 = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2, \qquad ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2$$

Cosmologies: FRW with matter+ Λ,

$$ds^2 = dt^2 - \mathbf{a}^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right), \qquad K = 0, \pm 1$$

$$\boxed{\dot{\mathbf{a}}^2 - \frac{\mathbf{a}^2}{3}(\Lambda + \rho) = -K} \quad \Rightarrow \quad \text{self-acceleration}$$

Solution for $f_{\mu\nu}$

$f_{\vartheta\vartheta} = U^2 = C^2 g_{\vartheta\vartheta}$ is already fixed, but f_{00}, f_{0r}, f_{rr} are still arbitrary functions of t, r. Let $U = U(t, r)$ and $T(t, r)$ be the new radial/time coordinates $\Rightarrow$

$$df^2 = f_{TT}\, dT^2 + 2 f_{TU} dT dU + f_{UU} dU^2 - U^2 d\Omega^2$$

This has to fulfill Einstein equations (B) with negative cosmological term $\tilde{\Lambda} = \tilde{\lambda}\, m^2 \sin^2 \eta < 0$. <u>Solution is the AdS</u>:

$$df^2 = \Delta dT^2 - \frac{dU^2}{\Delta} - U^2 d\Omega^2, \qquad \Delta = 1 - \frac{\tilde{\Lambda}}{3} U^2$$

Here $U = Cr$ for black holes, $U = C\mathbf{a}(t)r$ for cosmologies. $f_{\mu\nu}$ is flat when $\eta \to 0$ = <u>the bimetric RGT limit</u>

There remains to determine $T(t, r)$ and impose $\overset{(g)}{\nabla}_\mu T^\mu_\nu = 0$.

Imposing $\overset{(g)}{\nabla}_\mu T^\mu_\nu = 0$ for black holes

$$df^2 = (\theta^0)^2 - (\theta^1)^2 - U^2 d\Omega^2 = (\omega^0)^2 - (\omega^1)^2 - U^2 d\Omega^2$$ with

$$\theta^0 = \sqrt{\Delta} dT, \quad \theta^1 = \frac{dU}{\sqrt{\Delta}}, \quad \omega^0 = a\, dt + c\, dr, \quad \omega^1 = -c\, N^2\, dt + b\, dr.$$

One has to have

$$\omega^0 = \sqrt{1+\alpha^2}\theta^0 + \alpha\theta^1, \qquad \omega^1 = \sqrt{1+\alpha^2}\theta^1 + \alpha\theta^0,$$

since $U = Cr \Rightarrow$ $\qquad T = Ct - C \int \frac{\alpha}{\sqrt{1+\alpha^2}} \frac{N^2+\Delta}{N^2\Delta} dr$

$\overset{(g)}{\nabla}_\mu T^\mu_\nu = 0$ if $B = 0$ $\Rightarrow$ $\alpha = \frac{N^2 - \Delta}{2N\sqrt{\Delta}}$ $\Rightarrow$ solution is complete

Essentially the same as that of Isham and Storey' 78

Imposing $\overset{(g)}{\nabla}_\mu T^\mu_\nu = 0$ for cosmologies

One has $U = C\mathbf{a}(t)r$. Imposing $\overset{(g)}{\nabla}_\mu T^\mu_\nu \sim AB = 0$ via $B = 0$ gives a very complicated non-linear PDE. It is unclear if the solution exists.

Alternatively, one can require that $A = c_3 C - C - c_3 + 2 = 0$, but this constraints the values of c_3, c_4:

$$c_3 = \frac{C-2}{C-1}, \quad c_4 = -\frac{C^2 - 3C + 3}{(C-1)^2}, \quad \lambda = C - 1, \quad \tilde{\lambda} = \frac{1-C}{C},$$

$$\Rightarrow \qquad T(t,r) = -\int \frac{Cr\dot{\mathbf{a}}}{\Delta\sqrt{1-Kr^2}}\, dr$$

$\Rightarrow$ solution is complete, but only for special c_3, c_4

For $\eta \to 0$ becomes solution of the RGT model.

V. More exotic cosmologies

M.S.V. JHEP 1201 (2012) 035

Diagonal metrics

$$ds^2 = dt^2 - \mathbf{a}^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right), \qquad K = 0, \pm 1$$

$$df^2 = \alpha^2(t)dt^2 - \sigma^2(t)\mathbf{a}^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right).$$

Independent equations are

$$(a) \qquad G^0_0 = m^2 \cos^2\eta\, T^0_0 + \rho, \qquad \mathcal{G}^0_0 = m^2 \sin^2\eta\, \mathcal{T}^0_0,$$

$$(b) \qquad \dot{T}^0_0 + 3\frac{\dot{\mathbf{a}}}{\mathbf{a}}(T^0_0 - T^r_r) = 0, \qquad \dot{\rho} + 3\frac{\dot{\mathbf{a}}}{\mathbf{a}}(\rho + P) = 0$$

Setting $\boxed{\alpha = \mathbf{a}(\sigma\mathbf{a})^{\cdot}/\dot{\mathbf{a}}}$ and $\rho = \rho_0 \mathbf{a}^{-3-3/\gamma}$ (if $P = \gamma\rho$) solves Eqs.(b) and gives $\boxed{G^0_0 = \sigma^2 \mathcal{G}^0_0}$. Eqs.($a$) reduce to

Equations

$$(\ddagger) \qquad \boxed{\dot{\mathbf{a}}^2 + \mathrm{U}(\mathbf{a}) = -K} \qquad \text{with} \qquad \mathrm{U} = -\frac{\mathbf{a}^2}{3}\rho_{\rm tot}$$

$$(\star) \qquad \rho_{\rm tot} = \boxed{m^2\cos^2\eta\, T^0_0 + \rho = m^2\sin^2\eta\,\sigma^2\mathcal{T}^0_0}$$

where

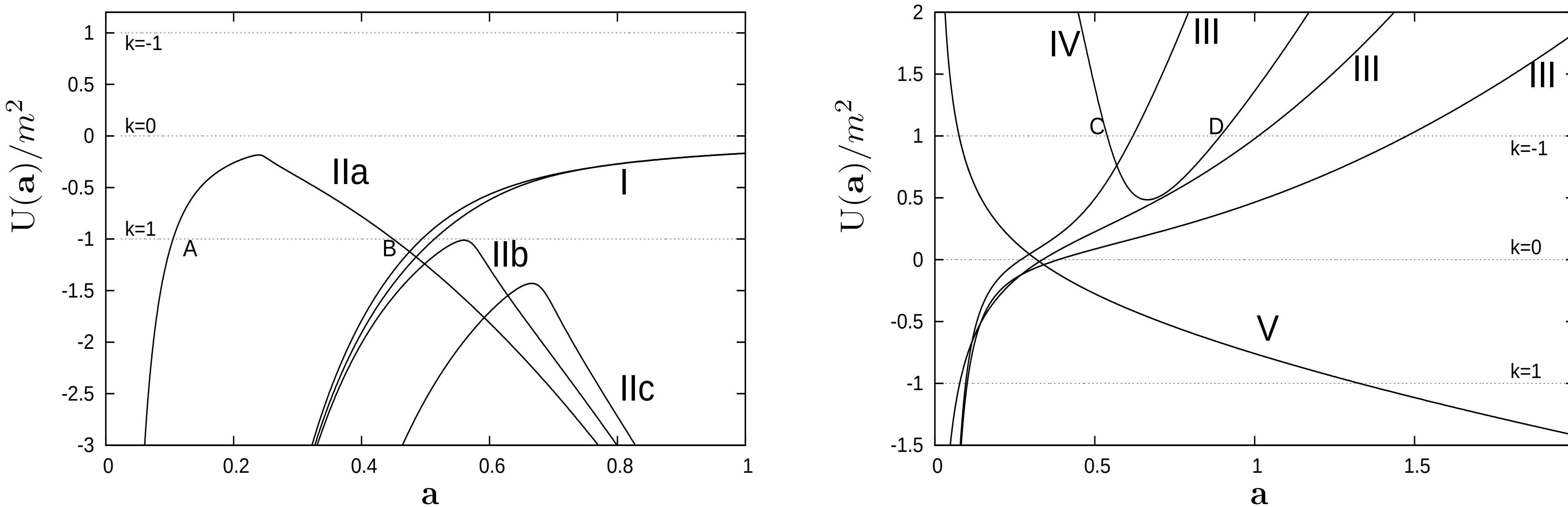
$$T^0_0 = (1-\sigma)((c_3+c_4)\sigma^2 + (3-5c_3-2c_4)\sigma + 4c_3 + c_4 - 6)$$

$$\mathcal{T}^0_0 = \frac{\sigma-1}{\sigma^3}(c_4\sigma^2 - (3c_3+2c_4)\sigma + c_4 + 3c_3 - 3)$$

Eq.$(\star)$ is a 4-th order algebraic equation for $\sigma = \sigma(\rho)$. Since $\rho = \rho(\mathbf{a}) \Rightarrow \sigma(\rho(\mathbf{a})) = \sigma(\mathbf{a}) \Rightarrow \mathrm{U} = \mathrm{U}(\mathbf{a})$.

$\exists$ several roots of $(\star) \Rightarrow$ several solution branches.

Physical and exotic solutions



- physical: total energy $\rho_{\rm tot} = m^2 \cos^2\eta\, T^0_0 + \rho \approx \rho$ as $\mathbf{a} \to 0$.
- exotic: $m^2 \cos^2\eta\, T^0_0 \approx -\rho$, $\rho_{\rm tot} \sim \rho^{2/3}$ can be negative $\Rightarrow$ solutions can be non-singular.
- $m^2 \sin^2\eta\, \mathcal{T}^0_0$ does not vanish for $\eta \to 0 \Rightarrow$ no RGT limit.

VI. Hairy black holes in the bigravity theory

M.S.V. arXiv:1202.6682

Static, diagonal metrics

$$ds^2 = Q^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2, \quad df^2 = a^2 dt^2 - \frac{U'^2}{Y^2} dr^2 - U^2 d\Omega^2$$

Q, N, Y, U, a are 5 functions of r, they fulfill 5 equations

$$\begin{aligned}
G^0_0 &= m^2 \cos^2\eta \, T^0_0, \\
G^r_r &= m^2 \cos^2\eta \, T^r_r, \\
\mathcal{G}^0_0 &= m^2 \sin^2\eta \, \mathcal{T}^0_0, \\
\mathcal{G}^r_r &= m^2 \sin^2\eta \, \mathcal{T}^r_r, \\
T^{r\prime}_r &+ \frac{Q'}{Q}(T^r_r - T^0_0) + \frac{2}{r}(T^\vartheta_\vartheta - T^r_r) = 0.
\end{aligned}$$

Equations

$$\frac{2NN'}{r} + \frac{N^2-1}{r^2} + m^2 \cos^2\eta\,\eta\left(\alpha_1 \frac{N}{Y} U' + \alpha_2\right) + \rho = 0,$$

$$\frac{2N^2Q'}{Qr} + \frac{N^2-1}{r^2} + m^2 \cos^2\eta\left(\alpha_1 \frac{a}{Q} + \alpha_2\right) - P = 0,$$

$$\{Y^2 - 1 + m^2 \sin^2\eta\,\alpha_3\}NU' + 2UNYY' + m^2 \sin^2\eta\,\eta\, Y\alpha_4 = 0,$$

$$\{a(Y^2-1) + m^2 \sin^2\eta\,\alpha_5\}U' + 2UY^2a' = 0,$$

$$\alpha_6 U' + \alpha_7 a' = 0,$$

where $\alpha_1 \ldots \alpha_7$ are

$$
\begin{aligned}
\alpha_1 &= 3 - 3c_3 - c_4 + \frac{2(c_4 + 2c_3 - 1)U}{r} - \frac{(c_4 + c_3)U^2}{r^2}, \\
\alpha_2 &= 4c_3 + c_4 - 6 + \frac{2(3 - c_4 - 3c_3)U}{r} + \frac{(c_4 + 2c_3 - 1)U^2}{r^2}, \\
\alpha_3 &= c_4 U^2 - 2(c_3 + c_4)rU + (c_4 + 2c_3 - 1)r^2, \\
\alpha_4 &= (3 - c_4 - 3c_3)r^2 - (c_4 + c_3)U^2 + (4c_3 + 2c_4 - 2)rU, \\
\alpha_5 &= [(a - Q)c_4 - Qc_3]U^2 + [2(2Q - a)c_3 + (Q - a)c_4 - Q]rU, \\
&+ [(2a - 3Q)c_3 + (a - Q)c_4 + 3Q - a]r^2, \\
\alpha_6 &= Q' N[(3c_3 + c_4 - 3)r^2 + (2(1 - c_4 - 2c_3))Ur + (c_4 + c_3)U^2], \\
&+ 2Q(Y - N)[(3 - c_4 - 3c_3)r + (c_4 + 2c_3 - 1)U], \\
&+ 2a(N - Y)[(1 - c_4 - 2c_3)r + (c_4 + c_3)U], \\
\alpha_7 &= Y[(3 - c_4 - 3c_3)r^2 + 2(c_4 + 2c3 - 1)Ur - (c_4 + c_3)U^2].
\end{aligned}
$$

Background black holes

$$\boxed{f_{\mu\nu} = C^2 g_{\mu\nu}}, \qquad ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2,$$

$$N^2 = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}, \qquad \Lambda = m^2 (C-1)(a_2 C^2 + a_1 C + a_0),$$

where C is a root of

$$(C-1)(b_3 C^3 + b_2 C^2 + b_1 C + b_0) = 0,$$

and a_k, b_s depend on c_3, c_4, η. If $\eta = 1$, $c_3 = 0.1$, $c_4 = 0.3$,

$$\begin{aligned} \{C_1, C_2, C_3, C_4\} &= \{1;\ -0.6458\,;2.6333\,;-8.5566\}, \\ \frac{\Lambda(C_k)}{m^2} &= \{0;\ -3.0559;\ -1.1812;\ +21.5625\}. \end{aligned}$$

$\Rightarrow$ Schwarzschild, SdS, SAdS

U, a backgrounds

$$
\begin{aligned}
N^2 &= 1 + m^2\cos^2\eta((1-2c_3-c_4)U^2 - \frac{2M}{r} \\
&+ (3c_3+c_4-3)Ur + (2-\frac{4}{3}c_3-\frac{1}{3}c_4)r^2), \\
\frac{Q}{N} &= a\frac{m^2\cos^2\eta}{2}\int_{r_1}^{r}\frac{dr}{xN^3}\mathcal{F}, \quad Y = \frac{m^2\sin^2\eta}{2U}\int_{r_2}^{r}\frac{dr}{N}\mathcal{F}, \\
\mathcal{F} &= (c_4-3+3c_3)x^2 + 2(1-2c_3-c_4)Ux + (c_3+c_4)U^2
\end{aligned}
$$

U, a, M, r_1, r_2 constants.

$g_{\mu\nu}$ approaches AdS as $r\to\infty$ in the leading order.

$f_{rr}=0 \Rightarrow f_{\mu\nu}$ is degenerate. If $U\to const$ as $r\to\infty$ then the proper volume is finite – spontaneous compactification.

Reduction of the equations

The 5 field equations contain a closed subsystem

$$
\begin{aligned}
N' &= \mathcal{D}N(r, N, Y, U, m, \eta, c_3, c_4), \\
Y' &= \mathcal{D}Y(r, N, Y, U, m, \eta, c_3, c_4), \\
U' &= \mathcal{D}U(r, N, Y, U, m, \eta, c_3, c_4)
\end{aligned}
$$

When a solution is known, one obtains Q via integrating

$$Q'/Q = \mathcal{F}(r, N, Y, U, m, \eta, c_3, c_4)$$

and a from algebraic relation

$$a/Q = \mathcal{A}(r, N, Y, U, m, \eta, c_3, c_4)$$

$\Rightarrow$ independent variables are N, Y, U.

Event horizon at $r = r_h$

$$N^2 = \sum_{n\geq 1} a_n(r-r_h)^n, \;\; Y^2 = \sum_{n\geq 1} b_n(r-r_h)^n, \;\; U = u r_h + \sum_{n\geq 1} c_n(r-r_h)^n$$

a_n, b_n, c_n depend on one free parameter u (and $\epsilon = \pm 1$).

- Horizon is common for both metrics
- Set of all black holes is one-dimensional and labeled by $u = U(r_h)/r_h$ = ratio of the even horizon radius measured by $f_{\mu\nu}$ to that measured by $g_{\mu\nu}$.
- Using the scaling symmetry $r \to \lambda r$, $N \to N$, $Y \to Y$, $U \to U/\lambda$, $m \to m/\lambda$ one can set $\boxed{r_h = 1}$.

Horizon temperatures

$$g_{00} = Q^2 = q^2\{r - r_h + \sum_{n\geq 2} c_n (r-r_h)^n\}, \qquad f_{00} = a^2 = q^2 \sum_{n\geq 1} d_n (r-r_h)$$

ξ – timelike Killing. Surface gravities $(T = \kappa/2\pi)$

$$\kappa_g^2 = -\frac{1}{2} g^{\mu\alpha} g_{\nu\beta} \overset{(g)}{\nabla}_\mu \xi^\nu \overset{(g)}{\nabla}_\alpha \xi^\beta = \lim_{r\to r_h} Q^2 N'^2 = \frac{1}{4} q^2 a_1 ,$$

$$\kappa_f^2 = -\frac{1}{2} f^{\mu\alpha} f_{\nu\beta} \overset{(f)}{\nabla}_\mu \xi^\nu \overset{(f)}{\nabla}_\alpha \xi^\beta = \lim_{r\to r_h} a^2 \left(\frac{Y}{U'}\right)'^2 = \frac{1}{4} q^2 \frac{d_1 b_1}{(c_1)^2} .$$

$$\boxed{\frac{\kappa_g^2}{\kappa_f^2} = \frac{T_g^2}{T_f^2} = \frac{a_1 (c_1)^2}{d_1 b_1} = 1}$$

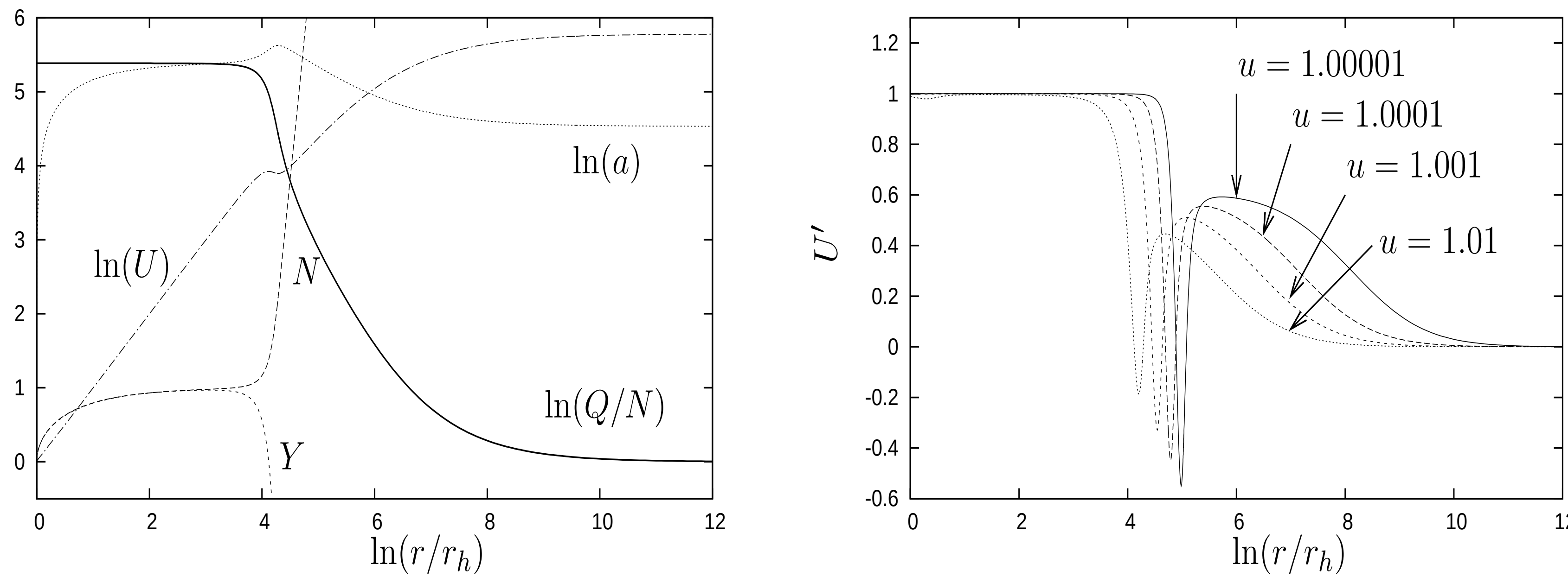
Deffayet, Jackobson '11

Strategy

- Solutions are obtained by integrating from the horizon for a given value of $u = U(r_h)$ towards large r.
- For $u = C_k$ they are the background black holes.
- For $u = C_k + \delta u$ they describe hairy deformations of the background black holes.

For $u = 1 + \delta u$ they describe hairy deformations of the Schwarzschild black hole.

Deforming Schwarzschild



- Close to Schwarzschild for $r < r_{\max}(u)$ but approaches U, a for $r \to \infty$.Deformations stay small close to horizon but are always large at infinity.
- $U' = 1$ if $u = 1$ but if $u > 1$ then $U' \to 0$ for $r \to \infty$
- Point-vice (non-uniform) convergence to Schwarzschild as $u \to 1$.

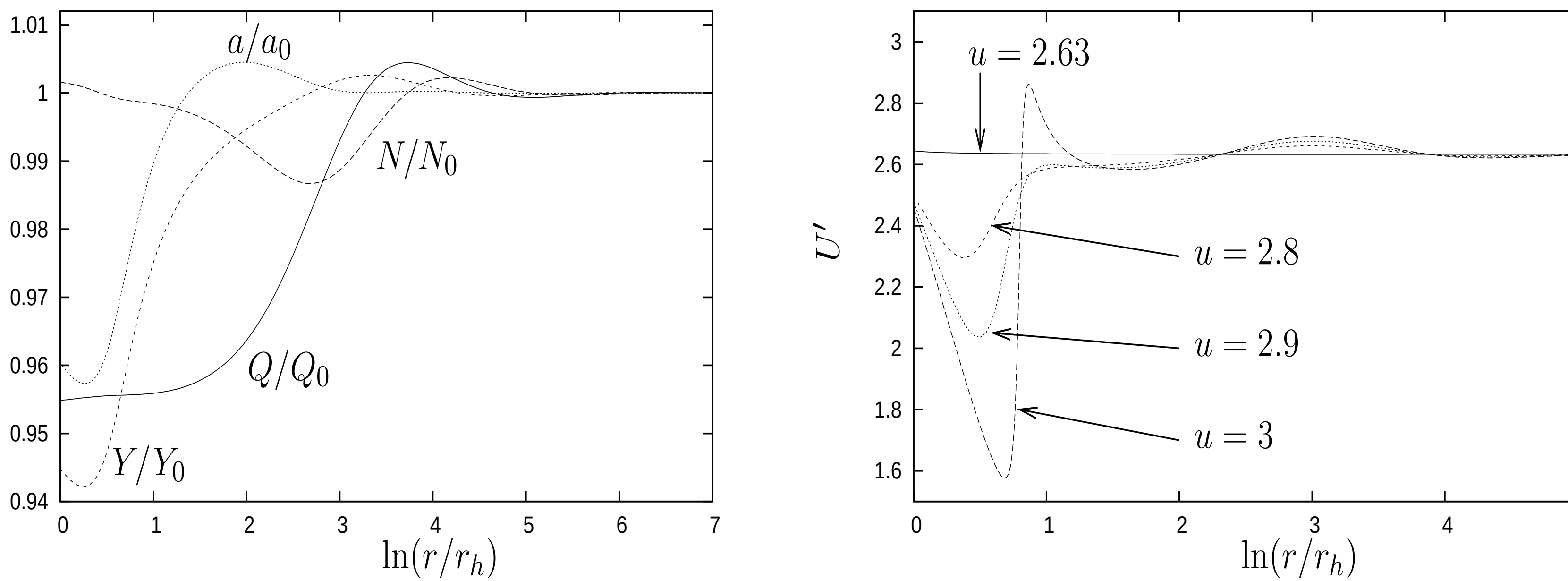
If it was asymptotically flat

$$
\begin{aligned}
N &= 1 - \frac{A \sin^2 \eta}{r} + B \cos^2 \eta \, \frac{mr + 1}{r} \, e^{-mr}, \\
Y &= 1 - \frac{A \sin^2 \eta}{r} - B \sin^2 \eta \, \frac{1 + mr}{r} \, e^{-mr} \\
U &= r + B \, \frac{m^2 r^2 + mr + 1}{m^2 r^2} \, e^{-mr},
\end{aligned}
$$

A, B two constants – Newton+VdVZ.
If u is fixed, not enough free parameters to fulfill tree matching conditions $\Rightarrow$ one cannot vary u continuously $\Rightarrow$ there can be no continuous asymptotically flat hairy deformations of Schwarzschild.
no ghost mode – less derivatives than in the AGS model
Isolated disjoint solutions are not yet excluded.

Deforming Schwarzschild-AdS

$u = C_k + \delta u$ $(k = 2, 3)$, deformations stay close to the horizon



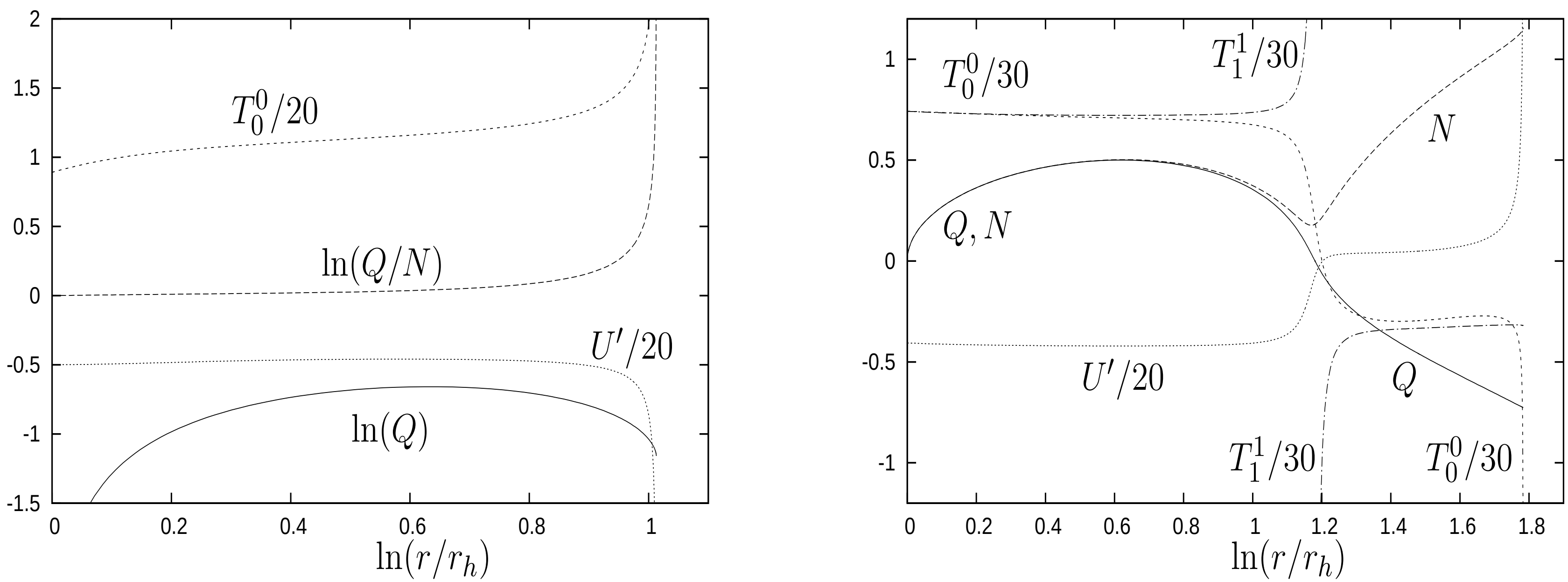
N_0, Q_0, Y_0, a_0 correspond to the background AdS. At large r there are 3 free parameters – enough for matching,

$$N/N_0 = A\sin^2\eta/r^3 + O(\delta U), \quad Y/Y_0 = A\sin^2\eta/r^3 + O(\delta U),$$

$$U/U_0 = B_1 e^{\lambda_1 r} + B_2 e^{\lambda_2 r}, \quad \Re(\lambda_1) < 0, \quad \Re(\lambda_2) < 0.$$

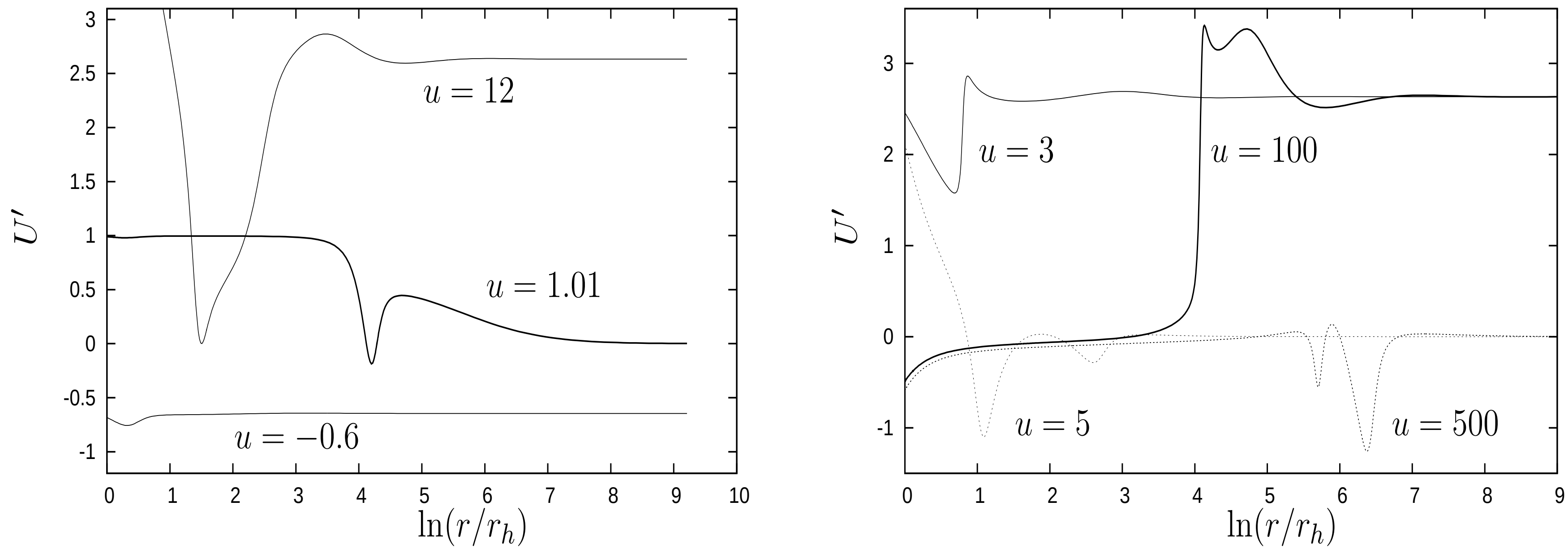
Deforming Schwarzschild-dS

$u = C_4 + \delta u$ with $\delta u < 0$ (left) and $\delta u > 0$ (right).



Deformations become singular at a finite distance from the horizon – solutions are compact 'bags of gold'.

Generic solutions – arbitrary u



U' tends either to zero or to the two AdS values. Solutions approach either AdS or U, a or they are ‘bags of gold’ – no new types of behaviour $\forall\ c_3, c_4, \eta > 0$.

The only asymptotically flat is pure Schwarzschild.

The only asymptotically dS is pure dS.

Special solutions for $\eta = 0$

$\mathcal{G}^{\mu}_{\nu} = 0 \Rightarrow f_{\mu\nu}$ is Ricci-flat, but it cannot be flat, since it has to have a horizon $\Rightarrow$ it is fixed and Schwarzschild

$$df^2 = Y^2(U)dt^2 - \frac{dU^2}{Y^2(U)} - U^2 d\Omega^2 \quad \text{with} \quad Y(U) = \sqrt{1 - \frac{u}{U}}$$

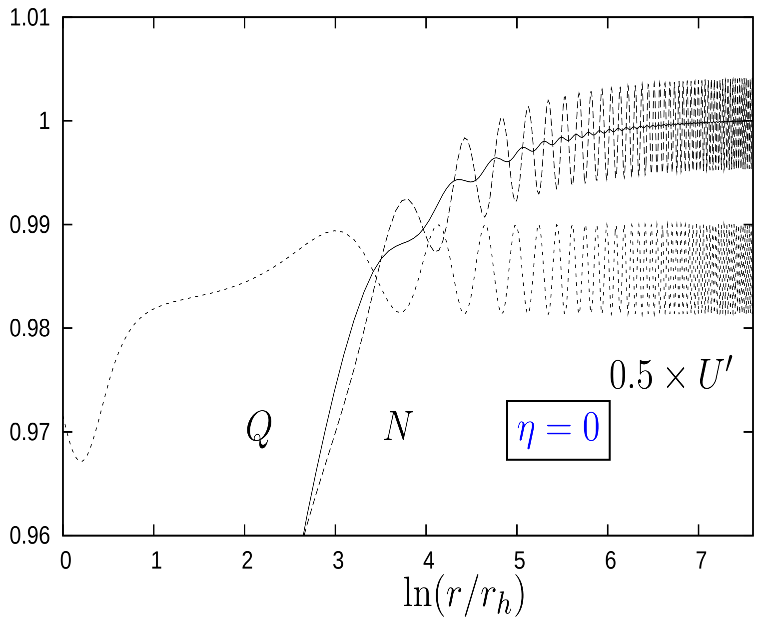
There rests to determine $U(r)$ and

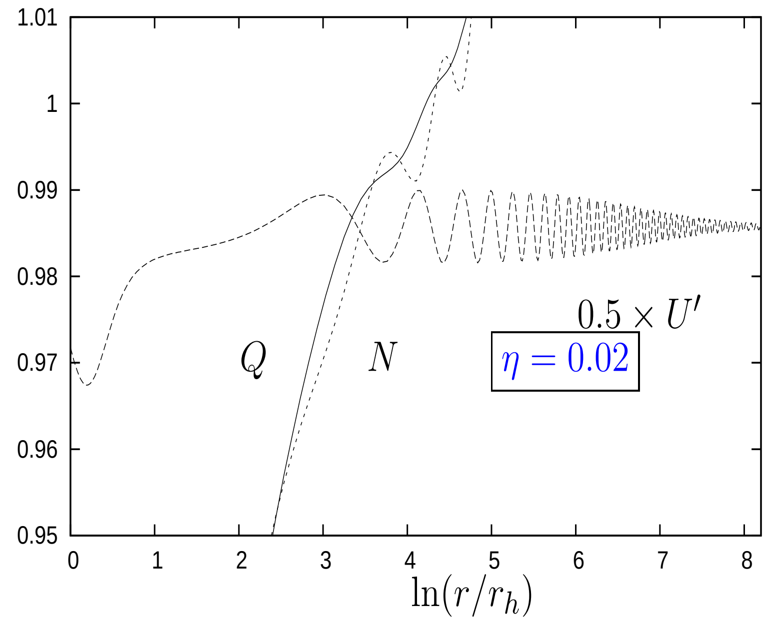
$$g_{\mu\nu}dx^\mu dx^\nu = Q^2(r)dt^2 - \frac{dr^2}{N^2(r)} - r^2 d\Omega^2$$

u is the free horizon parameter. $u = 1 \Rightarrow$ Schwarzschild $g_{\mu\nu} = f_{\mu\nu}$. For $u = C_\pm \Rightarrow$ new special Schwarzschilds

$$\boxed{g_{\mu\nu} = \frac{f_{\mu\nu}}{C_\pm^2}} \quad C_\pm = \frac{1}{2(c_3 + c_4)}\left(2c_4 + 5c_3 - 3 \pm \sqrt{12c_4 + 9(c_3 - 1)^2}\right)$$

Deforming special Schwarzschild





Tachyonic oscillations around flat metric at infinity

$$N = 1 + \delta N, \quad Q = 1 + \delta Q/r, \quad U = x + \delta U$$

$$\delta N \sim \delta Q \sim \delta U = \exp\{i\sqrt{2}m(r + \frac{1}{2}\ln(r))\}$$

VII. Lumps of pure gravity

M.S.V. arXiv:1202.6682

Regular center

No horizon. At small r one has

$$N = 1 + \left(m^2 \cos^2\eta\,(1 - \frac{3}{2}u + \frac{1}{2}u^2)\right) r^2 + O(r^4),$$

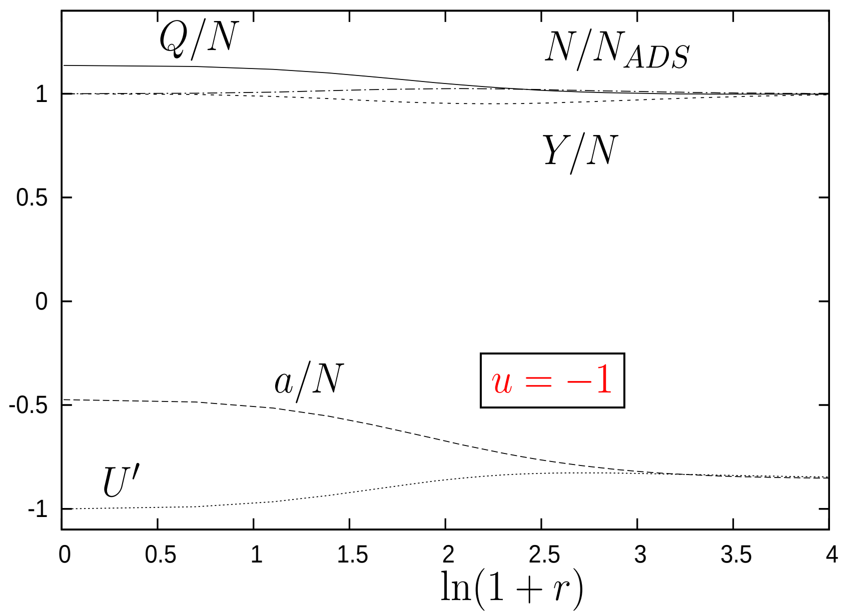
$$Y = 1 + m^2 \sin^2\eta\,\frac{u-1}{2u}\,x^2 + O(r^4), \qquad U = ur + O(r^3)$$

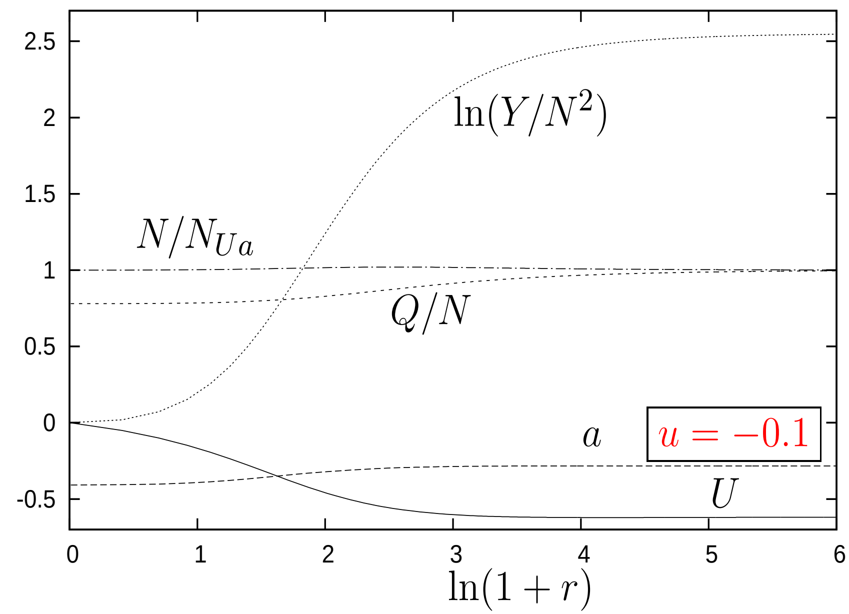
where $u = U'(0)$ is a free parameter $\Rightarrow$ the set of all solutions is one parametric, as in the black hole case.

One integrates the equations starting at $r = 0$ towards large r. Solutions has a regular core, while for large r they show the same behaviour as the black holes – asymptotically either AdS, or U, a, or singular at finite r.

Globally regular solutions – lumps

Asymptotically AdS (left) and asymptotically U, a (right)





The same asymptotic behaviour $\Rightarrow$ lumps can be viewed remnants of hairy black holes in the limit $r_h \to 0$.

VIII. Asymptotically flat stars and Vainstein mechanism

M.S.V. arXiv:1202.6682

Field equations

One adds $T^{(\mathrm{mat})\mu}_{\ \nu} = \mathrm{diag}(\rho, -P, -P, -P), \quad \rho(r) = \rho_\star(r - r_\star)$

One has

$$ds^2 = Q^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2, \quad df^2 = a^2 dt^2 - \frac{U'^2}{Y^2} dr^2 - U^2 d\Omega^2$$

and 6 equations for 6 functions of r: Q, N, Y, U, a, P

$$G^0_0 = m^2 \cos^2\eta\, T^0_0 + \rho, \quad G^r_r = m^2 \cos^2\eta\, T^r_r - P,$$

$$\mathcal{G}^0_0 = m^2 \sin^2\eta\, \mathcal{T}^0_0, \quad \mathcal{G}^r_r = m^2 \sin^2\eta\, \mathcal{T}^r_r,$$

$$T^{r\prime}_r + \frac{Q'}{Q}(T^r_r - T^0_0) + \frac{2}{r}(T^\vartheta_\vartheta - T^r_r) = 0,$$

$$P' = \frac{Q'}{Q}(\rho + P)$$

Regular center

At small r

$$N = 1 + \left(m^2 \cos^2 \eta \, (1 - \frac{3}{2} u + \frac{1}{2} u^2) - \frac{\rho_\star}{6} \right) r^2 + O(r^4),$$

$$Y = 1 + m^2 \sin^2 \eta \, \frac{u-1}{2u} x^2 + O(r^4),$$

$$U = ur + O(r^3)$$

$$a = a_0 + O(r^2),$$

$$P = P_0 + O(r^2),$$

$$Q = Q_0 + O(r^2)$$

where u, a_0, P_0, Q_0 are free parameters.

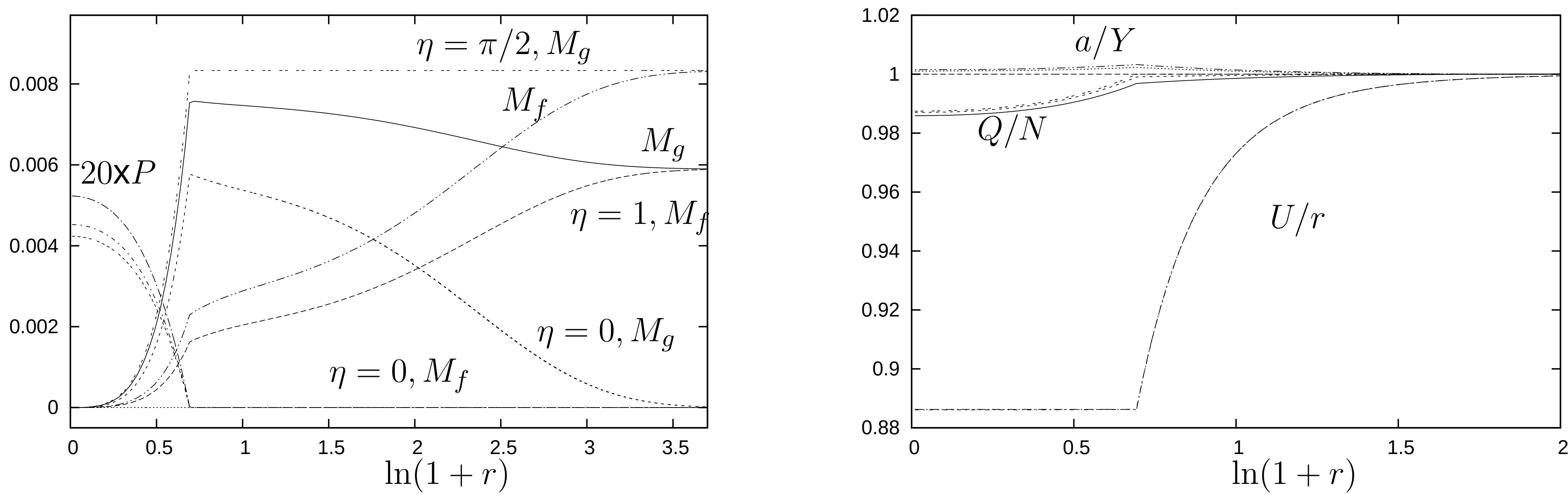
One should have $P(r) = 0$ for $r \geq r_\star$.

Asymptotically flat infinity

$$
\begin{aligned}
N &= 1 - \frac{A\sin^2\eta}{r} + B\cos^2\eta\,\frac{mr+1}{r}\,e^{-mr}, \\
Y &= 1 - \frac{A\sin^2\eta}{r} - B\sin^2\eta\,\frac{1+mr}{r}\,e^{-mr} \\
U &= r + B\,\frac{m^2r^2+mr+1}{m^2r^2}\,e^{-mr}, \\
Q &= 1 - \frac{A\sin^2\eta}{r} + \frac{2B\cos^2\eta}{r}\,e^{-mr}, \\
a &= 1 - \frac{A\sin^2\eta}{r} - \frac{2B\sin^2\eta}{r}\,e^{-mr}.
\end{aligned}
$$

A, B are 2 integration constants $\Rightarrow$ there are altogether $4+2=6 \Rightarrow$ enough for matching.

Solutions – Vainstein mechanism



$$g^{rr}=N^2=1-2M_g/r,\qquad f^{rr}=Y^2/U'^2=1-2M_f/r$$

$$(M_g)'=\frac{r^2}{2}(m^2\cos^2\eta\, T^0_0+\rho),\qquad (M_f)'=U'\,\frac{U^2}{2}\,m^2\sin^2\eta\,\mathcal{T}^0_0.$$

$0\leftarrow M_g, M_f\rightarrow A\sin^2\eta.$ If $m\rightarrow 0$ then $M_g\approx const\Rightarrow$ Vainstein

Summary

- We have studied black holes and cosmologies in the ghost-free bimetric massive gravity theory.
- For non-simultaneously diagonal metrics there are 'standard' Schwarzschild-dS black holes and self accelerating cosmologies.
- There are more exotic cosmological solutions for which the graviton contribution to the energy can be large and negative. They can be non-singular at $t = 0$.
- The theory admits also static hairy black holes of several types (AdS, Ua, compact). They are not asymptotically flat (apart from pure Schwarzschild) and reduce to lumps of pure gravity when $r_h \to 0$.
- There are also static asymptotically flat solutions with matter (stars) exhibiting the Vainstein mechanism.

Plan

- Massive gravity in D=4
- Checking Vainstein scenario
- Ghost-free theories
- Black holes and self-accelerating cosmologies
- More exotic cosmologies
- Hairy black holes
- Lumps of pure gravity
- Asymptotically flat stars and Vainstein mechanism