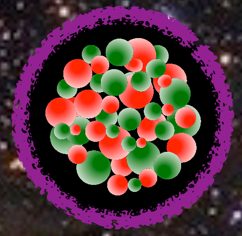
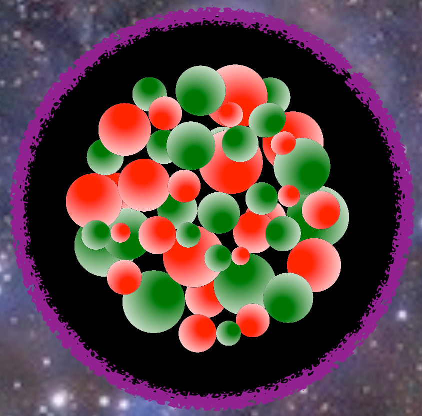
Resolving the Structure of Black Holes





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Recent work with:

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Outline

- Motivation: Solitons and Microstate Geometries
- Smarr Formula: "No Solitons without Horizons"
- Topological stabilization: "No Solitons without *Topology*"
- Microstates and fluctuations of microstate geometries
- New scales in microstate geometries/black holes
- Conclusions

Solitons versus Particles

Electromagnetism:

- Divergent self-energy of point particles ...
- Self-consistency/Completeness: Motion of particles should follow from action of electromagnetism ...

★ *Replace point sources by smooth "lumps" of classical fields*

$\Rightarrow$ Mie, Born-Infeld: Non-linear electrodynamics

Yang-Mills

- Non-abelian monopoles and Instantons

General Relativity

Non-linearities $\Rightarrow$ new classes of solitons?

Four dimensional GR, electromagnetism + asymptotically flat:

"No Solitons without horizons"

Nearest thing: Extreme, supersymmetric multi-black-hole solutions

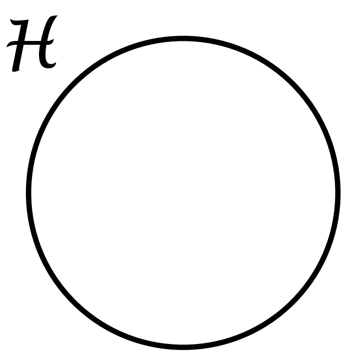
Hawking Radiation versus Unitary Evolution of Black Holes

Black hole uniqueness $\Rightarrow$ Universality of Hawking Radiation

Independent of details and states of matter that made the black hole

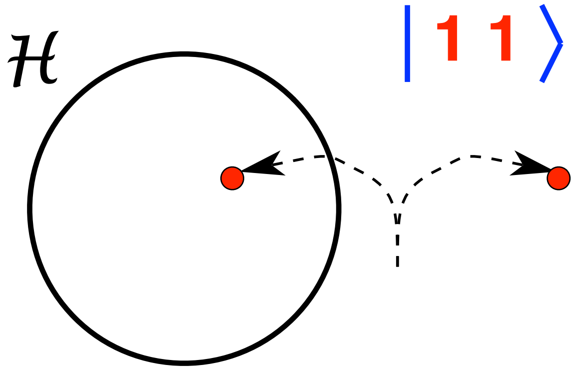
Complete evaporation of the black hole $\Rightarrow$

Loss of information about the states of matter that made the black hole



$|\,0\,0\,\rangle$

$|\,1\,1\,\rangle$



Entangled State of Hawking Radiation

$$\frac{1}{\sqrt{2}}\left(|\,0\,0\,\rangle \;+\; |\,1\,1\,\rangle\right)$$

Evaporation of the black hole:

Sum over internal states $\Rightarrow$

Pure state $\rightarrow$ Density matrix

Entanglement of N Hawking quanta with internal black hole state $= N \ln 2$

Complete evaporation + Entanglement $\Rightarrow$

Hawking radiation cannot be described by a simple wave function

Tension of Black hole uniqueness and Unitarity of Quantum Mechanics

Fix with small corrections to GR?

Entangled State of Hawking Radiation

$$\frac{1}{\sqrt{2}}\left(|\,0\,0\rangle + |\,1\,1\rangle\right) + \varepsilon\left(|\,0\,0\rangle - |\,1\,1\rangle\right) + \left(\varepsilon_1\,|\,0\,1\rangle + \varepsilon_2\,|\,1\,0\rangle\right)$$

Restore the pure state over vast time period for evaporation?

Mathur (2009): Corrections cannot be small for information recovery

⇒ There must be *O(1)* to the Hawking states at the horizon.

New physics at the horizon scale?

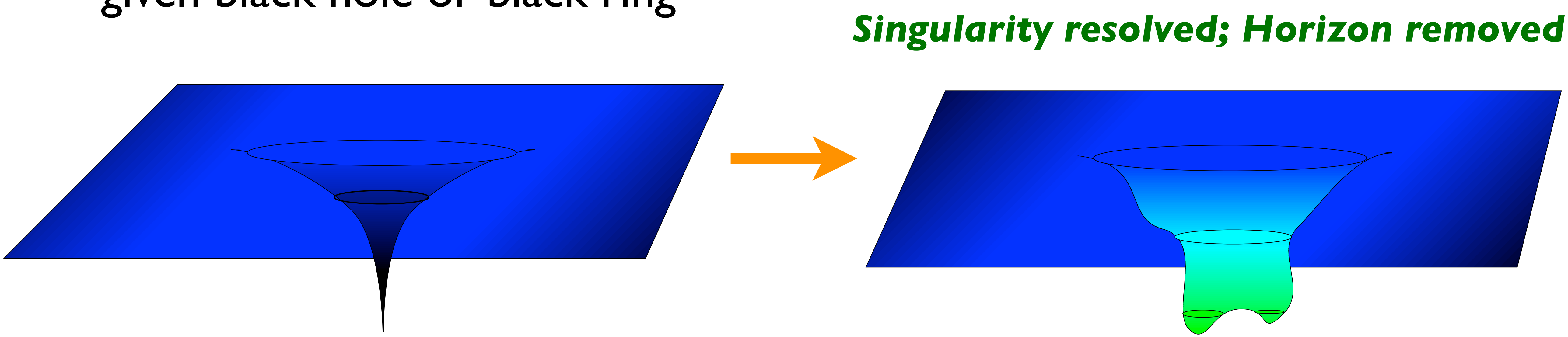
- Is there a way to avoid black holes and horizons in the low energy (massless) limit of string theory = supergravity?
- Can it be done in a manner that looks like a black hole on large scales in four dimensions?

Are there horizonless solitons?

Microstate Geometries:

Definition

- Solution to the bosonic sector of supergravity as a low energy limit of string theory
- Smooth, horizonless solutions with the same asymptotic structure as a given black hole or black ring



Simplifying assumption:

- Time independent metric (stationary) *and time independent matter*

Smooth, stable, end-states of stars in massless bosonic sector of string theory?

This is supposed to be impossible because of many no-go theorems:

"No Solitons without horizons"

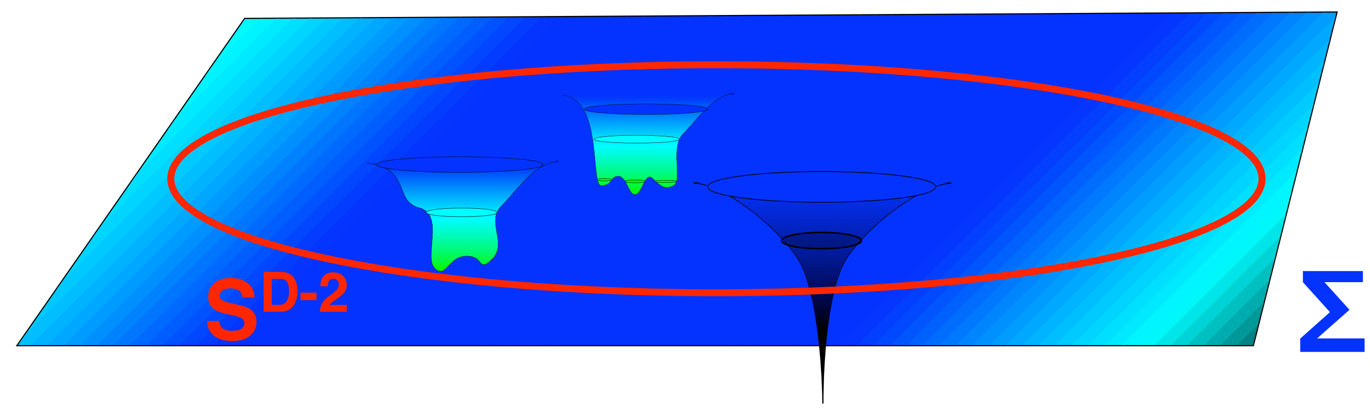
Intuition: *Massless fields travel at the speed of light ... only a black hole can hold such things into a star.*

The Komar Mass Formula

In a D-dimensional space-time with a Killing vector, K, that is time-like at infinity one has

$$M = -\frac{1}{16\pi G_D}\frac{(D-2)}{(D-3)}\int_{S^{D-2}} *dK \qquad K = \frac{\partial}{\partial t}$$

where S^{D-2} is (topologically) a sphere near spatial infinity in some hypersurface, Σ.



$$g_{00} = -1 + \frac{16\pi G_D}{(D-2)\,A_{D-2}}\frac{M}{\rho^{D-3}} + \dots \qquad *dK \approx -(\partial_\rho g_{00}) * (dt \wedge d\rho)$$

More significantly

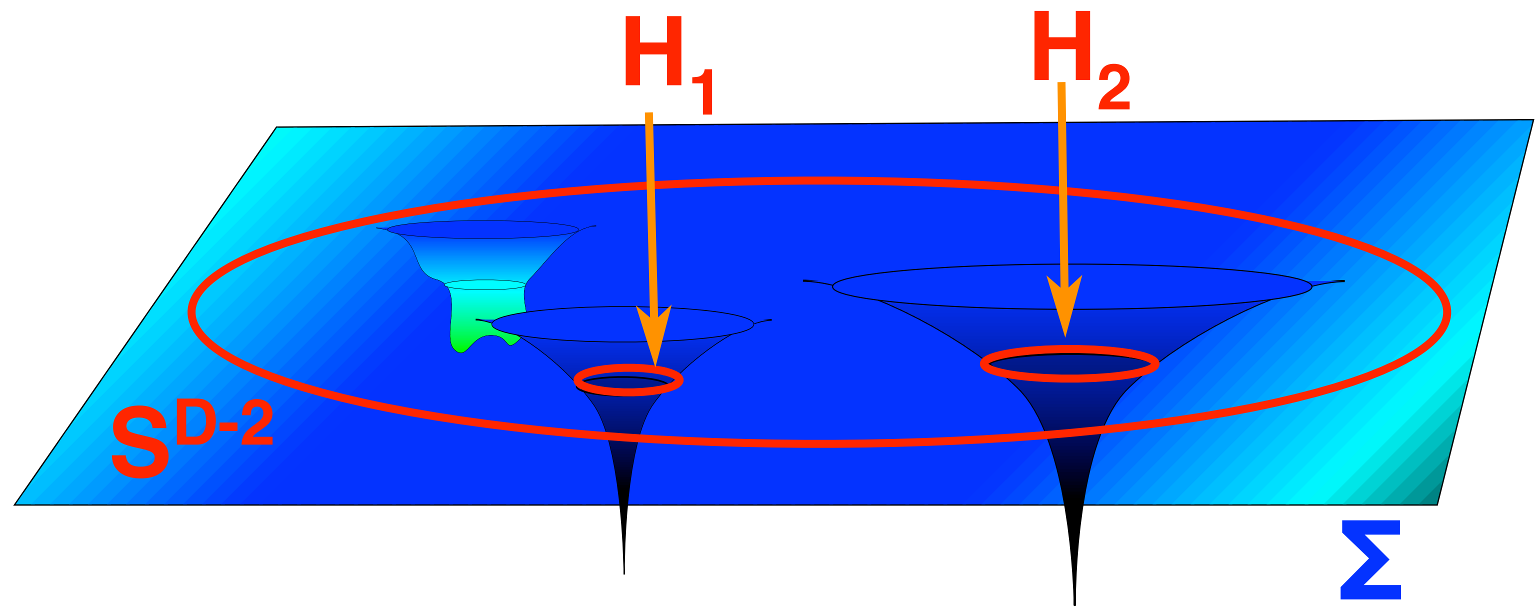
$$d * dK = -2 * (K^\mu R_{\mu\nu} dx^\nu) \qquad R_{\mu\nu} = 8\pi G_D \Big(T_{\mu\nu} - \frac{1}{(D-2)} T\, g_{\mu\nu}\Big)$$

If Σ is smooth with *no interior boundaries*:

$$M = \frac{1}{8\pi G_D}\frac{(D-2)}{(D-3)}\int_\Sigma K^\mu R_{\mu\nu}\, d\Sigma^\nu \xrightarrow{\textbf{linearized}} \approx \int_\Sigma T_{00}\, d\Sigma^0$$

Smarr Formula I

More generally, **Σ** will have *interior boundaries* that can be located at horizons, **H_J**.



Excise horizon interiors: $\Sigma \to \widetilde{\Sigma}$

$$\frac{8\pi G_D\,(D-3)}{(D-2)}\,M \;=\; \int_{\widetilde{\Sigma}} \cancelto{0}{R_{\mu\nu} K^{\mu}}\, d\Sigma^{\nu} \;+\; \tfrac{1}{2}\sum_{H_J}\int_{H_J} *dK$$

Null generators of Kerr-like horizons: $\xi \;=\; K \;+\; \vec{\Omega}_H \cdot \vec{L}_H$

Surface gravity of horizon, κ $\xi^a \nabla_a \xi^b \;=\; \kappa\, \xi^b \quad \Rightarrow \quad \tfrac{1}{2}\int_H *d\xi \;=\; \kappa\, \mathcal{A}$

$$\Rightarrow \quad \tfrac{1}{2}\sum_{H_J}\int_{H_J} *dK \;=\; \sum_{H_I}\left[\kappa_{H_I}\,\mathcal{A}_{H_I} \;+\; 8\pi G_D\,\vec{\Omega}_{H_I}\cdot \vec{J}_{H_I}\right]$$

Vacuum outside horizons:

$$\frac{8\pi G_D\,(D-3)}{(D-2)}\,M \;=\; \sum_{H_I}\left[\kappa_{H_I}\,\mathcal{A}_{\mathcal{H}_\mathcal{I}} \;+\; 8\pi G_D\,\vec{\Omega}_{H_I}\cdot \vec{J}_{H_I}\right]$$

Smarr Formula II: No Solitons Without Horizons

If Σ is smooth with *no interior boundaries*:

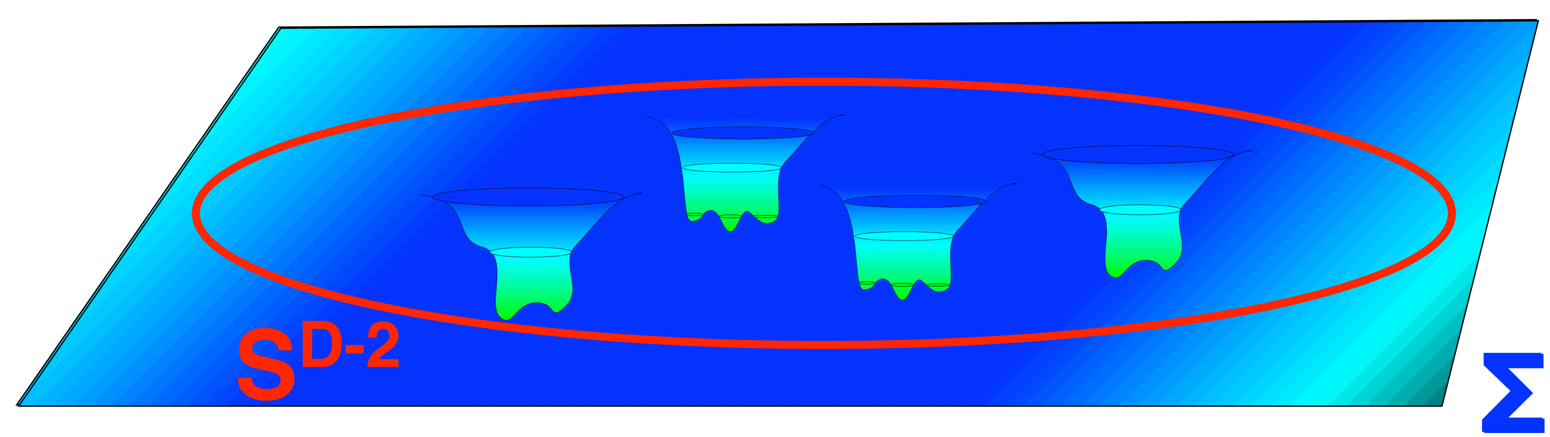
$$\frac{8\pi G_D\,(D-3)}{(D-2)}\,M \;=\; \int_\Sigma R_{\mu\nu} K^\mu \, d\Sigma^\nu$$

Goal: Show that $\int_\Sigma R_{\mu\nu} K^\mu \, d\Sigma^\nu$ = Boundary term

(with no contribution at infinity)

✦ *Not* true for *massive* fields … but (almost) true for *massless* fields

If Σ is a smooth space-like hypersurface populated only by smooth solitons (no horizons) the one must have:



$$M \equiv 0$$

Positive mass theorems with asymptotically flatness:

$\Rightarrow$ *Space-time can only be globally flat,* $\mathbf{R}^{4,1}$

$\Rightarrow$ "No Solitons Without Horizons" ....

It all comes down to:

$$M = \frac{1}{8\pi G_D} \frac{(D-2)}{(D-3)} \int_{\Sigma} *_D \left(K^\mu R_{\mu\nu} dx^\nu\right)$$

and "No solitons without horizons" requires showing that

$$*(K^\mu R_{\mu\nu} dx^\nu) = d(\gamma_{D-2})$$

for some global (D-2)-form, **γ**.

Bosonic sector of a generic massless supergravity

- Graviton, $g_{\mu\nu}$
- Scalars, Φ^A
- Tensor gauge fields, $F_{(p)}{}^K$

Scalar matrices in kinetic terms: $Q_{JK}(\Phi)$, $M_{AB}(\Phi)$

Bianchi: $d(F_{(p)}{}^K) = 0$

Equations of motion: $d{*}(Q_{JK}(\Phi)\, F_{(p)}{}^K) = 0$

Define: $G_{J,(D-p)} \equiv {*}\left(Q_{JK}(\Phi)\, F_{(p)}{}^K\right)$ and Q^{JK} by $Q^{IK} Q_{KJ} = \delta^I{}_J$

then: $d(F_{(p)}{}^K) = 0$ and $d(G_{J,(D-p)}) = 0$

Einstein equations:

$$
\begin{aligned}
R_{\mu\nu} &= Q_{IJ}\left[F^I{}_{\mu\rho_1\ldots\rho_{p-1}}\, F^J{}_\nu{}^{\rho_1\ldots\rho_{p-1}} - c\, g_{\mu\nu}\, F^I{}_{\rho_1\ldots\rho_p} F^{J\,\rho_1\ldots\rho_p}\right] \\
&\quad + M_{AB}\left[\partial_\mu\Phi^A\,\partial_\nu\Phi^B\right] \\
&= a\, Q_{IJ}\, F^I{}_{\mu\rho_1\ldots\rho_{p-1}}\, F^J{}_\nu{}^{\rho_1\ldots\rho_{p-1}} + M_{AB}\left[\partial_\mu\Phi^A\,\partial_\nu\Phi^B\right] \\
&\quad + b\, Q^{IJ}\, G_{I\,\mu\rho_1\ldots\rho_{D-p-1}}\, G_{J\nu}{}^{\rho_1\ldots\rho_{D-p-1}}
\end{aligned}
$$

for some constants a,b and c

Time Independent Solutions

Killing vector, K, is time-like at infinity

Assume time-independent matter: $\mathcal{L}_K F^I = 0\,, \quad \mathcal{L}_K \Phi^A = 0$

$$\Rightarrow \quad \mathcal{L}_K G_I = 0$$

$$K^\mu R_{\mu\nu} = a\, Q_{IJ}\, K^\mu\, F^I{}_{\mu\rho_1\dots\rho_{p-1}}\, F^J{}_\nu{}^{\rho_1\dots\rho_{p-1}} + M_{AB} \cancelto{0}{\left[K^\mu \partial_\mu \Phi^A \partial_\nu \Phi^B\right]}$$
$$+\, b\, Q^{IJ}\, K^\mu\, G_{I\,\mu\rho_1\dots\rho_{D-p-1}}\, G_{J\nu}{}^{\rho_1\dots\rho_{D-p-1}}$$

- $\mathcal{L}_K \Phi^A = 0 \;\Leftrightarrow\; K^\mu \partial_\mu \Phi^A = 0 \;\Rightarrow$ Scalars drop out of $R_{\mu\nu} K^\mu$
- Cartan formula for forms: $\cancelto{0}{\mathcal{L}_K \omega} = d(i_K(\omega)) + \cancelto{0}{i_K(d\omega)}$

  $d(F_{(p)}{}^I) = 0,\; d(G_{J,(D-p)}) = 0 \;\Rightarrow\; d(i_K(F_{(p)}{}^I)) = 0,\; d(i_K(G_{J,(D-p)})) = 0$
- Ignore topology: $i_K(F_{(p)}{}^I) = d\alpha_{(p-2)}{}^I\,, \quad i_K(G_{J,(D-p)}) = d\beta_{J,(D-p-2)}$
- Define (D-2)-form, $\gamma_{D-2} = a\, \alpha_{(p-2)}{}^J \wedge G_{J,(D-p)} + b\, \beta_{J,(D-p-2)} \wedge F_{(p)}{}^J$

  Then: $*(K^\mu R_{\mu\nu} dx^\nu) = d(\gamma_{D-2})$

$\Rightarrow$ **M = 0** $\Rightarrow$ "No Solitons Without Horizons" ....

Omissions:

- Topology
- Chern-Simons terms

Equations of motion in generic massless supergravity:

$$d*(Q_{JK}(\Phi)\ F_{(p)}{}^{K}) = \textit{Chern-Simons terms}$$

$$\Rightarrow\ d(G_{J,(D-p)}) = \textit{Chern-Simons terms}$$

$$\Rightarrow\quad *(K^{\mu}R_{\mu\nu}dx^{\nu}) \quad = \quad d(\gamma_{D-2}) \quad + \textit{Chern-Simons terms}$$

$$\Rightarrow\ \mathbf{M}\ \sim\ \textit{Topological contributions} + \textit{Chern-Simons terms}$$

Five Dimensional Supergravity

N=2 Supergravity coupled to two vector multiplets

Three Maxwell Fields, F^{I}, two scalars, X^{I}, $X^{1}X^{2}X^{3}$ = 1

$$
\begin{aligned}
S &= \int \sqrt{-g}\, d^5x \Big(R - \tfrac{1}{2} Q_{IJ} F^{I}_{\mu\nu} F^{J\mu\nu} - Q_{IJ}\partial_\mu X^I \partial^\mu X^J - \tfrac{1}{24} C_{IJK} F^{I}_{\mu\nu} F^{J}_{\rho\sigma} A^{K}_{\lambda}\, \bar{\epsilon}^{\mu\nu\rho\sigma\lambda} \Big) \\
Q_{IJ} &= \frac{1}{2}\, \mathrm{diag}\left((X^1)^{-2}, (X^2)^{-2}, (X^3)^{-2} \right)
\end{aligned}
$$

Einstein Equations:

$$
R_{\mu\nu} = Q_{IJ} \Big[F^{I}{}_{\mu\rho}\, F^{J}{}_{\nu}{}^{\rho} - \tfrac{1}{6}\, g_{\mu\nu}\, F^{I}{}_{\rho\sigma} F^{J\,\rho\sigma} + \partial_\mu X^I\, \partial_\nu X^J \Big]
$$

Generic stationary metric: $$ds_5^2 = -\, Z^{-2}\, (dt + k)^2 \;+\; Z\, ds_4^2$$

Four-dimensional spatial base slices, **Σ**:

- Assume simply connected
- Topology of interest: $H_2(\Sigma, \mathbb{Z}) \neq 0$

Simple connectivity

$$d(i_K(F^I)) = 0 \quad \Rightarrow \quad i_K(F^I) = d\lambda^I$$ for some functions, λ^I

$$K^\mu(Q_{IJ}\, F^I{}_{\mu\rho}\, F^J{}_\nu{}^\rho) = -\nabla_\rho(Q_{IJ}\,\lambda^I\, F^{J\rho\nu}) + \tfrac{1}{16}\, C_{IJK}\,\epsilon^{\nu\alpha\beta\gamma\delta}\,\lambda^I\, F^J{}_{\alpha\beta}\, F^K{}_{\gamma\delta}$$

= Boundary term + Chern-Simons contribution

Dual 3-forms: $G_I = *_5\, Q_{IJ}\, F^J$ Cartan: $d(i_K(G_I)) = -\,i_K(d(G_I))$

$$d(G_I) = d*(Q_{IJ}\, F^J) \sim C_{IJK}\, F^J \wedge F^K \quad \neq 0$$

Use $i_K F^J = d\lambda^J \quad \Rightarrow \quad i_K(C_{ILM}\, F^L \wedge F^M) \sim C_{ILM}\, d(\lambda^L F^M)$

Therefore $d\Big(i_K(G_I) + \tfrac{1}{2}\, C_{IJK}\,\lambda^J\, F^K\Big) = 0$

$$\Rightarrow \quad i_K(G_I) = d\beta_I - \tfrac{1}{2}\, C_{IJK}\,\lambda^J F^K + H_I$$

where β_I are global one-forms and H_I are *closed* but *not exact* two forms ...

$$K^\mu(Q^{IJ}\, G_{I\,\mu\rho\sigma}\, G_J{}^{\nu\rho\sigma}) = -2\,\nabla_\rho(Q^{IJ}\,\beta_{I\,\sigma}\, G_J{}^{\rho\nu\sigma}) - \tfrac{1}{4}\, C_{IJK}\,\epsilon^{\nu\alpha\beta\gamma\delta}\,\lambda^I\, F^J{}_{\alpha\beta}\, F^K{}_{\gamma\delta}$$

$$+ \boxed{Q^{IJ}\, H_I^{\rho\sigma}\, G_{J\,\rho\sigma\nu}}$$

Generalized Smarr Formula

$$R_{\mu\nu} = Q_{IJ}\left[\tfrac{2}{3}\, F^I{}_{\mu\rho}\, F^J{}_\nu{}^\rho + \partial_\mu X^I\, \partial_\nu X^J\right] + \tfrac{1}{6}\, Q^{IJ}\, G_{I\,\mu\rho\sigma}\, G_{J\,\nu}{}^{\rho\sigma}$$

$$K^\mu \partial_\mu X^I = 0$$

$$K^\mu\big(Q_{IJ}\, F^I{}_{\mu\rho}\, F^J{}_\nu{}^\rho\big) = -\nabla_\rho\big(Q_{IJ}\,\lambda^I\, F^{J\rho\nu}\big) + \tfrac{1}{16}\, C_{IJK}\,\epsilon^{\nu\alpha\beta\gamma\delta}\,\lambda^I\, F^J{}_{\alpha\beta}\, F^K{}_{\gamma\delta}$$

$$K^\mu\big(Q^{IJ}\, G_{I\,\mu\rho\sigma}\, G_J{}^{\nu\rho\sigma}\big) = -2\nabla_\rho\big(Q^{IJ}\,\beta_{I\,\sigma}\, G_J{}^{\rho\nu\sigma}\big) - \tfrac{1}{4}\, C_{IJK}\,\epsilon^{\nu\alpha\beta\gamma\delta}\,\lambda^I\, F^J{}_{\alpha\beta}\, F^K{}_{\gamma\delta} + Q^{IJ}\, H_I^{\rho\sigma}\, G_{J\,\rho\sigma\nu}$$

$$\Rightarrow\quad K^\mu R_{\mu\nu} = -\tfrac{1}{3}\,\nabla^\mu \underbrace{\big[2\,Q_{IJ}\,\lambda^I\, F^J{}_{\mu\nu} + Q^{IJ}\,\beta_I{}^\sigma\, G_{J\,\mu\nu\sigma}\big]}_{\textbf{boundary terms}} + \underbrace{\boxed{\tfrac{1}{6}\, Q^{IJ}\, H_I^{\rho\sigma}\, G_{J\,\rho\sigma\nu}}}_{\textbf{cohomology}}$$

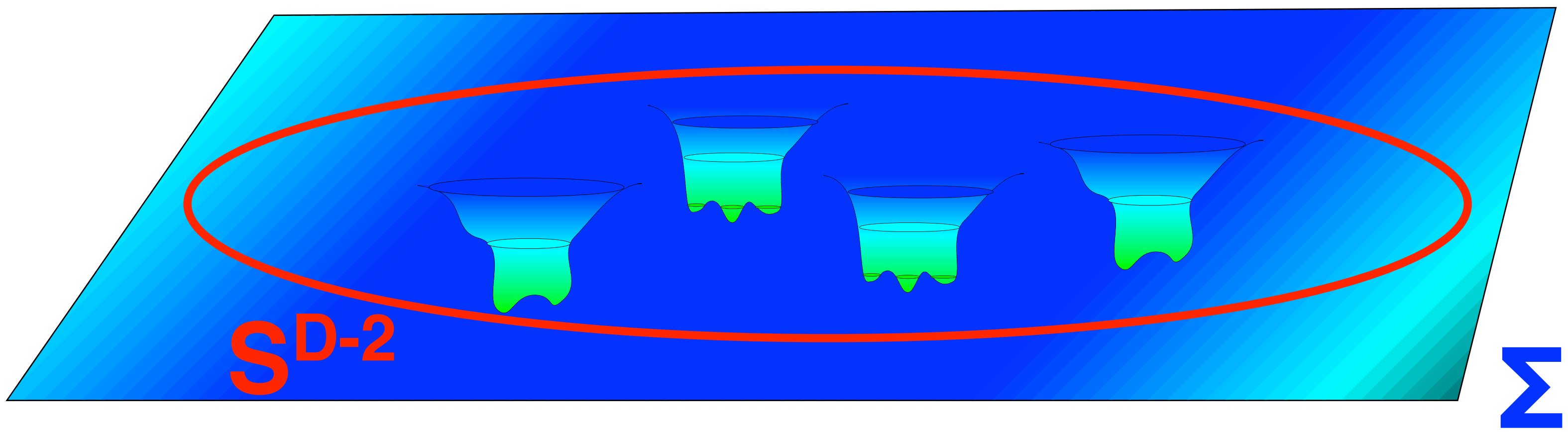
Chern-Simons contributions cancel!

$$\Rightarrow\quad M = \frac{3}{16\pi G_5}\int_\Sigma K^\mu R_{\mu\nu}\, d\Sigma^\nu = \frac{1}{16\pi G_5}\int_\Sigma H_J \wedge F^J$$

$$i_K(G_I) = d\beta_I - \tfrac{1}{2}\, C_{IJK}\,\lambda^J F^K + \boxed{H_I}$$

No Solitons without Topology

If Σ is a smooth hypersurface with no interior boundaries



$$M = \frac{3}{16\pi G_5} \int_\Sigma K^\mu R_{\mu\nu}\, d\Sigma^\nu = \frac{1}{16\pi G_5} \int_\Sigma H_J \wedge F^J$$

The mass can topologically supported by the cohomology $H^2(\Sigma, R)$

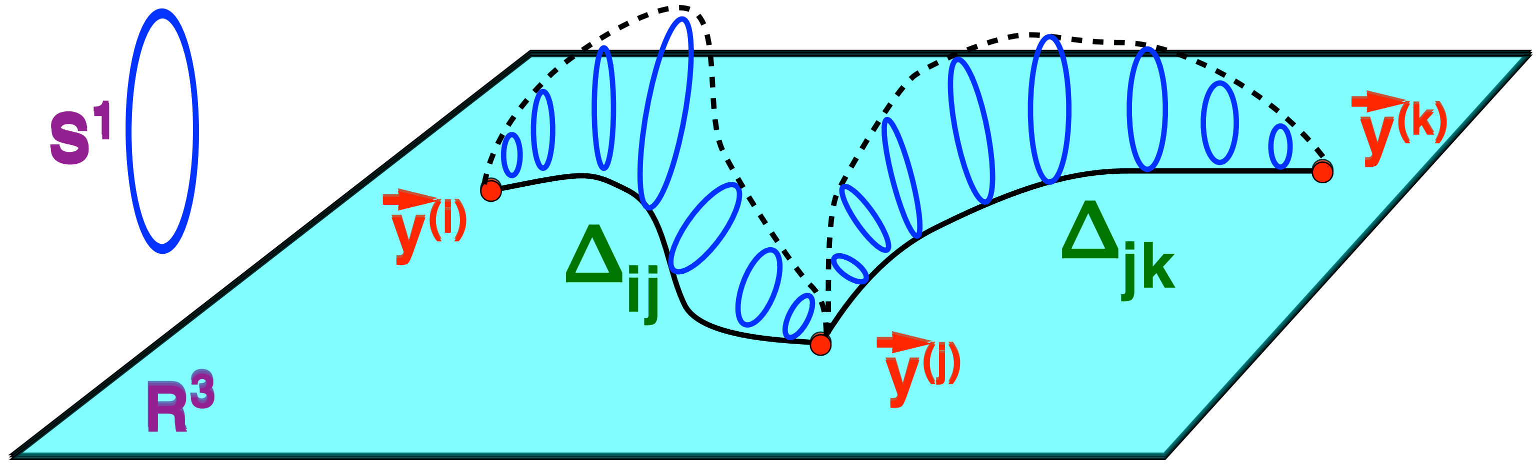
Stationary end-state of star held up by topological flux ...

- Black-Hole Microstate?
- A new object: *A Topological Star*

Only assumed time independence: ***Not simply for BPS objects***

A Class of BPS Examples

Large families of BPS solutions where the four-dimensional spatial base, **Σ**, is a circle fibration over flat R^3:

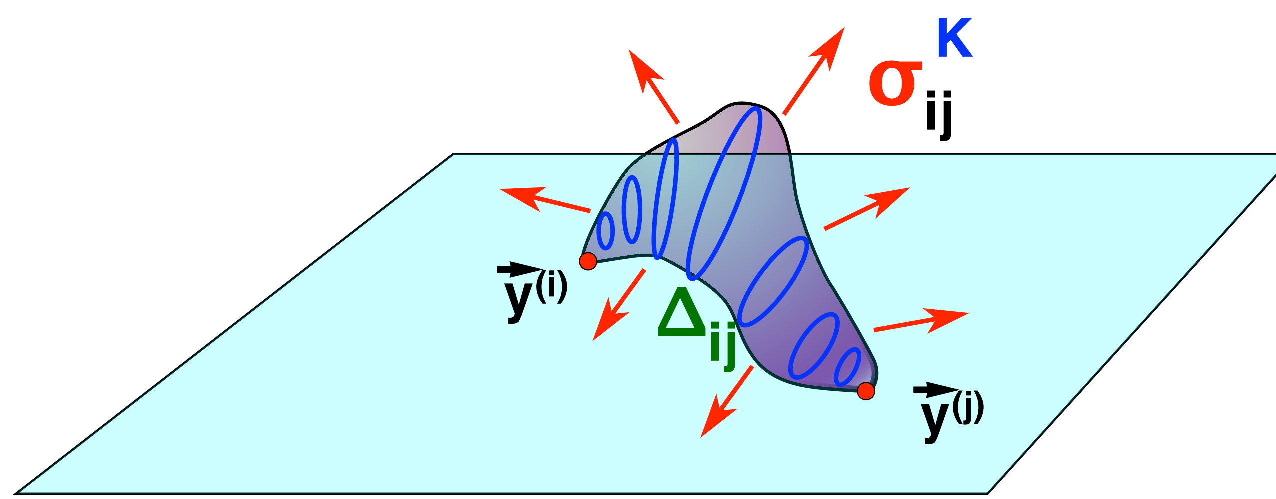


- Fiber pinches off at special points, $y = y^{(i)}$
- Non-trivial 2-cycles, Δ_{ij}: *S1* fiber along any curve from $y^{(i)}$ to $y^{(j)}$
- Intersection matrix computed from orientations at intersection points $y^{(i)}$

Fluxes:

$\sigma_A{}^K \equiv \sigma_{ij}{}^K$ = Flux of F^K through A^{th} cycle in $H^2(\Sigma,R)$

$$\sigma_{ij}{}^K \equiv \int_{\Delta_{ij}} F^K$$



Charge and Mass

Chern-Simons Interaction:

$$\nabla_\rho \left(Q_{IJ} F^{J\rho}{}_{\mu} \right) \;=\; \tfrac{1}{16} C_{IJK}\, \epsilon_{\mu\alpha\beta\gamma\delta}\, F^{J\,\alpha\beta}\, F^{K\,\gamma\delta}$$

Electric Charge, $\mathbf{Q_I}$ ~ Intersection of Magnetic fluxes $\mathbf{F^J} \wedge \mathbf{F^K}$

$$Q_I \;=\; -\, C_{IJK}\, \mathcal{I}^{AB}\, \sigma^J_A\, \sigma^K_B$$

$\mathcal{I}^{AB} \equiv$ Inverse of the Intersection Form

Mass:

$$M \;=\; \frac{1}{16\pi G_5} \int_\Sigma H_J \wedge F^J \;\xrightarrow{\text{BPS}}\; M \;=\; -\frac{1}{32\pi G_5}\, C_{IJK}\, \alpha^I \int_\Sigma F^J \wedge F^K$$

where $\alpha^I \;\equiv\; Z\,(X^I)^{-1}\big|_\infty$ = normalization of U(1) couplings

$\Rightarrow \quad \mathbf{M} \;=\; \boldsymbol{\alpha}^{\mathbf{I}}\, \mathbf{Q_I}$ BPS condition

Mass formula is more general for non-BPS examples

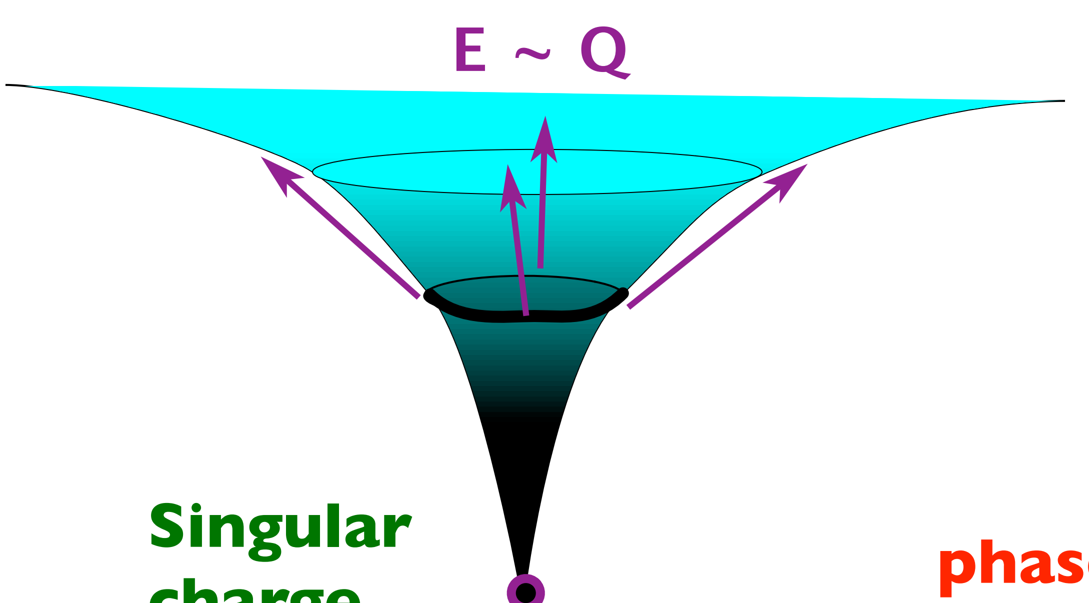
Resolving Black Holes by Geometric Transitions

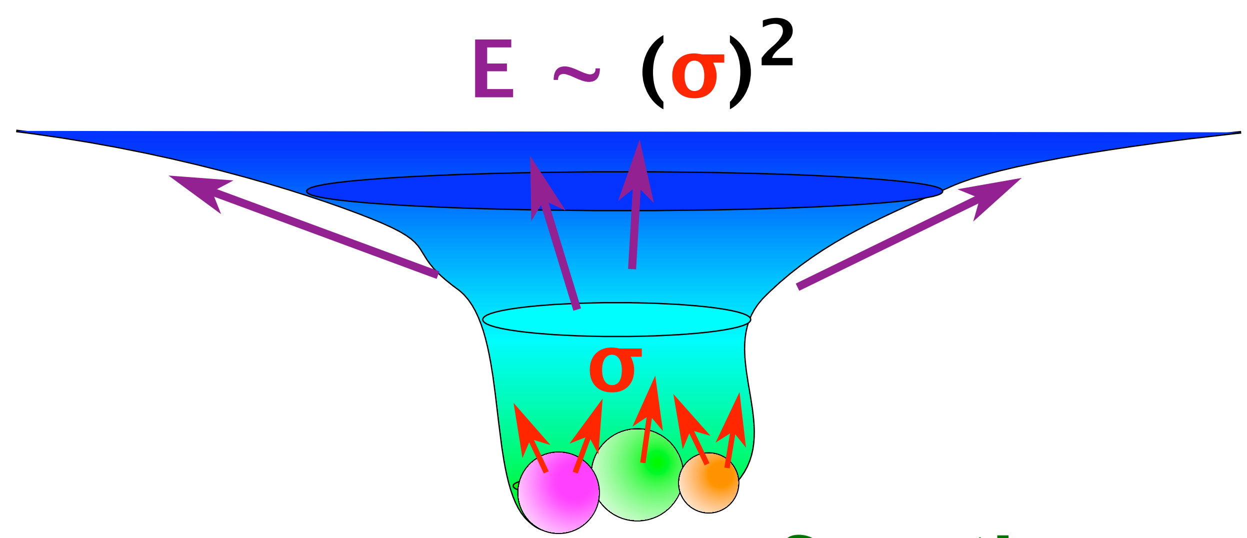
Standard description of black holes in string theory: Singular brane sources

Geometric transitions: Singular brane sources ⟶ Smooth fluxes

Electric Charge, $\mathbf{Q_I}$ ~ Magnetic fluxes $\sigma^{\mathbf{J}} \wedge \sigma^{\mathbf{K}}$

$$\nabla_\rho \big(Q_{IJ} F^{J\rho}{}_{\mu} \big) \;=\; \tfrac{1}{16}\, C_{IJK}\, \epsilon_{\mu\alpha\beta\gamma\delta}\, F^{J\,\alpha\beta}\, F^{K\,\gamma\delta}$$





Singular charge source

phase transition ⟶ **blowing up 2-cycles**

Smooth cohomological fluxes

New phase of black-hole physics?

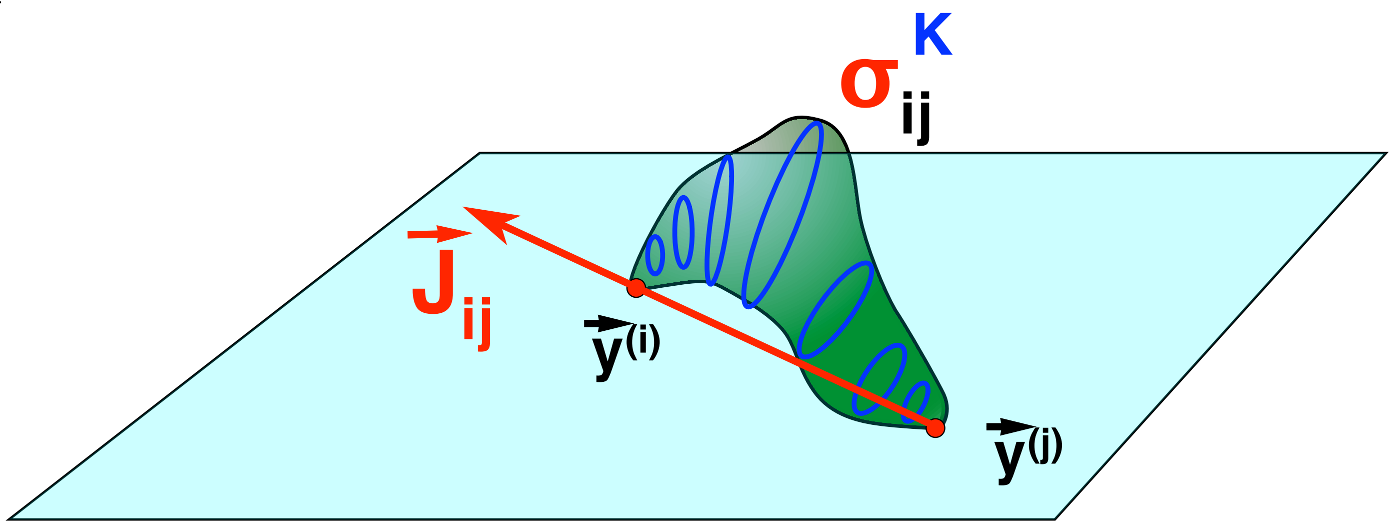
Chern-Simons Interaction is also the "dynamical key" to the geometric transition that resolves the singularity and removes the horizon of a black hole

Spin Systems and Bubble Equations

Each 2-cycle, or bubble, has an intrinsic angular momentum:

$$\Gamma_{ij} \equiv \pm \frac{1}{6} C_{IJK}\, \sigma^I_{ij}\, \sigma^J_{ij}\, \sigma^K_{ij}$$
$$\sim \left(Q_K\, \sigma^K\right)_{ij}$$

$$\vec{J}_{ij} = 8\, \Gamma_{ij}\, \frac{\vec{y}_i - \vec{y}_j}{|\vec{y}_i - \vec{y}_j|}$$



The Bubble Equations

No *closed time-like curves*
near special points, $y = y^{(i)}$

$$\sum_{j \neq i} \frac{\Gamma_{ij}}{|\vec{y}_i - \vec{y}_j|} = \sigma_i^\infty$$

Fixed fluxes + N points: (N-1) constraints on 3(N-1) variables, $\vec{y}^{(i)}$

(excluding center of mass).

$\Rightarrow$ 2(N-1) dimensional moduli space

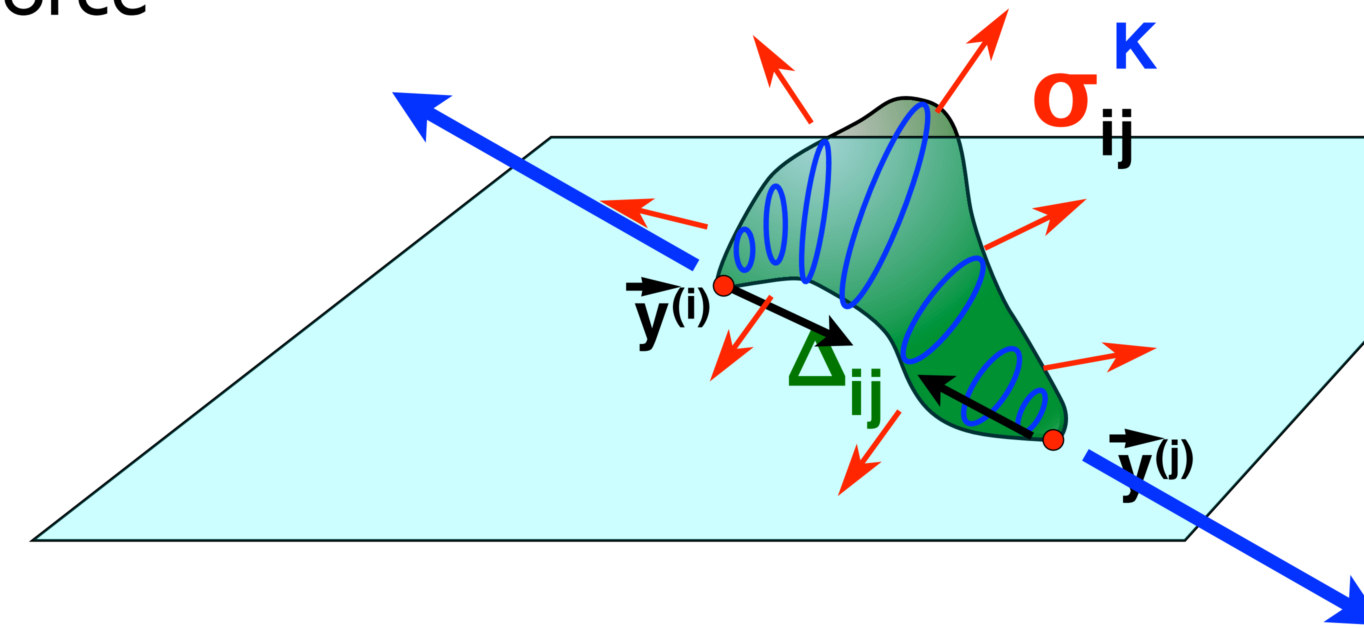
Rough picture of the classical moduli space

Gravity tends to try to collapse the 2-cycles ...

Bubbles + Flux $\Rightarrow$ Expansion force

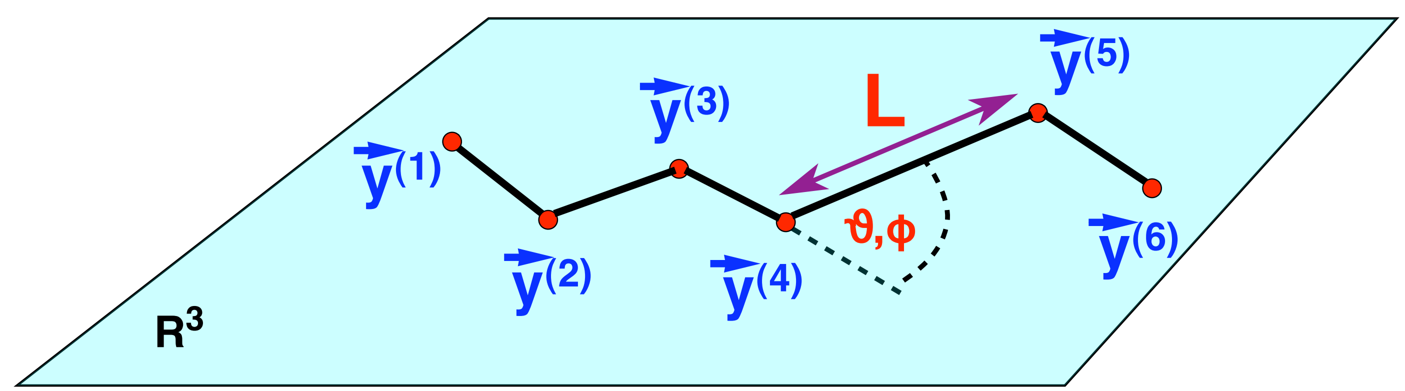
$\Rightarrow$ Equilibrium BPS Configuration

$$\sum_{j \neq i} \frac{\Gamma_{ij}}{|\vec{y}_i - \vec{y}_j|} = \sigma_i^\infty$$



Size of bubble =
Separation of points $y^{(i)}$ when attraction balances fluxes expansion

- Fluxes fix N-1 lengths, "**L**"
- 2(N-1) moduli: **θ, φ** ...

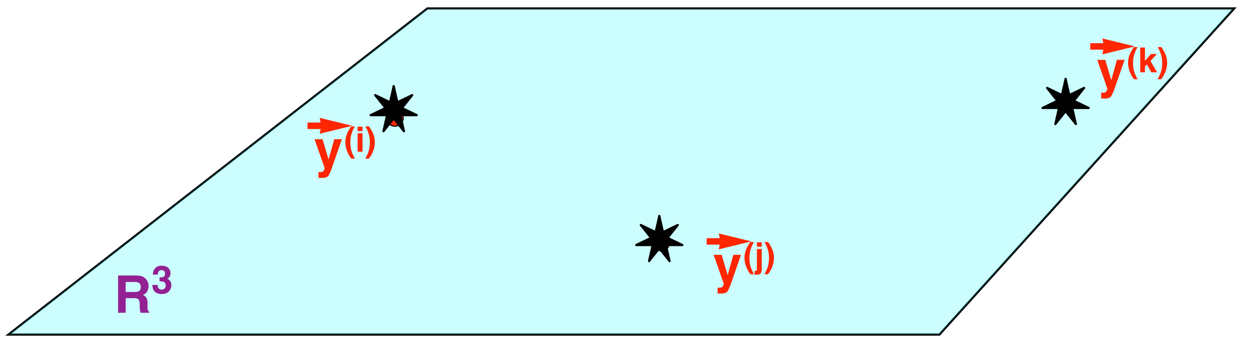


Four Dimensions: Multi-Black Hole Solutions

If the S^1 fiber scale remains finite at infinity then $\Sigma \sim R^3 \times S^1$ and one can compactify to an effective four-dimensional description

The fixed points, $y^{(i)}$, of the S^1 action are *singular* from a four-dimensional perspective.

$\Rightarrow$ *Multi-Black-Hole Solutions*



Denef: *Quiver Quantum Mechanics*

$$\sum_{j\neq i} \frac{\Gamma_{ij}}{|\vec{y}_i - \vec{y}_j|} = \sigma_i^{\infty}$$

"Integrability conditions"

Rich and complex moduli space:

- Classes of *marginally bound* configurations
- *Walls of marginal stability* where some black hole centers fly off to infinity
- *Wall crossing formulae* for degeneracies of states in dual quiver
- Classes of bound solutions with no walls of marginal stability

Higher-dimensional geometric significance was not appreciated/visible ...

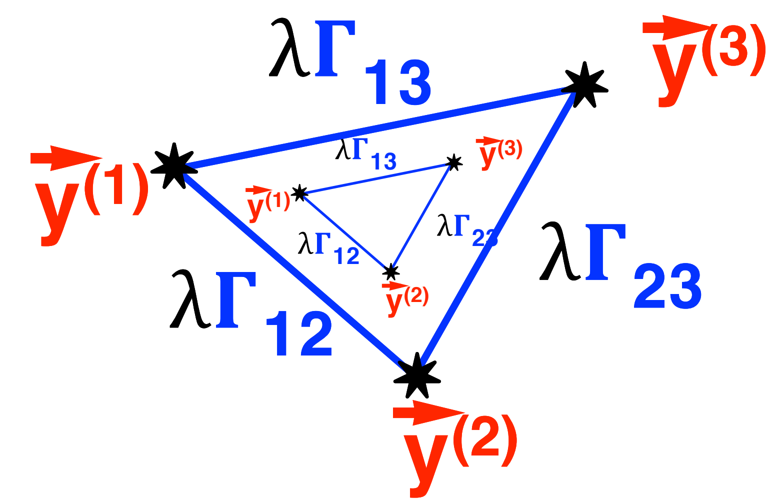
Scaling Solutions

$$\sum_{j\neq i} \frac{\Gamma_{ij}}{|\vec{y}_i - \vec{y}_j|} = \sigma_i^{\infty}$$

Very important class of solutions to bubble equations where one can have:

$$|\vec{y}_i - \vec{y}_j| \to 0$$

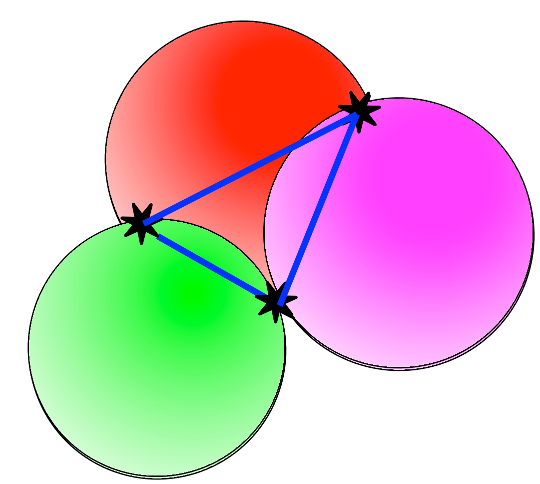
Simplest example: Three points where the Γ_{ij} satisfy the triangle inequalities:



$$|\Gamma_{13}| \le |\Gamma_{12}| + |\Gamma_{23}|$$ **+ permutations**

$$|\vec{y}_i - \vec{y}_j| \approx \lambda |\Gamma_{ij}|, \quad \lambda \to 0$$

Signs of Γ_{ij} cause λ^{-1} terms to cancel in bubble equations



More generally, clusters can scale to zero size in R^3: *Apparently very singular* ....

Homology cycles appear to be collapsing ...

Singular corners of moduli space?

Not from the five dimensional perspective ...

Five Dimensional Geometry of Scaling Solutions

Five-dimensional metric has warp factors:

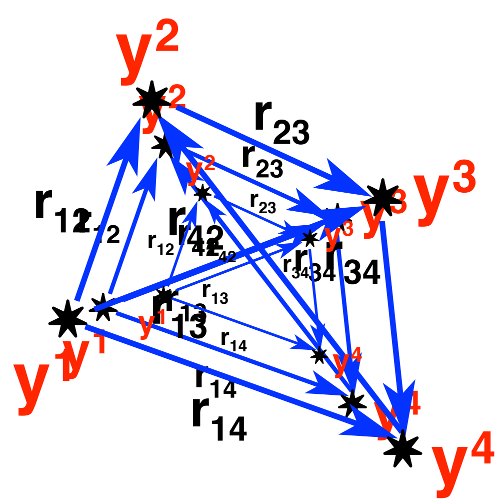
$$ds_5^2 \;=\; -\,Z^{-2}\,(dt+k)^2 \;+\; Z\,ds_4^2$$

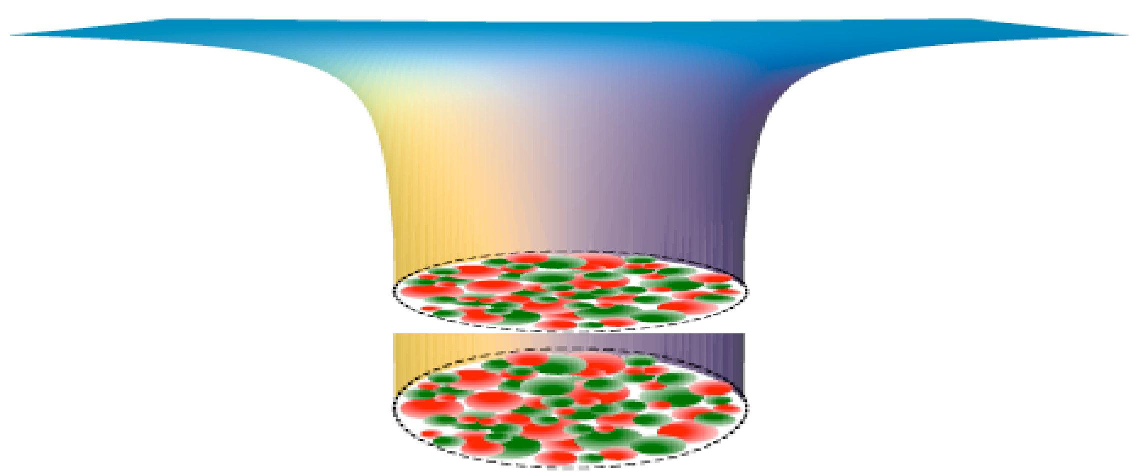
In scaling limits, Z diverges in precisely the correct manner to open up an $AdS_2 \times S^3$ (or $AdS_2 \times S^3$) throat

Closer scaling $\Leftrightarrow$ Deeper throat

Solutions to bubble equations with clusters of points converging:

$$r_{ij} \;\equiv\; |\vec{y}_i - \vec{y}_j| \;\to\; 0$$



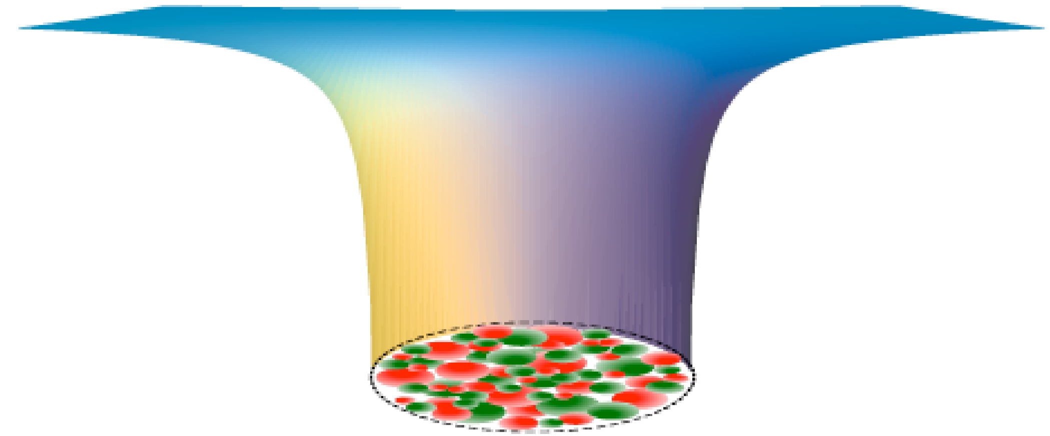


These solutions look exactly like an extremal black hole with a long AdS throat capped off by "bubbled geometry."

Homology cycles limit to a *fixed scale* determined by horizon area

Comments

★ Bound states, no walls of marginal stability:(?) Configurations are trapped in long AdS throat ...



★ Cycles/bubbles limit to finite size

★ The "foam" starts at/extends to the scale of the classical black hole horizon

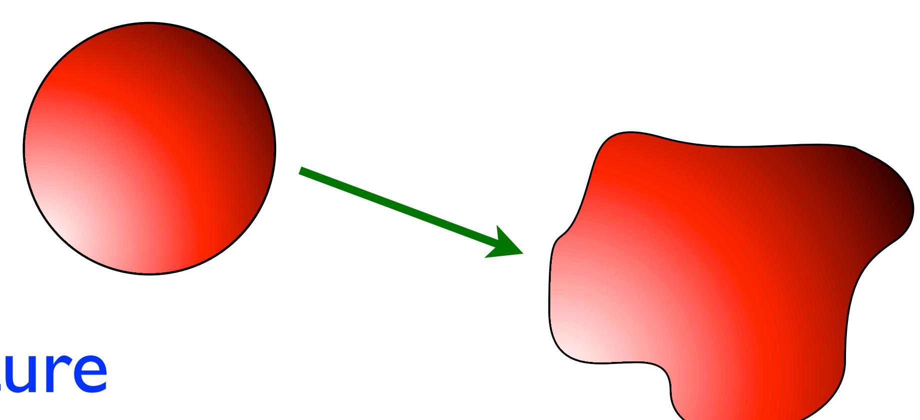
★ They look like classical black holes until one is very close to the horizon

★ Macroscopic solutions whose bubbles are much larger than String/Planck scale ⇒ Supergravity approximation is valid

★ Failure of Black-Hole Uniqueness

★ Apply AdS/CFT in throat: These geometries describe black-hole microstates. ⇒ "*Microstate Geometries*"

★ Far more general classes with bubbles whose shapes fluctuate as functions of the extra dimensions



⇒ Capture more black-hole microstate structure

New Parameters for Black Hole Physics?

Two new scales: *Classically free parameters*

★ The Transition scale, λ_T = Scale of a typical 2-cycle

~ Flux quanta on typical 2-cycle × ℓ_P

★ The Depth of the Throat ~ Maximum Red/Blue shift, z_{max}

Geometry/Holographic Field Theory:

Geometric transition represents a transition to a *new infra-red phase*

★ The Transition Scale:

- Fluxes = VEV of Order parameter of new phase
- Scale of Bubbles = New Dynamically Generated scale of field theory

e.g Holographic duals of N=1 gauge theories

Transitioned geometry → Confining phase; fluxes = gaugino condensate

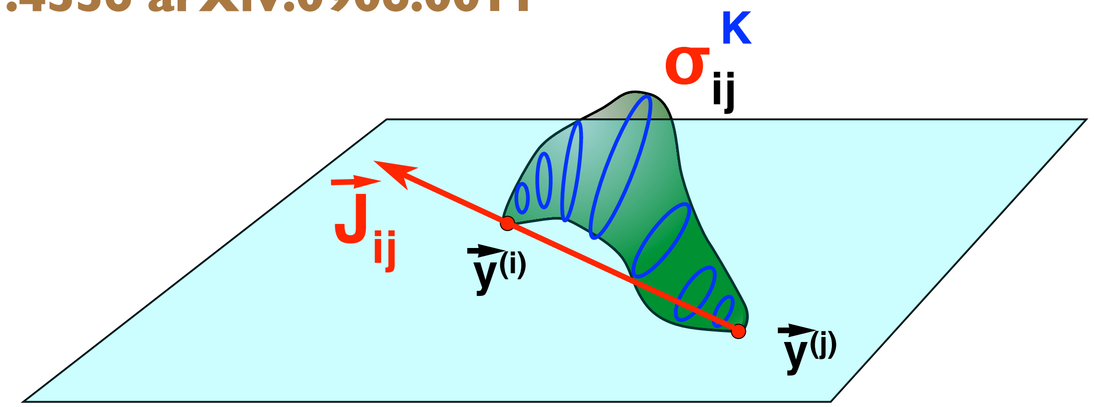
★ The Depth of the Throat: Determines Energy gap in dual Field theory

$E_{gap} \sim (\lambda_{gap})^{-1}$

λ_{gap} = *redshifted wavelength, at infinity of lowest mode of bubbles at the bottom of the throat.*

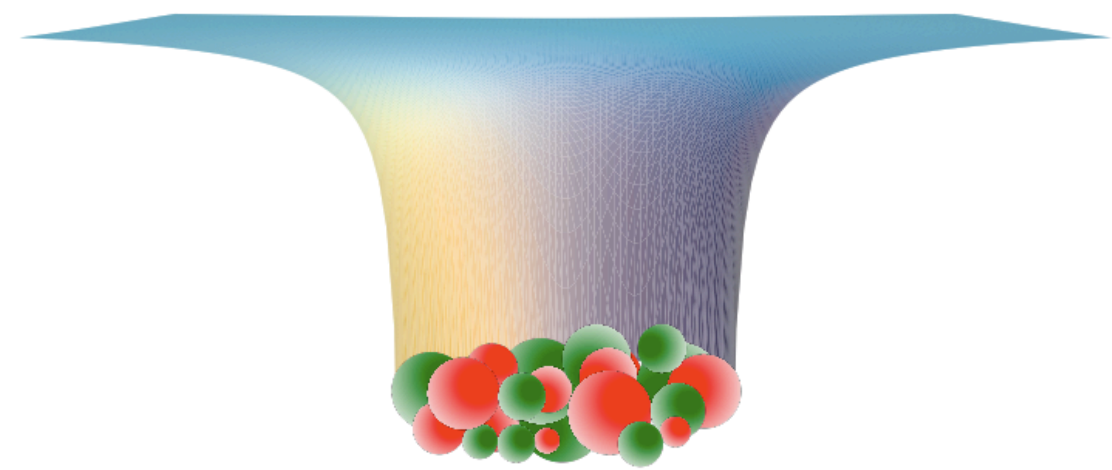
Quantization of Geometries: The Energy Gap

Bena, Wang and Warner, arXiv:0706.3786
de Boer, El-Showk, Messamah, Van den Bleeken, arXiv:0807.4556 arXiv:0906.0011



Each bubble has an intrinsic angular momentum

Semi-classical quantization of moduli space:
$\Leftrightarrow$ *quantizing these angular momenta*
$\Rightarrow$ The $y^{(i)}$ cannot be precisely localized



The depth of the AdS throat is a *very sensitive* function of the orientations of these angular momenta and quantization can make vast, *macroscopic* changes in geometry

$\Rightarrow$ Semi-classical quantization

- Limits throat depth: Fixes Maximum Red/Blue shift, z_{max}, and sets E_{gap} in the dual field theory
- Cuts off or "compactifies" phase-space volume of long throats.
- Can wipe out vast regions of smooth geometry in which curvature is small and supergravity is a good approximation

Non-BPS Microstate Geometries

Many examples of *extremal*, non-BPS microstate geometries

A handful of non-extremal microstate geometries ...

Jejjala, Madden, Ross and Titchener, hep-th/0504181

Non-extremal microstate geometries: A completely open problem ...

Smarr formula in five dimensions: Self-dual fluxes, σ^+, anti-self-dual fluxes, σ^-

$$M \;=\; |\sigma^+|^2 \;+\; |\sigma^-|^2 \qquad\qquad Q \;=\; |\sigma^+|^2 \;-\; |\sigma^-|^2$$

BPS $\Leftrightarrow$ purely self-dual or purely anti-self-dual cohomology ..

Many five-dimensional axi-symmetric BPS examples: $U(1)^2 \times R$ symmetry

Five-dimensional axi-symmetric, non-extremal solutions with $U(1)^2 \times R$ symmetry?

Effectively a two-dimensional problem: Can be reduced to a scalar coset

- Apply inverse scattering methods?
- Care with topology + Chern-Simons terms

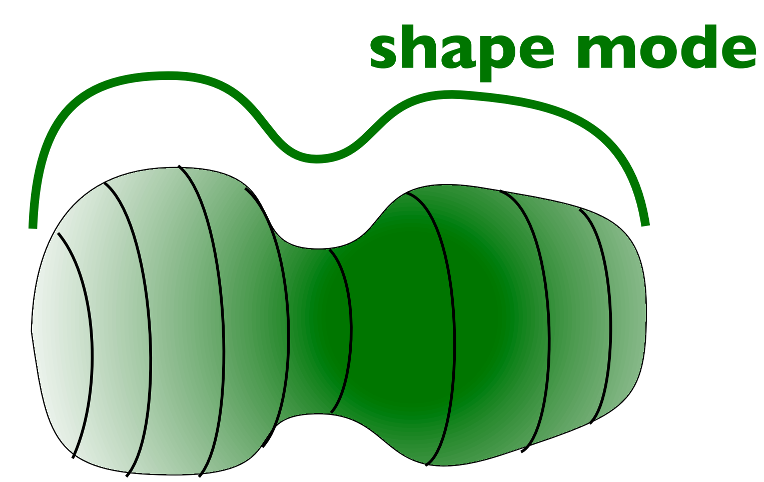
BPS Fluctuating Bubbled Geometries

The geometric transition stabilizes a fuzzball against gravity and makes microstate geometries possible ... this happens at scales ~ λ_T

Bubbled geometries can have *BPS shape fluctuations* that depend upon "transverse/internal dimensions." These shape fluctuations can go down to E_{gap} and/or the Planck scale, ℓ_P.

Huge amount of entropy lies in the shape fluctuations...
Is it enough to give a semi-classical picture of the black-hole entropy?

Extensive work in five-dimensions:
BPS shape fluctuations on 2-cycles depend upon functions of *one variable*:



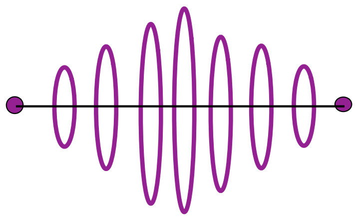
Expect entropy like that of a supertube

$$S \sim \sqrt{Q_1 Q_2} \sim Q$$

BPS black holes in five-dimesions: **$S \sim Q^{3/2}$**

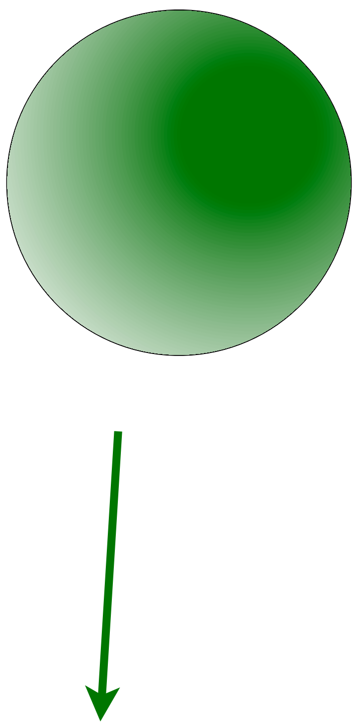
Such fluctuating geometries as *functions one variable* ***cannot*** capture the sufficient of dynamics underpinning the black hole entropy ...

BPS Microstate Geometries in Six Dimensions



Extra circle is now fibered over every five-dimensional 2-cycle $\Rightarrow$ 3-cycle.

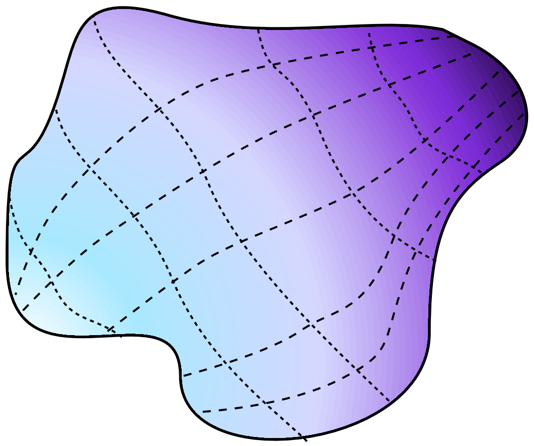
Make the fluctuating cycles in five-dimensions also depend upon new U(1) fiber ... and still be a BPS state?



Conjectured object **Bena, de Boer, Shigemori and Warner, 1107.2650**

The superstratum:

Completely ***new class*** of BPS soliton is six dimensions



- New class of solitonic *bound state* in string theory
- Completely smooth (microstate geometry)
- Defined by a topological 3-cycle fluctuates as *functions two variables*

Construction of examples? **$S \sim Q^{3/2}$???**

Final Comments

- Microstate Geometry program: Classify and study ***smooth, horizonless*** solutions to supergravity. *A much richer subject than previously expected*

  Miraculous existence through spatial ***topology*** and ***Chern-Simons terms***

- Emerge from geometric transitions:
  Singular brane sources → Smooth cohomological fluxes
  New phase of black hole ... bubbles start before horizon forms

- ***Mechanism*** for supporting matter before a horizon forms

- Generalized "no go" theorem for semi-classical solitons in string theory:
  If the space-time is even remotely classical, then only ***topological fuzz*** *at the horizon scale can support a soliton: No Solitons without Topology*

- Transition scale, λ_T = Scale of individual bubbles:
  Not fixed classically, large values entropically favored? $\lambda_T >> \ell_P$?

- Fluctuations of transitioned geometries: Scale E_{gap}. *Capture the entropy*?

- Multiple scales: The Horizon scale, M; The Transition scale, λ_T;
  The Energy Gap, $E_{gap} = (\lambda_{gap})^{-1}$; The Planck Length, ℓ_P.

Microstate Geometries give a beautiful geometric realization of these ideas