Representations of surface groups and beyond

Lille, 24-26 September 2018

Monday 24

10:00 ***Coffee & Welcome***

11:00 - 12:00: Daniele Alessandrini. ***Classification of real and complex projective structures with fixed holonomy***

Consider the following problem: given a fixed subgroup of PGL(n,**R**) or PGL(n,**C**), we want to classify all real and complex projective structures on some closed manifold whose holonomy is in the given subgroup. In this problem, the topology of the closed manifolds is not fixed in advance, and all possible topologies need to be determined.

We can answer this question in some special cases. For example, consider a Fuchsian subgroup of SL(2,**R**), embedded diagonally in SL(2n,**R**). We can classify all the $\mathbf{RP}^{2n-1}$ and the $\mathbf{CP}^{2n-1}$-structures on closed manifolds with holonomy contained there. Some obvious ones are the quotients of the domains of discontinuity. We can construct more via grafting, and we prove that all of them are of this form. This is joint work with Bill Goldman and Qiongling Li.

Lunch

13:50 - 14:50: Jean-Philippe Burelle. ***Positive representations are Schottky***

In 2003, Fock and Goncharov defined a "positive" subset of the character variety of a punctured surface into PSL(n,**R**) which generalizes the Teichmüller space for n=2. They prove, among other things, that these higher Teichmüller spaces are topological balls by defining a set of coordinates on them, and that positive representations are all faithful, discrete, and have a limit set in the flag variety of $\mathbf{R}^n$.

I will explain how these representations can be described by ping-pong dynamics using polyhedral subsets of the space of oriented flags, and how to construct a fundamental domain for their cocompact action on an open domain in projective space (when n is even) and spheres (when n = 3 mod 4). This is joint work with Nicolaus Treib.

15:00 - 16:00: Gabriele Link. ***Ergodic theory in locally symmetric spaces***

In this talk I want to state and explain the famous Hopf dichotomy well-known for locally symmetric spaces of rank one, and then discuss what is known and not known in higher rank. I will introduce all the necessary tools such as Patterson-Sullivan measures, Sullivan's shadow lemma and the different kinds of limit points which are relevant.

Coffee

16:30 - 17:30: Vincent Pecastaing. ***Conformal dynamics of semi-simple Lie groups***

The rigidity properties of semi-simple Lie groups and, in higher rank, of their lattices indicate that their dynamics on differentiable manifolds could be well understood, at least in some specific situations. This is roughly the idea of the Zimmer program. It is particularly interesting to consider the case where the action of the group preserves a geometric structure on the manifold, such as a metric tensor or a complex structure. Typically: Which group(s) can act on a certain type of structures? If they exist, can we classify the structures on which a given group can act?

In this talk, I will present the story of this subject in the case of conformal pseudo-Riemannian geometry, as well as recent results motivated by the previous questions. The key points of the proofs

rely on the hyperbolic dynamics of some well-chosen elements in the group. If time permits, I will also detail the relation with the recent breakthrough of Brown, Fisher and Hurtado on the Zimmer program.

Tuesday 25

9:30 - 10:30: Matthieu Gendulphe. ***What's wrong with the growth of simple closed geodesics on nonorientable hyperbolic surfaces?***
A celebrated result of Mirzakhani states that, if (S,m) is a finite area orientable hyperbolic surface, then the number of simple closed geodesics of length less than L on (S,m) is asymptotically equivalent to a positive constant times L raised to the power dim ML(S), where ML(S) denotes the space of measured laminations on S. We observed on some explicit examples that this result does not hold for nonorientable hyperbolic surfaces. The aim of this talk is to explain this surprising phenomenon.

Coffee

11:00 - 12:00: Jérémy Toulisse. ***Compact connected components of relative character varieties***
In a recent paper, Bertrand Deroin and Nicolas Tholozan introduced the notion of supra-maximal representations of the fundamental group of the s-punctured sphere into PSL(2,**R**). These representations have many surprising properties and form compact connected components of the relative character variety. In this talk, I will explain how the theory of parabolic Higgs bundles gives a way to construct similar representations into more general Hermitian Lie groups like SU(p,q). The corresponding components are compact and isomorphic to certain quiver varieties. This is joint work with Nicolas Tholozan.

Lunch

13:50 - 14:50: Olivier Glorieux. ***Topological mixing of the Weyl flow***
We will study the topological properties of the right action of the Weyl chamber flow on the space of Weyl chambers.
It corresponds to translation flows on the set of (parametrised) flats. One can see this dynamical system as a generalisation in higher rank of the geodesic flow. We will look at this family of flows, indexed on the unit sphere of a positive Weyl chamber and characterise precisely those for which the dynamics is topologically mixing. This is a joint work with T. Dang from Rennes.

15:00 - 16:00: Viveka Erlandsson. ***Determining the shape of a billiard table from its bounces***
Consider a billiard table shaped as a Euclidean polygon with labeled sides. A ball moving around on the table determines a bi-infinite "bounce sequence" by recording the labels of the sides it bounces off. We call the set of all possible bounce sequences the "bounce spectrum" of the table. In this talk I will explain why the bounce spectrum essentially determines the shape of the table: with the exception of a very small family (right-angled tables), if two tables have the same bounce spectrum, then they have to be related by a Euclidean similarity. The main ingredient in proving this fact is a technical result about Liouville currents for flat cone metrics. This is joint work with Moon Duchin, Chris Leininger, and Chandrika Sadanand.

Coffee

16:30 - 17:30: Bram Petri. ***Short geodesics on random hyperbolic surfaces***
Random Surfaces can be used to study the geometric properties of typical (hyperbolic) surfaces of

large genus. Moreover, they can sometimes be used in existence proofs. That is, sometimes the easiest way of proving that surfaces with certain properties exist is to prove that the probability that a random surface has these properties is non-zero. Of course there are multiple possible models of random surfaces. In this talk, a random surface will be a surface that is picked at random using the Weil-Petersson volume form on the moduli space of hyperbolic surfaces of a given genus. I will speak about the length spectrum of these random surfaces. This is joint work with Maryam Mirzakhani.

Conference dinner

Wednesday 26

9:30 - 10:30: Federica Fanoni. ***Curve graphs for infinite-type surfaces***
For surfaces of finite type, studying the action of the mapping class group on a graph, called the curve graph, has proved very useful to understand properties of the group itself. In the case of infinite-type surfaces (e.g. surfaces of infinite genus), the classical curve graph is not interesting from the coarse geometry viewpoint. I will discuss why and when we can (or can't) construct interesting graphs in the infinite-type case. Joint work with Matt Durham and Nick Vlamis.

Coffee

11:00 - 12:00: Peter Smillie. ***Constant curvature surfaces and flows on Teichmüller space***
Nontrivial actions of the fundamental group of a closed surface on Minkowski 3-space $\mathbf{M}^3$ are naturally in bijection with the total space of the tangent bundle to the Teichmüller space of the surface. Given such an action, it is well known that there is a unique convex spacelike surface in $\mathbf{M}^3$ that is invariant under the action and intrinsically hyperbolic (that is, with curvature -1). Taking the induced hyperbolic structure on this surface describes a sort of "exponential map" on Teichmüller space, which is connected to landslide flow and Einstein flow, which I will define. Recently, Bonsante and Seppi proved an analogue for the universal Teichmüller space. In joint work with Bonsante and Seppi, we extend this result beyond the universal case. In this talk, I will introduce at least one question related to this exponential map which is open even in the non-universal case, and point out the benefits of thinking beyond universal.

Lunch

13:50 - 14:50: Gye-Seon Lee. ***Exotic quasi-Fuchsian groups***
Let G be the isometry group of $(d+1)$-dimensional hyperbolic space. A subgroup H of G is quasi-Fuchsian if H is a convex cocompact discrete subgroup of G and the limit set of H is homeomorphic to the $(d-1)$-dimensional sphere. In this talk, I will explain how to construct examples of quasi-Fuchsian groups of G which are not quasi-isometric to the hyperbolic d-space using the Tits-Vinberg representation of Coxeter groups. Joint work with Ludovic Marquis.

15:00 - 16:00: Maxime Wolff. ***The parallelogram identity on groups and deformations of the trivial character in*** $\mathrm{SL}(2,\mathbf{C})$
For any finitely generated group Γ, we describe the set of all maps $\Gamma \to \mathbf{C}$ satisfying the parallelogram identity, $f(xy) + f(xy^{-1}) = 2\,f(x) + 2\,f(y)$. These maps form the Zariski-tangent space at the trivial character of the character variety $X(\Gamma,\mathrm{SL}(2,\mathbf{C}))$, and we explore further the algebraic deformations of this trivial character. This is joint work with Julien Marché.

Coffee