Topics in

$f(R)$ THEORIES OF GRAVITY

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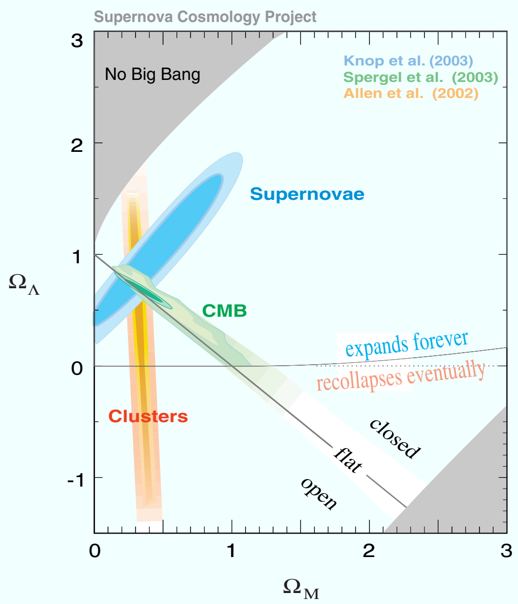
INTRODUCTION

The observed universe is well represented by a Friedmann-Lemaître spacetime the scale factor of which started to accelerate recently



$$ds^2 = g_{ij}dx^i dx^j = -dt^2 + a^2(t)d\sigma_{\mathrm{k}}^2$$

$$G_{ij} = \kappa T_{ij}^{\mathrm{total}} \quad ; \quad D_j T^{ij} = 0$$

$$H^2 + \frac{\mathrm{k}}{a^2} = \frac{\kappa}{3}\rho^{\mathrm{total}} \; ; \quad H = \frac{1}{a}\frac{da}{dt}$$

$$\left(\frac{H}{H_0}\right)^2 = \Omega^0_{\mathrm{rad}}\left(\frac{a_0}{a}\right)^4 + \Omega^0_{\mathrm{mat}}\left(\frac{a_0}{a}\right)^3 + \Omega^0_{\mathrm{k}}\left(\frac{a_0}{a}\right)^2 + \Omega^0_\Lambda$$

$$H_0 \approx 70 \; ; \; \Omega^0_{\mathrm{rad}} \approx 10^{-4} \; ; \; \Omega^0_{\mathrm{mat}} \approx 0.3 \; , \; \Omega^0_{\mathrm{k}} \approx 0$$

$\Omega^0_\Lambda \approx 0.7$: Dark Energy

$$\Longrightarrow \quad \kappa T^\Lambda_{ij} = -\Lambda g_{ij} \; , \; a(t) \to e^{\sqrt{\Lambda/3}\,t} \; , \; \Lambda = 3\Omega^0_\Lambda H_0^2$$

Origin of this acceleration

- An artefact of the averaging process ?

$G_{ij}(<g_{kl}>) = \kappa <T_{ij}>$ instead of $<G_{ij}(g_{kl})> = \kappa <T_{ij}>$
$G(<g_{kl}>) \neq <G(g_{kl})>)$ Ellis, 1971,..., see review Buchert, 2006 et seq.

- Exotic matter ? $(\rho_\Lambda + p_\Lambda \approx 0)$
  - Chaplygin gas : $p\rho = -A$, (Kamenshchik *et al.*, 2001)
  - Quintessence : R.G. plus φ with $V \propto 1/\varphi^n$ (Steinhardt *et al.*, 1997)

- "Modified" gravity ?
  - Λ : the simplest explanation, (Bianchi and Rovelli, Feb 2010)
  - MOND, see, e.g., Navarro and Acoleyen, 2005
  - "Branes" : DGP (2000), Deffayet (2001)
  - $f(R)$ lagrangian, instead of Hilbert's R

    C.D.T.T. (2003), Capozziello *et al.* (2003)

Outline of the talk

1. Introducing $f(R)$ theories of gravity

or : $f(R)$ theories as scalar-tensor theories of gravity

2. $f(R)$ cosmological models of dark energy
or : the search for viable models

3. $f(R)$ gravity and local tests
or : how to hide the scalar d.o.f. of gravity

4. Back to cosmological models
or : how to hide the scalar d.o.f. of gravity

5. Remarks on Black Holes in $f(R)$ theories

(uniqueness and thermodynamics)

1. Introducing $f(R)$ theories of gravity

- d.o.f. : gravity is described by a "graviton" and a "scalaron"

$$\bar{S}[g_{ij}] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\, f(R) + S_{\rm m}(\Psi\,; g_{ij})$$

(Weyl 1918, Pauli 1919, Eddington, 1924)

Metric variation yields a 4th order diff eqn for g_{ij} :

$$f'(R)\, G_{ij} + \frac{1}{2}(Rf' - f)g_{ij} + g_{ij}D^2 f' - D_{ij}f' = \kappa\, T_{ij} \quad (\Rightarrow \quad D_j T^{ij} = 0)$$

The trace :

$$3D^2 f' + (Rf' - 2f) = \kappa\, T$$

is a (2nd order) eom for R (or $f'(R)$), the "scalaron"(Starobinski, 1980)

Remark : "Palatini" variations yield different eom (Vollick, 2003 *et seq.*)

- Isolating the scalaron and coupling it to matter

Introduce a "Helmholtz" lagrangian :

$$\bar{S}[g_{ij}, s] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,[f'(s)R - (sf'(s) - f(s)] + S_{\rm m}(\Psi\ ; \tilde{g}_{ij} = e^{2C(s)}g_{ij})$$

(No reason for the scalaron not to couple to matter.)

hence TWO second order differential equations of motion :

$$f'(s)G_{ij} + \frac{1}{2}g_{ij}(sf'(s) - f(s)) + g_{ij}D^2f'(s) - D_{ij}f'(s) = \kappa T_{ij}$$

$$s = R - 2\kappa C'(s)T/f''(s) \qquad (\Rightarrow D_jT_i^j = TC'(s)\partial_i s)$$

$C(s) = 0$: standard $f(R)$ gravity ; $s = R$, same eom as before.

$C(s) \neq 0$: "detuned" $f(R)$ gravity (ND, Sasaki, Sendouda, 2007)

- Jordan *vs* Einstein frame description of $f(R)$ gravity

– The "Jordan frame" is the spacetime, $\tilde{\mathcal{M}}$, with metric $\tilde{g}_{ij} = e^{2C} g_{ij}$ to which matter is minimally coupled (that is : $\tilde{D}_j \tilde{T}^{ij} = 0$, *e.g.* $\tilde{\rho} \propto 1/\tilde{a}^3$).

In this frame the action is a Brans-Dicke type action (up to a divergence)

$$\tilde{S}[\tilde{g}_{ij}, \Phi] = \frac{1}{2\kappa} \int d^4x \sqrt{-\tilde{g}} \left[\Phi \tilde{R} - \frac{\omega(\Phi)}{\Phi} (\tilde{\partial}\Phi)^2 - 2U(\Phi) \right] + S_{\mathrm{m}}[\Psi; \tilde{g}_{ij}]$$

where $\quad \Phi(s) = f'(s) e^{-2C(s)} \quad , \qquad U(s) = \frac{1}{2}(s f'(s) - f(s)) e^{-2C(s)}$

and $\qquad \omega(s) = -\frac{3K(s)(K(s)-2)}{2(K(s)-1)^2}$ with $K(s) = \frac{d\ C}{d \ln \sqrt{f'}}$.

For standard $f(R)$ gravity, $C(s) = 0$; the Jordan frame is the original one. And $\omega = 0$; if $U \approx 0$, $f(R)$ gravity is ruled out since $\omega > 40000$ (Cassini) (see Damour Esposito-Farese, 1992, and below)

– The "Einstein frame" is the spacetime, $\mathcal{M}^*$, the metric of which, $g^*_{ij} = e^{-2k}\tilde{g}_{ij}$, makes the action for $f(R)$ gravity look like Einstein's :

$$S^*[g^*_{ij}, \varphi] = \tfrac{2}{\kappa}\int d^4x\sqrt{-g^*}\left[\tfrac{R^*}{4} - \tfrac{1}{2}(\partial^*\varphi)^2 - V(\varphi)\right] + S_{\mathrm{m}}[\Psi; \tilde{g}_{ij} = e^{2k(\varphi)}g^*_{ij}]$$

where $\varphi(s) = \sqrt{3}\ln\sqrt{f'(s)}$, $V(s) = \frac{sf'(s)-f(s)}{4f'^2(s)}$, $e^{2k(s)} = \frac{e^{2C(s)}}{f'(s)}$

– $\tilde{\mathcal{M}} \neq \mathcal{M}^*$ unless $\tilde{g}_{ij} = g^*_{ij}$, that is, $C(s) = \ln\sqrt{f'(s)}$,
(Magnano-Sokolewski, 1993, 2007)

– hence : $f(R)$ gravity is coupled quintessence

Ellis *et al.* (1989), Damour-Nordvedt-Polyakov (1993), Wetterich (1995), Amendola (1999), Copeland *et al.* (2006),...

• Jordan *vs* Einstein frames : an endless debate

– Einstein frame is the "physical" frame :

Magnano-Sokolewski (93, 07), Gunzig-Faraoni (98) (but see Faraoni (06))

("DEC does not hold in JF hence no positive energy theorem")

– Jordan frame is the "physical" frame : Damour Esposito-Farese (92) $\cdots$

"Jordan metric defines the lengths and times actually measured by laboratory rods and clocks (which are made of matter)" (Esposito-Farese Polarski, 2000)

– Jordan and Einstein frames are equivalent (classically) : Flanagan (04); Makino-Sasaki (91), Kaiser (95) (CMB anisotropies) ; Catena et al (06) (cosmo)

• Jordan *vs* Einstein frames : an example

– Capozziello et al, 10. FRW metric in JF ($ds^2 = -dt^2 + a^2(t)dx^2$).

Define $H(z)$ as : $H(t) \equiv \frac{\dot{a}}{a}$, $z(t) \equiv \frac{a_0}{a} - 1$

Define : $H_*(t) \equiv \frac{H_n^*}{a_*}\frac{da_*}{dt_*}$, $z_*(t) \equiv \frac{a_n^*}{a_*(t)} - 1$

(where $t_* = \int \sqrt{f'}dt$, $a_* = \sqrt{f'}\,a$.
and a_n^* and H_n^* such that $q_*(t_n) = q_0$ and $a_n^* \equiv a_*(t_n)$, $H^*(t_n) = H_0$.)

The $H(z)$ and $H_*(z_*)$ are different. (Correct.)

Since $H(z)$ and $H_*(z_*)$ are "Hubble laws", "the Jordan and Einstein frames are physically inequivalent". (Wrong.) Indeed :

– First, relate observable variables :

redshift ($Z = \frac{\nu}{\nu_0} - 1$) vs luminosity ($D = \sqrt{\frac{L}{4\pi l}}$ with $L = Nh\nu^2$)

where ν is the frequency of some atomic transition "there and then" ;
where ν_0 and l are the observed frequency and apparent luminosity.

In the JF, matter is minimally coupled, the EEP holds and ν is the same as in the lab now. Hence, as in GR : $Z = \frac{a_0}{a} - 1$, $D = (1+Z)\int_0^Z \frac{dZ}{H}$

– Second, recall that matter is not minimally coupled in EF :

In the EF, the interaction of ϕ with matter implies $m_* = m/\sqrt{f'}$ (Damour Gef, 92). Now $\nu_* \propto m_*$ and the frequency "there and then" (ν_*) is NOT the frequency measured in the lab now (ν).

Hence find : $Z_* \equiv \frac{\nu}{\nu_0^*} - 1 = Z$ and $D_* = D$. (Catena et al,. ND Sasaki) :

Relationships between observables do not depend on the frame.

• Hamiltonian structure of $f(R)$ gravity

In a nutshell :

– Extra dof : either K ($\dot{g}_{\mu\nu}$) : "Odstrogradsky formulation", (Buchbinder-Lyahovich 87, Querella 99, Esawa et al 99-09) or R ($\ddot{g}_{\mu\nu}$), Boulware 84.

– the action can be written in the Jordan or the Einstein frame

ALL variables are related by (non-linear) canonical transformations.
(N.D., Sendouda,Youssef 09, N.D., Sasaki, Sendouda, Yamauchi 09)

– Equivalence at the quantum level ? At linear order, yes (CMB), otherwise ?

• Junction conditions in $f(R)$ gravity

In a nutshell :

– Do not impose the continuity of 1st, 2nd and 3rd order derivatives of JF $\tilde{g}_{ij}$

– Impose continuity of 1st and 2nd order derivatives of $\tilde{g}_{ij}$ and of R and its 1st derivative (Teyssandier-Tourrenc 83, ND Sasaki, Sendouda, 07)

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2. $f(R)$ cosmological models of Dark Energy

• The Carroll-Duvvuri-Trodden-Turner
and Capozziello-Carloni-Troisi proposal (2003)

$$f(R) = R - \frac{\mu^{2(1+n)}}{R^n}\ (n > 0)\ ; \quad \mu^2 \sim 10^{-33}\mathrm{eV} \quad \text{or} \quad \mu^2 = \frac{1}{\ell^2} \text{ with } \ell \sim H_0^{-1}$$

Late time Einstein frame Friedmann equations when matter has become negligible (φ large, > 2, say) :

$$3H_*^2 - \dot\varphi^2 - 2V(\varphi) \approx 0, \quad \ddot\varphi + 3H_*^2\dot\varphi + \frac{dV}{d\varphi} \approx 0 \quad \text{with} \quad V(\varphi) \propto e^{-\frac{(n+2)\varphi}{2\sqrt{3}(n+1)}}$$

Solution : $a_*(t) \propto t^q \quad (q \to 3\ ,\ w^*_{\mathrm{DE}} \to -0.77$ for large n), $\varphi \sim \sqrt{3}\,p \ln t$

Jordan frame scale factor : $d\tilde s^2 = t^{-2p} ds_*^2 = -d\tilde t^2 + \tilde a^2(\tilde t) dx^2$

hence : $\tilde a(\tilde t) \propto \tilde t^{\frac{2}{3(1+\tilde w_{\mathrm{DE}})}}$
with $\quad \tilde w_{\mathrm{DE}} = -1 + \frac{2(n+2)}{3(n+1)(2n+1)} \to -1 \qquad$ for large n (2,3,4 is enough)

• A first flaw

Amendola, Polarski, Tsujikawa *et al.*, 2003 onwards

Friedmann equations when matter dominates over DE ($\rho_* = e^{-4\varphi/\sqrt{3}}\tilde{\rho}$) :

$$3H_*^2 - \dot{\varphi}^2 \approx \kappa\rho_* \ , \quad \ddot{\varphi} + 3H_*^2\dot{\varphi} \approx \frac{\kappa\rho_*}{2\sqrt{3}} \ , \quad \dot{\rho}_* + H_*\rho_* = -\frac{\dot{\varphi}}{\sqrt{3}}\rho_*$$

Hence : $\tilde{a}(\tilde{t}) \propto \tilde{t}^{1/2}$ instead of $\tilde{t}^{2/3}$ (for $\varphi \approx 0$, V is negligible, not $\dot{\varphi}$)

the CDTT model is ruled out

• Conditions for a standard matter era followed by late acceleration

Introduce the dynamical variables : $x_1 = -\frac{\dot{f}'}{Hf'}$, $x_2 = -\frac{f}{6Hf'}$, $x_3 = -\frac{R}{6H^2}$

Define $r(R) = -\frac{Rf'}{f}$ and $m(R) = \frac{Rf''}{f'}$ (Copeland *et al.* 1997, 2006)

Write the Friedmann equations as $\frac{\dot{x}_3}{H} = -\frac{x_1 x_3}{m} - 2x_3(x_3 - 2)$ *etc* ;
Find the fixed points such that $\dot{x}_i = 0$: $P = (x_1(m), x_2(m), x_3(m))$

The "phase space trajectory" $m = m(r)$ can connect a (saddle) matter point

$$P_{\mathrm{M}} = (r \approx -1, m \approx 0+) \quad \text{with} \quad dm/dr|_{-1} > -1$$

to a stable fixed point corresponding to

– either exponential acceleration, $P_{\mathrm{S}} \in r = -2$, if $0 < m(-2) \leq 1$

– or $\tilde{a} \propto \tilde{t}^r, r > 1$, $P_{\mathrm{A}} \in m = -(1+r)$, if $\frac{dm}{dr} < -1$ and $\frac{\sqrt{3}-1}{2} < m < 1$

Example :

$$f(R) = R \pm \frac{\mu^{2(1+n)}}{R^n} \quad \text{with} \quad -1 < n < 0$$

Developments (Capoziello-Tsujikawa, 2007) :

– $w_{\mathrm{DE}} < -1$ for $z < z_b$ "crossing of the phantom boundary"

– w_{DE} diverges at $z = z_c$, z_b and $z_c \to \infty$ for $n \to -1$

3. $f(R)$ gravity and local tests

- A ninety year old mistake

CDTT, 2003 : “Many solar system tests of gravity theory depend on the Schwarzschild (de Sitter) solution, which Birkhoff’s theorem ensures is the unique, static, spherically symmetric solution (...) Astrophysical tests of gravity will be unaffected by the modification we have made.”

Weyl (1918), Pauli (1919) and Eddington (1924) had said the same...

Pechlaner-Sexl (1966), Havas (1977) : $f(R)$ field equations are fourth order differential equations ; they possess extra-runaway solutions : there is no Birkhoff theorem ; the solution outside an extended source is not the Schwarzschild metric and depends on its equation of state.

• One-scale $f(R)$ models of DE are $\omega = 0$ Brans-Dicke theory

Chiba (2003), Olmo (2005), Erickcek et al (2006), Navarro-Acoleyen (2006),...

The short answer :

Recall that $f(R)$-gravity is of Brans-Dicke type with $\omega = 0$ and a potential

$$U = \frac{1}{2}(Rf'(R) - f(R))$$

Teyssandier-Tourrenc, 1983

Can U be neglected when studying gravity in the solar system ? Yes

Indeed $f(R) = R - H_0^4/R$, yields $U = \mathcal{O}(H_0^2) = \mathcal{O}(1/\ell^2)$ with $\ell \gg L_{\mathrm{SS}}$.

Therefore, see $e.g$ Will or Damour-Esposito Farese, ω must be large to comply with solar system observations : $\omega > 40000$ (Cassini).

Hence, all one-scale $f(R)$ models of dark energy are ruled out.

The details :

The equations of motion are :

$$D_*^2\varphi - \frac{dV}{d\varphi} = \frac{4\pi G_*}{\sqrt{3}} T^* \ , \ G_{ij}^* - 2\partial_i\varphi\partial_j\varphi + g_{ij}^*\left[(\partial^*\varphi)^2 + 2V(\varphi)\right] = 8\pi G_* T_{ij}^*$$

with $\quad g_{ij}^* = f'\tilde{g}_{ij} \quad , \quad \rho_* = \tilde{\rho}/f'^2 \quad , \quad f' = e^{2\varphi/\sqrt{3}}$

Linearize : $\varphi = \varphi_c + \varphi_1$, $g_{ij}^* = f_c'(\eta_{ij} + h_{ij})$ with $V_c = \mathcal{O}(\kappa\rho_c) = \mathcal{O}(1/\ell^2)$.

scalaron : $\triangle\varphi_1 - m^2\varphi_1 \approx -\frac{4\pi G_{\rm eff}}{\sqrt{3}}\tilde{\rho}_\odot \qquad (\triangle_* = \triangle/f_c' \ , \ G_{\rm eff} = G_*/f_c')$

$$m^2 = \frac{f_c' d^2V}{d\varphi^2}\Big|_c = \mathcal{O}(1/\ell^2) \text{ whereas } \triangle = \mathcal{O}(1/L_{\rm SS}^2) \text{ hence } \varphi_1 \approx \frac{G_{\rm eff}\tilde{M}_\odot}{\sqrt{3}r}$$

metric : $ds_*^2/f_c' \approx -(1 - 2G_{\rm eff}\tilde{M}/r)dt^2 + (1 + 2G_{\rm eff}\tilde{M}/r)d\vec{x}^2$

$$d\tilde{s}^2 \approx \left(1 - \frac{2\phi_1}{\sqrt{3}}\right)\frac{ds_*^2}{f_c'} \approx -\left(1 - \frac{2\tilde{G}\tilde{M}}{r}\right)dt^2 + \left(1 + \frac{2\gamma\tilde{G}\tilde{M}}{r}\right)d\vec{x}^2 \ , \ \tilde{G} = \frac{4G_{\rm eff}}{3} \ , \ \gamma = 1/2$$

A scalar curvature "locked" at its cosmological, small, value :

Equation of motion for the scalar curvature (or scalaron, that is, φ) :

$$3D^2 f' + (Rf' - 2f) = \kappa T \qquad \text{(not } R = \kappa T \text{ as in GR !)}$$

Linearize : $R = R_c + R_1$ so that $D^2 R_1 - m^2 R_1 = \frac{\kappa T}{3 f''_c}$, $m^2 = \frac{(f' - R f'')|_c}{3 f''_c}$

Solve with source being, say, a constant density star—or numerically,
Multamaki-Vilja, Kanulainen et al, (2007)
and find that, if $m r_\odot \ll 1$, then $R \approx R_c$ outside and inside the star.

**

On the other hand, if $m r_\odot \gg 1$, find Schwarzschild (hence $\gamma = 1$) :

$$d\tilde{s}^2 \approx -\left(1 - \frac{2GM_\odot(1-\epsilon)}{r}\right) dt^2 + \left(1 + \frac{2GM_\odot(1+\epsilon)}{r}\right) d\vec{x}^2 \ , \ \epsilon = \frac{e^{-m(r-r_\odot)}}{2m^2 r_\odot^2} \approx 0$$

but... $R \approx -\kappa T \gg R_c$ inside the star : linear approximation breaks down

- Where do we stand ?

– the original CDTT model $f(R) = H_0^2(RH_0^{-2} - 1/(RH_0^{-2})^n)$ failed : final acceleration but no matter era

– various one-scale cosmologically viable models had been proposed, e.g. : $f(R) = H_0^2(RH_0^{-2} + (RH_0^{-2})^n)$ with $0 < n < 1$

– all badly failed to comply with local gravity constraints because the scalar curvature is locked at its cosmological value $R = \mathcal{O}(H_0^2) = \mathcal{O}(\kappa\rho_{\rm cosmo})$ everywhere, even inside the Sun where, instead, GR gives $R = \mathcal{O}(\kappa\rho_\odot)$

– however, if the scalaron could be given a heavy mass, the linear approximation (violated in that limit) indicates that the gravity field of the Sun would be the same as in GR

– hence : look for two-scale $f(R)$ models and solve the full non-linear scalaron eom

- Conventional wisdom...:

e.g. Starobinski, 2007

linearise the scalaron eom on the SS background : $\frac{dV}{d\varphi}|_{\rm SS} \propto \kappa\rho_\odot$:

$$\triangle\varphi_1 - m^2\varphi_1 \approx 0 \quad \text{with} \quad m^2 = \frac{f' - Rf''}{3f''}$$

If, for $R \gg 1/\ell^2$, *e.g.* $R \sim \kappa\rho_\odot$: $f' \to 1$ and $m^2_{\rm SS} \approx \frac{1}{3f''|_{\rm SS}}$ is positive and large ($(mL)_{\rm SS} \gg 1$) then deviations from Einstein's GR are small.

CDTT model : $m^2_{\rm SS}$ is large and negative (Dolgov-Kawasaki, 2003)

- ...and the "Chameleon" effect

Khoury-Weltman (2003)
Navarro-Acoleyen (2006), Capozziello-Tsujikawa (2007)

or : how can the scalar curvature become equal to the local matter density

Chameleon details (Takami-Tsujikawa 2008)

(For a review see Brax et al., P. Brax ihes seminar)

Once again, look at scalaron eom : $D_*^2\varphi - \frac{dV}{d\varphi} = \frac{\kappa}{2\sqrt{3}}T_*$

In $\approx$ flat background : $\triangle_*\varphi = \frac{dV_{\rm eff}}{d\varphi}$ with $\frac{dV_{\rm eff}}{d\varphi} = \frac{dV}{d\varphi} - \frac{\kappa\rho_*}{2\sqrt{3}}$

Contrarily to V, $V_{\rm eff}$ may have minima for $\varphi = \varphi_\odot$ inside the Sun and for $\varphi = \varphi_e$ outside. One looks for $\varphi|_{e,\odot} = \varphi(R_{e,\odot})$ such that $R_{e,\odot} \approx \kappa\rho_{e,\odot}$.

Solution outside : $\varphi = \varphi_e + C\frac{r_\odot}{r}$ when $m_e r_\odot \ll 1$ with $m_e^2 = \frac{d^2V}{d\varphi^2}|_e$

Solution inside : $\varphi \approx \varphi_\odot$ up to $r = r_1$, then interpolation to φ outside.

if $r_1 \approx r_\odot$, then $C = 3\beta C_{\rm lin}$ with $\beta = \frac{\varphi_e - \varphi_\odot}{GM_\odot/r_\odot} \approx \frac{\varphi_e}{GM_\odot/r_\odot}$

One needs $\beta < 10^{-5}$ to have $\gamma - 1 < 10^{-5}$ (Cassini)

- A new family of $f(R)$ models of dark energy

Hu-Sawicki (2007), Starobinski (2007), Odintsov et al (2008), ...

$$f(R) = R + \lambda R_c \left[\frac{1}{(1+R^2/R_c^2)^n} - 1 \right],\ etc$$

$\frac{dV}{d\varphi} - \frac{\kappa \rho_e}{2\sqrt{3}} = 0$ gives $\varphi_e = \mathcal{O}\left(\rho_c/\rho_e\right)^{2n+1}$

Now, $\beta \approx \frac{\varphi_e}{GM_\odot/r_\odot} \approx 10^6 \varphi_e < 10^{-5}$; hence $\varphi_e < 10^{-11}$

For $\rho_e = \rho_{\rm galaxy} = 10^5 \rho_c$ then $\varphi_e = \mathcal{O}(10^{-5(2n+1)})$ and hence the models evade Local Gravity Constraints as soon as $n > 1/2$

In a nutshell : the Chameleon effect relies on (1) "locking" φ, that is, the scalar curvature, on its Einstein value inside the Sun, and (2) on a local environment much denser than the asymptotic, cosmological, value.

- Beyond flat background : $f(R)$ dense star models

Frolov (2008); Kobayashi-Maeda (2008) : $R_{central} \to \infty$?

Babichev-Langlois (2009) : NO, if $\rho - 3P > 0$.

4. Back to cosmology : "cured viable" models

- An uncontrolable scalaron instability ?

The new family of models yield cosmological scale factors which are undistinguishable from Λ-CDM until after the end of the matter era and tend to a de Sitter regime $a \propto e^{H\tilde{t}}$.

... However, because the mass of the scalaron is chosen to be high in high matter density environment (in order to comply with Local gravity Constraints), the cosmological perturbations of the scalaron diverge in the early matter era unless their amplitude is tuned to a very small value.

Starobinski (2007), Tsujikawa (2007)

... but this problem can be cured too :

$$f(R) = R - \mu\, R_c \frac{(R/R_c)^{2n}}{(R/R_c)^{2n}+1} + \frac{R^2}{6M^2}$$

Appleby, Battye and Starobinsky (2009)

.... Better than Λ ???

- $f(R)$ models of Dark Energy, summary

– Failed attempt ? technically no, observationaly, not yet...

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LETTERS

Confirmation of general relativity on large scales from weak lensing and galaxy velocities

Reinabelle Reyes[1], Rachel Mandelbaum[1], Uros Seljak[2,3,4], Tobias Baldauf[2], James E. Gunn[1], Lucas Lombriser[2] & Robert E. Smith[2]

tational lensing effect[3]. Here we report that $E_G = 0.39 \pm 0.06$ on length scales of tens of megaparsecs, in agreement with the general relativistic prediction of $E_G \approx 0.4$. The measured value excludes a model[1] within the tensor–vector–scalar gravity theory[4,5], which modifies both Newtonian and Einstein gravity. However, the relatively large uncertainty still permits models within $f(R)$ theory[6], which is an extension of general relativity. A fivefold decrease in uncertainty is needed to rule out these models.

— ...aesthetically... ?

... What **is** then the origin of the present acceleration of the universe ?...

- "Modified" gravity ?
  - $f(R)$-gravity : standard models : not convincing ; "detuned" $f(R)$?
  - MOND ? "Branes" ?
- Exotic matter ?
  - Chaplygin gas ? Quintessence ? (see Efstathiou *et al.* 2007)
- An artefact of the averaging process ?

  $G_{ij}(<g_{kl}>) = \kappa <T_{ij}>$ instead of $<G_{ij}(g_{kl})> = \kappa <T_{ij}>$
- ... or "simply" Λ ?

  usual objections : why so small, and why now ? Bianchi-Rovelli (2010) :
  - Λ is no "blunder"
  - no strong probability argument against "now"
  - if 120 orders of magnitude off HEP prediction: HEP problem !

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   or : how to hide the scalar d.o.f. of gravity
4. Back to cosmological models
   or : how to hide the scalar d.o.f. of gravity

5. Remarks on Black Holes in $f(R)$ theories

   (uniqueness and thermodynamics)

4. Remarks on Black Holes in $f(R)$ theories

- On uniqueness theorems

T. Damour and G. Esposito-Farese (1992)
In vacuum $f(R)$ is GR+ϕ. Black holes have no scalair hair.
Hence Kerr (de Sitter) is the unique solution.

Whitt (1984)
showed the unicity of Kerr (de Sitter) for $f(R) = R + aR^2$ if $a > 0$.
NB: the method applies to all $f(R)$ such that $f'(2f - Rf') > 0$ in DOC.

Mignemi-Wiltshire (1992)
showed the unicity of Schwarzschild for $f(R) = R + \sum a_n R^n$ with $a_2 > 0$
(spherically symmetric case when asymptotic flatness is requested).

Nothing much since then it seems...

- Bekenstein entropy and $f(R)$ BH thermodynamics

– The only ingredients necessary to describe the Penrose process are : a metric (e.g. Kerr with parameters M, $J = Ma$ in the Jordan Frame) geodesic motion (implying EEP). IF (big if !) $E = \delta M$ and $L = \delta J$:

– Result : $\delta M - \Omega \delta J \geq 0$, that is, $\delta F(A) \geq 0$, applies to $f(R)$ theories.
(Ω is the angular velocity of the BH, A its area.)

– The (more general) area theorem, $\delta A \geq 0$ also applies to $f(R)$ theories.

– Hence the Bekenstein entropy : $S_B = \alpha A$ s.t. $T dS_B = dM - \Omega dJ$ in $f(R)$ theories as in GR. ($T = \frac{\kappa}{8\pi\alpha}$, κ being the BH surface gravity)

– As for α it is obtained e.g. by analytical continuation of quantum fields across the horizon : $\alpha = 1/4$ in $f(R)$ theories as in GR.

If all true Bekenstein entropy and first law generalize to all $f(R)$ theories....

where M and J are the gravitational mass and angular momentum of the BH.

- Wald entropy and "Global charges" of $f(R)$ spacetimes

Entropy as a Noether charge, Wald (1993) : $S_W = f'(R_h)S$.

Various methods to define global charges : Euclidean methods plus :

– Start with eg Kerr (M, a, Jordan Frame). Go to the Einstein frame (where no geodesic motion) : at ∞, $M \to \sqrt{f'_\infty} M$, $\xi_t = \sqrt{f'_\infty}(1,0,0,0)$. Obtain the global charges as $M_i = f'_\infty M$, $J_i = f'_\infty J$.

– Hamiltonian formulation (ADM, 1959)
Vary $f(R)$ action with a YGH term in JF, keep track of boundary terms

$$\bar{S} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,[f'(s)R - (sf'(s) - f(s)] + \frac{1}{\kappa}\int_{\partial\mathcal{V}} d^3x\sqrt{|h|}K\, f'(s)$$

Obtain :

$$E_{\rm ADM} \equiv H_{\rm on\ shell} = -\frac{1}{\kappa}\int_{\mathcal{S}} d^2x\sqrt{\sigma}(f'(s)k + f'' r^a\partial_a s) = f'_\infty M$$

NB : OK with Deser-Tekin (2002), not Deser-Tekin (2007)

– Ashtekar-Magnon (1984) method (see also, ND, J Katz, 2006) applied to $f(R)$ in JF yields $M_{conf} = f'_\infty M$ (Koga et al, 2005).

begin(• The Katz Bičàk Lynden-Bell method (1985, 1997)

– Nœther identities, conserved current, superpotential and charge :
Vary $\hat{\mathcal{L}} \equiv \sqrt{-g}\, L +$ surface term for $x^\mu \to x^\mu + \xi^\mu$; get (on shell) :

$$\partial_\mu \hat{j}^\mu = 0 \quad \Longrightarrow \quad \partial_\mu \hat{j}^{[\mu\nu]} = \hat{j}^\nu$$

$$\Longrightarrow \quad q \equiv \int_S d^{D-2}x\ \hat{j}^{[01]}$$ is constant in time.

– Choice for the vector ξ^μ :

Translation (mass) and rotation (angular momenta) Killing vectors, “appropriately” normalized

– Regularization (“zero point energy”, background) :

$$Q \equiv \int_S d^{D-2} \hat{J}^{[01]} \qquad ; \qquad \hat{J}^{[\mu\nu]} \equiv -\frac{1}{16\pi}(\hat{j}^{[\mu\nu]} - \overline{\hat{j}^{[\mu\nu]}})$$

– Choice for surface term :

$\hat{\mathcal{L}} \equiv \hat{L} + D_\mu \hat{k}^\mu$ where the vector k^μ is chosen “appropriately”

– Extensions : ND-Katz-Ogushi (03) ; ND-Katz (04); ND-Morisawa (05)

- KBL mass in $f(R)$ theories (ND, 2007)

$$\delta \int_{\mathcal{M}} d^D x \, (\hat{L} + D_\mu \hat{k}^\mu) = \int_{\partial\mathcal{M}} d^{D-1}x \, n_\mu(\hat{V}^\mu + \delta\hat{k}^\mu) \qquad \text{(on shell)}$$

with $\quad L = f(R) \quad$ and $\quad V^\mu = -(\otimes^{\mu\nu\rho}\delta g_{\nu\rho} - \otimes_\sigma^{\mu\nu\rho}\delta\Gamma^\sigma_{\nu\rho})$

$\delta g_{\mu\nu} = 2D_{(\mu}\xi_{\nu)}$, find : $\hat{j}^{[\mu\nu]} = 2(2\xi^{[\mu}D^{\nu]}f'(R) + D^{\mu[}\xi^{\nu]}f'(R) + \xi^{[\mu}\hat{k}^{\nu]})$

choose $\quad k^\mu = (g^{\mu\nu}\Delta^\rho_{\nu\rho} - g^{\nu\rho}\Delta^\mu_{\nu\rho})f'(R) \quad$ where $\quad \Delta^\mu_{\nu\rho} \equiv \Gamma^\mu_{\nu\rho} - \overline{\Gamma^\mu_{\nu\rho}}$

Hence $\quad Q = -\frac{1}{8\pi}\int_{S_\infty} d^{D-2}x \left(D^{[0}\hat{\xi}^{1]} - \overline{D^{[0}\hat{\xi}^{1]}} + \xi^{[0}\hat{k}^{1]} \right)$ ("Komar +").

choose ξ^μ : timelike KV $\xi^\mu = (1,0,0,0)$

find, for Kerr (de Sitter) solution : $M_i = f'(R_\infty)M$

Note that for purely quadratic theories : $f'(R_\infty) = f'(0) = 0...$
(Deser-Tekin 2007)

end)

• back to BH thermodynamics

Wald's entropy : $S_W = f'(R_{horizon})S_B$; ($S_B = A/4$)

Global charges : $M_i = f'(R_\infty)M$, $J_i = f'(R_\infty)J$

First law of BH thermodynamics holds : $T\,dS_W = dM_i - \Omega dJ_i$;

because $f'(R_{horizon}) = f'(R_\infty)$

Remarks :
– Could this condition be a way to prove uniqueness of Kerr for all $f(R)$ theories ?
– $M_i \neq M$: indication that SEP is violated ? If yes, what is the link between M_i and $M_{inertial}$ as defined in PPN formalism ??
(where $M_{inertial} - M$ is tested with Nordvedt η parameter :
$\eta = 4\beta - \gamma - 3 = 4 - 1/2 - 3 = 1/2$ if $f(R)$ potential can be ignored (Damour Esposito-Farese, 92)

When question marks begin to accumulate it is wiser to stop...

Thank you for your attention