Quantum gravity at one-loop and AdS/CFT

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(mostly) based on
S. Bhattacharyya, A. Grassi, M.M. and A. Sen,
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The AdS/CFT correspondence is supposed to provide a dual, gauge theory description of quantum gravity/string theory/M-theory on certain backgrounds.

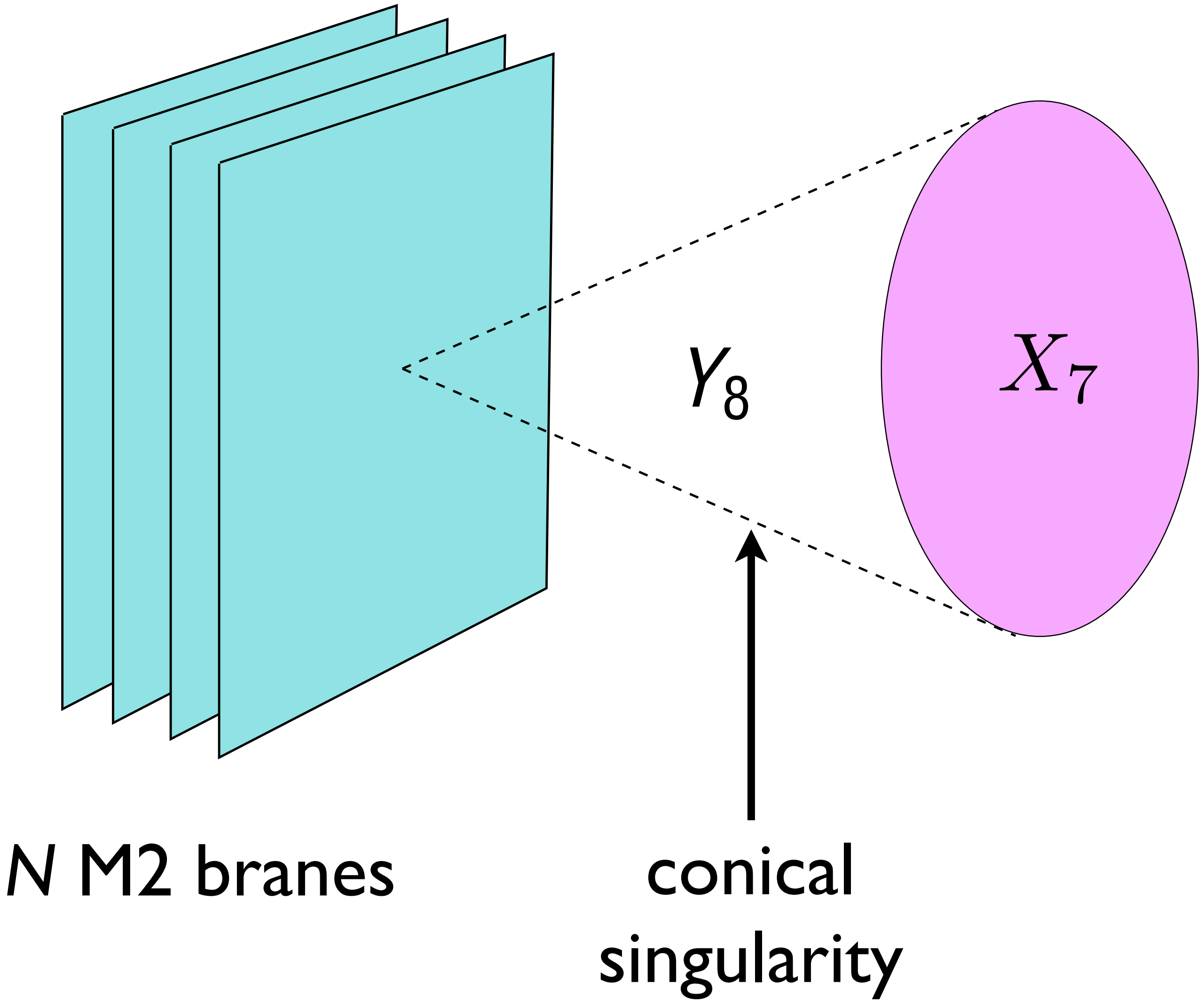
Most of the tests however have focused on the classical aspects of the correspondence. In the gauge theory side, this corresponds to the planar or large N limit

In this talk I will present a test of the correspondence beyond the planar limit. On the gauge theory side, we will deal with ABJM theory and its generalizations. On the AdS side, the test involves a one-loop computation in quantum (super)gravity

AdS4/CFT3

In the last four years or so, we have learned that superconformal Chern-Simons-matter theories describe M-theory backgrounds of the form

$$\mathrm{AdS}_4 \times X_7$$



X_7

Sasaki-Einstein: N=2

Tri-Sasaki: N=3

$\mathbb{S}^7/\mathbb{Z}_k$: N=6

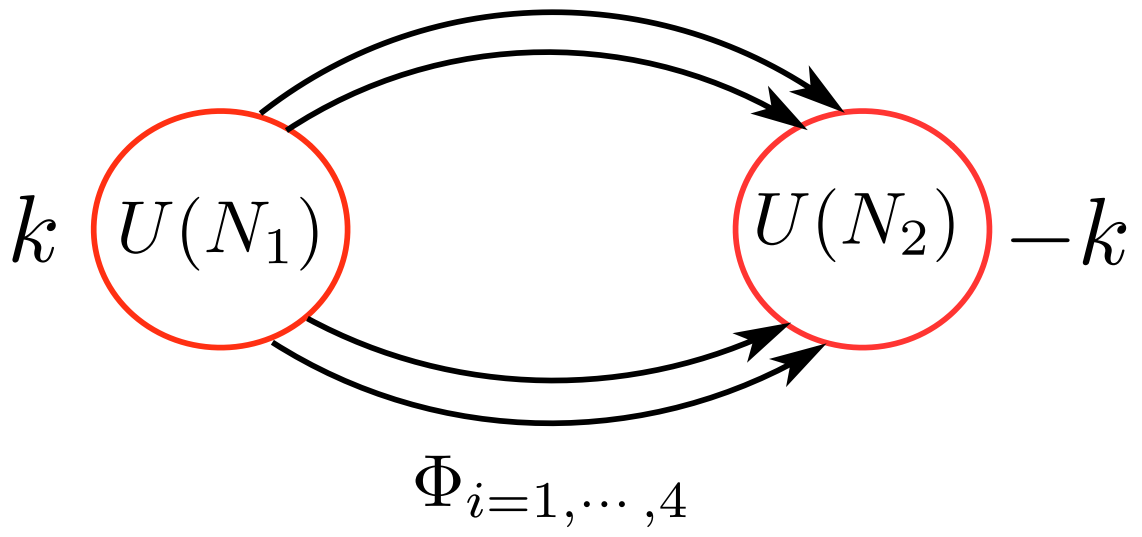
Basic building block of CFT3s: *Chern-Simons theory* and its supersymmetric extensions

CS level (must be an integer)

$$S_{\mathrm{CS}} = -\frac{k}{4\pi}\int_M \mathrm{tr}\Big(A\wedge \mathrm{d}A + \frac{2\mathrm{i}}{3}A\wedge A\wedge A\Big)$$

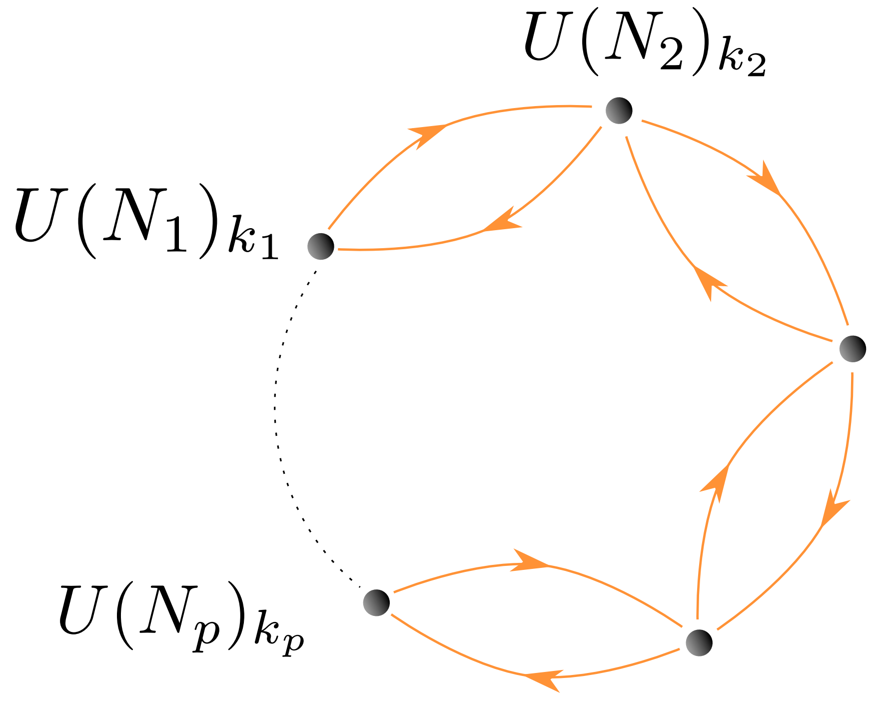
Susy extensions use the standard *N*=2, 3d vector multiplet, built on the gauge connection, and the *N*=2, 3d matter hypermultiplet. Many examples can be constructed by using *quivers*

Example: *ABJM theory*
2 CS theories +
4 hypers in the bifundamental
N=6 SUSY



N=3 "necklace quivers":
p nodes, CS theory at each node with gauge group $U(N_a)$ and levels k_a, $a = 1, \cdots, p$

$$\sum_{a=1}^{p} k_a = 0$$



[Jafferis-Tomasiello]

Here, the cone in the AdS dual is a hyperKahler manifold determined by the CS levels.

To simplify the discussion, we will mostly assume that all nodes have equal rank

$$N_a = N$$

M-theory duals

These are Freund-Rubin backgrounds with metrics of the form

$$\mathrm{d}s^2 = L^2\left(\frac{1}{4}\mathrm{d}s^2_{\mathrm{AdS}_4} + \mathrm{d}s^2_{X_7}\right)$$

The common rank *N* is related to the radius *L* by

$$6\,\mathrm{vol}(X_7)\left(\frac{L}{2\pi\ell_p}\right)^6 = N$$

$L/\ell_p \simeq 1$

L/ℓ_p

$L/\ell_p \gg 1$

Planckian sizes, strong quantum gravity effects

N small

weak curvature, classical SUGRA is a good approximation

N large: "thermodynamic limit"

The natural expansion in M-theory is in powers of ℓ_p/L

This leads to the *M-theory expansion* of CS-matter theories: a *1/N* expansion at fixed k_a. This is *not* the *'t Hooft expansion*, which is an expansion in powers of *1/N* at *fixed* 't Hooft parameter

$$\lambda = \frac{N}{k} \qquad\qquad k_a = n_a k, \quad k \gg 1$$

By reduction on a circle, these M-theory backgrounds lead to type IIA string backgrounds. The 't Hooft expansion corresponds to the *genus expansion* of the string. This is an expansion in powers of the string coupling constant, at a given curvature radius.

Going beyond tree-level in an effective theory

On the string side, testing AdS/CFT beyond the planar limit involves calculating higher genus string amplitudes. This is a hard but presumably well-defined problem

However, testing the subleading *1/N* terms in the M-theory expansion directly in M-theory is more delicate. This is because we only know the low-energy limit of M-theory, i.e. 11d SUGRA. This theory is an *effective theory* and it is *not* renormalizable [Deser-Seminara, Bern et al., ...]

A popular philosophy is to state that the gauge theory "defines" M-theory on these backgrounds. But then there is nothing to test. Testing AdS/CFT means that we can make sense of both sides

Of course, we can test AdS/CFT, at least classically, since an effective field theory like supergravity always makes sense at tree level. But can we go beyond tree-level in an effective, non-renormalizable field theory?

The answer is: *yes* -provided we introduce new parameters corresponding to the new counterterms, as in chiral perturbation theory [Weinberg, Gasser-Leutwyler, ...]. These can then be fixed by comparison to experiment or to the microscopic theory

pion
Lagrangian

$$\mathcal{L} = \frac{F_\pi^2}{4} \underset{\substack{\big\uparrow \\ \text{tree level}}}{\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)} + \ell_1 \underset{\substack{\big\uparrow \\ \text{generated} \\ \text{at one-loop}}}{\left[\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)\right]^2} + \cdots$$

But more is true: in the amplitudes computed in effective field theories, there are terms with a specific functional dependence which only depend on the tree-level effective Lagrangian and receive contributions at one-loop only.

Example: coefficients of logs in the $\pi\pi \to \pi\pi$ scattering amplitude

$$\mathcal{M} = -\frac{1}{96\pi^2 F_\pi^4}\left[3s^2 \log\left(\frac{-s}{\mu_1^2}\right) + t(t-u)\log\left(\frac{-t}{\mu_2^2}\right) + u(u-t)\log\left(\frac{-u}{\mu_2^2}\right)\right] + \cdots$$

Generically, non-analytic (log) terms in the amplitudes, coming from the infrared region of the loop integration, depend only on the parameters of the tree-level Lagrangian. Therefore, *they can be computed reliably, independently of the UV completion* [cf. Donoghue]

This philosophy can be applied to ordinary gravity, regarded as an effective theory. For example, the Newton potential between two large masses has a well-defined quantum correction which receives only one-loop corrections [Donoghue, Bjerrum-Bohr, Holstein, ...]

$$V(r) = -\frac{GM_1M_2}{r}\left[1 + \underset{\Big\uparrow}{\xi}\left(\frac{\ell_p}{r}\right)^2 + \cdots\right]$$

constant calculable
at one-loop

Log terms in the black hole entropy

The free energy and entropy of a black hole can be computed from the partition function of Euclidean quantum (super)gravity. At tree level one finds the standard Bekenstein-Hawking result [Gibbons-Hawking]

$$\frac{1}{k_B}S = \frac{A}{4\ell_p^2}$$

Can we compute quantum corrections to the entropy by using just low-energy, Euclidean (super)gravity? Following the intuition of effective field theory, we should focus on logarithmic corrections of the form

$$\frac{1}{k_B}S = \frac{A}{4\ell_p^2} + c\log\left(\frac{A}{\ell_p^2}\right) + \cdots$$

It is not difficult to show, by a simple estimate of diagrams, that the *n*-loop correction to vacuum graphs scales as

$$\left(\frac{\ell_p}{L}\right)^{(D-2)(n-1)}$$

$L :$ characteristic scale (BH radius, AdS radius)

$D :$ spacetime dimension

We conclude that neither higher loops, nor higher terms in the Lagrangian, can contribute to this log correction: *the coefficient c of the log correction to the black hole entropy can be computed reliably and at one-loop in (super)gravity* [Sen+Banerjee, Gupta, Mandal]

This is useful even if we have an UV completion in terms of superstring theory, since calculations in SUGRA are usually easier than in the full-fledged superstring

D-brane, microscopic counting in string theory makes precise predictions for the coefficient of the log term in the black hole entropy. It has been verified by Sen and collaborators that, in all cases where this can be tested, one finds agreement with the low-energy or “macroscopic” calculation

Partition functions and AdS/CFT

In this talk we will look at the *Euclidean partition functions of M-theory on the Freund-Rubin backgrounds*

$$\mathrm{AdS}_4 \times X_7$$

AdS/CFT gives us their microscopic description, in terms of the partition functions of *N=3* Chern-Simons-matter theories on the three-sphere (boundary of AdS4).

In the last 2-3 years, a lot of information has been found about these partition functions on the gauge theory side, by combining many different techniques. I will now summarize what is known about them

The partition function *at all orders in 1/N,* and up to non-perturbative corrections, is given by an Airy function:

$$Z(N,k_a) \propto \mathrm{Ai}\left[C(k_a)^{-1/3}(N-B(k_a))\right]$$
$$+\,\mathcal{O}\left(\mathrm{e}^{-\sqrt{kN}}, \mathrm{e}^{-\sqrt{N/k}}\right)$$

membrane instantons $\sim \mathcal{O}\left(\mathrm{e}^{-L^3/\ell_P^3}\right)$

worldsheet instantons $\sim \mathcal{O}\left(\mathrm{e}^{-L^2/\ell_s^2}\right)$

$B(k_a), C(k_a)$ calculable functions

$$C(k_a) \propto \mathrm{vol}(X_7)$$

[Drukker-M.M.-Putrov, Herzog et al., Fuji-Hirano-Moriyama, M.M.-Putrov]

Much recent progress on non-perturbative effects...

Example: ABJM theory

$$Z_{\text{ABJM}}(N,k) \propto \text{Ai}\left[\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right]$$

This expression resums the *full perturbative, quantum gravity partition function of M-theory/11d SUGRA* and includes the full perturbative series in ℓ_p/L

Tree level: classical gravity at large *N*

The leading large *N* result corresponds to the Euclidean quantum gravity partition function at tree level:

$$\log Z(N, k_a) \equiv F(N, k_a) \approx -\frac{2}{3} C(k_a)^{-1/2} N^{3/2}$$

(regularized) gravitational
action on-shell

This is the famous 3/2 scaling first found in the gravity side by [Klebanov-Tseytlin] and first explained microscopically in [Drukker-M.M.-Putrov]

One-loop: a universal log term

Going to next-to-leading order in the *1/N* expansion one finds

$$F(N,k_a) \approx -\frac{2}{3}C(k_a)^{-1/2}N^{3/2} - \frac{1}{4}\log N + \cdots$$

After using the AdS/CFT dictionary, this is of the form

$$\boxed{-\frac{3}{2}\log\left(\frac{L}{\ell_p}\right)}$$

Notice that it is *universal*: it does not depend on the X_7 compactification. The arguments presented before can be immediately adapted to conclude that *we should be able to reproduce this log contribution with a one-loop calculation in 11d SUGRA*

One-loop: heat kernel + zero modes

A generic one-loop contribution to a partition function factorizes into an integral over the *non-zero modes* of the relevant kinetic operator *A*, and an integral over its zero modes

The contribution of non-zero modes can be calculated in terms of the heat kernel

$$K(\tau) = \mathrm{e}^{-\tau A}$$

which has the short-distance De Witt-Seeley expansion

$$\mathrm{Tr} K(\tau) = \frac{1}{(4\pi)^{d/2}} \sum_{n=0}^{\infty} \tau^{n-d/2} \int \mathrm{d}^d x \sqrt{g}\, a_n(x)$$

This makes possible to extract the *log L* contribution from non-zero modes as

$$-\frac{1}{2}\ln\det{}'A = \left(\frac{1}{(4\pi)^{d/2}}\int \mathrm{d}^d x\sqrt{g}\,a_{d/2}(x) - \underset{\uparrow}{n_A^0}\right)\log\,L + \cdots$$

number of zero modes of *A*

Zero-modes are typically due to asymptotic symmetries, i.e. gauge transformations generated by non-normalizable parameters. The integration over them leads to a factor in *Z*

$$L^{\pm\beta_A n_A^0}$$

β_A depends on the type of field and the dimension of spacetime

Zero-modes in gravity calculations are subtle, and sometimes they are not properly taken into account. Fortunately, a detailed treatment has been given by Sen and collaborators in their computation of log corrections to black hole entropy.

In our calculation they are *the only source of contributions*, since in 11d the coefficient of the heat kernel *vanishes*

Are there zero modes for the 11d SUGRA fields on the Freund-Rubin background?

Graviton + ghost vector: no zero modes

Gravitino + ghost spinor: no zero modes

3-form:
no zero modes

4 ghost scalars: no zero modes

3 ghost one-forms: no zero modes

2 ghost two-forms:
one zero mode each in
Euclidean AdS4!
[Camporesi-Higuchi]

The contribution of these zero-modes to the free energy is

$$-(\beta_2 - 2)\log L$$

Independent of X_7 : explains universality!

A simple calculation of the scaling properties of the path-integral measure for two-forms gives

$$\beta_2 = \frac{D}{2} - 2$$ in *D* dimensions

This is because, in terms of an *L*-independent metric, the path integral measure is defined by

$$\int [\mathcal{D}B_{\mu\nu}] \exp\left[-L^{D-4}\int \mathrm{d}^D x\sqrt{g^{(0)}}g^{(0)\mu\nu}g^{(0)\alpha\beta}B_{\mu\alpha}B_{\nu\beta}\right] = 1$$

We then find in *D*=*11*

$$-\left(\frac{D}{2} - 4\right)\log L = -\frac{3}{2}\log L$$ *yes!*

Conclusions

- I have argued that certain quantum corrections can be calculated in ordinary (super)gravity. *Any* reasonable UV completion should reproduce these corrections. These has been successfully applied to log corrections to black hole entropy in string theory (note: loop quantum gravity does not seem to work)

- A similar situation appears in AdS/CFT: the microscopic, gauge theory side, should reproduce log terms in the partition function. Indeed it does, as we have seen here in the context of *N=3* Chern-Simons-matter theories

Prospects

- We have done the test in 11d. What happens in type IIA SUGRA in10d? The calculation is much more difficult (in progress) but should test in a non-trivial way the relationship between M-theory and type IIA strings

- There are by now many one-loop predictions in these theories (Wilson loops, membrane and worldsheet instantons...). Can we test them?