Slava Rychkov and his collaborators solved a 40-year-old problem posed by the Parisi-Sourlas conjecture

Slava Rychkov and his collaborators, Apratim Kaviraj and Emilio Trevisani, have recently published a series of four articles that answer a question which has puzzled physicists for more than four decades. They are thus finally shedding light on the role played by disorder in some fundamental models in statistical physics, such as the Random Field Ising Model and the branched polymers model.

Real systems are inherently disordered. Even in the purest of crystalline solids, defects consisting of irregularities and impurities appearing at random positions impact the properties of the material in ways that we have not yet fully understood. To investigate the role that disorder might play in real systems, physicists add small extra terms of disorder to well-known models, and study how their behavior is affected.

One of the best-known models in statistical physics is the Ising model. The Random Field Ising Model (RFIM) is obtained by adding a magnetic field with random intensity and direction but constant in time to each lattice site in the pure Ising model, and despite having been considered one of the simplest ways to account for disorder, its understanding has posed many challenges.

The question that Rychkov, Kaviraj and Trevisani set out to solve concerns the Parisi-Sourlas conjecture, which physicists Giorgio Parisi, a professor at Sapienza University of Rome, and Nicolas Sourlas, a colleague of Slava Rychkov at École normale supérieure, formulated in the late 1970s to relate the RFIM to the pure Ising model.

By arguing that the fixed points in a disordered system are characterized by supersymmetric properties, Parisi and Sourlas were able to show that something called "dimensional reduction" occurred: the critical exponents for the disordered case in dimension d were the same as the ones of its pure (i.e. non-disordered) correspondent in d-2.

Despite being able to explain a series of exact and numerical results obtained on this model, the conjecture showed at least as many limitations, as some of its implications, such as for example the absence of a phase transition for the RFIM in d=3, are clearly not true.

Slava Rychkov became very interested in this problem in 2017, at a conference on Disordered Systems organized at La Sapienza University in Rome. At that time, the spin-glass community had been stuck for decades in an attempt to understand the conjecture.

Fascinated by the long-lasting debate spurred by the Parisi-Sourlas conjecture, Prof. Rychkov promptly teamed up with Apratim Kaviraj and Emilio Trevisani, who at the time were both working as postdocs at École normale supérieure in Paris. After more than four years of hard work and sophisticated calculations, they have recently published four papers that promise to give an answer to this four-decade-old conundrum.

They divided the problem into several smaller ones, which they solved separately. In a first paper posted in December 2019, they used conformal bootstrap methods to prove that the existence of supersymmetric fixed points implies dimensional reduction [1].

The idea to solving the second part of the problem was provided by an article published in 1985 by physicist John Cardy, currently professor emeritus at the University of Oxford and professor at the University of California, Berkeley. In the paper, Cardy obtains supersymmetry and dimensional reduction for the RFIM by neglecting terms irrelevant at high dimension. In a second article [2], through a series of complex and subtle calculations, Rychkov and collaborators identified two perturbations that could not be neglected below a certain dimension $d_c \approx 4.5$, thus invalidating the Parisi-Sourlas hypothesis that the fixed points in the RFIM are characterized by supersymmetric properties at $d<d_c$. In a third paper [3] Kaviraj and Trevisani extended their reasoning to the case of branched polymers and were able to explain why the conjecture seems to hold in all dimensions for this class of systems. A fourth paper [4] by Rychkov, Kaviraj and Trevisani summarizes their results and offers a greater picture.

The papers on RFIM have been published by the Journal of High Energy Physics and by Physical Review Letters, and have been positively welcomed by the physics community.

Giorgio Parisi commented: *"Slava Rychkov and his collaborators have done magnificent work by understanding in greater detail the origin of the so-called 'Parisi-Sourlas' supersymmetry and the reasons for its success and its failure. It is an important step forward in this very difficult problem that has attracted my attention for more than forty years."*

John Cardy said: *"This work is what I should have tried to do years ago, but at the time the task looked too daunting - so many different terms to account for. I'm pleased that it seems to resolve in a plausible scenario".*

Edouard Brézin, a physicist and a professor emeritus at École normale supérieure, said: *"At last, after more than forty years of bewilderment, the long-standing puzzle on why and when supersymmetry implies a dimensional reduction is understood. The beautiful supersymmetric (SUSY) arguments of Parisi-Sourlas, for the random field Ising model, was known to fail in three dimensions, whereas no failure was apparent for the model of branched polymers in which Parisi and Sourlas had found a similar SUSY, and a similar dimensional reduction. Rychkov and his collaborators have discovered the origin of the problem. The presence of a SUSY breaking operator, becoming relevant near five dimensions, explains why the dimensional reduction fails for the RFIM in four or three dimensions. Furthermore, the same authors have shown that the branched polymer problem is free of any relevant SUSY breaking".*

If they are right, Slava Rychkov and his collaborators might have finally found the conditions under which the Parisi-Sourlas conjecture is valid, thus answering a question that physicists have been struggling with for 40 years.

[1] A. Kaviraj, S. Rychkov and E. Trevisani, *Random field Ising model and Parisi-Sourlas supersymmetry. Part I. Supersymmetric CFT*, *JHEP* **04** (2020) 090 [arXiv:1912.01617]
[2] A. Kaviraj, S. Rychkov, and E. Trevisani, *Random Field Ising Model and Parisi-Sourlas Supersymmetry II. Renormalization Group*, *JHEP* **03** (2021) 219 [arXiv:2009.10087v3]
[3] A. Kaviraj and E. Trevisani, *Random Field φ^3 Model and Parisi-Sourlas Supersymmetry,* [arXiv:2203.12629]
[4] A. Kaviraj, S. Rychkov and E. Trevisani, *Parisi-Sourlas Supersymmetry in Random Field Models*, *Phys. Rev. Lett.* **129**, 045701 [arXiv:2112.06942]

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