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Mathematical Physics

My parents were philologists (and so is my brother, Tzvetan), thus, as a child, I did not benefit from an early exposure to exact sciences. I inherited instead an interest in the history of science and a taste for communication between science and humanities.

It has been always fascinating to discover the same mathematical law governing apparently different phenomena. Instances of Felix Klein's pre-established harmony between mathematics (a game of pure thought) and natural phenomena seem even more impressive. The realization that numbers govern the sounds of music was a leading theme for Pythagoras and his school, the founders of mathematics as a science. Here is a more recent (and less popular) example of this type. After years of intensive study, following previous theoretical and experimental work of many physicists, Max Planck wrote in 1900 his formula for the energy distribution of black-body radiation that marked the beginning of quantum theory. Nobody seems to have noticed, however, that the small frequency (or high temperature) expansion of Planck's formula gives rise to the

Bernoulli numbers. Jacob Bernoulli (1654–1705) introduced his numbers in the context of probability theory. In the 19th century they were related to modular forms that are basic to analytic number theory. The integral coefficients in the Fourier expansion of such forms, which play a prominent role in the context of number theory, appear as multiplicities in the interpretation of statistical mechanics.

My young collaborator N. M. Nikolov and I pointed out (in a paper completed during my recent stay at the IHES, following a stimulating discussion with Maxim Kontsevitch) that the unique, normalized, modular form of weight four reproduces Planck's black-body energy distribution in conformally compactified space-time.

I also recall with pleasure a paper by Ya. Stanev and myself, initiated at the IHES (benefiting from the advice of another visitor, B. B. Venkov), in which the Galois group for the roots of unity was used to solve the Schwarz (finite monodromy) problem for the Knizhnik-Zamolodchikov equation, providing another link between number theory and, this time, conformal field theory models.

The legendary Alexandre Grothendieck (to whom the IHES owes much of its fame of the 1960s) explains in his "Recoltes et semailles" that a major stimulus for his abstract approach to algebraic geometry was the desire to find a common ground for the geometry of the continuum and the discrete «geometry of numbers». Alain Connes provides another unifying view of the discrete and the continuous—within noncommutative geometry. I vividly remember the series of seminar talks of Dirk Kreimer on his Hopf algebra approach to renormalization when the discussion (often continuing at lunch) revealed a close link with the Hopf algebra Connes and Moscovici had introduced, studying the transverse index theorem in non-commutative geometry. The visionary talk of Pierre Cartier at the 40th anniversary of the IHES, in which he dreamed of a unification of the ideas of Grothendieck, Connes-Kreimer and Kontsevitch, seems nowadays closer to reality than eight years ago.

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