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Knowing, Understanding, Discovering

A great mystery which presents itself to each one of us is that we are conscious of two distinct entities—our thought and the outside world. "I think, therefore I am," said Descartes. However, all of us (or nearly all of us) admit that "the other" also exists. A reality that each of us yearns, in one way or another, to know, in order to master it or simply to understand it. But what does it mean, "to understand"? This word has many meanings. For the scientific mind it means first of all to classify phenomena and pinpoint the relationships between them. The classification and the relationships exist. This is an observable fact that distinguishes outside reality from that which we construct sometimes in our sleep. The second stage is to group these relationships in a more general law of which they are the consequences and, finally, to transpose this law in a model constructed by our thought. Perhaps to understand is always to find the adequacy between our thought and a part of the reality of the outside world.

In scientists' construction of models, mathematics is a fundamental tool. It has long been useful; it is now indispensable for the formulation of observed facts. Reality has been revealed to be much richer and stranger than what our senses can perceive. It is difficult for a physicist to describe with exactitude the quantic phenomena using everyday words, but ever more elaborate mathematical models can translate their properties.... For the mathematician, these models become reality itself. It is marvellous to see that mathematics, a tool of thought and generator of concepts, possesses such an adequacy with reality. However, I think that no model can completely exhaust reality. I hope that new tools and observable facts that we cannot foresee will reserve surprises for future generations. The extraordinary complexity of biological systems now also requires mathematical modelization. This in turn initiates the creation of new branches and attempts to construct a model of thought itself. However, will this be enough to explain the "I am" of Descartes's aphorism?

Having expressed these reserves on the capacity of mathematics to find a final solution to every-

thing, I shall now sing its praises. It is a universal language where truths are absolute and unquestionable even if their verification is sometimes arduous. With any other language it is difficult to convey completely a relatively subtle thought, even with one's mother tongue, let alone to translate it in another language. Mathematics is a precious tool to construct models of experimental realities but it is also a marvellous reality in itself, which all intelligences on planet Earth (and maybe on other planets) can consult if it interests them. Many physicists searching for models of physical phenomena have discovered new mathematical concepts which have fascinated them. The mathematical beings thus created have lives of their own and have engendered others. A lot has been written about the mutual fertilization of mathematics and physics, and I will say no more about it.

Let me move on to more personal considerations. For me, working in mathematics is an escape route into an ideal world, where the journey is limited only by one's own limitations. The country is full of truths to be learned or truths to be discovered; I love to learn, but it is fantastic to find a new truth, even a very small one. A mathematical result concerning a model coming from physics has a very special flavour, because it presages a yet unknown property of the inexhaustible reality we are immersed in. How pleased the researcher is, even if the discovery is a very minor one! I love the mathematician's trade, a mixture of reasoned vision and the skilled work of calculations. One cannot make calculations without a guiding thread concerning their structure and aim, neither with the human mind

nor the computer. Certain results—sometimes surprising—are obtained only after long calculations which one cannot always circumvent in later proofs.

I will end by saying that the mathematical universe exists through the community of mathematicians who create it—or who discover it, if the reader prefers that philosophy. It is a great joy for a mathematician to belong to this community of citizens of the same ideal country. Specialists of the same discipline, whatever their nationality, share a certain number of truths and curiosity for unresolved problems. Their common knowledge and interests unite them more than rivalries of priority could divide them. Exchanges of points of view are stimulating and enriching. Work in collaboration is particularly gratifying. Mathematical affinities sometimes turn into real friendship, the salt of life.

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