BOLTZMANN
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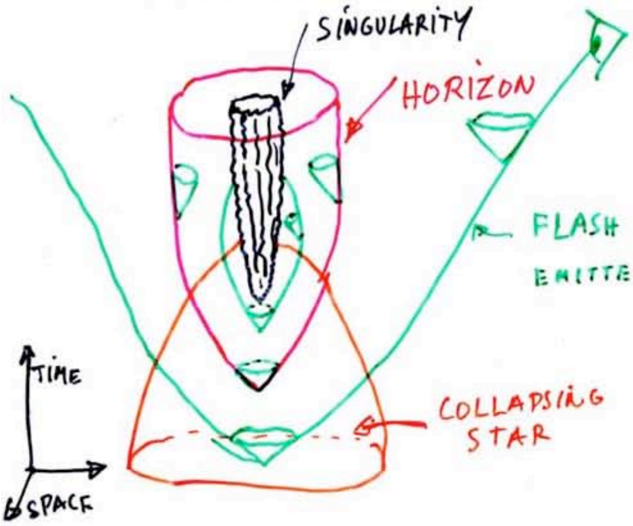
THE ENTROPY OF BLACK HOLES

Thibault DAMOUR

INSTITUT DES HAUTES ETUDES SCIENTIFIQUES

BLACK HOLE (Oppenheimer, Snyder '39 Kruskal '60 ; Penrose '60 ; ...)

COLLAPSE OF A STAR IN GENERAL RELATIVITY



EXTERNAL METRIC (Schwarzschild '16)

$R_{\mu\nu} = 0$

SPHERICAL SYMMETRY

$$\boxed{ds^2_{SCHW} = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)}$$

"SCHWARZSCHILD RADIUS": $r_s = \frac{2GM}{c^2}$ $\leftrightarrow$ "HORIZON": REGULAR HYPERSURFACE

$r = r_s : \mathcal{U}(v, \theta, \varphi)$

Eddington-Finkelstein coordinates:

$(t, r, \theta, \varphi) \to (v, r, \theta, \varphi)$ $\quad \boxed{v \equiv t + r_*}$

$$\boxed{r_* \equiv \int \frac{dr}{1 - 2GM/c^2 r} \equiv r + \frac{2GM}{c^2} \ln\left(\frac{r}{\frac{2GM}{c^2}} - 1\right)}$$

$$\boxed{ds^2_{SCHW} = -\left(1 - \frac{2GM}{c^2 r}\right) dv^2 + 2\, dv\, dr + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)}$$

$r = \frac{2GM}{c^2}$: REGULAR NULL HYPERSURFACE  (SINGULARITY: $r = 0$ SPACELIKE)

BLACK HOLE = PHYSICAL OBJECT: 'GRAVITATIONAL SOLITON'

- 'NO HAIR THEOREMS' (Israel, Carter, ...)

ISOLATED BLACK HOLE RELAXES (ON TIME SCALE ~ GM/c^3) TO A (CLASSICAL) STATE CHARACTERIZED BY A FEW GLOBAL PARAMETERS: $\underset{\text{MASS}}{M}$, $\underset{\text{ANGULAR MOMENTUM}}{J}$, $\underset{\text{ELECTRIC CHARGE}}{Q}$

UNIQUE (CLASSICAL) EXTERNAL GEOMETRY: $\underset{\text{KERR-NEWMAN}}{ds^2}(M, J, Q)$

- INEQUALITY ($c = 1$)

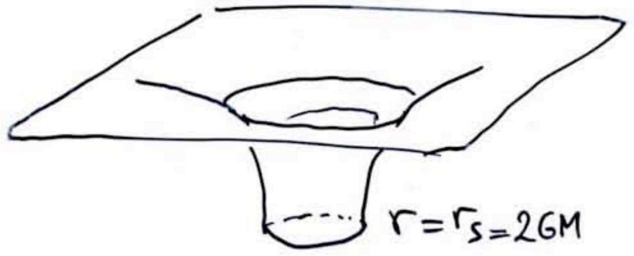
$$\boxed{(GM)^2 \geqslant \left(\frac{J}{M}\right)^2 + GQ^2}$$

EXTREMAL BHs: $(GM)^2 = \left(\frac{J}{M}\right)^2 + GQ^2$

GEOMETRY OF EXTERNAL SPATIAL SECTIONS

NON-EXTREMAL (SCHWARZSCHILD)

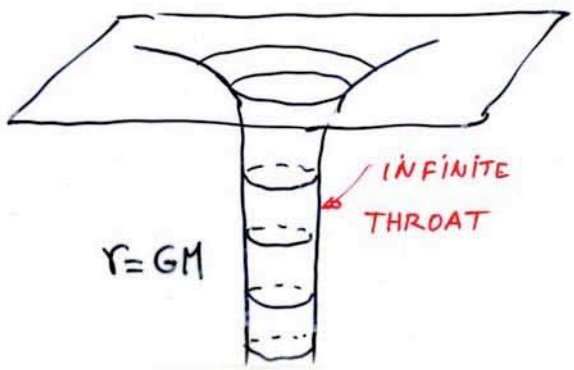
$$t = cst: \quad ds^2 = \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



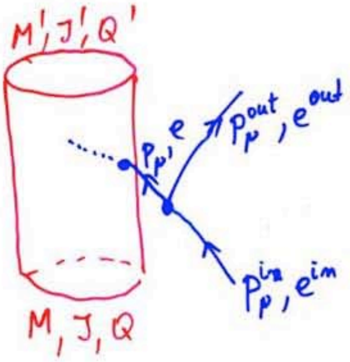
EXTREMAL REISSNER-NORDSTRØM

$$\sqrt{G}\, M = |Q|$$

$$ds^2 = -\left(1 - \frac{GM}{c^2 r}\right)^2 c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{GM}{c^2 r}\right)^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



BLACK HOLE ENERGETICS (Penrose, Christodoulou-Ruffini)



$$M' = M + \delta M \quad ; \quad \delta M = E = -p_t$$
$$J' = J + \delta J \quad ; \quad \delta J = p_\varphi$$
$$Q' = Q + \delta Q \quad ; \quad \delta Q = e$$

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = -\mu^2$$

$$\boxed{\delta M - \frac{a\delta J + r_+ Q\delta Q}{r_+^2 + a^2} = \frac{r_+^2 + a^2\cos^2\theta}{r_+^2 + a^2}\,|p^r|_{\mathcal{H}} \geqslant 0}$$

$\begin{pmatrix} c=1 \\ G=1 \end{pmatrix}$ $\quad a \equiv \frac{J}{M}$ $\quad r_+ \equiv M + \sqrt{M^2 - a^2 - Q^2}$ 'radius of horizon'

'radial kinetic energy' on the HORIZON

INEQUALITY
$$\boxed{\delta M \geqslant \frac{a\delta J + r_+ Q\delta Q}{r_+^2 + a^2}}$$

REVERSIBLE TRANSFORMATIONS
$$\delta M = \frac{a\delta J + r_+ Q\delta Q}{r_+^2 + a^2}$$

$(\delta M, \delta J, \delta Q)$
$+ (\delta' M, \delta' J = -\delta J, \delta' Q = -\delta Q)$
$\Rightarrow (M + \delta M + \delta' M, J + \delta J + \delta' J, Q + \delta Q + \delta' Q)$
$= (M, J, Q)$

IRREVERSIBLE TRANSFORMATIONS
$$\delta M > \frac{a\delta J + r_+ Q\delta Q}{r_+^2 + a^2}$$

ONE CAN INTEGRATE A SEQUENCE OF REVERSIBLE TRANSF.
(Christodoulou-Ruffini)

$$dM = \left(\frac{a}{r_+^2 + a^2}\right)(J,Q)\,dJ + \left(\frac{r_+ Q}{r_+^2 + a^2}\right)(J,Q)\,dQ = d\big(M(J,Q)\big)$$

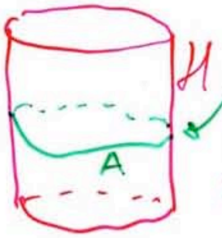
IRREVERSIBILITY IN BLACK HOLE PHYSICS

Christodoulou-Ruffini
(MASS FORMULA)

$$\boxed{M^2 = \left(M_{irr} + \frac{Q^2}{4M_{irr}} \right)^2 + \frac{J^2}{4M_{irr}^2}}$$

WITH

$$\boxed{\delta M_{irr} \geqslant 0 \quad \begin{matrix} \nearrow \delta M_{irr} = 0 \text{ REVERSIBLE} \\ \searrow \delta M_{irr} > 0 \text{ IRREVERS.} \end{matrix}}$$



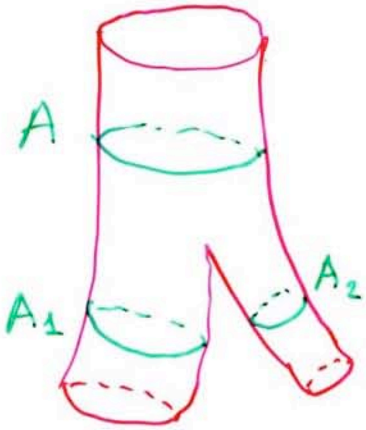
AREA OF SPATIAL SECTION OF HORIZON

$$A = 4\pi(r_+^2 + a^2) = 16\pi M_{irr}^2$$

hence

$$\boxed{\delta A \geqslant 0}$$

MORE GENERALLY (Hawking)



$$\boxed{\delta \left(\sum_a A_a\right) \geqslant 0}$$

BLACK HOLE THERMODYNAMICS

If one defines

$$\boxed{S_{BH} \equiv \alpha A}$$

the following ANALOGIES hold

- "1^{ST} LAW OF BH THERMODYNAMICS"

$$\boxed{dM = \Omega\, dJ + \Phi\, dQ + T_{BH}\, dS_{BH}}$$

$\Omega = \dfrac{a}{r_+^2+a^2}$ $\quad \Phi = \dfrac{Q r_+}{r_+^2+a^2}$

$(\ell^\mu \partial_\mu)_\mu = (\partial_t + \Omega \partial_\varphi)_\mu$

SURFACE GRAVITY

$$\boxed{\ell^\nu \nabla_\nu \ell^\mu = \kappa\, \ell^\mu \qquad \kappa = \frac{\sqrt{M^2 - a^2 - Q^2}}{r_+^2 + a^2}}$$

SURFACE GRAVITY

$$\boxed{T_{BH} = \frac{1}{\alpha}\left(\frac{\partial M}{\partial A}\right)_{J,Q} = \frac{\kappa}{8\pi\alpha}}$$

SCHWARZ.: $\kappa = \dfrac{GM}{r_s^2} = \dfrac{1}{4M}$

$\boxed{\text{EXTREMAL BH}: \ \kappa = 0}$

- "0^{TH} LAW OF BH THERMODYNAMICS" (Carter)

$$\boxed{T_{BH} \text{ is UNIFORM OVER } \mathcal{H}}$$

- "2^{ND} LAW OF BH THERMODYNAMICS"

$$\boxed{\delta S_{BH} \geqslant 0}$$

- LOCAL VERSIÒN OF 2^{ND} LAW (Damour)

'FLUID OF GENERATORS'

$dS_{BH} = \alpha dA$

SHEAR

$$\boxed{\frac{d}{dt}(dS_{BH}) - \tau \frac{d^2}{dt^2}(dS_{BH}) = \frac{dA}{T_{BH}}\left[2\eta\, \sigma_{AB}\sigma^{AB} + \zeta\theta^2 + \rho\left(\vec{J}_{BH} - \sigma_{BH}\vec{v}\right)^2\right]}$$

JOULE EFFECT

$\tau = \kappa^{-1}$

SHEAR VISCOSITY $\eta = \frac{1}{16\pi}$

BULK VISCOS. $\zeta = -\eta$

SURFACE RESISTIVITY $\rho = 4\pi = 377\,\text{OHM}$

BEKENSTEIN'S BLACK HOLE ENTROPY ('72)

SEVERAL ARGUMENTS:

- ? REVERSIBLE TRANSF.: $\underbrace{\delta M - \Omega \delta J - \Phi \delta Q}_{\propto \delta M_{irr} \propto \delta A} = \frac{r_+^2 + a^2\cos^2\theta}{r_+^2 + a^2} |p^r|_{\mathcal{H}}$

  NEED $p^r = 0$ @ $r = r_+$

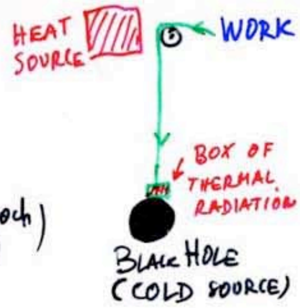
  CONFLICT WITH HEISENBERG: $[r, p_r] = i\hbar$

  → QUANTUM LIMITATION: $\delta A \geq \mathcal{O}(\hbar)$ WHEN ABSORBING ONE PARTICLE

  i.e. LOOSING ONE BIT OF INFORMATION

  → SUGGESTS $\boxed{S_{BH} = \hat{\alpha} \frac{c^3}{\hbar G} A = \hat{\alpha} \frac{A}{\ell^2_{PLANCK}}}$ WITH $\boxed{\hat{\alpha} = \mathcal{O}(1)}$

- QUANTUM LIMITATIONS ON EFFICIENCY OF CARNOT CYCLES



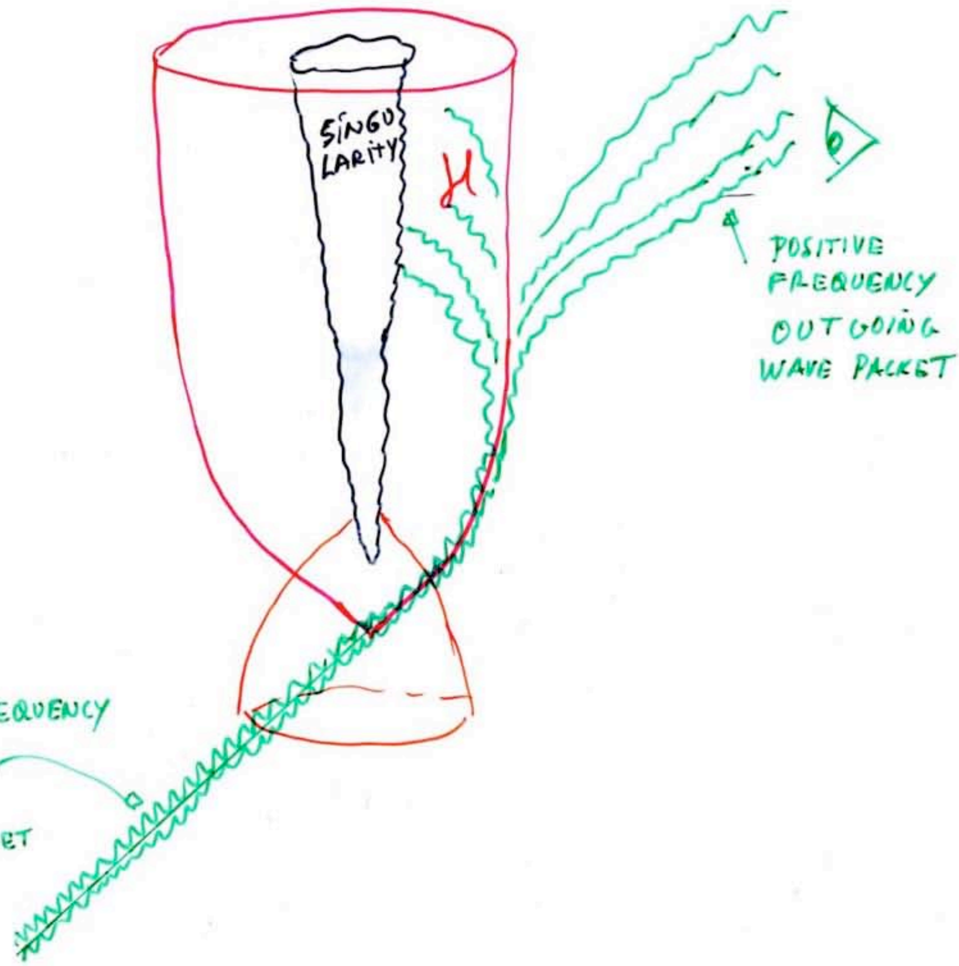
(Geroch '71)

→ $\boxed{T_{BH} \sim \frac{\hbar}{c} \kappa}$

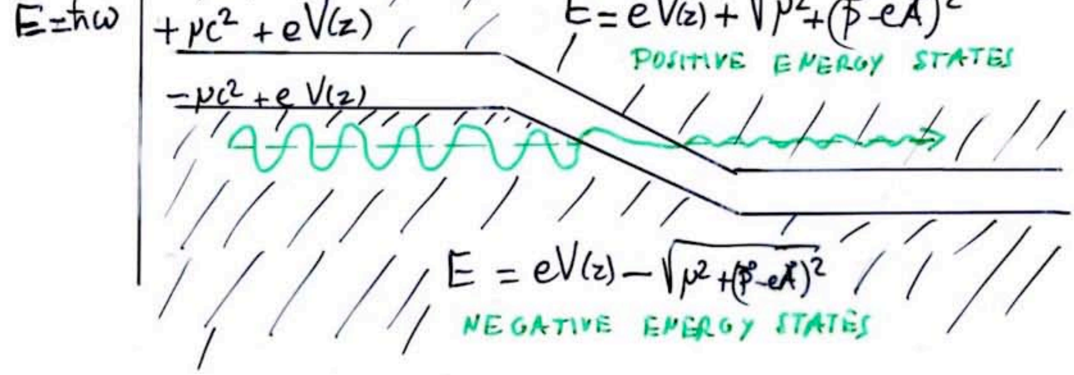
- SUGGESTION OF ∃ 'GENERALIZED SECOND LAW'

$$\boxed{\delta (S_{BH} + S_{EXTERIOR}) \geq 0}$$

HAWKING RADIATION ('74)



SIMILAR TO 'KLEIN PARADOX'



CREATION OF PARTICLE-ANTIPARTICLE PAIR $\propto$ | TRANSITION '$E<0$' $\rightarrow$ '$E>0$' $|^2$

MASSLESS SCALAR FIELD IN SCHWARZSCHILD SPACETIME

$$0 = \Box_g \varphi \equiv \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g}\, g^{\mu\nu} \partial_\nu \varphi \right)$$

$$\varphi_{\omega,\ell,m}(t,r,\theta,\varphi) = \frac{e^{-i\omega t}}{\sqrt{2\pi|\omega|}} \frac{u_{\omega\ell m}(r)}{r} Y_{\ell m}(\theta,\varphi)$$

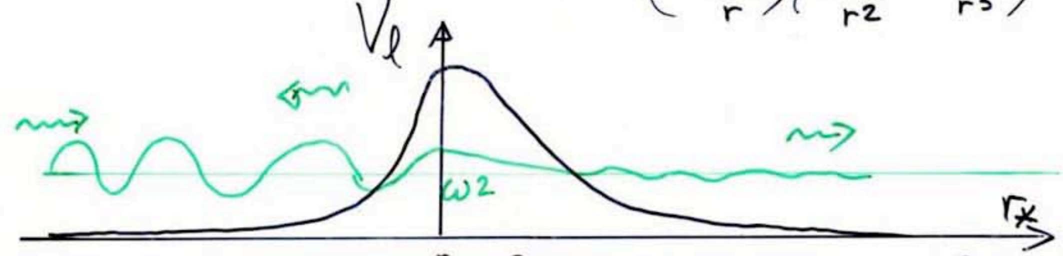
$$\boxed{r_* \equiv \int \frac{dr}{1-\frac{2M}{r}} = r + 2M \ln\left(\frac{r}{2M} - 1\right)}$$

$$-\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-2M/r} = \left(1-\frac{2M}{r}\right)\left[-dt^2 + dr_*^2\right]$$

$$\frac{\partial^2 u}{\partial r_*^2} + \left(\omega^2 - V_\ell[r(r_*)]\right) u = 0$$

EFFECTIVE RADIAL POTENTIAL

$$V_\ell(r) = \left(1-\frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)$$



$r_* \to -\infty$	$r_* \sim 0$	$r_* \to +\infty$
HORIZON $r \to 2M$	$r \sim 3M$	$r \to +\infty$
		INFINITY

NEAR HORIZON

$u_\omega^{\mathcal{H}} = e^{-i\omega(t - r_*)}$ $\Leftarrow$ MODULO PENETRABILITY FACTOR OF POTENTIAL BARRIER ('GREY BODY FACTOR' $\Gamma_\ell(\omega)$) $\Leftarrow$ POSITIVE FREQUENCY AT ∞: $u_\omega(r) \sim e^{-i\omega(t - r_*)}$

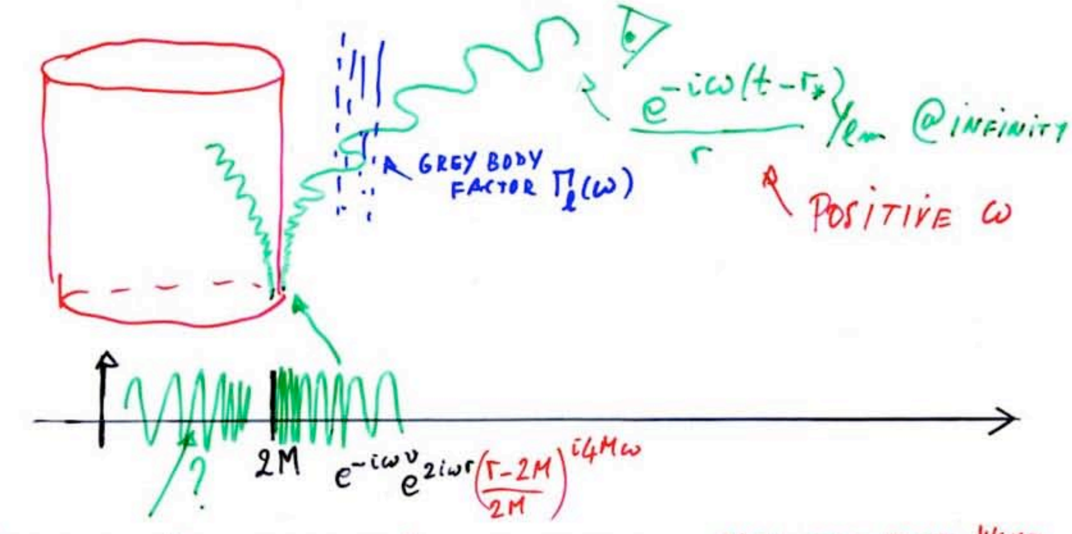
USE EDDING.-FINK COORDS

$$v = t + r_* \Rightarrow \quad t - r_* \equiv v - r_*$$

$$\left[\varphi_\omega^{out}(v,r)\right]_{\substack{\text{NEAR} \\ \text{BUT} \\ \text{OUT} \\ \mathcal{H}}} \propto e^{-i\omega(v - 2r_*)} = e^{-i\omega v} e^{+2i\omega r} \left(\frac{r-2M}{2M}\right)^{i4M\omega}$$

CRUCIAL FACTOR

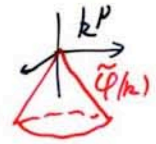
HAWKING TUNNELLING

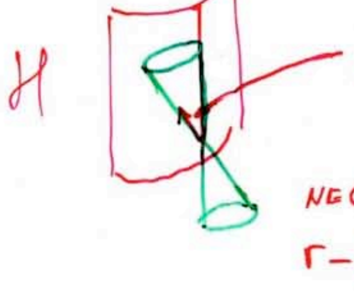


? CONTINUATION BELOW $r = 2M$ SO AS TO HAVE A NEGATIVE FREQ. WAVE PACKET

LOCAL (FLAT SPACE) CRITERION FOR '$\omega < 0$'

$$\varphi_-(x) = \int_{C_-} d^4k\, \tilde{\varphi}(k)\, e^{i k_\mu x^\mu} \leftrightarrow \varphi_-(x^\mu + i y^\mu) \text{ EXISTS}$$





$r \to r - \varepsilon$ IS FUTURE DIRECTED

$\Downarrow$

NEG. FREQ. WAVE PACKET DEFINED BY $r - 2M \to r - 2M - i\varepsilon$ ANALYTIC CONTINUATION

$$n_\omega(v,r) = N_\omega\, \varphi_\omega^{out}(v, r - i\varepsilon) \propto (r - 2M - i\varepsilon)^{i4M\omega}$$

$$= N_\omega \left[\theta(r-2M)\, \varphi_\omega^{out}(r-2M) + e^{4\pi M\omega}\, \theta(2M-r)\, \varphi_\omega^{out}(2M-r) \right]$$

$$(e^{-i\pi})^{i4M\omega} = e^{+4\pi M\omega}$$

THEN

$$\boxed{|N_\omega|^2 = \frac{1}{e^{8\pi M\omega} - 1}}$$

BOLTZMANN-PLANCK FACTOR

HAWKING TEMPERATURE

RATE OF PARTICLE CREATION BY BLACK HOLE

$$\boxed{\begin{aligned}\frac{d}{dt}\langle N\rangle &= \int \frac{d\omega}{2\pi} \sum_{l,m} |N_\omega|^2\, \Gamma_l(\omega) \\ &= \sum_{l,m} \int \frac{d\omega}{2\pi} \frac{1}{e^{8\pi M\omega}-1}\, \Gamma_l(\omega)\end{aligned}}$$

PLANCK SPECTRUM

GREY BODY FACTOR: TRANSMISSION COEFFICIENT THROUGH $V_l(\omega)$

$=$ ABSORPTION CROSS SECTION $\sigma_l(\omega)$

TEMPERATURE

$$\boxed{T_{BH} = \frac{1}{8\pi M} = \frac{1}{2\pi}\frac{\hbar}{c}\,\kappa}$$

SCHWARZSCHILD

GENERAL BLACK HOLE

SURFACE GRAVITY $l^\nu \nabla_\nu l^\mu = \kappa l^\mu$

$$dM = \dots + \frac{\kappa\, dA}{8\pi}$$

KERR-NEWMAN: $\quad \kappa = \dfrac{\sqrt{M^2-a^2-Q^2}}{r_+^2+a^2} \Rightarrow$ $\boxed{T_{BH}=0 \text{ FOR EXTREMAL BH's}}$

BEKENSTEIN-HAWKING ENTROPY

$$\boxed{S_{BH} = \frac{1}{4}\frac{c^3}{\hbar G} A = \frac{1}{4}\frac{A}{l^2_{PLANCK}}}$$

$\frac{1}{4}$ VALID IN ALL DIMENSIONS

CHALLENGES AND PUZZLES

- PHYSICAL MEANING OF $S_{BH} = \frac{1}{4}\frac{A}{\ell_P^2}$?

$$S_{BH} = \ln\left(\#\ \text{microscopic states}\right) \ ?$$

UNIVERSALITY OF $\frac{1}{4}$ → CONSISTENCY CONDITION ON ANY 'QUANTUM COMPLETION' OF GR

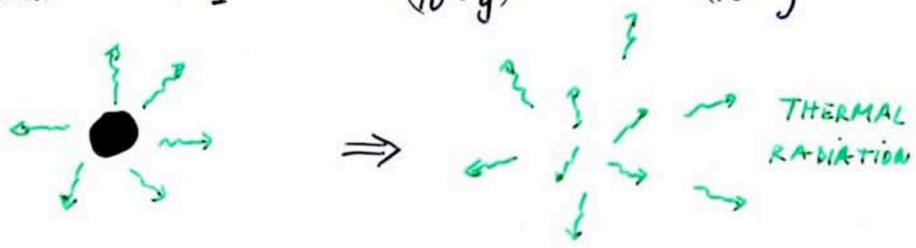
- ? GENERALIZED 2^{ND} LAW ? $\delta\left(S_{EXT} + \frac{1}{4}\frac{A}{\ell_P^2}\right) \geqslant 0$?

- FINAL STATE OF EVAPORATION PROCESS ?

SCHW: $$\frac{dM}{dt} \sim -A\,T_{BH}^4 \sim -(M)^2\left(\frac{1}{M}\right)^4 \sim -M^{-2}$$

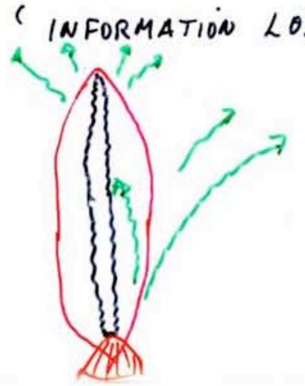
$$\Rightarrow \quad M^3 \sim M_0^3 - (t - t_0)$$

$$t_{\text{EVAPORATION}} \sim t_P\left(\frac{M}{M_P}\right)^3 \sim 10^{-44}_{s}\left(\frac{M}{10^{-5}g}\right)^3 \sim 10^{10}yr\left(\frac{M}{10^{14}g}\right)^3$$

IF 

- ? PURE STATE → THERMAL STATE ? UNITARITY VIOLATION ?

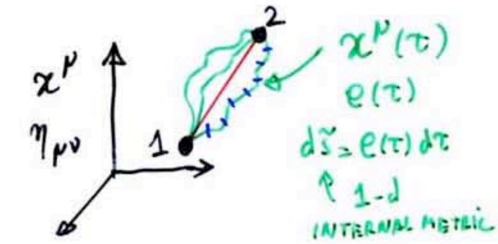
- ? 'INFORMATION LOSS' ?



? IS THERE A REMNANT ?

? WHAT HAPPENS TO THE CORRELATED MODES THAT FALL TOWARDS THE BH SINGULARITY ?

QUANTUM RELATIVISTIC PARTICLES AND QUANTUM FIELD THEORY



$$S[x^\mu(\tau), e(\tau)] = \frac{1}{2}\int d\tau \left[\frac{1}{e(\tau)}\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} - e(\tau)\, m^2\right]$$

CLASSICAL EOM (GAUGE $e=1$)

$$\begin{cases} \dfrac{d^2 x^\mu}{d\tau^2} = 0 \rightarrow x^\mu = x^\mu_0 + p^\mu \tau \\ \eta_{\mu\nu}\dfrac{dx^\mu}{d\tau}\dfrac{dx^\nu}{d\tau} = -m^2 \rightarrow \eta^{\mu\nu} p_\mu p_\nu = -m^2 \end{cases}$$

FIRST QUANTIZATION:

EITHER FEYNMAN: $$\mathcal{A}(2;1) = \int \frac{\mathcal{D}x^\mu(\tau)\,\mathcal{D}e(\tau)}{\text{diff}} e^{iS[x,e]} \propto \int_0^\infty dT \int \frac{d^Dp}{(2\pi)^D} e^{ip_\mu(x_2^\mu - x_1^\mu)} e^{-\frac{i}{2}T(p^2+m^2)}$$

$$= \int \frac{d^Dp}{(2\pi)^D} \frac{e^{ip\cdot(x_2-x_1)}}{i(p^2+m^2)} = G_F(x_2^\mu - x_1^\mu)$$

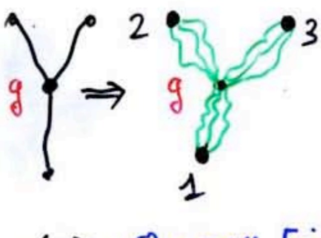
OR HEISENBERG: $\hat{x}^\mu(\tau) = \hat{x}^\mu_0 + \hat{p}^\mu \tau$ WITH $[\hat{x}^\mu, \hat{p}_\nu] = i\hbar\,\delta^\mu_\nu$

SINGLE PARTICLE STATES: $|p_\mu; m\rangle$

$$\hat{p}_\mu\,|p_\mu; m\rangle = p_\mu\,|p_\mu; m\rangle$$

CONSTRAINT: $(\eta^{\mu\nu}\hat{p}_\mu\hat{p}_\nu + m^2)|p_\mu; m\rangle = 0$

SECOND QUANTIZATION: SPLITTING WORLDLINES $\Longleftrightarrow$ INTERACTING MULTI-PARTICLE STATES

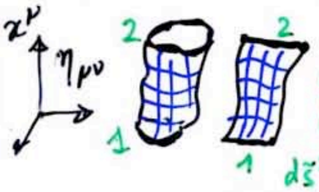


$$\Rightarrow \mathcal{A}(2,3;1) = \int_{\substack{\text{SPLITTING} \\ \text{WORLDLINES}}} \mathcal{D}x\,\mathcal{D}e\, e^{iS} = \int d^Dx\; G_F(x_2 - x)\, G_F(x_3 - x)\, G_F(x - x_1)$$

$\Longleftrightarrow$ QUANTUM FIELD THEORY $$\hat{S} = \int d^Dx \left[-\frac{1}{2}(\partial\hat{\varphi})^2 - \frac{m^2}{2}\hat{\varphi}^2_{(x)} - \frac{g}{3!}\hat{\varphi}^3_{(x)}\right]$$

$$_{\text{OUT}}\langle p_2, p_3 \cdots | p_1 \cdots \rangle_{\text{IN}} = \prod_i (\Box_i - m^2)\, \mathcal{A}(x_2^\mu x_3^\mu \cdots; x_1^\mu, \cdots)$$

QUANTUM RELATIVISTIC STRINGS AND STRING THEORY



$X^\mu(\tau,\sigma)$

$h_{ab}(\sigma^c)$

$ds^2 = h_{ab}(\sigma)\, d\sigma^a d\sigma^b$

2-d INTERNAL METRIC

$a,b = 0,1 : \sigma^a = (\sigma^0, \sigma^1) = (\tau,\sigma)$

$$\boxed{S[X^\mu_{(\tau,\sigma)}, \sqrt{h}\, h^{ab}_{(\tau,\sigma)}] = -\frac{1}{2} T \iint d\tau\, d\sigma \sqrt{h}\, h^{ab}_{(\sigma)} \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} \eta_{\mu\nu}}$$

STRING TENSION $\boxed{T = \frac{1}{2\pi\alpha'}}$ $\boxed{\alpha' = l_s^2 = \frac{1}{m_s^2}}$

STRING LENGTH

CLASSICAL EOM (GAUGE $\sqrt{h}\, h^{ab} = \eta^{ab}$)

$$\begin{cases} \partial_\tau^2 X^\mu - \partial_\sigma^2 X^\mu = 0 \\ T_{ab} = \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \eta_{ab} \eta^{cd} \partial_c X^\mu \partial_d X_\mu = 0 \end{cases}$$

FIRST QUANTIZATION

OPEN STRING (FREE ENDS)

$$\boxed{\hat{X}^\mu(\tau,\sigma) = \underbrace{\hat{x}^\mu + 2 l_s^2 \hat{p}^\mu \tau}_{\text{CENTER OF MASS}} + i\sqrt{2}\, l_s \underbrace{\sum_{n\neq 0} \frac{\hat{\alpha}^\mu_n}{n} e^{-in\tau} \cos n\sigma}_{\text{INTERNAL OSCILLATIONS}}}$$

$\sin n\sigma$ FOR FIXED ENDS: (DIRICHLET) BOUNDARY CONDITIONS

$$[\hat{x}^\mu, \hat{p}^\nu] = i\hbar \eta^{\mu\nu} \qquad [\hat{\alpha}^\mu_m, \hat{\alpha}^\nu_n] = m\, \delta^0_{m+n}\, \hbar\, \eta^{\mu\nu}$$

USUAL HARMONIC OSCILLATORS:

$$m>0: \; a^\mu_n \equiv \frac{\alpha^\mu_n}{\sqrt{n}} \; ; \; (a^\mu_n)^\dagger = \frac{\alpha^\mu_{-n}}{\sqrt{n}}$$

$$[a^\mu_m, a^{\dagger\nu}_n] = \eta^{\mu\nu} \delta_{mn} \hbar$$

CONSTRAINTS : $\hat{T}_{ab}^{(>0 \text{ FREQ})} |\Psi\rangle = 0$

FREE (SINGLE) STRING STATES

$$|p_\mu; M_n; \zeta_{\dots}\rangle = \zeta_{\mu\nu\dots;\rho\dots;\dots}\, a_1^{+\mu} a_1^{+\nu}\dots a_2^{+\rho}\dots a_n^{+\lambda}\dots \;|p_\mu; 0\rangle$$

p_μ: CoM MOMENTUM; M_n: TOWER OF MASSES FOR CoM MOTION; $\zeta_{\dots}$: 'HELICITY TENSOR' LABELLING INTERNAL EXCITATION STATE; $|p_\mu;0\rangle$: GROUND STATE OF ALL OSCILLATORS

$$a_n^\mu |0\rangle = 0$$

CONSTRAINTS: $\forall n \geq 0$

$$\hat{L}_n\, |p; M; \zeta\rangle = 0$$

$$\& \quad \hat{L}_n \equiv \frac{1}{2}\sum_m \eta_{\mu\nu}\, \hat{\alpha}_m^\mu\, \hat{\alpha}_{n-m}^\nu$$

with $\hat{\alpha}_0^\mu \equiv \sqrt{2}\, l_s\, \hat{p}^\mu$

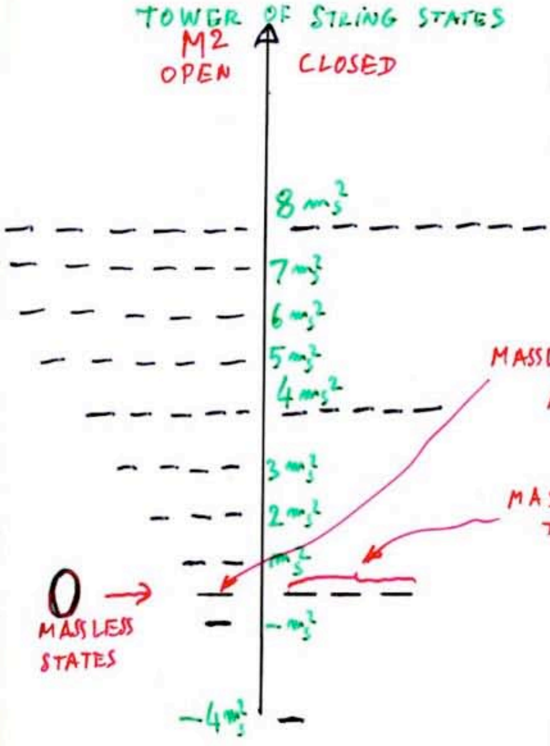
$\hat{L}_0 \Rightarrow$ MASS-SHELL CONDITION

$$\boxed{\frac{M_n^2}{m_s^2} = -1 + \sum_{n=1}^{\infty} n\, \underbrace{(\hat{a}_n^\mu)^+ \hat{a}_n^\nu\, \eta_{\mu\nu}}_{\hat{N}_n}}$$

$\hat{N}_n$ = OCCUPATION NUMBER OPERATOR OF $\cos n\sigma$ HARMONIC

OPEN: $M_n^2 = m_s^2\left[-1 + \sum_{n=1}^{\infty} n\, N_n\right]$

CLOSED: $M_n^2 = 4 m_s^2\left[-1 + \frac{1}{2}\left(\sum_{n\geq 1} n\, N_n + \sum_{n\geq 1} n\, \tilde{N}_n\right)\right]$



MASSLESS VECTOR

$$A_\mu(x) = \zeta_\mu e^{ip.x} + h.c. \qquad |p_\mu; 0; \zeta_\mu\rangle = \zeta_\mu\, a_1^{+\mu}\, |p; 0\rangle$$

MASSLESS GENERAL TENSOR

$$|p; 0; \zeta_{\mu\nu}\rangle = \zeta_{\mu\nu}\, a_1^{+\mu} \tilde{a}_1^{+\nu}\, |p; 0\rangle$$

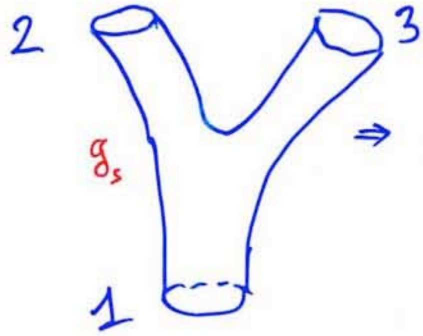
$$\zeta_{\mu\nu} = \zeta^{TT}_{(\mu\nu)} + \zeta^{TS}_{(\mu\nu)} + \zeta^{A}_{[\mu\nu]}$$

$\zeta^{TT}_{(\mu\nu)}$: GRAVITON STATE, $h_{\mu\nu}(x) = \zeta^{TT}_{(\mu\nu)} e^{ip.x} + \dots$

$\zeta^{TS}_{(\mu\nu)}$: SCALAR STATE, $\Phi(x)$

$\zeta^{A}_{[\mu\nu]}$: ANTISYMMETRIC TENSOR, $B_{[\mu\nu]}(x)$

STRING INTERACTIONS : 'SPLITTING STRINGS'

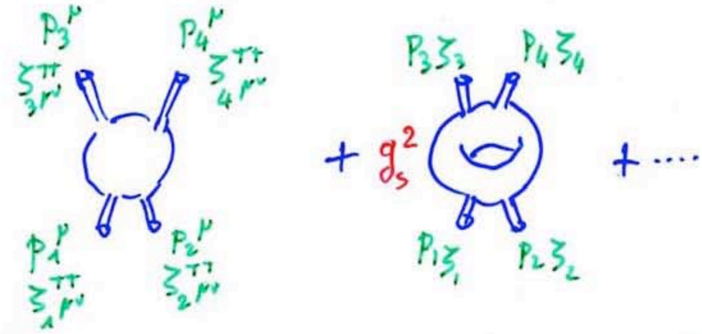


$$\Rightarrow \mathcal{A}(2,3;1) = \langle p_2 \xi_2 ; p_3 \xi_3 \mid p_1 \xi_1 \rangle$$

$$= g_s \langle \hat{V}(p_2 \xi_2)\, \hat{V}(p_3 \xi_3)\, \hat{V}(p_1 \xi_1) \rangle$$

↑ STRING COUPLING CONSTANT

FOR INSTANCE : 2 GRAVITONS → 2 GRAVITONS .

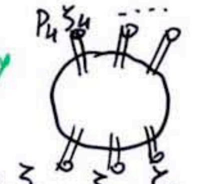


- CONSISTENCY OF STRING THY ⇒ D = FIXED < 26 BOSONIC / 10 SUPERSTRING

- CONSISTENCY OF STRING THY ⇒ GAUGE INVARIANCE: $\xi'_{\mu\nu} = \xi_{\mu\nu} + p_\mu b_\nu + a_\mu p_\nu$

  ⇒ ASYMPTOTIC GR INVARIANCE : $h'_{\mu\nu}(x) = h_{\mu\nu}(x) + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x)$

UNIQUENESS OF G.R. ⇒ LOW-ENERGY LIMIT OF



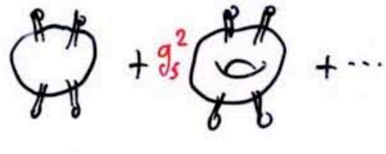
≡ EINSTEIN'S PREDICTIONS FROM $\mathcal{L} = \sqrt{g} R = (\partial h)^2 + h \partial h \partial h + h^2 \partial h \partial h$

$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$

- $\boxed{l_s^2 \text{ : FUNDAMENTAL LENGTH}}$ → ~ UV REGULATOR

  CURES THE UV PROBLEMS OF G.R.

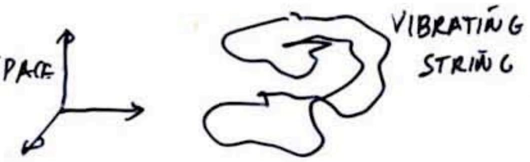
AND DEFINES A PERTURBATIVE QUANTUM GRAVITY THY :



SOME NON-PERTURBATIVE ASPECTS OF STRING THEORY

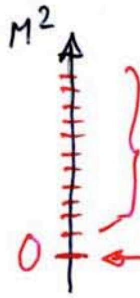
'OLD' PERTURBATIVE STRING THEORY:

QUANTIZE



~ QUANTIZE

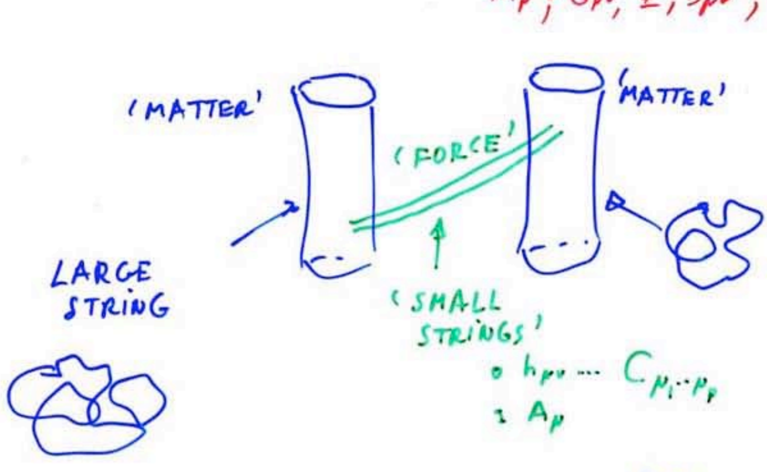
⇒ ∞ TOWER OF QUANTUM STATES = QUANTUM PARTICLES = QUANTUM FIELDS



MASSIVE STATES ~ MATTER

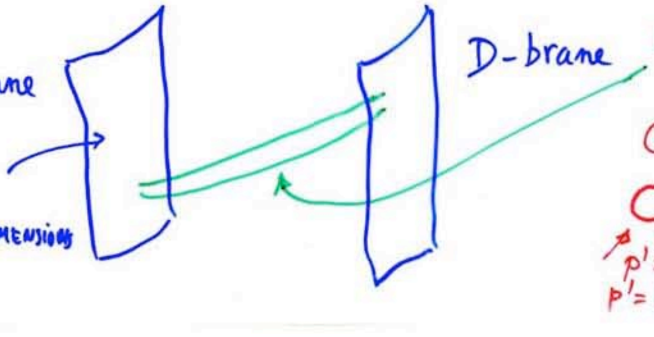
LONG-RANGE FORCES: A_μ; $G_{\mu\nu}$, Φ, $B_{\mu\nu}$; $C_{\mu_1 \cdots \mu_p}$ IN SUPERSTRING ('RR FIELDS')

GENERALIZATION OF MAXWELL A_μ BUT DO NOT COUPLE TO STRING?



$\ni h_{\mu\nu} \cdots C_{\mu_1 \cdots \mu_p}$
$\ni A_\mu$

∃ ALSO NON-PERTURBATIVE OBJECTS: 'D-BRANES' (Polchinski)



EXTENDED IN p SPATIAL DIMENSIONS (+ time)

LONG-RANGE FORCES: SMALL CLOSED STRINGS:

$G_{\mu\nu}(x)$, Φ, $B_{\mu\nu}$

$C_{\mu_1 \cdots \mu_{p'}}(x)$

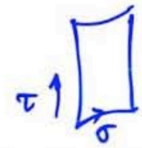
$p' = p+1$ FOR 'ELECTRIC'

$p' = 10-3-p$: 'MAGNETIC'

MULTI - D-BRANE STATES IN ($\boxed{\mathrm{II\,B}}$) SUPERSTRING THY

MASSLESS BOSONIC INTERACTIONS IN II B SUPERSTRING THY $\quad G_{\mu\nu},\ \Phi,\ B_{[\mu\nu]};\ C,\ \boxed{C_{[\mu\nu]}},\ C_{[\mu\nu\rho\sigma]}$

COUPLES ELECTRICALLY TO A D1-BRANE = D-STRING



$$\int C_{\mu\nu}\, dX^\mu \wedge dX^\nu$$

AND MAGNETICALLY TO A D5-BRANE

$$\int \tilde{C}_{\mu_1\cdots\mu_6}\, dX^{\mu_1}\wedge\cdots\wedge dX^{\mu_6} \quad \text{WITH} \quad \underbrace{d\tilde{C}_6}_{7} \sim \underset{\varepsilon^{\mu_1\cdots\mu_{10}}}{\underset{\uparrow}{*}} \underbrace{dC_2}_{3}$$

E.G. D-STRING

THE ELASTIC FLUCTUATIONS OF D-STRING ARE DESCRIBED BY OPEN STRINGS ENDING ON IT

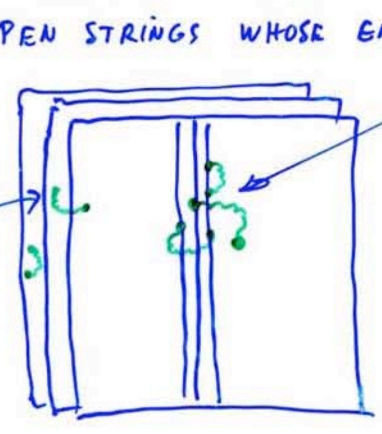
A VERY MASSIVE OBJECT:

$$\text{TENSION} \quad T_{D1} = \frac{m_s^2}{g_s}$$

INFINITELY HEAVY AS $g_s \to 0$

MORE GENERALLY THE (QUANTIZED) ELASTIC FLUCTUATIONS OF AN ASSEMBLY OF D-BRANES ARE DESCRIBED BY QUANTUM OPEN STRINGS WHOSE ENDS ARE ATTACHED TO THE BRANES

E.G.



N_5 D5-branes

N_1 D1-branes

VARIOUS TYPES OF EXCITATIONS:

1 - 1 STRINGS

5 - 5 STRINGS

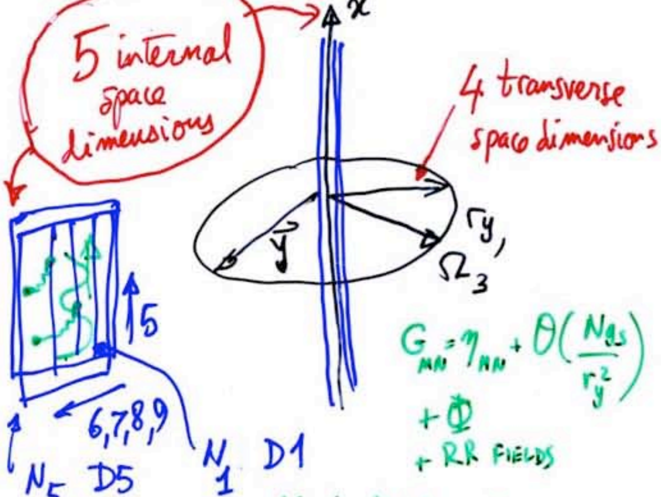
$1\to5$ and $5\to1$ STRINGS

SUPERSYMMETRIC D-BRANE SYSTEMS AND EXTREMAL BLACK HOLES

[Sen '95] Strominger-Vafa '96, Callan-Maldacena '96, ...

e.g. D1-D5 BRANE CONFIGURATION IN IIB

$\boxed{N g_s \ll 1}$



$$G_{MN} = \eta_{MN} + \mathcal{O}\left(\frac{N g_s}{r_y^2}\right)$$

$+ \Phi$

+ RR FIELDS

+ MOMENTUM $P_5 = \frac{N}{R_5}$ in direction x^5

DISTRIBUTE TOTAL $P_5 = \frac{N}{R_5}$

AMONG

$4\, N_1 N_5$ BOSONIC EXCITATIONS ($\begin{smallmatrix}1-5\\5-1\end{smallmatrix}$ OPEN STRINGS)

$+\, 4\, N_1 N_5$ FERMIONIC ONES

COUNTING HOW MANY WAYS:

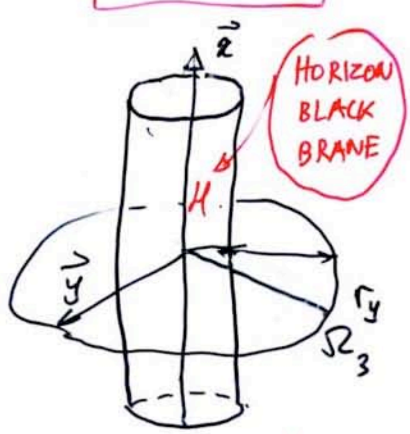
$$N = \sum_{i=1}^{4N_1N_5} \sum_m \left\{ n\, (a_m^i)^\dagger a_m^i + n\, (b_m^i)^\dagger b_m^i \right\}$$

DEGENERACY

$$\mathcal{D}(N) \simeq \exp\left[2\pi \sqrt{4 N_1 N_5 \left(1 + \tfrac{1}{2}\right) \frac{N}{6}}\,\right]$$

$$\boxed{\log \mathcal{D}(N) \simeq 2\pi \sqrt{N_1 N_5 N}}$$

$\boxed{N g_s \gg 1}$



CURVED SPACETIME

$$ds^2_{\text{string}} = H_1^{-1/2} H_5^{-1/2} \left[-dt^2 + dx_5^2 + H_n (dt + dx_5)^2 \right] + H_1^{1/2} H_5^{1/2}\, d\vec{y}^{\,2} + H_1^{1/2} H_5^{-1/2} \left(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2 \right)$$

$$e^{-2\Phi} = H_5 / H_1$$

$$C_{05} = -\tfrac{1}{2} \left(H_1^{-1} - 1 \right)$$

$$(dC)_{ijk} = \varepsilon_{ijkl}\, \partial_l\, H_5$$

$$\begin{cases} H_1 = 1 + c_1 \dfrac{g_s N_1}{r_y^2} \\ H_5 = 1 + c_5 \dfrac{g_s N_5}{r_y^2} \\ H_n = c_n \dfrac{g_s^2 N}{r_y^2} \end{cases}$$

AREA OF $(4+1)$-d EINSTEIN METRIC HORIZON $(r_y = 0)$

$$A = 2^6 \pi^7 g_s^2 \ell_s^8 \sqrt{N_1 N_5 N}\, ; \quad G_5 = 8 \pi^6 g_s^2 \ell_s^8$$

$$\boxed{\frac{A}{4 G_5} = 2\pi \sqrt{N_1 N_5 N}}$$

COUNTING DEGENARACIES OF STRING EXCITATIONS

$$\hat{N} = \sum_{i=1}^{g} \sum_{n \in \mathbb{N}} \left\{ n\, \hat{a}^{i\dagger}_n \hat{a}^i_n + n\, \hat{b}^{i\dagger}_n \hat{b}^i_n \right\}$$

BOSONIC OSCILLATORS $[a, a^\dagger] = 1$ FERMIONIC OSCILL. $\{b, b^\dagger\} = 1$

PARTITION FUNCTION (CANONICAL ENSEMBLE)

$$Z(\beta) = \mathrm{Tr}\, e^{-\beta \hat{N}} = \prod_{i=1}^{g} \prod_n \mathrm{Tr}\, e^{-\beta n a^{\dagger i}_n a^i_n - \beta n b^{\dagger i}_n b^i_n}$$

$$= \prod_n \left(\mathrm{Tr}\, e^{-\beta n a^\dagger_n a_n}\right)^g \prod_n \left(\mathrm{Tr}\, e^{-\beta n b^\dagger_n b_n}\right)^g$$

$$= \left(\prod_{n=1}^{\infty} \frac{1}{1-e^{-\beta n}}\right)^g \left(\prod_n (1+e^{-\beta n})\right)^g$$

$$\ln Z(\beta) = g \sum_n \ln\left[(1-e^{-\beta n})^{-1}\right] + g \sum_n \ln(1+e^{-\beta n})$$

CONTINUOUS APPROXIMATION

$$\ln Z(\beta) \simeq -\frac{g}{\beta} \int_0^\infty dx \ln(1-e^{-x}) + \frac{g}{\beta} \int_0^\infty dx \ln(1+e^{-x}) = \underbrace{\frac{g}{\beta}\frac{\pi^2}{6}}_{\text{BOSONS}} + \underbrace{\frac{g}{\beta}\frac{\pi^2}{12}}_{\text{FERMIONS}}$$

LEGENDRE TRANSFORM: $Z(\beta) = \sum_N D(N) e^{-\beta N} = \sum_N e^{S(N) - \beta N} \simeq e^{S(\bar{N}) - \beta \bar{N}}$

$$\boxed{\ln D(N) = S(N) \simeq 2\pi \sqrt{\frac{g + \frac{1}{2} g}{6} N}}$$

(Hardy-Ramanujan, Cardy)

IN BRIEF: RELATIVISTIC GAS IN $d = 1$ DIMENSION $E = P = \frac{n}{R}$

IN ANY d: $E \sim T g V_d T^d \sim g R^d T^{d+1}$; $S \sim g V_d T^d \sim g R^d T^d$

$$\boxed{S \sim g \left(\frac{ER}{g}\right)^{\frac{d}{d+1}} \underset{d=1}{\sim} g \sqrt{\frac{ER}{g}} \sim \sqrt{g E R} \sim \sqrt{g N}} \qquad \boxed{E = \frac{N}{R}}$$

D BRANES AND BPS EXTREMAL BLACK HOLES

FOLLOW SUPERSYMMETRIC (BPS) QUANTUM STATES AS $g_s \nearrow$

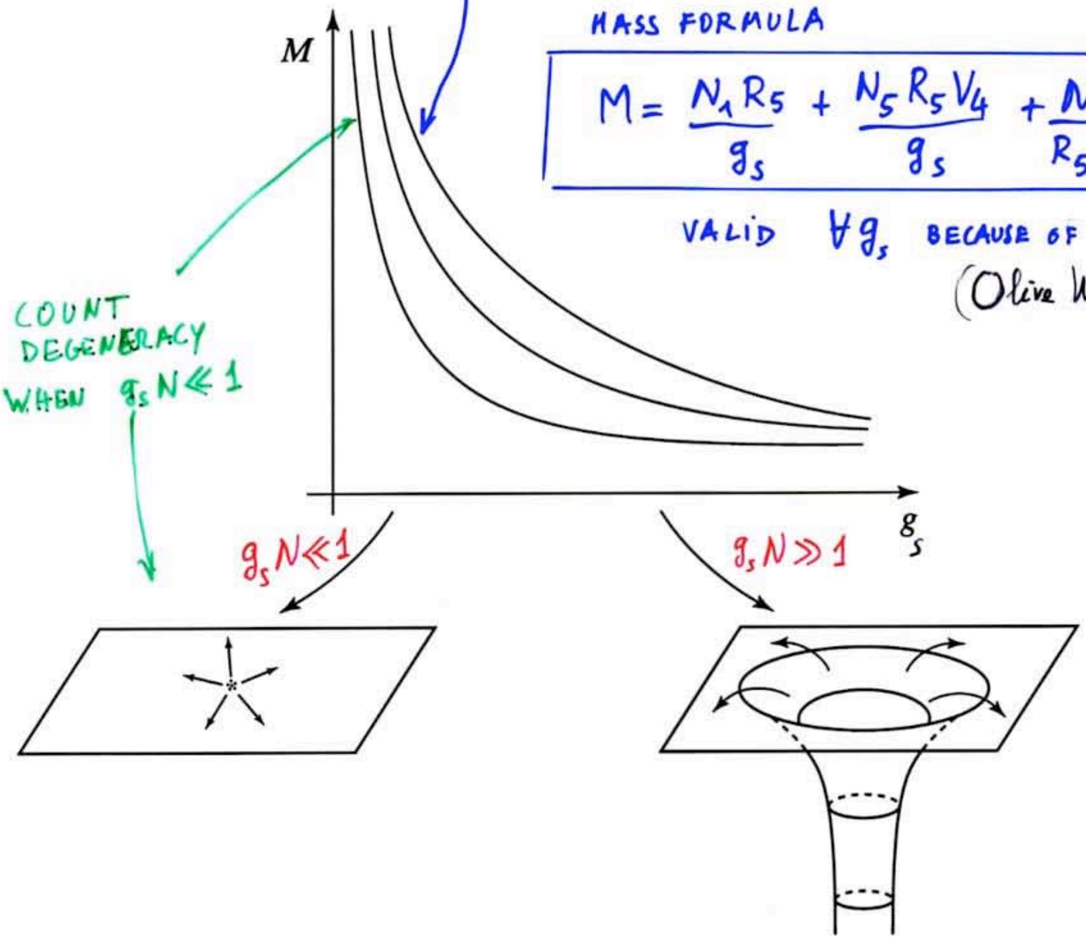
MASS FORMULA

$$\boxed{M = \frac{N_1 R_5}{g_s} + \frac{N_5 R_5 V_4}{g_s} + \frac{N}{R_5}}$$

VALID $\forall g_s$ BECAUSE OF BPS

(Olive Witten)

COUNT DEGENERACY WHEN $g_s N \ll 1$



COMPUTE

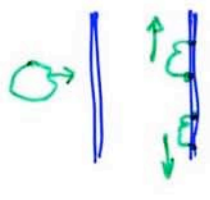
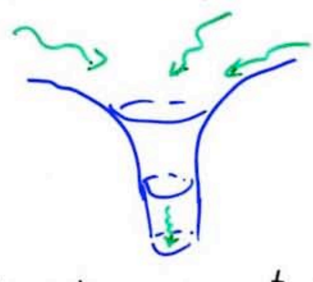
$\frac{A}{4G}$ WHEN $g_s N \gg 1$

STRING THEORY AND BLACK HOLES

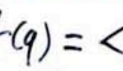
- THE AGREEMENT $\mathcal{D}(N) \simeq e^{S_{BH}}$ IS A DEEP CONFIRMATION OF THE ABILITY OF STRING THY TO DESCRIBE QUANTUM GRAVITATIONAL EFFECTS
$$\left(S_{BH} \propto \frac{1}{\hbar G} \right)$$

- AGREEMENT checked for MANY BPS BLACK HOLES:
  - in $D=5$
  - in $D=4$ (Maldacena Strominger, Johnson Khuri Myers, ...)
  - BEYOND THERMODYNAMICAL LIMIT: FINITE N_i's (Ooguri Strominger Vafa)
  - INCLUDING $R + \ell_s^p R^n$ HIGHER-ORDER CURVATURE CORRECTIONS (Sen ...)
  - INCLUDING MULTI-CENTER BH SOLUTIONS (Denef ...)

- AGREEMENT REMARKABLY EXTENDED TO NEAR-EXTREMAL BH's
  - ENTROPY
  - RADIATION AND ABSORPTION (Callan Maldacena, Das Mathur, ..., Wadia ...)

EQUIVALENCE:  AND 

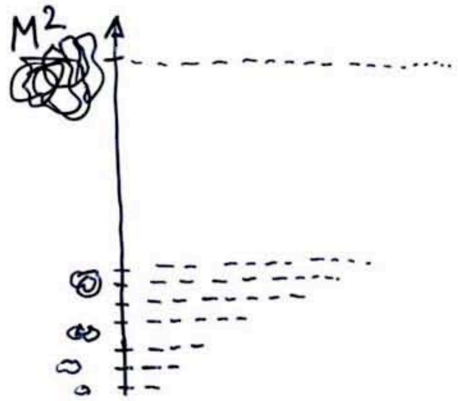
- → AdS/CFT (Klebanov, Maldacena, ..) which, in turn, suggests that BH entropy and evaporation could, in principle, be described within SYM_4

- HOWEVER: NO UNDERSTANDING OF TRANSITION
$$\mathcal{Z}(q) = \langle q | \quad \rangle \longrightarrow \langle q | \quad \rangle$$
 

- POOR UNDERSTANDING OF VERY NON-SUSY STATES SUCH AS SCHWARZSCHILD BLACK HOLES + ASSOCIATED FINAL STATE + UNITARITY?

MASSIVE STRING STATES AND SCHWARZSCHILD BL. HOLES B22

(Susskind '93 ; Horowitz-Polchinski '97 '98 ; Damour-Veneziano '00)



? DO THEY BECOME BLACK HOLES AS $M \nearrow$

? FOR WHICH MASS M ?

STRING ENTROPY

EG OPEN BOSONIC

$$\boxed{\begin{aligned} M^2 &= m_s^2 \left[-1 + N \right] \\ \hat{N} &= \sum_{n=1}^{\infty} \sum_{i=1}^{D-2} n \left(\hat{a}^i_n\right)^\dagger \hat{a}^i_n \end{aligned}}$$

BOSONIC OSCILLATORS

$$\left[a^i_n , \left(a^j_m\right)^\dagger\right] = \delta^{ij} \delta_{nm}$$

EIGENVALUES OF EACH $\hat{a}^\dagger \hat{a} = 0, 1, 2, 3, \ldots$

DEGENERACY OF MASS LEVEL M :

$$\mathcal{D}(M) = \exp\left[2\pi \sqrt{\frac{D-2}{6} N}\right] \simeq \exp\left[2\pi \sqrt{\frac{D-2}{6}} \frac{M}{m_s}\right]$$

STRING ENTROPY

$$\boxed{S_{\text{STRING}}(M) \equiv \ln \mathcal{D}(M) \simeq 2\pi \sqrt{\frac{D-2}{6}} \frac{M}{m_s}}$$

STRING ENTROPY VS BLACK HOLE ENTROPY

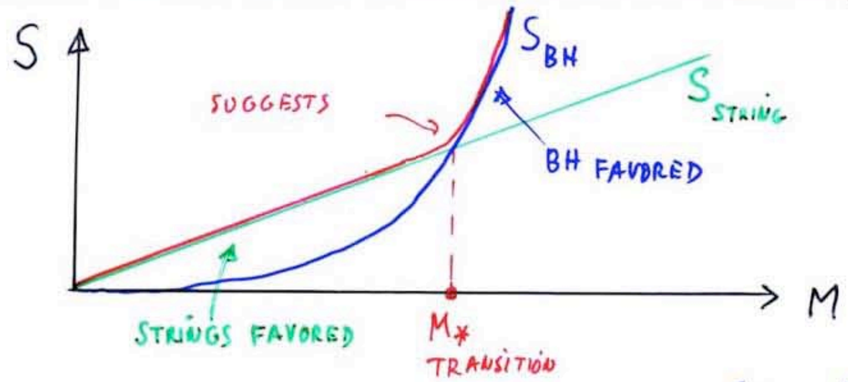
IN UNCOMPACTIFIED SPACE DIMENSION d (SPACETIME $D \equiv d+1$)

$$S_{BH}(M) \sim \frac{A}{G_d} \sim \frac{R_{BH}^{d-1}}{G_d}$$

RADIUS OF BH: $-g_{00} = 1 - c\,\frac{G_d M}{r^{d-2}}$

$$\rightarrow \quad R_{BH} \sim (G_d M)^{\frac{1}{d-2}}$$

$$\boxed{S_{BH}(M) \sim \frac{(G_d M)^{\frac{d-1}{d-2}}}{G_d} \sim G M^2 \text{ in } d=3}$$



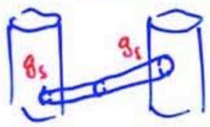
EXPECT TRANSITION STRING $\rightleftarrows$ BH WHEN $\frac{(G_d M_*)^{\frac{d-1}{d-2}}}{G_d} \sim \frac{M}{m_s}$

$$(G_d M_*)^{\frac{1}{d-2}} \sim l_s$$

$$\boxed{R_{BH}^{*} \sim l_s}$$

USING

$$\boxed{G_d \sim g_s^2\, l_s^{d-1}}$$



$$\boxed{g_s^2\, M_* \sim m_s}$$

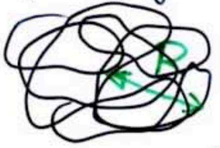
M_* LARGER THAN PLANCK MASS

$$M_P \sim \frac{1}{G_d^{\frac{1}{d-1}}} \sim \frac{m_s}{g_s^{\frac{2}{d-1}}}$$

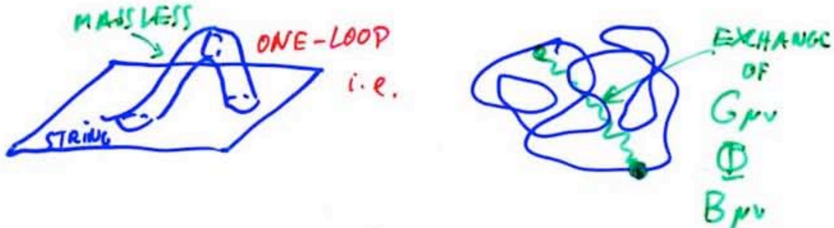
SELF-GRAVITY OF STRINGS AND A DYNAMICAL STUDY OF THE TRANSITION STRING $\longrightarrow$ BLACK HOLE

Define characteristic size of string state

$$\boxed{R^2 \equiv \frac{1}{d}\left\langle :\left(\hat{X}^{i}_{\perp_{pp}}(\tau,\sigma)\right)^2: \right\rangle = \frac{2}{d}\, \ell_s^2 \sum_{n=1}^{\infty} \frac{a_n^{i\dagger} a_n^i}{n}} \qquad i=1,\dots,d$$



EFFECT OF SIZE ON THE MASS OF A STRING STATE



$$\int d\tau\, \delta M^2 = 64\pi\, G\, T^2 \iint d^2\sigma_1 d^2\sigma_2 \frac{d^D k}{(2\pi)^D} e^{ik\cdot(X_1 - X_2)} \left(\partial_+ X_1^\mu \cdot \partial_+ X_{2\mu}\right)\left(\partial_- X_1^\nu \partial_- X_{2\nu}\right)$$

@ σ_1 @ σ_2

$$\boxed{M = M_0 - c_d \frac{G_d M^2}{R^{d-2}}}$$

'BARE MASS' when $g_s \to 0$ adiabatically

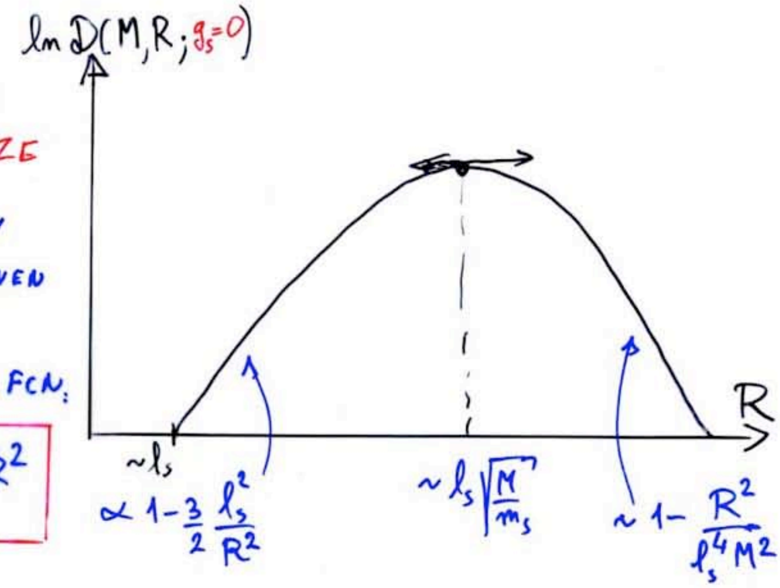
MASS SHIFT depends on SIZE OF STATE

DISTRIBUTION OF FREE STRING STATES BY SIZE

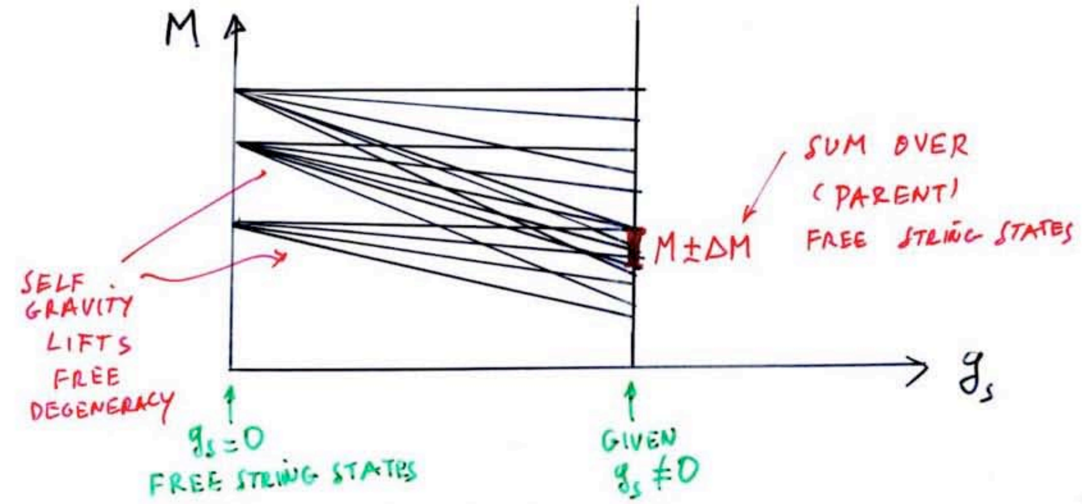
MODIFIED COUNTING BY CONSTRAINT $R^2 =$ GIVEN

$\rightarrow$ MODIFIED 'GRAND CANONICAL' PARTITION FCN:

$$\boxed{Z(\beta,\gamma) = \mathrm{Tr}\, e^{-\beta \hat{N} - \gamma \hat{R}^2}}$$



ENTROPY OF SELF-GRAVITATING STRINGS



DEGENERACY OF SELF-GRAVITATING STRING STATES

$$\mathcal{D}(M,R) = e^{S(M,R)}$$

$$S(M,R;g_s^2) \simeq a_0\, M\left(1-\frac{1}{R^2}\right)\left(1-\frac{R^2}{M^2}\right)\left(1+\frac{g_s^2 M}{R^{d-2}}\right)$$

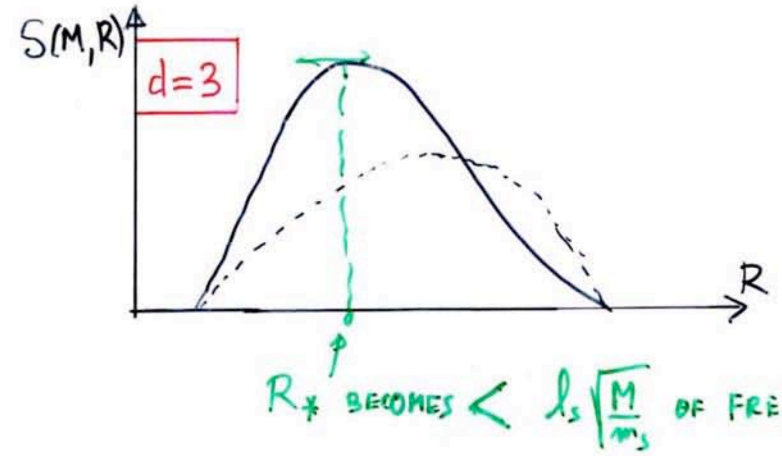
IN UNITS $l_s \sim 1$

USUAL STRING ENTROPY

LESS STATES IF R SMALL BECAUSE OF CONSTRAINT ON SIZE

MORE STATES IF R SMALL BECAUSE $M_0 > M$

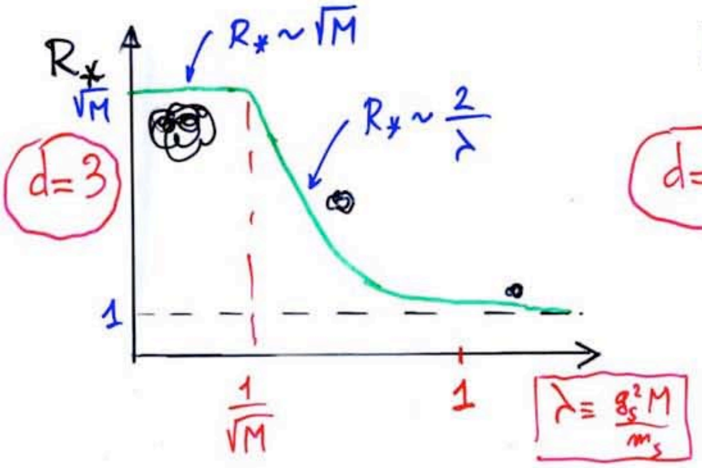
MOST PROBABLE SIZE OF SELF-GRAVITATING STRING

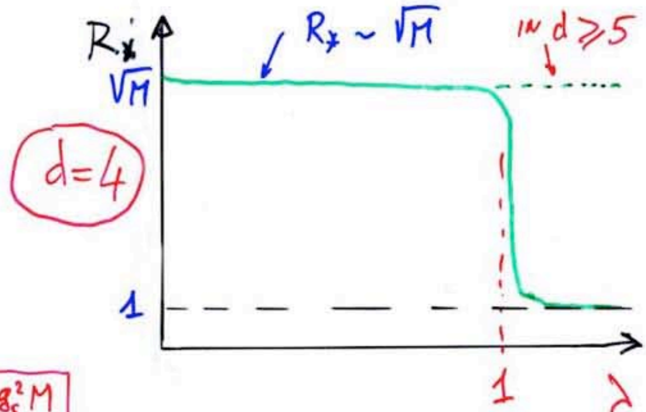


R_* BECOMES $< \; l_s\sqrt{\frac{M}{m_s}}$ OF FREE STRINGS

RANDOM WALK

SHRINKING OF STRING SIZE WITH SELF-GRAVITY



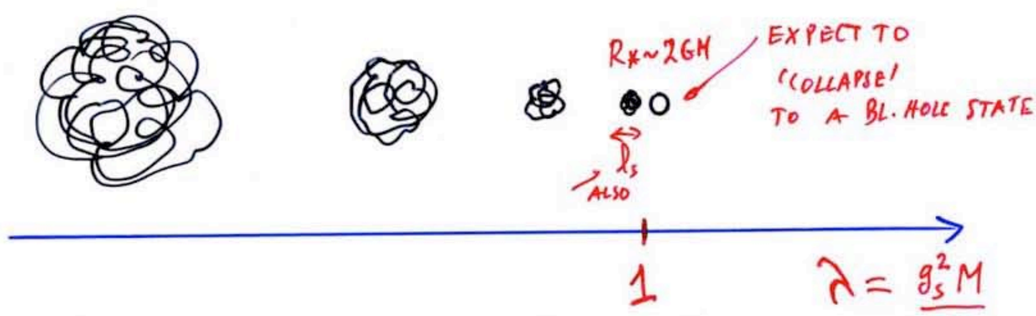


IMPORTANCE OF SELF GRAVITY (d=3)

$$\text{'COMPACTNESS'} \equiv \frac{2GM}{R_*} \simeq \frac{2\lambda}{(2/\lambda)} \simeq \lambda^2$$

$$\boxed{\text{WHEN } \lambda \to 1 \;:\; R_* \to 2GM \to \text{EXPECT STRING TO BECOME A BLACK HOLE}}$$

IN OTHER WORDS



DYNAMICALLY PREDICTS THAT A STRING $\to$ BLACK HOLE

WHEN

$$\boxed{g_s^2 M \simeq m_s}$$

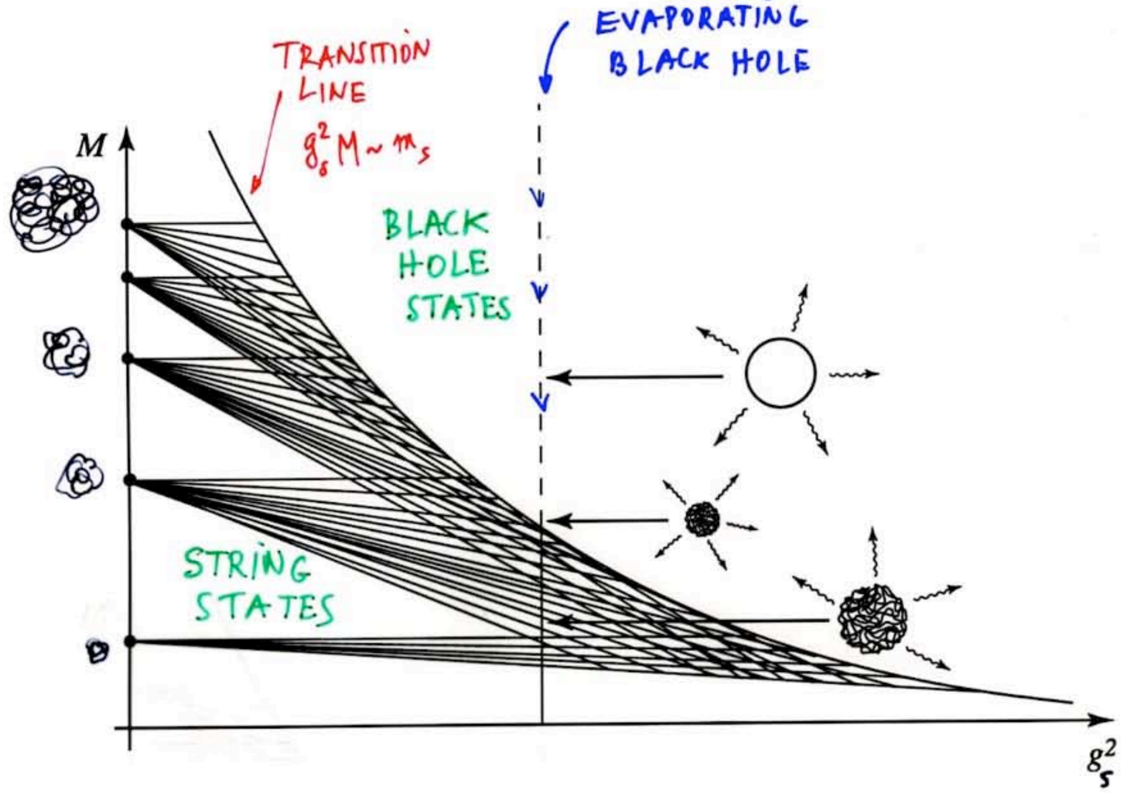
i.e. PRECISELY WHEN

$$\boxed{S_{STRING}(M) \simeq S_{BH}(M)}$$

STRING ENTROPY

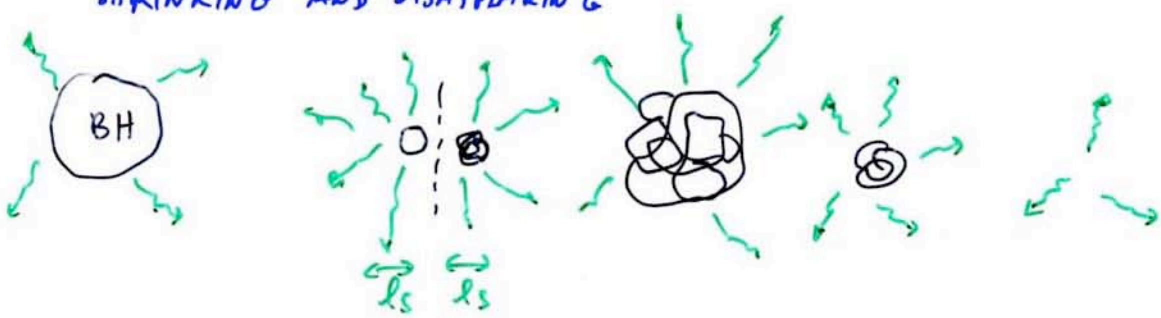
BLACK HOLE ENTROPY

PHYSICAL PICTURE

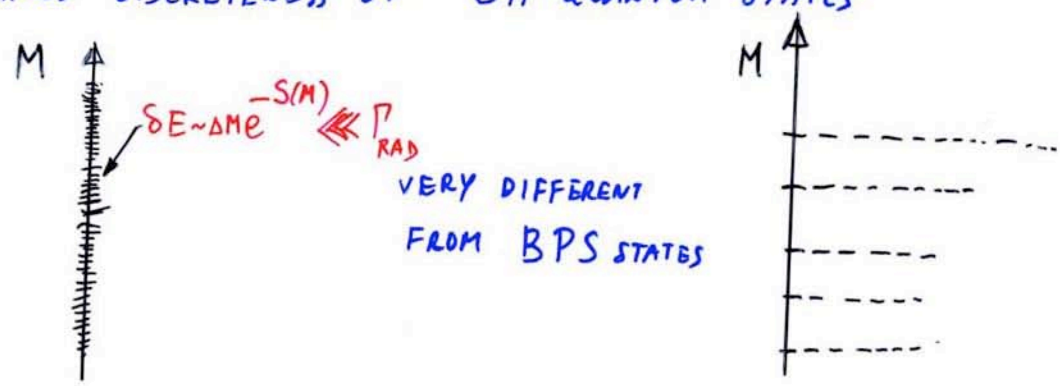


- THIS CONFIRMS THE STATISTICAL MEANING OF $S_{BH}(M)$ AS $S_{STRING}(M)$ WHEN $g_s^2 M = m_s$, BUT DOES NOT HELP TO EXPLAIN STATISTICAL MEANING WHEN $g_s^2 M > m_s$

- SUGGESTS THAT AN EVAPORATING BH TRANSFORMS INTO A VERY TIGHT STRING STATE WHEN $M = \frac{m_s}{g^2} \gg M_{Planck}$ THEN IT EXPANDS AGAIN TO SIZE $R \gg l_s$ BEFORE SHRINKING AND DISAPPEARING



- BLURRED DISCRETENESS OF BH QUANTUM STATES



CONCLUSIONS

- THE STRING THEORY CALCULATIONS (IN MANY CASES, SOME WITH GREAT COMPLEXITY AND STRUCTURE) THAT

$$\ln \mathcal{D}(N)_{\text{BPS STATES}} = S_{\substack{\text{EXTREMAL} \\ \text{BPS BH} \\ \text{SAME GLOBAL CHARGES}}}$$

INCLUDING FACTOR $\frac{1}{4}$, OR EVEN HIGHER-ORDER CURVATURE CORRECTIONS,

IS A DEEP CONFIRMATION OF THE ABILITY OF STRING THY TO DESCRIBE QUANTUM GRAVITATIONAL EFFECTS

- HOWEVER, NO UNDERSTANDING OF THE EVOLUTION OF THE QUANTUM STATE:

$$\Psi_{\text{BPS}}(q) = \langle q | \quad \rangle \longrightarrow \; ? \; \Psi_{\text{BH}}(q) \overset{?}{=} \langle q | \quad \rangle \, ?$$

- THE CHECK THAT A GENERIC MASSIVE STRING STATE SHRINKS UNDER ITS OWN WEIGHT AND BECOMES $\simeq$ SCHWARZSCHILD BLACK HOLE $\left(R_* \sim (G_d M)^{\frac{1}{d-2}} \right)$ WHEN (IN ORDER OF MAGNITUDE) $S_{\text{STRING}}(M) \sim S_{\text{BH}}(M)$ IS THE BEGINNING OF THE UNDERSTANDING OF THE EVOLUTION OF THE QUANTUM STATE:

$$\Psi_{\text{STRING}}(q) = \langle q | \quad \rangle \longrightarrow \Psi_{\substack{\text{TIGHT} \\ \text{STRING}}}(q) = \langle q | \quad \rangle$$

- HOWEVER, THERE IS STILL NO TECHNICALLY PRECISE DESCRIPTION OF ANY $\left\{ \begin{array}{l} \text{HIGHLY-DEGENERATE} \\ \text{OR WITH HIGH STATE DENSITY} \end{array} \right\}$ ('BLACK HOLE QUANTUM STATE')

  AND THEREFORE NO TECHNICALLY PRECISE, AND UNITARY, DESCRIPTION OF THE QUANTUM EVAPORATION OF BLACK HOLES

- IT IS PROBABLE THAT THE PROCESS IS UNITARY AND THAT THE END POINT OF THE EVAPORATION OF SCHWARZS. BH IS: BH $\longrightarrow$ TIGHT STRING $\longrightarrow$ FINAL DECAY INTO MASSLESS QUANTA

- AdS/CFT DEFINES IN PRINCIPLE THE WHOLE PROCESS: FORMATION + EVAPORATION OF BH AS A UNITARY EVOLUTION IN SYM_4 FOR $g^2_{YM} N \gg 1$ BUT 'DETAILS ARE LACKING'