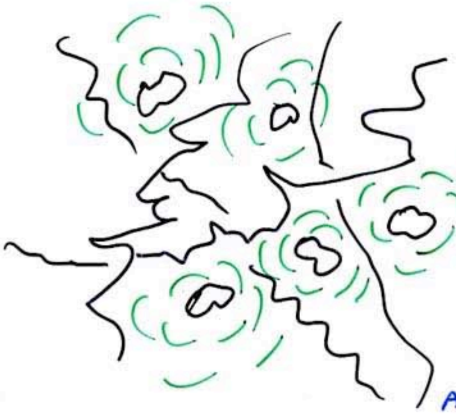
COSMIC STRINGS AND
FUNDAMENTAL STRINGS
IHP, PARIS, 22-27 SEPT 05

GRAVITATIONAL WAVE EMISSION
(ESPECIALLY BURSTS)
FROM COSMIC (SUPER)STRINGS

Thibault DAMOUR

IHES

COSMIC STRINGS AND GRAVIT. WAVES



NETWORK OF COSMIC STRINGS:

- LONG STRINGS
- LOOPS

OSCILLATING LOOPS OF STRING ARE EFFICIENT GW EMITTERS (Vilenkin '81)

→ ∃ STOCHASTIC BACKGROUND OF GRAVIT. WAVES

Vilenkin 81
Hogan, Rees 84
Vachaspati, Vilenkin 85
Bennett, Bouchet 88
Caldwell, Allen '92
Caldwell, Battye, Shellard 96

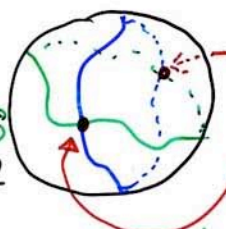
CUSP FORMING IS GENERIC Turok 84

TIME GAUGE
$X^0 = \tau$

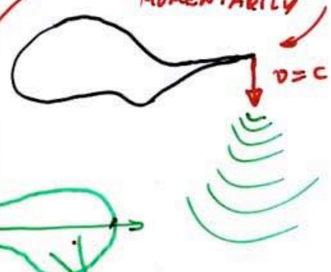
$\vec{X}(\tau,\sigma) = \frac{1}{2}\left[\vec{a}(\tau-\sigma) + \vec{b}(\tau+\sigma)\right]$

$\dot{\vec{a}}^2 = 1$
$\dot{\vec{b}}^2 = 1$
$\langle\dot{a}\rangle = \langle\dot{b}\rangle = \vec{0}$

Kibble, Turok 82

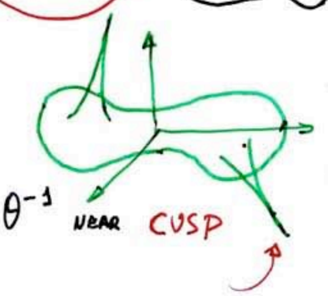


WHEN $\dot{\vec{a}}(\tau-\sigma) = \dot{\vec{b}}(\tau+\sigma)$ A CUSP FORMS MOMENTARILY



GRAVIT. WAVES FROM CUSP

BEAM OF HIGH FREQUENCY GW's



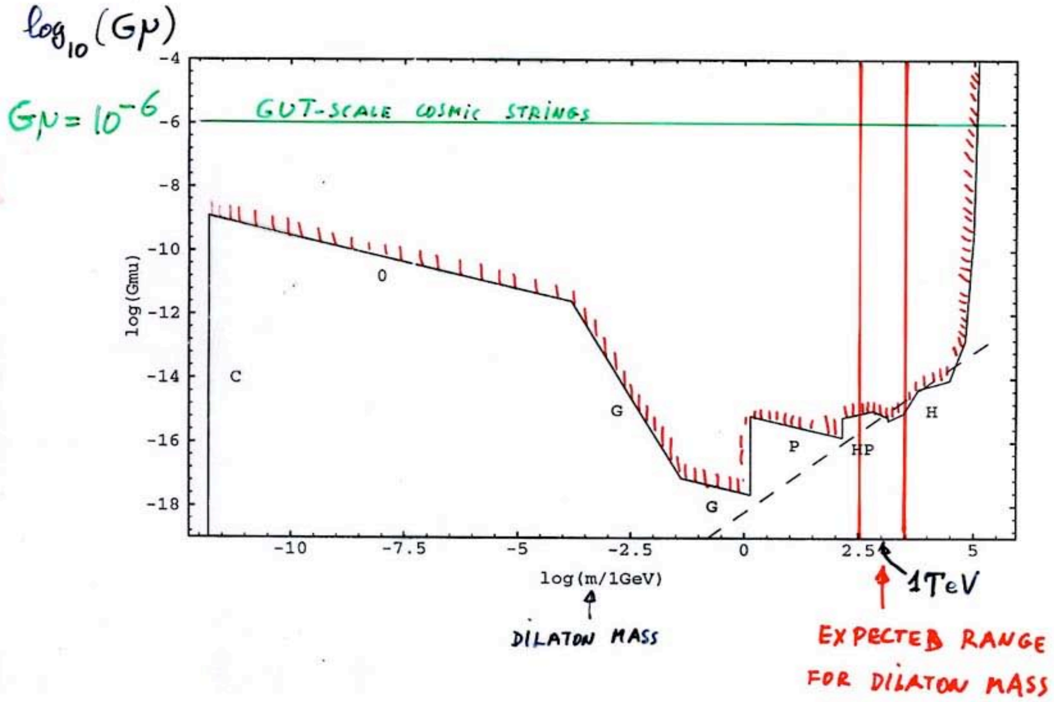
Vachaspati, Vilenkin '85 $\frac{d\dot{E}}{d\Omega} \propto \theta^{-1}$ NEAR CUSP

COSMIC STRINGS AND THE STRING DILATON

(T.D., A. VILENKIN '97)

HERE USUAL ASSUMPTION : $\alpha_{\text{DILATON}} \sim 1$, $m_\varphi \neq 0$

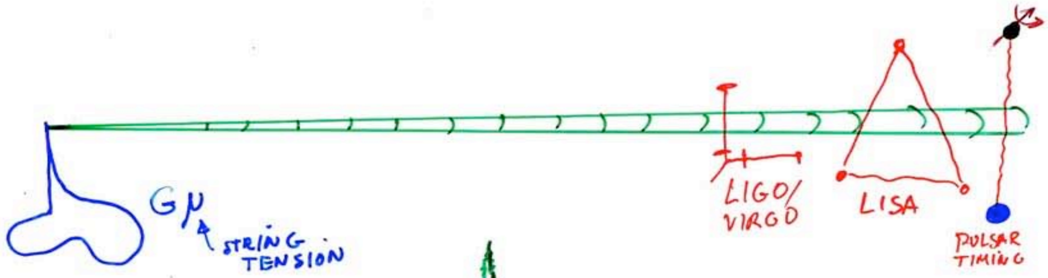
Figure 1: Constraints on $\log_{10}(G\mu)$ versus $\log_{10}(m_\phi/1\text{Gev})$. The region above the solid curve is forbidden. Labels indicate the source of the various constraints: Cavendish experiments (C), $\Omega_\phi h^2 < 1(O)$, gamma-ray background (G), photodissociation (P), combined hadroproduction and photodissociation (HP), hadroproduction (H). The dashed line indicates the condition of validity (17). [The constraints apply only above the dashed line.]

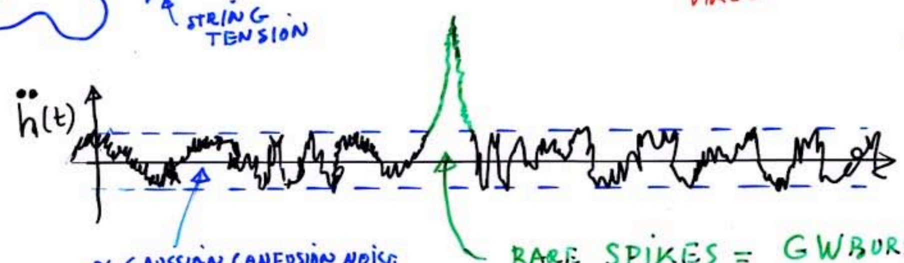


GRAVITATIONAL WAVE BURSTS FROM CUSPS

UNTIL 2000, IT WAS TACITLY ASSUMED THAT THE STRING-GENERATED STOCHASTIC GW BACKGROUND WAS NEARLY GAUSSIAN

Damour, Vilenkin '00, '01 GW BACKGROUND IS STRONGLY NON GAUSSIAN, IT CONTAINS SHARP GW BURSTS (FROM CUSPS) STANDING ABOVE A NEARLY GAUSSIAN "CONFUSION NOISE"





- GW BURSTS MIGHT BE DETECTABLE IN LIGO OR LISA FOR STRING TENSIONS AS SMALL AS $G\mu \sim 10^{-13}$
- NEED TO REVISIT PULSAR-TIMING LIMITS ON $G\mu$ BECAUSE USUAL $h_{RMS}(f)$ OVERESTIMATES $h_{CONFUSION}(f)$

RECENT DEVELOPMENTS

- CMB OBSERVATIONS DISFAVOR "STRUCTURE FORMING" ($\sim$ GUT) STRINGS WITH $G\mu \sim 10^{-6}$

  E.G. $G\mu < 6.1 \times 10^{-7}$ Pogosian et al 04, Jeong, Smoot '04

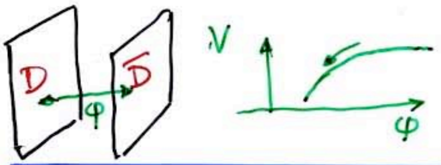
- AN ANALYSIS OF 17 YEAR OF PULSAR TIMING DATA HAS CLAIMED A STRONG LIMIT

  $\Omega_g h^2 < 2 \times 10^{-9}$ Lommen '02

- RENEWED INTEREST IN THE POSSIBILITY THAT FUNDAMENTAL STRINGS OF SUPERSTRING THEORY MAY BE COSMIC STRINGS

  Witten '85, Sarangi, Tye '02, ... Tye ..., KKLMMT '03, Dvali, Vilenkin '04, Copeland, Myers, Polchinski '04, Jackson, Jones, Polchinski '04

END OF BRANE INFLATION
Dvali, Tye '99



EXPECT $\boxed{10^{-11} \lesssim G\mu \lesssim 10^{-6}}$

- HOWEVER, CONTRARY TO ORDINARY COSMIC STRINGS WITH $p \simeq 1$ RECONNECT — Shellard 88, Matzner 88, Mccracken 88

  F (OR D) STRINGS $\Rightarrow$ $\boxed{\underset{(0.1)}{10^{-3}} \lesssim p \lesssim 1}$

- \+ Siemens, Olum '01, Siemens, Olum, Vilenkin '02 ARGUED THAT GRAV. RAD. IS LESS EFFICIENT THAN THOUGHT BEFORE IN DAMPING SMALLSCALE WIGGLES

  $\Rightarrow \boxed{\alpha \equiv \frac{\ell_{wiggle}(t)}{t} \overset{?}{\ll} 50\, G\mu}$

STRING DYNAMICS: $X^\mu(\tau,\sigma)$

$$S_{\text{NAMBU}} = -\mu \int d\tau\, d\sigma \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2 (X')^2}$$

$\dot{X}^\mu = \partial_\tau X^\mu$, $X'^\mu = \partial_\sigma X^\mu$, $\dot{X}\cdot X' \equiv g_{\mu\nu}(X)\dot{X}^\mu X'^\nu$

$$S_{\text{POLYAKOV}} = -\frac{1}{2}\mu \int d\tau\, d\sigma \sqrt{h}\, h^{ab} g_{\mu\nu}(X)\, \partial_a X^\mu \partial_b X^\nu$$

CONFORMAL GAUGE: $\sqrt{h}\, h^{ab} = \eta^{ab}$

$\Rightarrow$ CONSTRAINTS $\quad \dot{X}^2 + X'^2 = 0 \quad ; \quad \dot{X}\cdot X' = 0$

EQ. OF MOTION: $\boxed{\ddot{X}^\mu - X''^\mu + \Gamma^\mu_{\alpha\beta}(X)\,(\dot{X}^\alpha \dot{X}^\beta - X'^\alpha X'^\beta) = 0}$

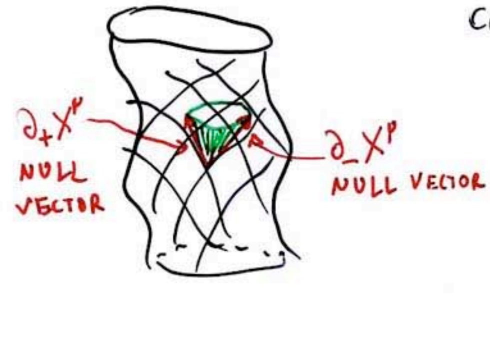
IN FLAT SPACE: $\quad \ddot{X}^\mu - X''^\mu = 0$

NULL WORLD-SHEET COORDS:

$$\sigma_\pm = \tau \pm \sigma$$

EOM: $\quad \frac{\partial}{\partial\sigma_+}\frac{\partial}{\partial\sigma_-} X^\mu = 0$

$$\boxed{X^\mu(\tau,\sigma) = \frac{1}{2}[\underbrace{X^\mu_+(\sigma_+)}_{\text{LEFT-MOVING}} + \underbrace{X^\mu_-(\sigma_-)}_{\text{RIGHT-MOVING}}]}$$



CONSTRAINTS: $\boxed{\begin{aligned} (\partial_+ X^\mu_+)^2 &= 0 \\ (\partial_- X^\mu_-)^2 &= 0 \end{aligned}}$

CUSPS

TIME GAUGE : $X^0(\tau,\sigma) = \tau = \frac{1}{2}[\sigma_+ + \sigma_-]$

IN CENTER-OF-MASS FRAME : $X^i_\pm(\sigma_\pm) =$ PERIODIC $\Rightarrow \langle \dot{X}^i_\pm(\sigma_\pm) \rangle = 0$

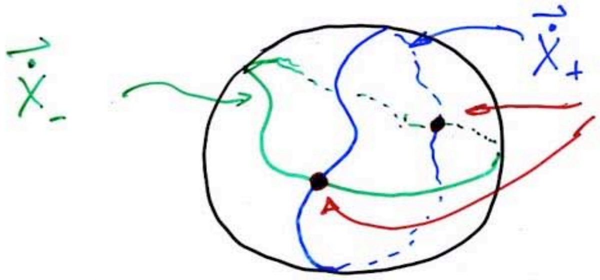
CONSTRAINTS : $(\partial_\pm X^\mu_\pm)^2 = -(\partial_\pm \underbrace{X^0_\pm}_{\sigma_\pm})^2 + (\partial_\pm X^i_\pm)^2 = 0$

$$-1 + (\dot{X}^i_\pm)^2 = 0$$

$$(\vec{\dot{X}}_+)^2 = 1 = (\vec{\dot{X}}_-)^2$$

$\vec{\dot{X}}_+$ AND $\vec{\dot{X}}_-$ ARE PERIODIC (WITH ZERO AVERAGE) ON UNIT SPHERE

Kibble, Turok '82



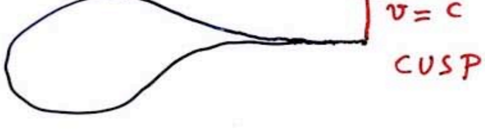
GENERICALLY EXPECT

$\exists$ 2 INTERSECTIONS

Turok '84

INTERSECTION : $\partial_+ X^\mu_+ = \partial_- X^\mu_- = \ell^\mu$

IN SPACE :



GW BURSTS FROM CUSPY STRINGS

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \tfrac{1}{2} h \, \eta_{\mu\nu}$$

$$\Box \bar{h}_{\mu\nu} = -16\pi \, G \, T_{\mu\nu} \qquad \partial^\nu \bar{h}_{\mu\nu} = 0$$

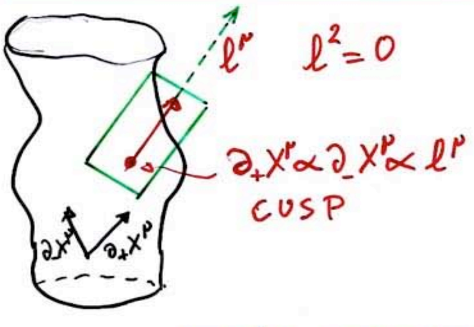
HARMONIC GAUGE

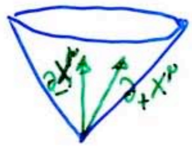
STRING STRESS-ENERGY TENSOR

$$T^{\mu\nu}(x^\lambda) = \mu \int d\tau \, d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta^{(4)}\!\left(x^\lambda - X^\lambda(\tau,\sigma)\right)$$

- USE LEFT – RIGHT DECOMPOSITION : $\boxed{\sigma_\pm \equiv \tau \pm \sigma}$

$$\boxed{X^\mu(\tau,\sigma) = \tfrac{1}{2}\left[X_+^\mu(\sigma_+) + X_-^\mu(\sigma_-) \right] \qquad \begin{array}{l} (\partial_+ X_+^\mu)^2 = 0 \\ (\partial_- X_-^\mu)^2 = 0 \end{array}}$$





- USE FOURIER TRANSFORM

$$\boxed{T^{\mu\nu}(k^\lambda) = \frac{\mu}{T_\ell} \int_{\Sigma_\ell} d\tau \, d\sigma \; \dot{X}_+^{(\mu} \, \dot{X}_-^{\nu)} \, e^{-\frac{i}{2} k \cdot (X_+ + X_-)}}$$

- STAY POINCARÉ COVARIANT

GW AMPLITUDE FROM STRINGS

$$\boxed{\bar{h}_{\mu\nu}(t,\vec{x}) = \frac{\kappa_{\mu\nu}(t-r,\vec{n})}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)}$$

$$\kappa_{\mu\nu}(t-r,\vec{n}) = \sum_{\omega=\pm m\frac{2\pi}{T_\ell}} 4G\, e^{-i\omega(t-r)}\, T_{\mu\nu}(\omega, \vec{k}=\omega\vec{n})$$

$T_\ell = \frac{\ell}{2}$ ∧ invariant length $\ell = \frac{E_0}{\mu}$

$$\boxed{T^{\mu\nu}(\vec{k}_m,\omega_m) = \frac{\mu}{\ell}\, I_+^{(\mu} I_-^{\nu)}}$$

$$\boxed{I_\pm^\mu \equiv \int_0^\ell d\sigma_\pm\, \dot{X}_\pm^\mu(\sigma_\pm)\, e^{-\frac{i}{2}k_m\cdot X_\pm}}$$

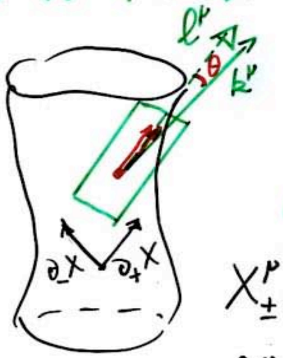
LOGARITHMIC FOURIER TRANSFORM OF WAVE FORM (HIGH-FREQ PART)

$$\kappa^{\mu\nu}(f,\vec{n}) \equiv |f| \int dt\, e^{2\pi i f t}\, \kappa^{\mu\nu}(t-r,\vec{n})$$

$$\boxed{\kappa^{\mu\nu}(f,\vec{n}) = 2G\mu |f|\, I_+^{(\mu}(\omega,\omega\vec{n})\, I_-^{\nu)}(\omega,\omega\vec{n})}$$

LEFT-RIGHT FACTORIZATION OF GW AMPLITUDE $\kappa^{\mu\nu}(k)$

GW AMPLITUDE FROM CUSPS



AT CUSP

$$\partial_+ X^\mu \propto \partial_- X^\mu \propto \ell^\mu \leftarrow \text{NULL VECTOR}$$

NEAR CUSP

$$X^\mu_\pm(\sigma_\pm) = X^\mu_c + \ell^\mu \sigma_\pm + \tfrac{1}{2}\ddot{X}^\mu_\pm \sigma_\pm^2 + \tfrac{1}{6}\dddot{X}^\mu_\pm \sigma_\pm^3 + \dots$$

$$\dot{X}^\mu_\pm(\sigma_\pm) = \ell^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \tfrac{1}{2}\dddot{X}^\mu_\pm \sigma_\pm^2 + \dots$$

$$\kappa^{\mu\nu}(f) \propto I_+^{(\mu} I_-^{\nu)} \qquad \omega_\ell = \frac{2\pi}{T_\ell} = \frac{4\pi}{\ell}$$

$$I^\mu_\pm = \int_{\sigma_0}^{\sigma_0+\ell} d\sigma_\pm \, (\ell^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \dots)\, e^{+\frac{i}{12} m\omega_\ell \ddot{X}_\pm^2 \sigma_\pm^3 + \dots}$$

CAN BE GAUGED AWAY (WHEN $\theta = 0$)

$m \to \pm\infty$

$\sim \theta\sigma_\pm^2 + \theta^2\sigma_\pm$ WHEN $\theta \neq 0$

$$\boxed{\kappa^{\mu\nu}(f, \vec{n}_{cusp}) = -C \frac{G\mu}{(2\pi|f|)^{1/3}} e^{2\pi i f t_c} A_+^{(\mu} A_-^{\nu)} + \text{GAUGE}}$$

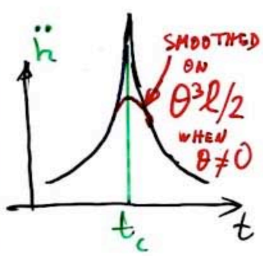
i.e. FOR $\theta = 0$

$$C = 4\pi(12)^{4/3}/[3\Gamma(\tfrac{1}{3})]^2 \qquad A^\mu_\pm = \ddot{X}^\mu_\pm / |\ddot{X}_\pm|^{4/3}$$

TIME DOMAIN

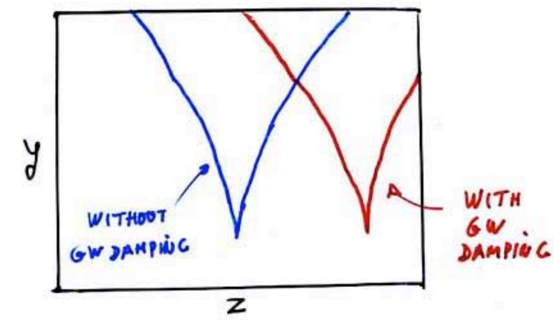
SIGNAL ROBUST UNDER $\exists$ SMALL-SCALE WIGGLES (Siemens, Olum '03)

$$\boxed{\begin{aligned} \kappa(t) &\propto |t - t_c|^{1/3} \\ \ddot{\kappa}(t) &\propto |t - t_c|^{-5/3} \end{aligned}}$$

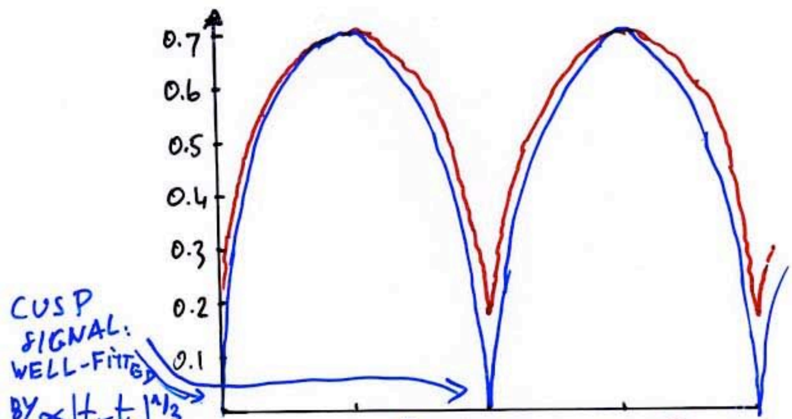


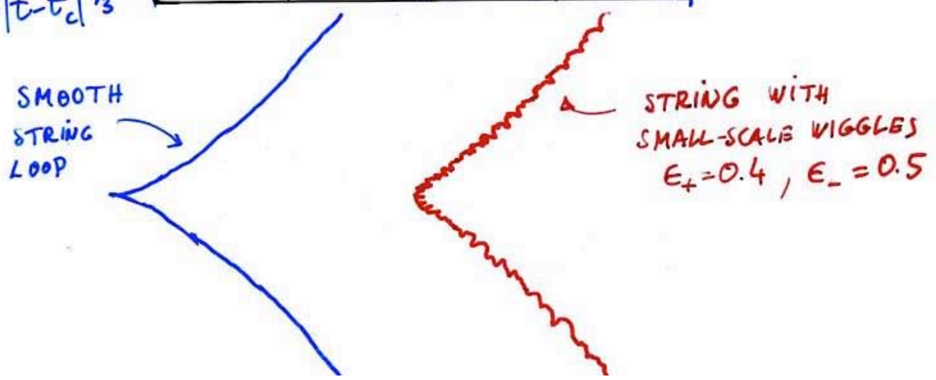
ROBUSTNESS OF CUSPS AND CUSP-WAVEFORMS

- GRAVITATIONAL RADIATION DAMPING : Quashnock, Spergel '90

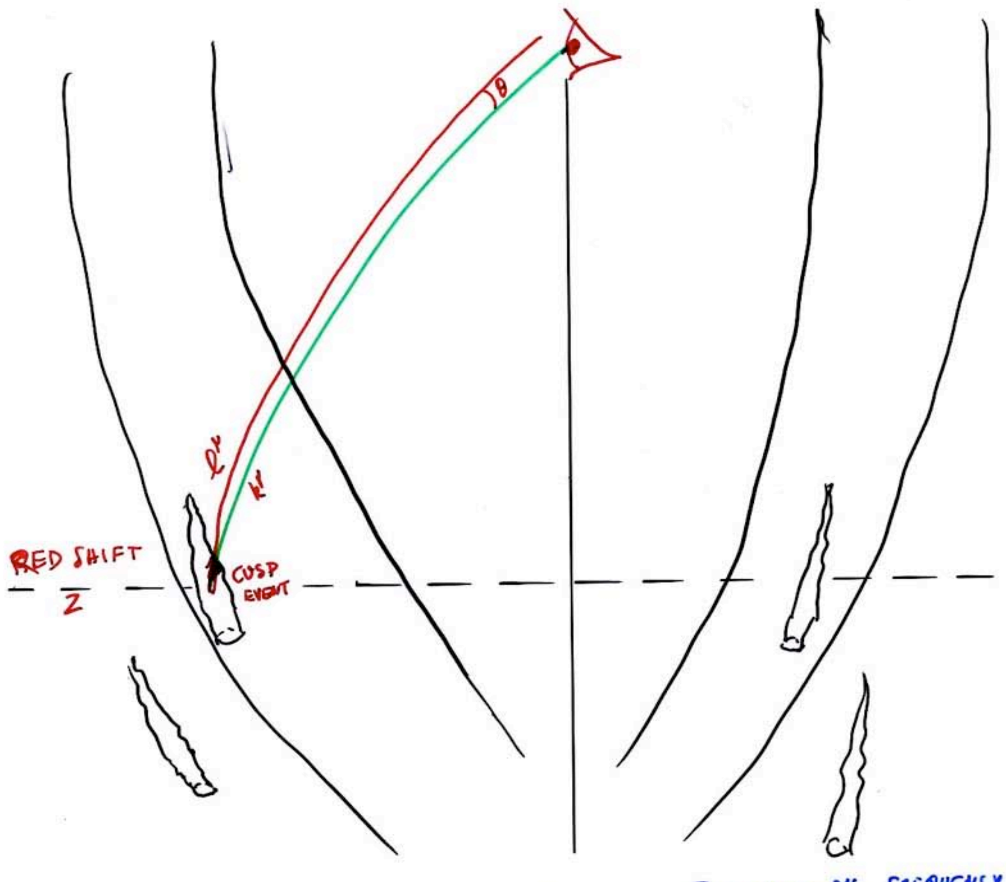


- SMALL-SCALE STRUCTURE (WIGGLES) Siemens, Olum '03





GW BURSTS FROM COSMOLOGICAL STRING NETWORK



- EFFECT OF COSMOLOGICAL EXPANSION ON $\bar{h}_{\mu\nu}(f)$ — ON FREQUENCY / ON AMPLITUDE
- NUMBER OF CUSP EVENTS PER UNIT SPACE-TIME VOLUME

$$\boxed{\nu(t) \sim c\, n_\ell(t)/(\ell/2)}$$

$c \equiv$ # cusp events per loop period

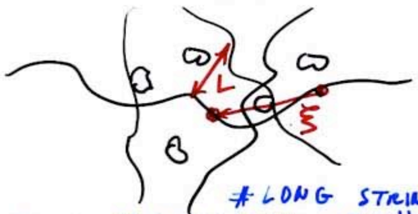
$n_\ell(t)$ = loop density

- BEAMING FRACTION WITHIN $\boxed{\theta_m \equiv \left[(1+z)\, f\, \ell/2\right]^{-1/3}}$

LOOP NUMBER DENSITY $n_\ell(t)$

"STANDARD SCENARIO"

ASSUMES $P_{\text{RECONNECT}} = 1$; $\ell_{\text{WIGGLES}} \sim \Gamma G\mu\, t$ (Bennet Bouchet 88)



COHERENCE LENGTH (ALONG STRING) $\xi^{st}(t) \sim t$

TYPICAL DISTANCE BETWEEN STRINGS $L^{st}(t) \sim \xi^{st} \sim t$

\# LONG STRINGS ~ 1 PER HUBBLE VOLUME

THEN # LOOPS FORMED PER HUBBLE VOL. + HUBBLE TIME: $N_\ell \sim \frac{1}{\Gamma G\mu}$

COSMIC SUPERSTRINGS : $p < 1$, POSSIBLY $p \ll 1$

EXPANSION STRAIGHTENS OUT STRINGS ON SUB-HORIZON SCALES : EXPECT $\boxed{\xi(t) \sim t}$

$p \ll 1 \Rightarrow$ # LONG STRINGS PER HUBBLE VOLUME MUST BE $\sim p^{-1}$ FOR SCALING

THIS CORRESPONDS TO $\boxed{L(t) \sim p^{1/2}\, t}$

OK WITH NON-EXPANDING SIMULATIONS (Sakellariadou, Vilenkin '90)

IF $\boxed{\ell_{\text{WIGGLES}}(t) \sim \alpha t \sim \ell_{\substack{\text{LOOPS}\\ \text{JUST}\\ \text{FORMED}}}(t)}$ WHERE, MAYBE $\boxed{\alpha \equiv \epsilon\, \Gamma G\mu \ll \Gamma G\mu}$

\# LOOPS FORMED PER HUBBLE VOL, PER HUB. TIME $N_\ell \sim \frac{p^{-1} t}{\ell(t)} \sim \frac{1}{p\alpha}$

LIFETIME OF LOOPS $\tau \sim \frac{\alpha}{\Gamma G\mu}\, t \Rightarrow$ $\boxed{n_\ell(t) \sim \frac{\tau}{t}\frac{N_\ell}{t^3} \sim \frac{1}{p\,\Gamma G\mu\, t^3}}$

$\epsilon \equiv \alpha/\Gamma G\mu$ HAS DISAPPEARED FROM n_ℓ BUT REMAINS IN $\ell(t)$

GW OBSERVABLES FROM STRING NETWORK

- TYPICAL GW AMPLITUDE AROUND FREQUENCY f FROM CUSPS AT REDSHIFT Z

$$\boxed{h^{cusp}(f,z) \sim G\mu\, \alpha^{2/3} (f t_0)^{-1/3}\, \varphi_h(z)\, \Theta\left[1-\theta_m(\alpha,f,z)\right]}$$

$$\alpha = \epsilon\Gamma G\mu$$

$$\varphi_h(z) = z^{-1}(1+z)^{-1/3}(1+z/z_{eq})^{-1/3}\,; \quad \theta_m(\alpha,f,z) = (\alpha f t_0)^{-1/3}(1+z)^{1/6}(1+z/z_{eq})^{1/6}$$

- RATE OF GWB's ORIGINATING FROM $\sim$ Z, AND OBSERVED AT f

$$\boxed{\dot{N}(f,z) \sim 100\, \frac{c\,\epsilon}{p}\, t_0^{-1} \alpha^{-8/3} (f t_0)^{-2/3}\, \varphi_n(z)}$$

$$\varphi_n(z) \simeq z^3 (1+z)^{-7/6} (1+z/z_{eq})^{11/6}$$

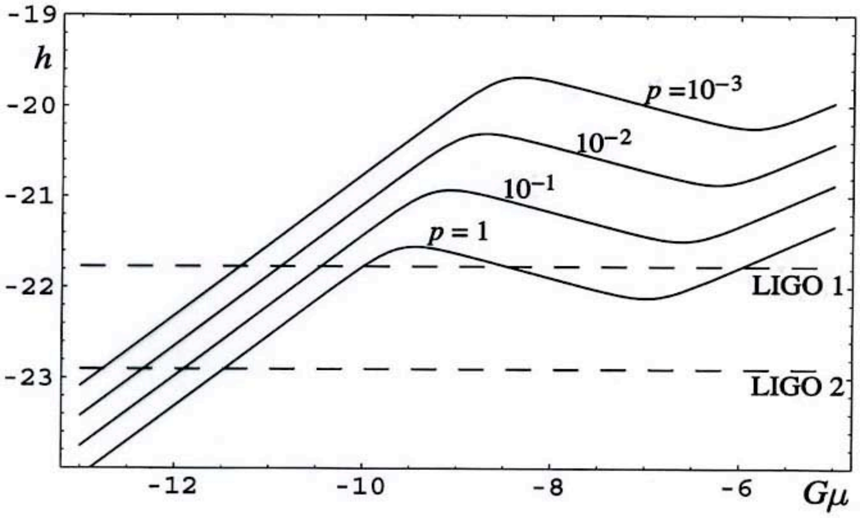
- TYPICAL GWB AMPLITUDE, AROUND f, THAT ONE CAN EXPECT TO DETECT WITH OCCURRENCE RATE $\dot{N}$

$$\boxed{h_{\dot{N}}(f) \sim \mathcal{F}\left[f, \dot{N}, p, \epsilon, G\mu, c\right]}$$

- CONFUSION NOISE (SUMMING OVER OVERLAPPING BURSTS ONLY)

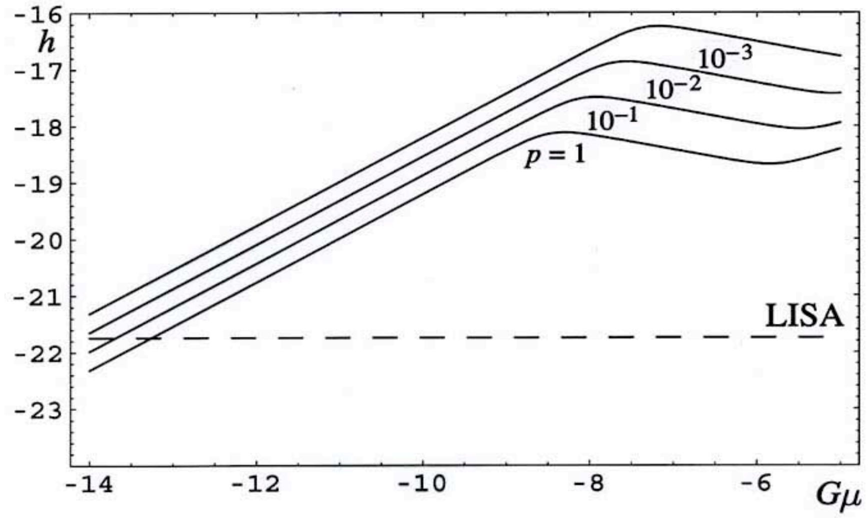
$$\boxed{h^2_{confusion}(f) = \int \frac{dz}{z}\, \frac{\dot{N}(f,z)}{|f|}\, h^2_{cusp}(f,z)\, \Theta\left[\frac{\dot{N}(f,z)}{|f|} - 1\right]}$$

GWB AMPLITUDES RECURRING ONCE PER YEAR EMITTED BY COSMIC STRING CUSPS (WITH $C=1$) IN THE LIGO-VIRGO FREQUENCY BAND ($f_c \sim 150$ Hz) VERSUS THE STRING TENSION $G\mu$ ($\epsilon = \frac{\alpha}{\Gamma G\mu} = 1$; p VARYING)



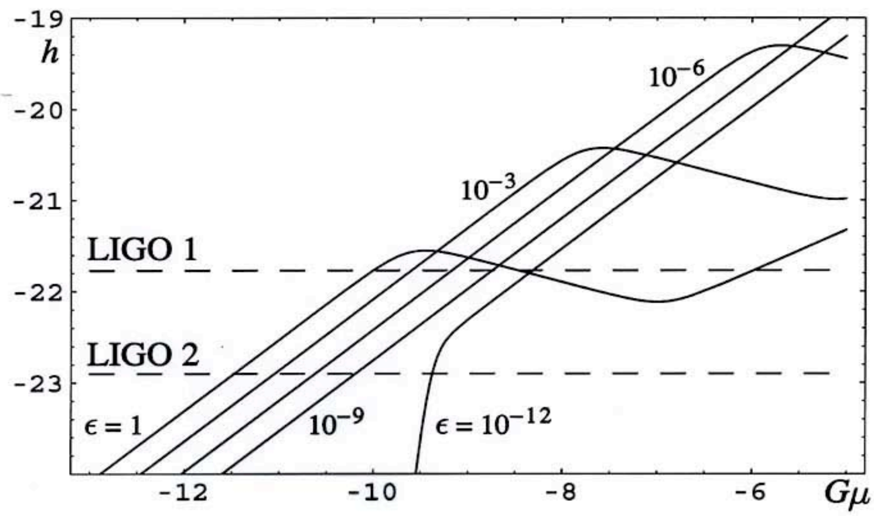
GWB AMPLITUDES RECURRING ONCE PER YEAR IN LISA FREQUENCY BAND ($f_c \sim 3.88 \times 10^{-3}$ Hz)

($\epsilon = \frac{\alpha}{\Gamma G\mu} = 1$; VARYING p)



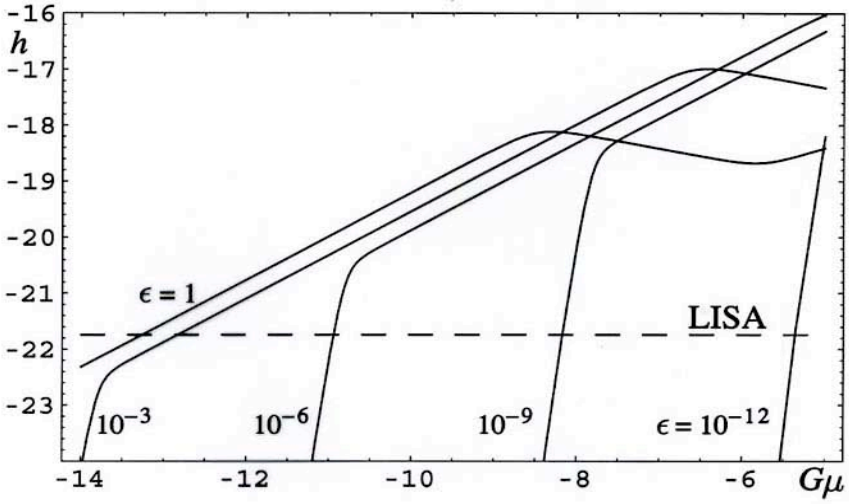
GWB AMPLITUDES RECURRING ONCE PER YEAR IN LIGO FREQUENCY BAND

($p = 1$; EFFECT OF $\epsilon \equiv \frac{\alpha}{\Gamma G\mu} \ll 1$)

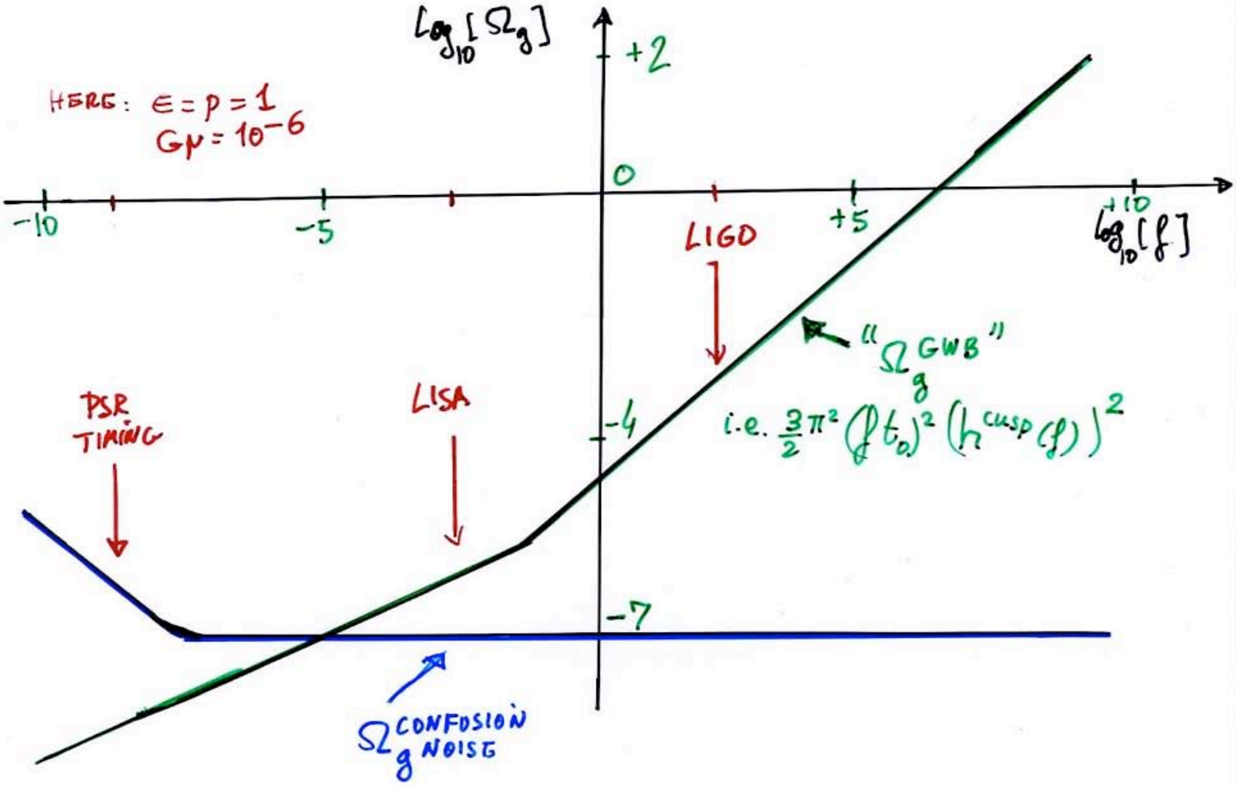


GWB AMPLITUDES RECURRING ONCE PER YEAR IN LISA FREQUENCY BAND

$\left(p = 1 ;\ \text{EFFECT OF } \epsilon \equiv \frac{\alpha}{\Gamma G\mu} \ll 1 \right)$

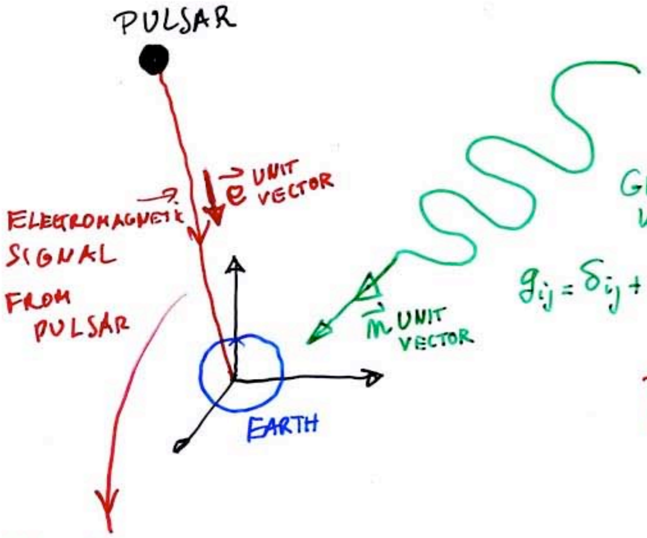


STOCHASTIC GW BACKGROUND AND PULSARS



IN THE LOW-FREQUENCY RANGE APPROPRIATE FOR PULSAR TIMING EXPERIMENTS, THE GW BURSTS ARE DROWNED IN THE CONFUSION NOISE COMING FROM THE SUPERPOSITION OF ALL GW'S FROM STRINGS.

STOCHASTIC GW BACKGROUND AND PULSAR TIMING



GRAVITATIONAL WAVE

$$g_{ij} = \delta_{ij} + h_{ij}^{TT}\left(t - \frac{\vec{m}\cdot\vec{x}}{c}\right)$$

TT GAUGE

$h_{00} = h_{0i} = 0$

$\partial_j h_{ij}^{TT} = 0 \, , \quad h_{ss}^{TT} = 0$

ALONG PULSAR SIGNAL: $0 = ds^2 = -c^2 dt^2 + \left(\delta_{ij} + h_{ij}\left(t - \frac{\vec{n}\vec{x}}{c}\right)\right) dx^i dx^j$

$$c\,dt = |d\vec{x}| + \frac{1}{2} h_{ij}\left(t - \frac{\vec{n}\vec{x}}{c}\right) \frac{dx^i}{d\sigma}\frac{dx^j}{d\sigma} d\sigma \qquad d\sigma = |d\vec{x}|$$

IN TT GAUGE x^i_{PSR} AND x^i_{EARTH} ARE CONSTANT

ONE-WAY TIME DELAY: $\int_{PULSAR}^{EARTH} dt = \frac{1}{c}\int_P^E d\sigma + \frac{1}{2c}\int_P^E h_{ij}\left(t - \frac{\vec{n}\vec{x}}{c}\right)\dot{x}^i \dot{x}^j d\sigma$

$$\boxed{t_{PULSAR\to EARTH} = \frac{\sigma_{P\to E}}{c} + \frac{1}{2}\frac{1}{1-\vec{n}\cdot\vec{e}}\, e^i e^j \left[H_{ij}(t) - H_{ij}\left(t - (1-\vec{n}\cdot\vec{e})\frac{\sigma}{c}\right)\right]}$$

CONSTANT

FLUCTUATING PART

DIRECTION DEPENDENT

$H_{ij}(t) \equiv \int dt\, h_{ij}(t)$

ANTI-DERIVATIVE OF $h_{ij}(t)$

FLUCTUATING PULSAR TIMING RESIDUALS

$$\Delta(t) = \sum_{\substack{\text{RANDOM}\\ \text{ENSEMBLE}\\ \text{OF GWs}}} \frac{1}{2}\frac{1}{1-\vec{m}\cdot\vec{e}}\, e^i e^j \left[H_{ij}(t) - H_{ij}\left(t - (1-\vec{m}\cdot\vec{e})\frac{\sigma}{c}\right)\right]$$

↑ GW RED NOISE, ON TOP OF OTHER PULSAR NOISES

Sahzin '78, Detweiler '79, Mashhoon, Grishchuk '80, Bertotti et al '83 ....

RANDOM $h_{ij}(t) \rightarrow$ POWER-SPECTRUM $\rho_{GW}(f) \sim \frac{\pi}{4G} f^2 h^2_{rms}(f)$

OR $\Omega_{gw}(f) \equiv \frac{\rho_{GW}(f)}{\rho_{CLOSURE}} = 6\pi G t_0^2 \rho_{GW} \sim \frac{3\pi^2}{2}(f t_0)^2\, h^2_{rms}(f)$

$$\boxed{\Delta^2(f) = \frac{H_0^2}{8\pi^4}\,\frac{\Omega_{gw}(f)}{f^4}\, B\!\left(2\pi f\frac{\sigma}{c}\right)}$$

= 1 WHEN $2\pi f\sigma/c \gg 1$, AS IN PULSAR TIMING

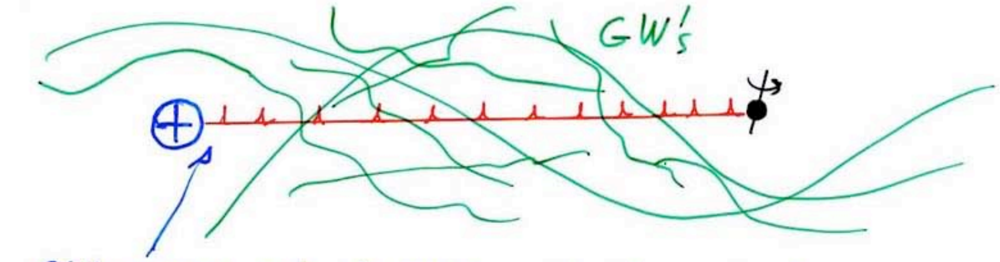
REDDENING

LOGARITHMIC POWER-SPECTRUM OF TIMING RESIDUALS

$$\boxed{\begin{aligned}\Omega_{gw}(f) &\sim 30\, G\mu\, c\, p^{-1} \epsilon^{-1/3} \left(\Gamma G\mu f t_0\right)^{-1/3} \\ &\sim 10^{-2.46} \frac{c}{h_{100}^2} (G\mu)^{2/3} p^{-1} \epsilon_{eff}^{-1/3} \left(\frac{f}{1/10\,yr}\right)^{-1/3}\end{aligned}}$$

(Damour, Vilenkin '05)

PULSAR TIMING AND GW BACKGROUND



GW's INTRODUCE NOISE IN THE REGULARITY OF ARRIVAL TIMES OF PULSES
(Sahzin '78, Detweiler '79, ...

THE DISCOVERY OF THE VERY STABLE "MILLISECOND" PULSARS HAS ALLOWED TO SET STRINGENT LIMITS ON

$$\Omega_g(f_{PSR}) \equiv \frac{d\rho_{GW}/d\ln f}{\rho_{CRITICAL}} \qquad \rho_c \equiv \frac{3H_0^2}{8\pi G}$$

$$f_{PSR} \sim \frac{1}{T_{OBSERVING}} \sim \frac{1}{10\,yr}$$

Kaspi, Taylor, Ryba '94
8 years of PSR observations

$$\boxed{\Omega_g h^2 < 6 \times 10^{-8}}$$

[or 9.3×10^{-8} McHugh et al 96]

Pulsar Timing Array
Lommen, Backer '01
Lommen '02
17 years of PSR observations pieced together

$$\boxed{\Omega_g h^2 < 2 \times 10^{-9}} \quad ?$$

$$(\Omega_g h^2)_{\substack{NEYMAN-\\PEARSON\\TEST}} < 2.8 \times 10^{-6}$$

"COULD THE LARGE SCATTER IN THE GREEN BANK DATA BE DUE TO A TRANSIENT GRAVIT. WAVE BURST ACTIVITY ?"

Damour, Vilenkin '04

Lommen '01

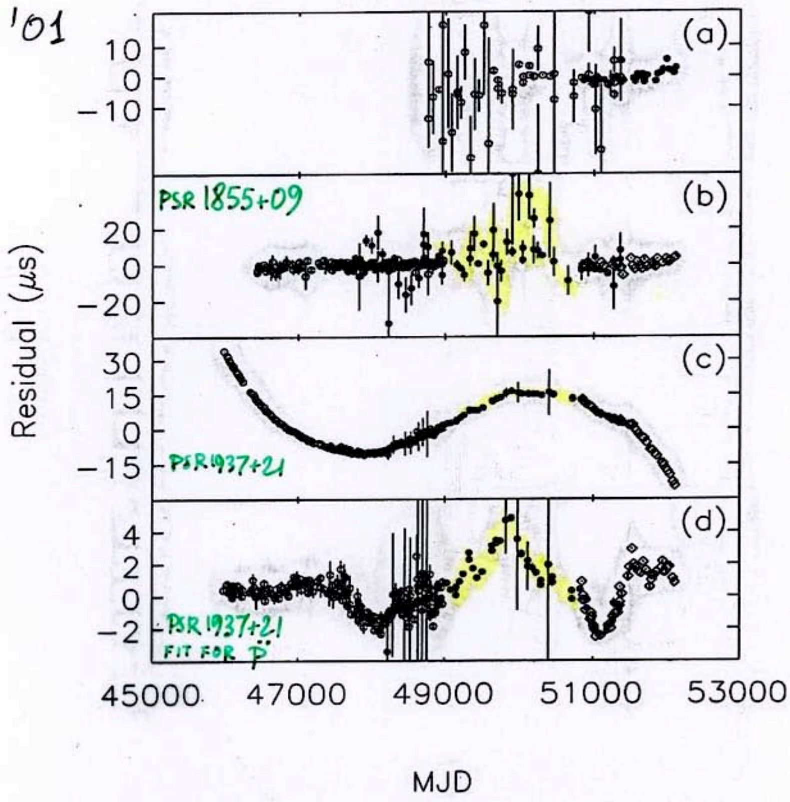


Figure 5.9 (a) PSR J1713+0747, (b) PSR B1855+09, (c) PSR B1937+21 and (d) PSR B1937+21 fit for $\ddot{P}$. The open circles are KTR94 data, the filled circles are Green Bank data, and the open diamonds are ABPP data.

ANALYTICAL ESTIMATE OF CONFUSION NOISE

$$h^2_{\text{confusion}}(f) = \int \frac{dz}{z} \frac{\dot{N}(f,z)}{|f|} \, h^2_{\text{cusp}}(f,z) \, \Theta\left(1-\theta_m(\alpha,f,z)\right) \Theta\left[\frac{\dot{N}(f,z)}{|f|}-1\right]$$

RESTRICTS TO $\theta_m < 1$ i.e. OSCILLATION MODE NUMBER $m > 1$

RESTRICTS TO OVERLAPPING BURSTS

EXCLUDE RARE, INTENSE BURSTS, WHICH ARE OBSERVATIONALLY IRRELEVANT

$$\Omega_g^{\text{confusion}}(f) \sim \frac{3}{2}\pi^2 (f t_0)^2 h^2_{\text{confusion}}(f)$$

LOW-FREQUENCY PART OF SPECTRUM

$\langle \# \rangle$ CUSP EVENTS PER PERIOD

$$\boxed{\Omega_g^{\text{confusion}}(f) \sim 30\, G\mu \frac{c}{p\,\epsilon^{1/3}} \left(\Gamma G\mu f t_0\right)^{-1/3}}$$

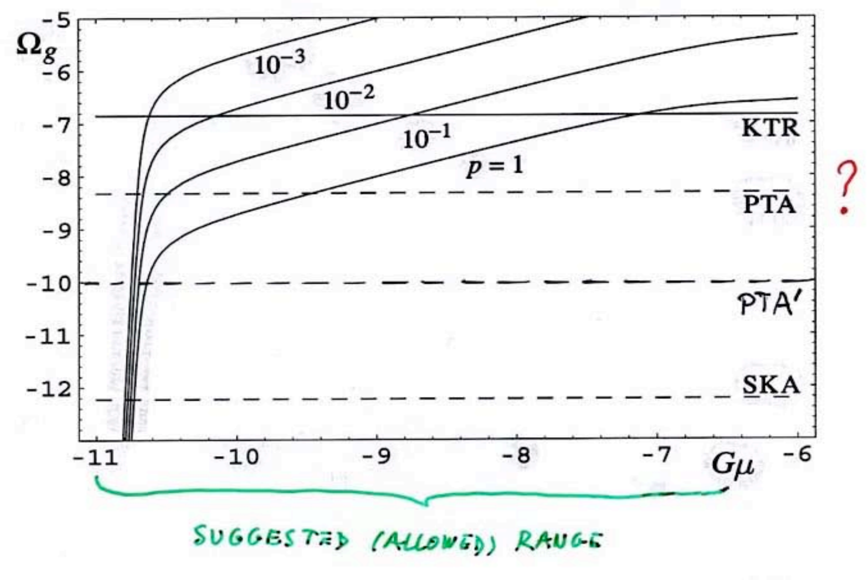
$$\Omega_g h^2 < 6\times 10^{-8} \Rightarrow G\mu < 10^{-7} c^{-3/2} p^{3/2} \epsilon^{1/2}$$

$$\Omega_g h^2 < 2\times 10^{-9} \Rightarrow G\mu < 3.6\times 10^{-10} c^{-3/2} p^{3/2} \epsilon^{1/2} \quad ?$$

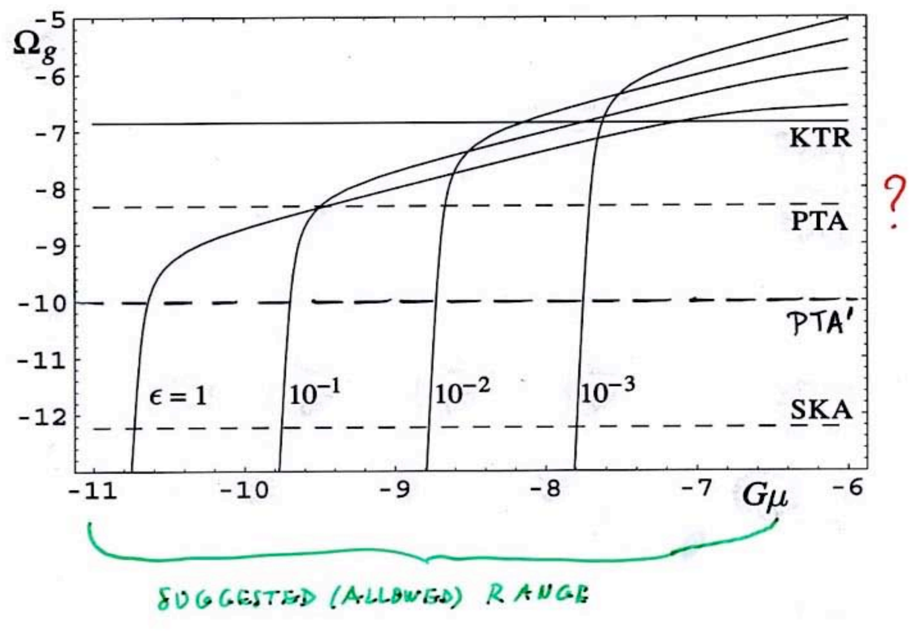
SQUARE KILOMETER ARRAY COULD REACH $\Omega_g h^2 \simeq 10^{-12.6}$

APPLY IF $\alpha = 50\,\epsilon\, G\mu > 10^{-9}$

T21



Ω_g
-5
-6
-7
-8
-9
-10
-11
-12
10^{-3}
10^{-2}
10^{-1}
$p = 1$
KTR
PTA
?
PTA'
SKA
-11
-10
-9
-8
-7
-6
$G\mu$
SUGGESTED (ALLOWED) RANGE



Σ DISAPPEARING SIGNAL IF $50 \epsilon G\mu < 10^{-9}$

CONCLUSIONS

- GRAVITATIONAL WAVE BURSTS FROM CUSPY COSMIC (SUPER)STRING OFFER THE EXCITING POSSIBILITY OF DETECTING (SUPER)STRINGS WITH $G\mu \gtrsim 10^{-13}$ WITH LIGO OR LISA
- PULSAR TIMING MIGHT DETECT A STOCHASTIC GW BACKGROUND FROM STRING WITH $G\mu \gtrsim 10^{-11}$. EXISTING PSR TIMING DATA MIGHT CONTAIN GWB SIGNALS.
- THE PREDICTIONS FOR LIGO OR LISA ARE QUITE ROBUST UNDER CHANGING p (WHICH ONLY IMPROVES THE SIGNAL) OR $\epsilon = \alpha/\Gamma G\mu$: AT LEAST WHEN $\epsilon \gtrsim 10^{-11}$ FOR LIGO
  $\epsilon \gtrsim 10^{-7}$ FOR LISA

  HOWEVER, FOR SMALLER ϵ'S THE GW SIGNAL MIGHT DISAPPEAR WHEN $G\mu \sim 10^{-10}$
- THE PULSAR TIMING SIGNAL IS IMPROVED BY $p < 1$ BUT IS RATHER SENSITIVE TO A DECREASE OF ϵ
  THE SIGNAL DISAPPEARS IF $50\epsilon G\mu < 10^{-9}$
- IT IS URGENT TO DEVELOP A NEW GENERATION OF STRING NETWORK SIMULATION ABLE TO DETERMINE HOW THE CRUCIAL LOOP-LENGTH PARAMETER $\alpha \equiv \ell/t$ DEPENDS ON $G\mu$ (AND p), AND TO CONFIRM THAT $n_\ell(t) \sim (p\,\Gamma G\mu\, t^3)^{-1}$, AND TO DETERMINE C
- IT IS ALSO IMPORTANT TO ASSESS THE VIABILITY OF MIXED F-, D-STRING NETWORKS