EUROSTRINGS
3–6 APRIL 2006

E_{10}

AND

M-THEORY

Thibault Damour
IHES

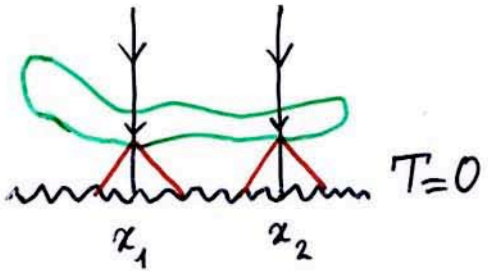
WORK WITH: M. Henneaux, A. Kleinschmidt, H. Nicolai, ...

OTHER APPROACHES: E_{11} : P.C. West '01, I. Schnakenburg, P.C. West '01, ...

E_{10} : J. Brown, O. Ganor, C. Helfgott '04

OTHER HINTS OF E_{10} : N. Obers, B. Pioline, E. Rabinovici '98 ; T. Banks, W. Fischler, L. Motl '99

NEAR SPACELIKE SINGULARITY LIMIT AND A $SUGRA_{11}/[E_{10}/K(E_{10})]_1$ CORRESPONDENCE

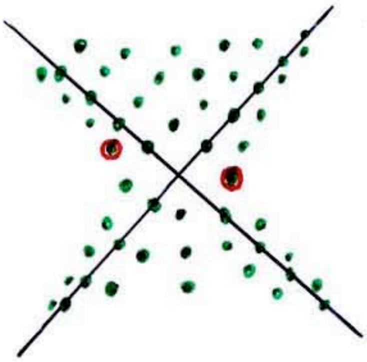


GRADIENT EXPANSION (BKL)

($\sim$ SMALL TENSION EXPANSION : $\alpha' \to \infty$)

$$\partial_{x^1}^{k_1}\partial_{x^2}^{k_2}\cdots\partial_{x^{10}}^{k_{10}} \ll \partial_T^{k_1+k_2+\cdots+k_{10}}$$

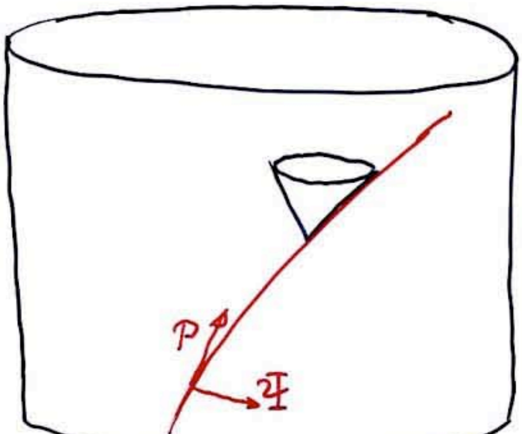
$$\Updownarrow$$



HEIGHT EXPANSION IN KAC-MOODY ALGEBRA

ROOT : $\alpha = n_0\alpha_0 + n_1\alpha_1 + \cdots + n_9\alpha_9$

CONJECTURED CORRESPONDENCE

SUGRA$_{11}$ (OR M-THEORY)		MASSLESS SPINNING PARTICLE ON COSET $E_{10}/K(E_{10})$
$G_{\mu\nu}(t,\vec{x})$ $A_{\mu\nu\lambda}(t,\vec{x})$ $\Psi_\mu(t,\vec{x})$	$\longleftrightarrow$	

$$S_{11} = \int d^{11}x \Big\{ \frac{E}{4} R(G) - \frac{E}{48}(dA_3)^2 + \frac{2}{(12)^4} F_4 \wedge F_4 \wedge A_3 - \frac{i}{2}\bar{\Psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \Psi_\rho - \frac{i}{96}\left(\bar{\Psi}_\mu \Gamma^{\mu\alpha\beta\gamma\delta\nu}\Psi_\nu + 12\,\bar{\Psi}^\alpha \Gamma^{\beta\gamma}\Psi^\delta\right) F_{\alpha\beta\gamma\delta} + \dots \Big\}$$

\+ LOOP CORRECTIONS

$\longleftrightarrow$

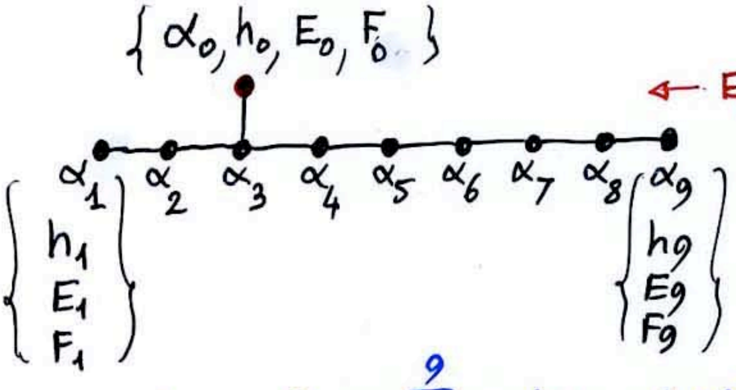
$$S_1^{COSET} = \int dt \Big\{ \frac{1}{4n}\langle P(t) | P(t) \rangle - \frac{i}{2}\left(\Psi(t) \,|\, \overset{vs}{D}\, \Psi(t)\right)_{vs} + \left(\chi(t) \,|\, P(t) \odot \Psi(t)\right)_s \Big\}$$

T.D., Henneaux, Nicolai '02 ; T.D., Kleinschmidt, Nicolai '06 ; de Buyl, Henneaux, Paulot '06

E_{10}

rank 10 : $\dim \mathfrak{h} = 10$ AND $\exists$ 10 basic raising gators E_{α_i}

10 SIMPLE ROOTS

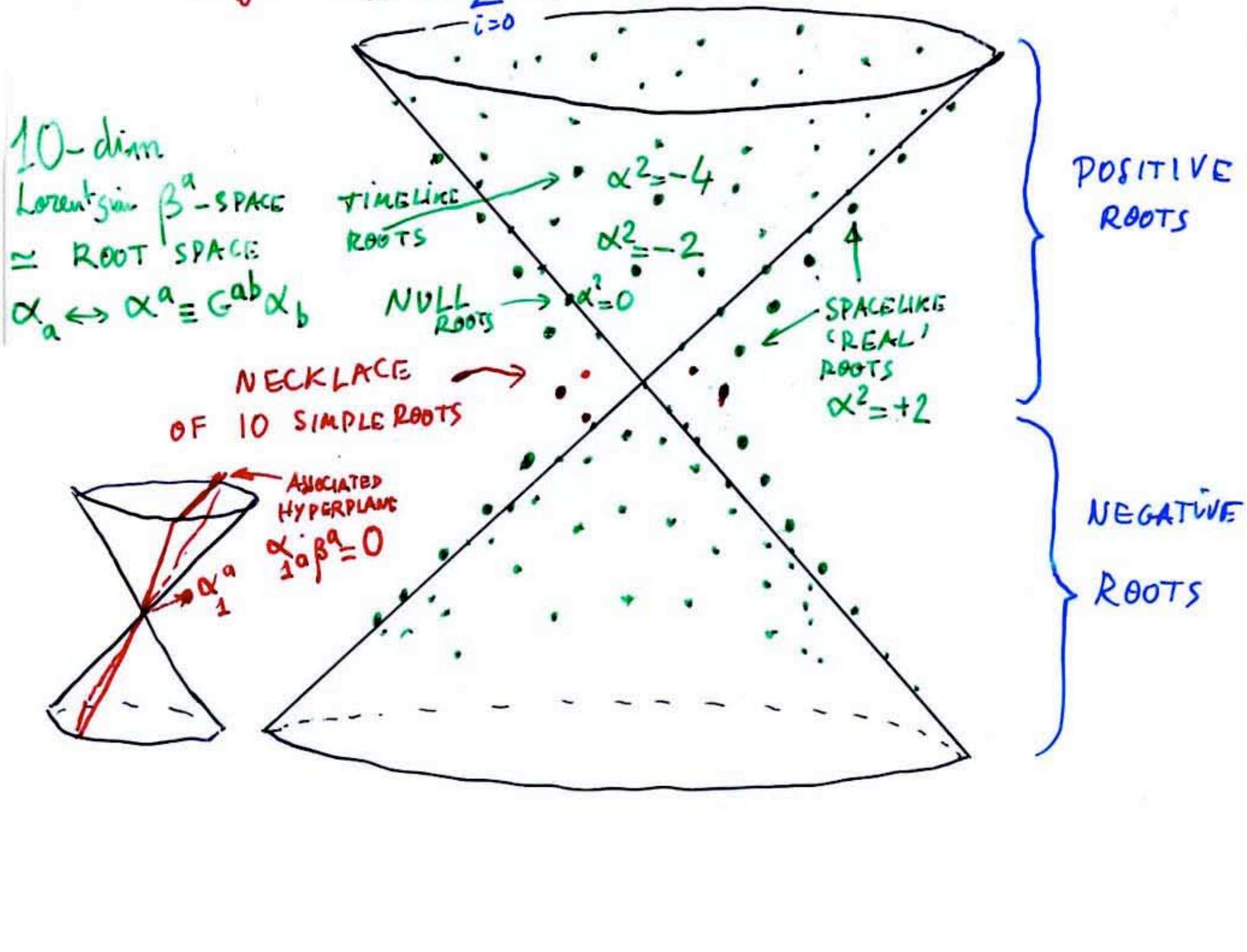


$\leftarrow E_0 = E^{123} \in E^{[abc]}$; $F_0 = F_{123} \in F_{[abc]}$

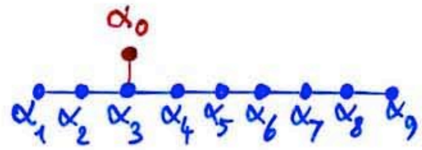
with h_0 DEFINES GL_{10} SUBALGEBRA

$$\{\text{ALL ROOTS}\} = \{\underbrace{\alpha = \sum_{i=0}^{9} n^i \alpha_i \,;\, n^i \in \mathbb{N}}_{\text{POSITIVE ROOTS}}\} \cup \{\underbrace{\alpha = -\sum_{i=0}^{9} n^i \alpha_i \,;\, n^i \in \mathbb{N}}_{\text{NEGATIVE ROOTS}}\}$$

height $ht[\alpha] = \sum_{i=0}^{9} n_i$



DECOMPOSING E_{10} W.R.T. $GL(10)$ SUBALGEBRA



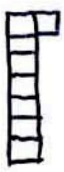
← $SL(10) = A_9$ SUBALGEBRA

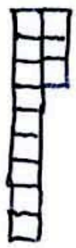
"LEVEL" ℓ : $\quad \alpha = \ell\,\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j$

$\ell = 0$ $\quad GL(10)$ GENERATORS $K^a{}_b$ $\quad [K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b$

$\ell = \pm 1$ $\quad E^{[a_1 a_2 a_3]}$, $F_{[a_1 a_2 a_3]}$  3 INDICES

$\ell = \pm 2$ $\quad E^{[a_1 \cdots a_6]}$, $F_{[a_1 \cdots a_6]}$  6 INDICES

$\ell = \pm 3$ $\quad E^{[a_0 | a_1 \cdots a_8]}$, $F_{[a_0 | a_1 \cdots a_8]}$  9 INDICES

$\ell = \pm 4$  $\oplus$  12 INDICES

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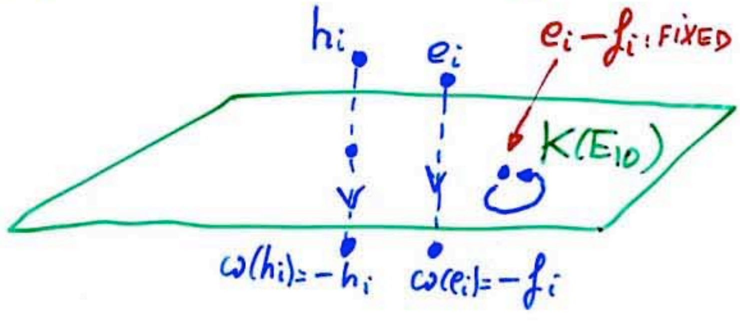
•

$K(E_{10})$: MAXIMAL COMPACT SUBGROUP OF THE CANONICAL REAL FORM OF E_{10}

FIXED SET OF CHEVALLEY INVOLUTION ω



GL(10) DECOMPOSITION OF $K(E_{10})$

$J^{ab} = K^a{}_b - K^b{}_a$: $SO(10)$ $\quad [J^{ab}, J^{cd}] = 4\,\delta^{[b}_{[c}\, J^{a]}{}_{d]}$

$J^{a_1a_2a_3} = E^{a_1a_2a_3} - F_{a_1a_2a_3}$ $\quad [J^{a_1a_2a_3}, J^{b_1b_2b_3}] = J^{a_1a_2a_3b_1b_2b_3} - 18\,\delta^{a_1b_1}\delta^{a_2b_2}J^{a_3b_3}$

$J^{a_1a_2\cdots a_6} = E^{a_1\cdots a_6} - F_{a_1\cdots a_6}$ $\quad [J^3, J^6] = J^9 + J^3$

$J^{a_0|a_1\cdots a_8} = E^{a_0|a_1\cdots a_8} - F_{a_0|a_1\cdots a_8}$ $\quad [J^3, J^9] = J^{12} + J^6$

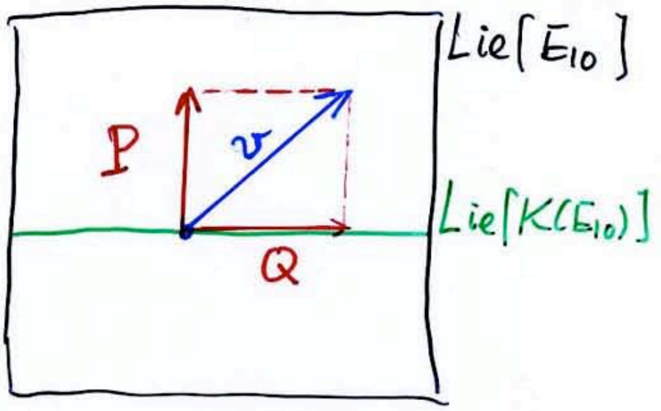
$\vdots$

$D=1$ $E_{10}/K(E_{10})$ COSET MODEL

GROUP ELEMENT $g(t) \in E_{10}$

$Lie[E_{10}]$ 'VELOCITY': $\boxed{v \equiv \frac{dg}{dt} g^{-1}}$

DECOMPOSING v



$$\underset{Lie(E_{10})}{v} \equiv \underset{Lie(K(E_{10}))^{\perp}}{P} + \underset{Lie(K(E_{10}))}{Q}$$

'VERTICAL' 'HORIZONTAL'

COSET ACTION

BOSONIC PART

'VERTICAL' PART OF VELOCITY $v = \dot{g} g^{-1}$

$$\boxed{S_{1\,\mathrm{BOS}}^{\mathrm{COSET}} = \int dt \, \frac{1}{4n} \langle P(t) \,|\, P(t) \rangle}$$

UNIQUE INVARIANT QUADRATIC FORM OF $Lie(E_{10})$

SIGNATURE: $- + + + + + + + + +$; $+ + + + + \cdots\cdots$; $\cancel{- - - - - \cdots}$
CARTAN ; $K(E_{10})^{\perp}$; $\cancel{K(E_{10})}$

SYMMETRY: $g(t) \longrightarrow \underset{\text{LOCAL } K(E_{10})}{k(t)} \, g(t) \, \underset{\text{GLOBAL } E_{10}}{g_0}$

$$P(t) \longrightarrow k(t) \, P(t) \, k(t)^{-1}$$

$$Q(t) \longrightarrow k(t) \, Q(t) \, k^{-1}(t) + \partial_t k \, k^{-1}$$

HORIZONTAL VELOCITY: $K(E_{10})$ CONNECTION

FERMIONIC PART

$$\boxed{S_{1\,\mathrm{FERM}}^{\mathrm{COSET}} = -\frac{i}{2} \int dt \left(\Psi(t) \,|\, \overset{vs}{D} \Psi(t) \right)_{vs} + \int dt \left(\chi(t) \,|\, P(t) \circ \Psi(t) \right)_{s}}$$

'VECTOR-SPINOR REPRES. OF $K(E_{10})$': $\Psi = (\psi_a , \dots)$

EXPLICIT PARAMETRIZATION OF $E_{10}/K(E_{10})$

$$g(t) = e^{h^a{}_b(t) K^b{}_a} \; e^{\frac{1}{3!} A_{a_1a_2a_3}(t) E^{a_1a_2a_3} + \frac{1}{6!} A_{a_1\cdots a_6} E^{a_1\cdots a_6} + \frac{1}{9!} A_{a_0|a_1\cdots a_8} E^{a_0|a_1\cdots a_8} + \cdots}$$

$GL(10): K^a{}_b$

h^a_b

$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c$

$A_{a_1a_2a_3}$

$A_{a_1\cdots a_6}$

$A_{a_0|a_1\cdots a_8}$

$+ \cdots$

indices raised by g^{ab}

$$\boxed{S_1^{E_{10}/K(E_{10})} = \int \frac{dt}{n(t)} \left[\frac{1}{4} \left(g^{ac} g^{bd} - g^{ab} g^{cd} \right) \dot{g}_{ab} \dot{g}_{cd} + \frac{1}{2}\frac{1}{3!} \dot{A}_{a_1a_2a_3} \dot{A}^{a_1a_2a_3} + \frac{1}{2}\frac{1}{6!} DA_{a_1\cdots a_6} DA^{a_1\cdots a_6} + \frac{1}{2}\frac{1}{9!} DA_{a_0|a_1\cdots a_8} DA^{a_0|a_1\cdots a_8} + \cdots \right.}$$

$$DA_{a_1\cdots a_6} = \dot{A}_{a_1\cdots a_6} + 10\, A_{[a_1\cdots a_3} \dot{A}_{a_4\cdots a_6]}$$

$$DA_{a_0|a_1\cdots a_8} = \dot{A}_{a_0|a_1\cdots a_8} + 42\, A_{\langle a_1\cdots a_3} \dot{A}_{a_4\cdots a_9\rangle} - 42\, \dot{A}_{\langle a_1\cdots a_3} A_{a_4\cdots a_9\rangle} + 280\, A_{\langle a_1\cdots a_3} A_{a_4\cdots a_6} \dot{A}_{a_7\cdots a_9\rangle}$$

$\langle \cdots \rangle$ = projection on Young

CORRESPONDENCE $E_{10}/K(E_{10})$ COSET $\Longleftrightarrow$ $SUGRA_{11}$

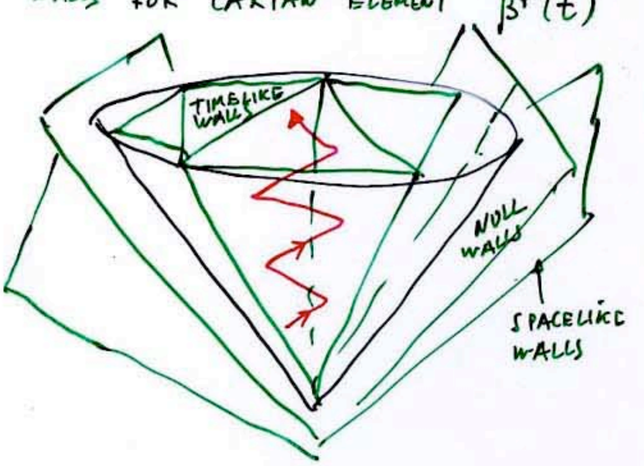
$$\mathcal{L}_1^{E_{10}} \sim (g^{-1}\dot{g})^2 + (\dot{A}_3)^2 + (\dot{A}_6 + A_3\dot{A}_3)^2 + (\dot{A}_9 + A_6\dot{A}_3 + A_3A_3\dot{A}_3)^2 + \dots$$

$$\mathcal{F}_4 = dA_3$$

$$\mathcal{L}_{11}^{SUGRA} = \int d^{11}x \left[\sqrt{-G}\left[R(G) - \frac{(dA_3)^2}{48}\right] + \frac{1}{(12)^4}\mathcal{F}_4 \wedge \mathcal{F}_4 \wedge A_3\right]$$

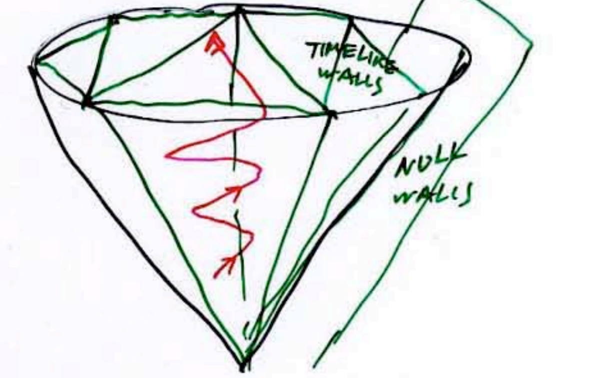
$\Downarrow$

BILLIARD WITH INFINITE NUMBER OF EXPONENTIAL WALLS FOR CARTAN ELEMENT $\beta^i(t)$



$\Downarrow$

BILLIARD WITH LARGE BUT FINITE # OF EXPONENTIAL WALLS FOR $\beta^a(t,x)$, DIAGONAL PART OF $G_{ij}(t,x)$ IN IWASAWA DECOMP.



$$H_1 = \frac{1}{2}G^{\mu\nu}\pi_\mu\pi_\nu + \sum C_\alpha(q,p)\, e^{-2\alpha(\beta)}$$

$$\alpha(\beta) = \sum_i m^i \alpha_i(\beta) \qquad m^i \in \mathbb{N}$$

SIMPLE ROOTS

$$H_{11} = \frac{1}{2}G^{\mu\nu}\pi_\mu\pi_\nu + \sum_A C_A(Q, P, \partial_x\beta, \partial_x Q, \dots) \times e^{-2w_A(\beta)}$$

$$w_A(\beta) = \sum_i m^i w_i(\beta)$$

DOMINANT WALLS

$$\boxed{\text{DICTIONARY}}$$

$$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c = G^{ab}(t, \vec{x}_0)$$

WRT A SPECIAL FRAME $\theta^a(x) = e^a_i(x)\, dx^i$

$$\dot{A}_{a_1a_2a_3}(t) = \mathcal{F}_{0a_1a_2a_3}(t, \vec{x}_0)$$

$$DA^{a_1\dots a_6}(t) = g^{a_1a_1'} \cdots g^{a_6a_6'}\left[\dot{A}_{a_1'\dots a_6'} + 10\, A_{[3}\dot{A}_{3]}\right] = -\frac{1}{4!}\varepsilon^{a_1\dots a_6 b_1\dots b_4}\, \mathcal{F}_{b_1\dots b_4}(t, \vec{x}_0)$$

$$DA^{b|a_1\dots a_8}(t) = g^{\cdot\cdot}\left[\dot{A}_9 + 42 A_3\dot{A}_6 + 280 A_3A_3\dot{A}_3\right] = +\frac{3}{2}\varepsilon^{a_1\dots a_8 b_1 b_2}\, C^b{}_{b_1b_2}(\vec{x}_0)$$

$$d\theta^a = \frac{1}{2}C^a{}_{bc}\,\theta^b \wedge \theta^c$$

THE CORRESPONDENCE WORKS FOR ALL TERMS OF HEIGHT $\leqslant 29$

$$\sum_i m^i \leqslant 29 \qquad\qquad \sum_i m^i \leqslant 29$$

HIGHER-ORDER M-THEORY CORRECTIONS AND E_{10}

(Damour, Nicolai, 2005)

$$S_M = \int \frac{d^{11}x \sqrt{-G}}{\ell_P^9} \Big[\underbrace{R - F^2 + AFF}_{\text{TWO DERIVATIVES } g\partial^2 g + \partial^2 A} \Big] + \text{HIGHER-ORDER CORRECTIONS}$$

FROM
- ONE STRING-LOOP CORRECTIONS TO $\overset{D=10}{\mathrm{II}A}$ AMPLITUDES Green Schwarz '82, Sakai, Tanii '87
- DIVERGENCES IN $D=11$ SUGRA Deser, Seminara '99, '00
- M-THEORY LOOPS Green Vanhove '97, Green Gutperle Vanhove '97
- ANOMALY IN 5-BRANE Duff, Liu, Minasian '95

+ Tseytlin '00, Peeters, Vanhove, Westerberg '01

$\nu > 0$

$$S_1 = \frac{\nu}{\ell_P^3} \int d^{11}x \sqrt{-G} \Big[t_8 t_8 R^4 - \frac{1}{4} \varepsilon_8 \varepsilon_8 R^4 \quad \leftarrow \text{EXPECTED FROM ONE-LOOP (IIA) LIGHT-CONE 4-PT AMPLITUDES}$$

$$+ \frac{2}{4} \varepsilon_8 \varepsilon_8 R^4 \quad \leftarrow \text{CHANGES THE SIGN OF } \bar{E}_8$$

$$- 4 \varepsilon_{11} A_3 \left[\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2 \right] \quad \leftarrow \text{CHERN-SIMONS TYPE}$$

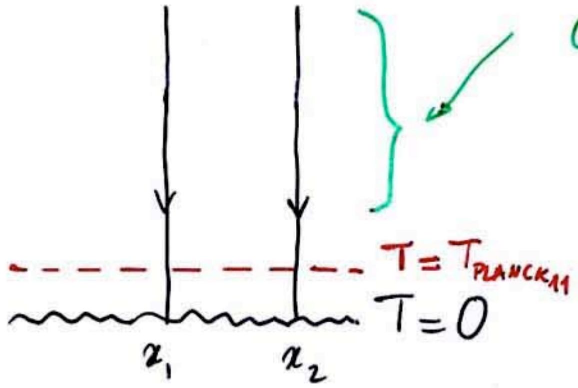
$$+ \underbrace{R^2 (DF)^2 + R (DF)^3 + (DF)^4}_{\text{NOT KNOWN IN NICE FORM}} + \underbrace{\cdots + F^8}_{\text{NOT KNOWN}} \Big]$$

$$t_8 M^4 \equiv t_8^{\mu_1 \cdots \mu_8} \underbrace{M_{\mu_1\mu_2}}_{\text{ANTISYMMETRIC}} M_{\mu_3\mu_4} M_{\mu_5\mu_6} M_{\mu_7\mu_8} \equiv 24\, \mathrm{tr} M^4 - 6 \left(\mathrm{tr} M^2 \right)^2$$

$$\varepsilon_8 \varepsilon_8 R^4 \equiv \delta^{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6\nu_7\nu_8}_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7\mu_8} R^{\mu_1\mu_2}{}_{\nu_1\nu_2} R^{\mu_3\mu_4}{}_{\nu_3\nu_4} R^{\mu_5\mu_6}{}_{\nu_5\nu_6} R^{\mu_7\mu_8}{}_{\nu_7\nu_8}$$

$$\equiv \bar{E}_8 = \text{EULER-LOVELOCK DENSITY}$$

EFFECT OF $\mathcal{R}^4 + \ldots$ TERMS IN THE NEAR SPACELIKE SINGULARITY LIMIT



CONSIDER INTERMEDIATE ASYMPTOTICS

$$T_{PLANCK} \ll T \ll T_{INITIAL}$$

WITH $\partial_i \ll \partial_0$ ALREADY

IN THIS INTERMEDIATE ASYMPTOTICS :

2-DERIVATIVE S_{SUGRA}, PROJECTED IN 'β SPACE'

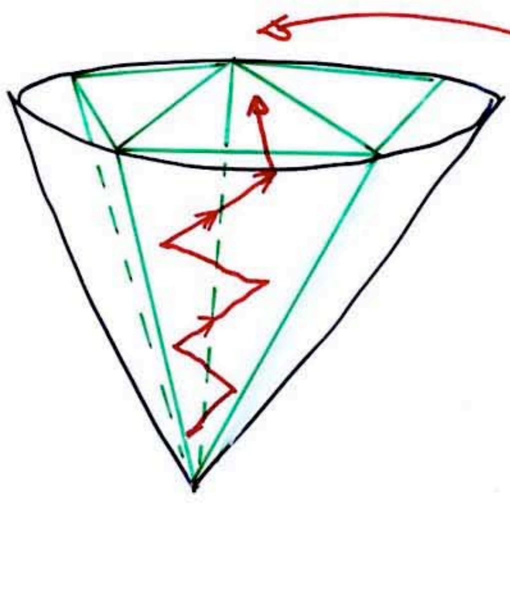
SPATIAL ZEHNBEIN:

$$\boxed{e^a_\mu = e^{-\beta^a} \mathcal{N}^a_\mu}$$

REVEALS THE BEGINNING OF THE E_{10} ROOT STRUCTURE

UPPER TRIANGULAR FREEZES AS $T \to 0$

β - SPACE



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EFFECT OF $\mathcal{R}^4 + \ldots$

AS $\beta \nearrow$

i.e. $T \searrow$ TOWARD T_{PLANCK}

LEADING COMPONENTS OF $\mathcal{R}$, F , DF

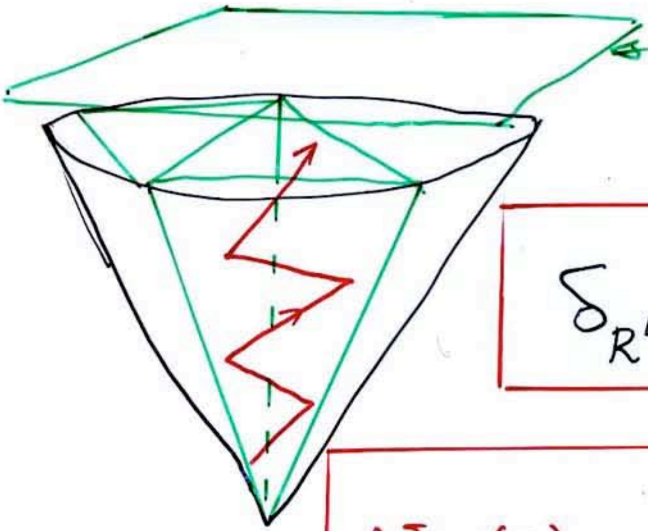
$v_a \equiv \partial_t \beta^a$

$$\boxed{\sigma \equiv \sum_{a=1}^{10} \beta^a}$$

$$\boxed{\begin{aligned} R_{0a0a} &\simeq e^{2\sigma}\, v_a\, \bar{v}_a \\ R_{abab} &\simeq e^{2\sigma}\, v_a\, v_b \\ D_0 F_{0abc} &\simeq e^{2\sigma} e^{-(\beta^a+\beta^b+\beta^c)}\, \varepsilon_{abc} \end{aligned}}$$

$\bar{v}_a \equiv \partial_t \sigma - \partial_t \beta^a$

LEADING CORRECTION TO HAMILTONIAN FOR β-DYNAMICS



SPACELIKE 'WALL' ASSOCIATED TO R^4

$$\boxed{\delta_{R^4} \mathcal{H}(\beta, \pi_\beta) = c\, e^{-2 w_{R^4}(\beta)}}$$

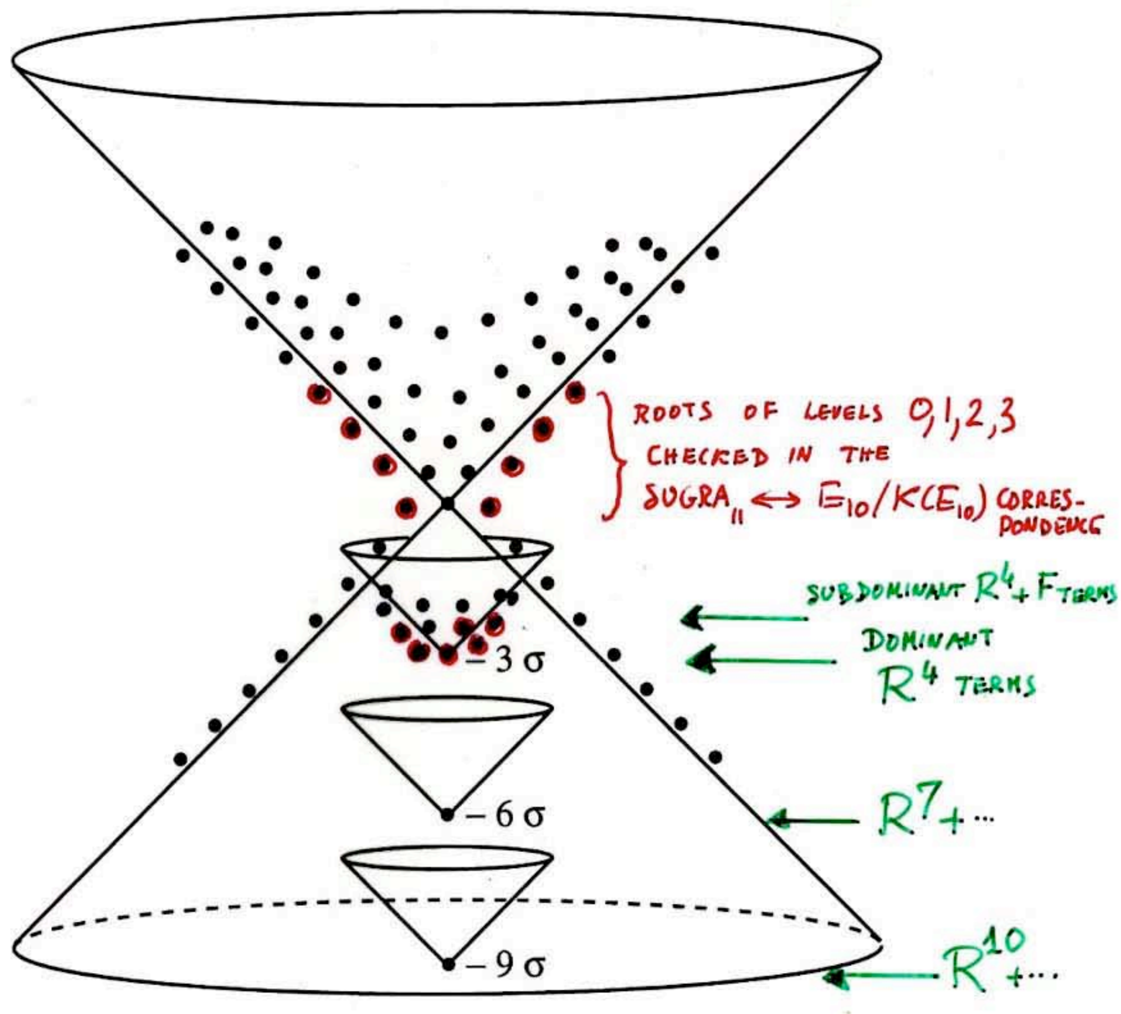
$$\boxed{w_{R^4}(\beta) = -3\sigma = -3 \sum_{a=1}^{10} \beta^a}$$

DOES CORRESPOND TO A ROOT OF E_{10}

IMAGINARY: $\alpha^2 = -10$; $\ell = -10$; $ht = -115$

w_{R^4} PERMUTATION SINGLET OF $GL(10)$ $\longleftrightarrow$ $\alpha_{R^4} \leftrightarrow$ SINGLET OF $GL(10)$

ROOTS OF E_{10}


ROOTS OF LEVELS 0,1,2,3
CHECKED IN THE
$SUGRA_{11} \leftrightarrow E_{10}/K(E_{10})$ CORRESPONDENCE
SUBDOMINANT R^4 + F TERMS
DOMINANT R^4 TERMS
R^7 + ...
R^{10} + ...
-3σ
-6σ
-9σ

SIGN OF $\delta_{R^4}\mathcal{H} = c\, e^{-2 w_{R^4}(\beta)}$

$$c = c_1 + c_2$$

c_1: $t_8 t_8 R^4 - \frac{1}{4}\varepsilon_8\varepsilon_8 R^4$

c_2: $+\frac{2}{4}\varepsilon_8\varepsilon_8 R^4$

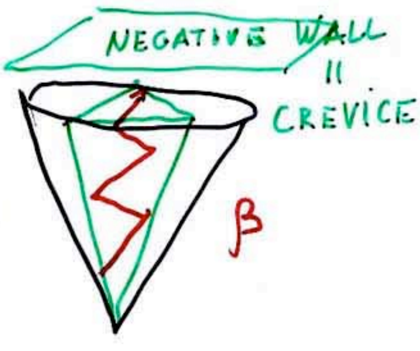
$$c_1 = \sum_{a,b}{}' v_a^2 v_b^2 \left(v_a\bar{v}_a + v_b\bar{v}_b - \bar{v}_a\bar{v}_b\right)^2 + \frac{1}{3}\sum_{a,b,c}{}'' v_a^2 v_b^2 v_c^2 \left(v_a + v_b + v_c\right)^2 \geq 0$$

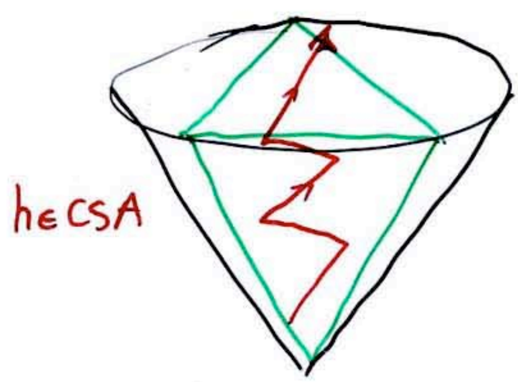
AND $\mathcal{O}(1) \times \left(\sum_a v_a\right)^8$

$$c_2 = -60 \sum_{a_1 < \cdots < a_8}{}' v_{a_1} v_{a_2} \cdots v_{a_8} : \quad -0.0138 \leq \frac{c_2}{\left(\sum_a v_a\right)^8} \leq 0.0391$$

$$\boxed{\text{PROBABLY} \quad c = c_1 + c_2 \geq 0}$$

NO-BOUNCE IN LOOP-CORRECTED SUGRA $\xleftrightarrow{\text{OK}}$ NO-BOUNCE IN COSET MODEL





'E_{10} PREDICTIONS' FOR HIGHER-LOOP CORRECTIONS IN M-THEORY

- ONLY $R^4, R^7, R^{10}, \ldots, R^{3k+1}, \ldots$ ARE COMPATIBLE WITH E_{10} AMONG R^N TERMS

  [OK WITH Russo, Tseytlin '97]
- SIMILARLY ONLY $R^{3k+1-n}(DF)^n$ IS OK WITH E_{10}

- EACH R^{3k+1} CORRECTION MUST BE ASSOCIATED WITH A $SL(10)$ SINGLET GENERATOR OF E_{10} AT LEVEL $\ell = -10k$

- E_{10} MIGHT ENCODE INFORMATION ABOUT THE ALGEBRAIC STRUCTURE OF LOOP CORRECTIONS

CONCLUSIONS

- THE NEAR SPACELIKE SINGULARITY LIMIT $[\approx \alpha' \to \infty]$ SUGGESTS $\exists$ HIDDEN $E_{10}(\mathbb{R})$ SYMMETRY OF M-THEORY

- SUGGESTS A CORRESPONDENCE BETWEEN
  M-THEORY $\longleftrightarrow$ MASSLESS SPINNING PARTICLE ON $E_{10}/K(E_{10})$
- CONFIRMATIONS: BOSONIC AND FERMIONIC SECTORS $SUGRA_{11}$, R^4 + ... CORRECTIONS
- WITH PRESENT TOOLS, IT IS UNCLEAR HOW TO EXTEND THE DICTIONARY SUGRA $\Leftrightarrow$ COSET BEYOND ITS CURRENT DEFINITION MAYBE BECAUSE $\nexists$ NO COMMON DOMAIN OF VALIDITY OF THE TWO DESCRIPTIONS

- INTERESTING TO SPECULATE THAT, AS ONE APROACHES A COSMOLOGICAL SINGULARITY, SPACE 'DE-EMERGES' AND ONE MUST REPLACE $SUGRA_{11} \longrightarrow$ 1-DIM $E_{10}/K(E_{10})$ COSET
  [THE 10-DIM SPATIAL EXTENSION BEING REPLACED BY THE INFINITE NUMBER OF COORDINATES IN COSET $E_{10}/K(E_{10})$]