

GALILEO GALILEI INSTITUTE
19-21 SEPTEMBER 2005
FLORENCE

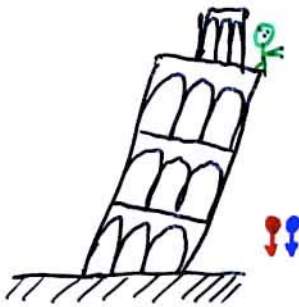
EXPERIMENTAL TESTS OF GRAVITY:

WERE GALILEO AND EINSTEIN 100% RIGHT ?

Thibault DAMOUR

I H E S

GALILEO AND THE UNIVERSALITY OF FREE FALL



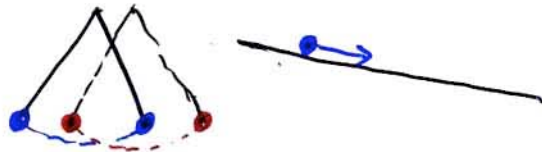
GALILEO'S MYTHICAL EXPERIMENT

BUT: A SAMPLE OF HISTORY

1553 BENEDETTI



~ 1600 GALILEO IN PISA



1634 JESUITS CABEO, RICCIOLI DROPPED WEIGHTS FROM THE TOP OF A TOWER (~80 FEET HIGH) IN THE JESUIT CHURCH OF FERRARA

1638 GALILEO PUBLISHES HIS DISCORSI ON TWO NEW SCIENCES (LEIDEN)

1641 VINCENZO RINIERI WRITES TO GALILEO THAT "A JESUIT (CABEO) WRITES THAT THEY COME DOWN AT THE SAME TIME". $\Rightarrow$ TRIGGERS HIM TO DROP BODIES FROM PISA'S LEANING TOWER.

HE FINDS THAT BODIES DO NOT EXACTLY FALL TOGETHER

$\Rightarrow$ GALILEO THEN EXPLAINS TO HIM THAT THE EXPERIMENTAL RESULTS MUST BE INTERPRETED!

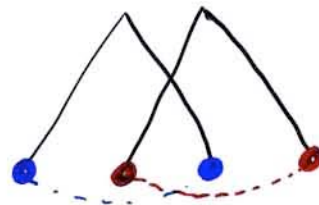
GALILEO'S DISCORSI (1638, LEIDEN)

- AFTER DISCUSSING THE RELATIVE IMPORTANCES OF WEIGHT AND RESISTANCE FROM THE MEDIUM:

"... veduto, dico questo, cascai in opinione che si levasse totalmente la resistenza del mezzo, tutte le materie descenderebbero con eguali velocità"

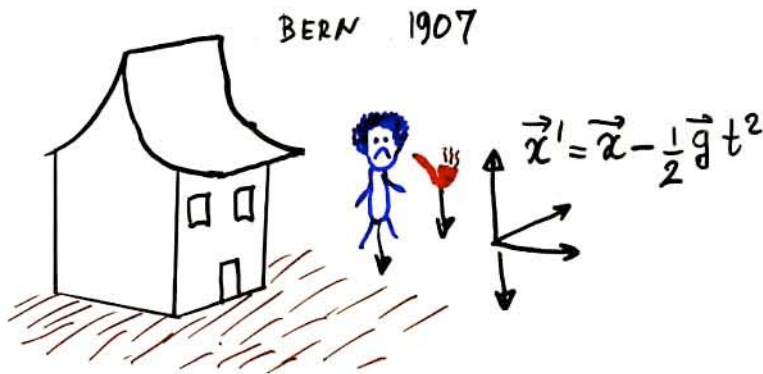
"... having observed this I came to the conclusion that, if one could totally remove the resistance of the medium, all substances would fall at equal speeds"

- PRECISE (?) PENDULA EXPERIMENTS

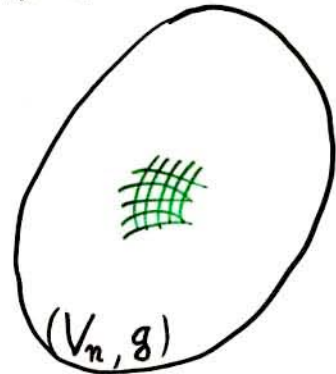


"Accordingly, I took two balls, one of lead and one of cork, the former more than a hundred times heavier than the latter, and suspended them by means of two equal fine threads, each four or five cubits long the heavy body maintains so nearly the period of the light body that neither in a hundred nor even in a thousand will the former anticipate the latter by as much as a single moment, so perfectly do they keep step."

EINSTEIN'S VISION



RIEMANN ~ 1856



LOCAL EFFACEMENT OF $\vec{g}$

$$\exists x'^\mu; g'_{\mu\nu}(x') = \eta_{\mu\nu} + O[(x' - x_0)^2]$$

LOCAL EFFACEMENT OF $\Gamma_{\mu\nu}^\lambda \sim \partial g$

UNIVERSALITY OF FREE FALL $\longleftrightarrow$ UNIVERSAL COUPLING OF MATTER
 TO ONE $g_{\mu\nu}(x)$
 ['HYPOTHESIS OF EQUIVALENCE']
 ['EQUIVALENCE PRINCIPLE']

$$S_{\text{TOT}} = \frac{c^4}{16\pi G} \int \sqrt{g} \frac{d^4x}{c} R(g) + S_{\text{MATTER}}[\phi, A, H; g_{\mu\nu}]$$

TWO SORTS OF EXPERIMENTAL TESTS

• MATTER-COUPPLING TESTS (RHS)

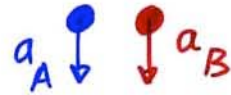
• TESTS OF THE DYNAMICS OF $g_{\mu\nu}$ (LHS)



TESTS OF THE COUPLING MATTER & GRAVITY

"EQUIVALENCE PRINCIPLE" $S_{\text{MATTER}}[\psi, A, H; g_{\mu\nu}]$

• UNIVERSALITY OF FREE FALL



Adelberger's group $\left(\frac{\Delta a}{a}\right)_{\text{Fe-Si}} = (3.6 \pm 5.0_{\text{STAT}} \pm 0.7_{\text{SYST}}) \times 10^{-13}$

Lunar Laser Ranging's group $\left(\frac{\Delta a}{a}\right)_{\oplus \oplus} = (-1.0 \pm 1.4) \times 10^{-13}$

• CONSTANCY OF "CONSTANTS" $\alpha_{\text{EM}} \equiv \frac{e^2}{\hbar c}$

Atomic Clock Tests

Harmon '03; Bize '03; Fischer '05

$$\frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} = (-0.9 \pm 2.9) \times 10^{-15} \text{ yr}^{-1}$$

Oklo's natural fission reactor

Khlykhter '76; Damour-Dyson '96; Fujii '00

$$\left\langle \frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} \right\rangle = (-0.9 \pm 5.9) \times 10^{-17} \text{ yr}^{-1}$$

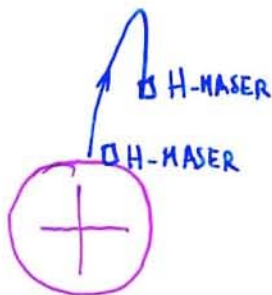
Quasar absorption lines

Quast '04; Srianand '04

$$\left\langle \frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} \right\rangle = (-0.7 \pm 1.9) \times 10^{-16} \text{ yr}^{-1}$$

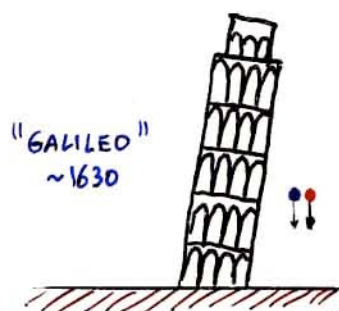
• GRAVITATIONAL REDSHIFT

Vessot, Levine '79

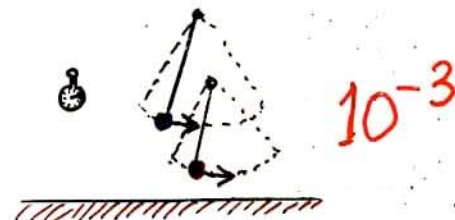


$$\frac{\Delta \nu}{\nu} = (1 \pm 10^{-4}) \frac{\Delta U}{c^2}$$

UNIVERSALITY OF FREE FALL



NEWTON
≤ 1686



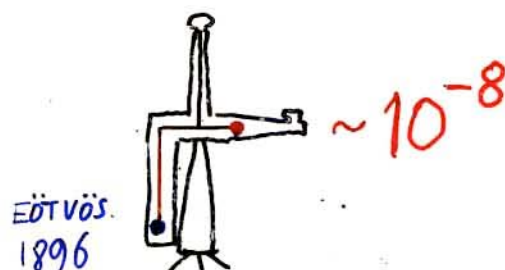
$\sim 10^{-7}$

LAPLACE
≤ 1805



[POINCARÉ 1906

$\sim 10^{-8}$]



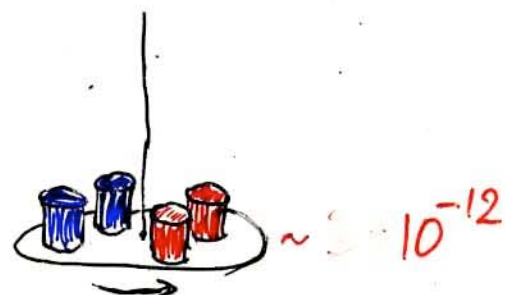
DICKE ET AL.

1962

BRAGINSKY AND PANOV 1972



$\sim 3 \times 10^{-11}$



ADELBERGER ET AL.

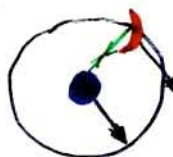
1989-1994, 2001-

LUNAR
LASER
RANGING

1969 - NOW

DICKEY ET AL. '94

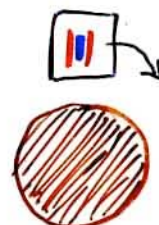
WILLIAMS ET AL. '96



$\sim 10^{-13}$

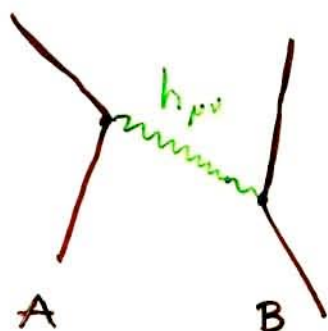


SPACE
TESTS
OF THE
EQUIVALENCE PRINCIPLE



MICROSCOPE 2009 $\sim 10^{-15}?$
STEP 200? $\sim 10^{-18}?$

DYNAMICS OF THE GRAVITATIONAL FIELD: WEAK FIELD REGIME



"ONE - GRAVITON EXCHANGE"

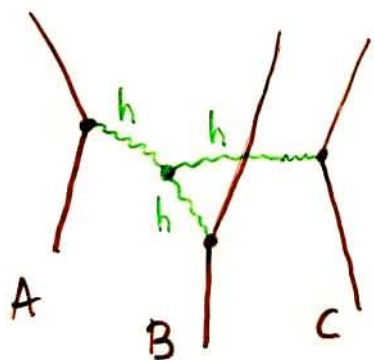


LINEARIZED EINSTEIN'S EQUATIONS

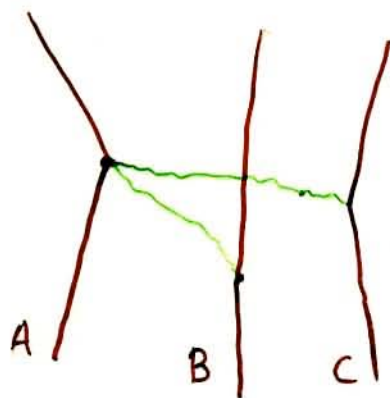
$$\square h_{\mu\nu} = -\frac{16\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} T \gamma_{\mu\nu} \right)$$

$$S_{INT} = 2G \iint ds_A ds_B m_A u_A^\mu u_A^\nu P_{\mu\nu}^{\rho\sigma} D[x_A^{\rho} - x_B^{\rho}] m_B u_B^\rho u_B^\sigma$$

$$L^{2-body} = \frac{1}{2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[1 + \frac{3}{2c^2} (\vec{v}_A^2 + \vec{v}_B^2) - \frac{7}{2c^2} \vec{v}_A \cdot \vec{v}_B - \frac{1}{2c^2} (\vec{n}_{AB} \cdot \vec{v}_A)(\vec{n}_{AB} \cdot \vec{v}_B) + O\left(\frac{1}{c^4}\right) \right]$$



+



$$L^{3-body} = -\frac{1}{2} \sum_{B \neq A \neq C} \frac{G^2 m_A m_B m_C}{r_{AB} r_{AC} c^2} + O\left(\frac{1}{c^4}\right)$$

TESTS OF THE DYNAMICS OF THE GRAV. FIELD

SOLAR-SYSTEM TESTS :

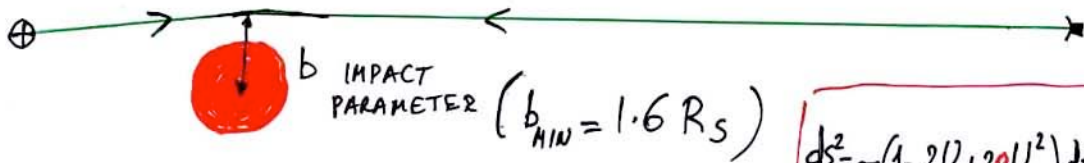
WEAK ($h_{\mu\nu} < 10^{-6}$) **AND QUASI-STATIC** ($\frac{\partial h}{c \partial x} \sim \frac{v}{c} \lesssim 10^{-4}$) **FIELDS**

- PERIHELION ADVANCE OF MERCURY
I. Shapiro '90, assuming $J_2 \sim 2 \times 10^{-7}$
$$\Delta \dot{\omega} = \overset{\dot{\omega}^{GR}}{\underset{||}{42.98}} (1.000 \pm 0.001)$$
- LIGHT DEFLECTION (VLBI)
S. S. Shapiro... '04
$$\Delta \theta = \Delta \theta^{GR} (1 + (-0.9 \pm 2.2) \times 10^{-4})$$
- ORBITAL MOTION OF THE MOON
(Nordtvedt '68) LUNAR LASER RANGING
Williams... '04
$$(\Delta r_{\oplus \odot})_{SYNODIC} = (3 \pm 4) \text{ mm } \cos D$$
- VARYING FREQUENCY SHIFT OF
RADIO LINKS : CASSINI SPACECRAFT
(Bertotti, Iess, Tortora '03)
$$\frac{\Delta \nu / \nu}{(\Delta \nu / \nu)^{GR}} = 1 + (1.0 \pm 1.1) \times 10^{-5}$$

QUASI-STATIC, WEAK-FIELD EINSTEIN GRAVITY OK AT

10^{-5} LEVEL

VARYING FREQUENCY SHIFT OF RADIO LINKS WITH THE CASSINI SPACECRAFT (Bertotti, Iess, Tortora '03)



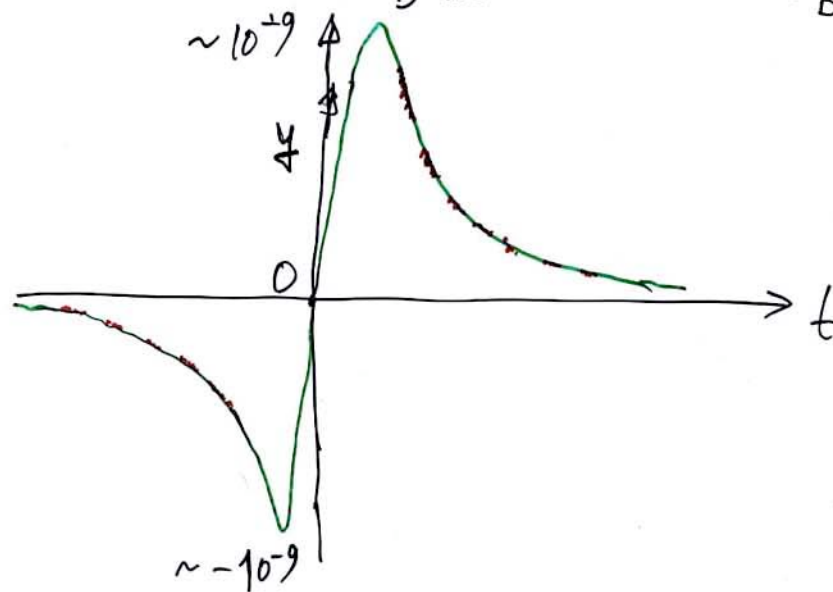
TWO-WAY RELATIVISTIC
FREQUENCY SHIFT

$$ds^2 = -\left(1 - \frac{2U}{c^2} + \frac{2\beta U^2}{c^4}\right) dt^2 + \left(1 + \frac{2\gamma U}{c^2}\right) d\vec{x}^2$$

$$\gamma_{GR} \equiv 1$$

$$\gamma \equiv \left(\frac{\Delta\nu}{\nu}\right)_{\text{GRAV.}}^{\text{TWO-WAY}} = -4(1+\gamma) \frac{GM_{\text{SUN}}}{c^3 b} \frac{db}{dt} = -0.2 \mu\text{sec} (1+\gamma) \frac{1}{b} \frac{db}{dt}$$

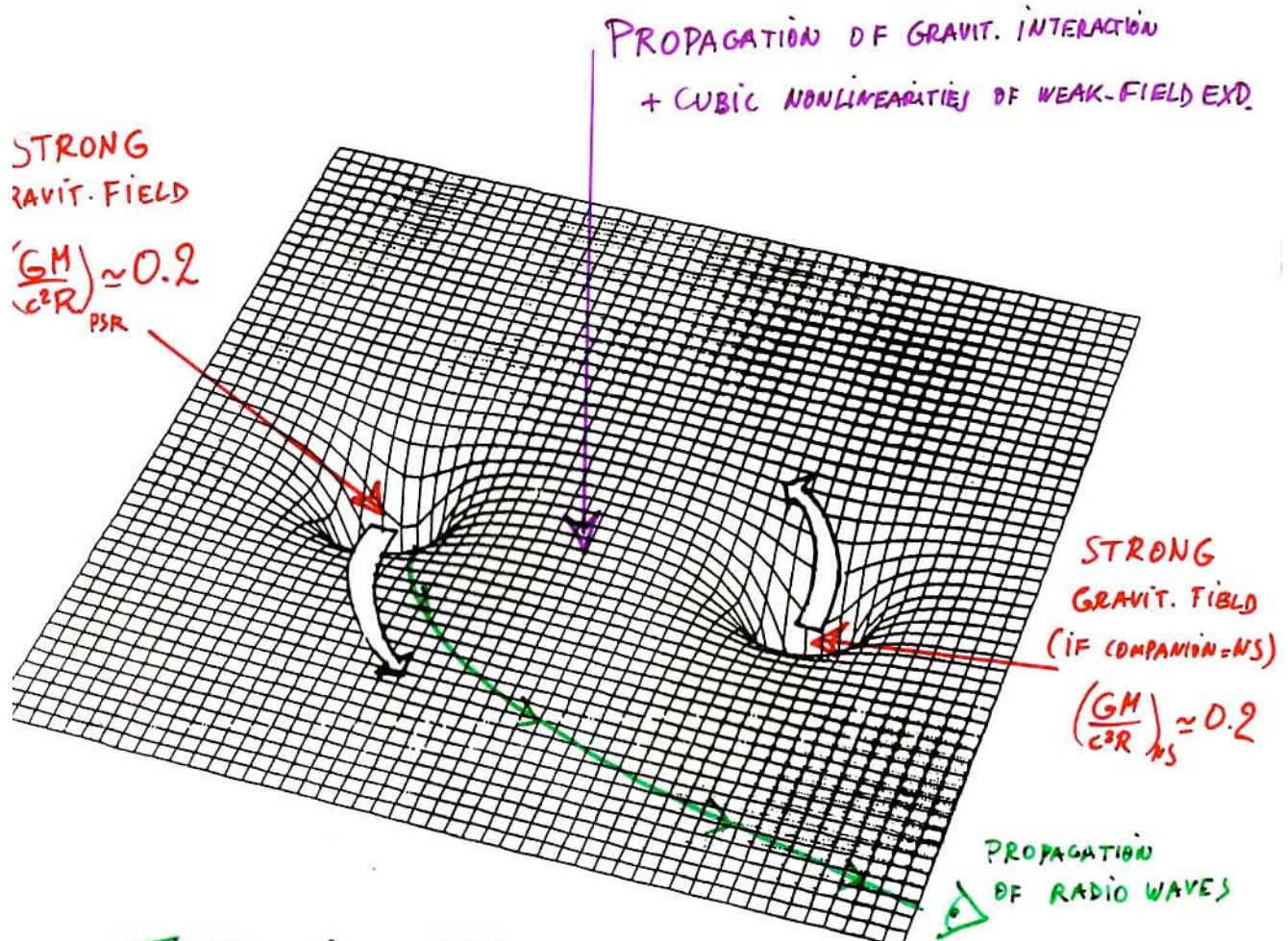
AS EARTH MOVES



$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

BINARY PULSARS: FIRST POSSIBILITY OF PROBING THE FULL STRUCTURE OF RELATIVISTIC GRAVITY

- RADIATIVE EFFECTS [FIELD PROPAGATION]
- HIGHLY NON-LINEAR EFFECTS [STRONG FIELDS]



THEORETICAL ASPECTS OF BINARY PULSARS:

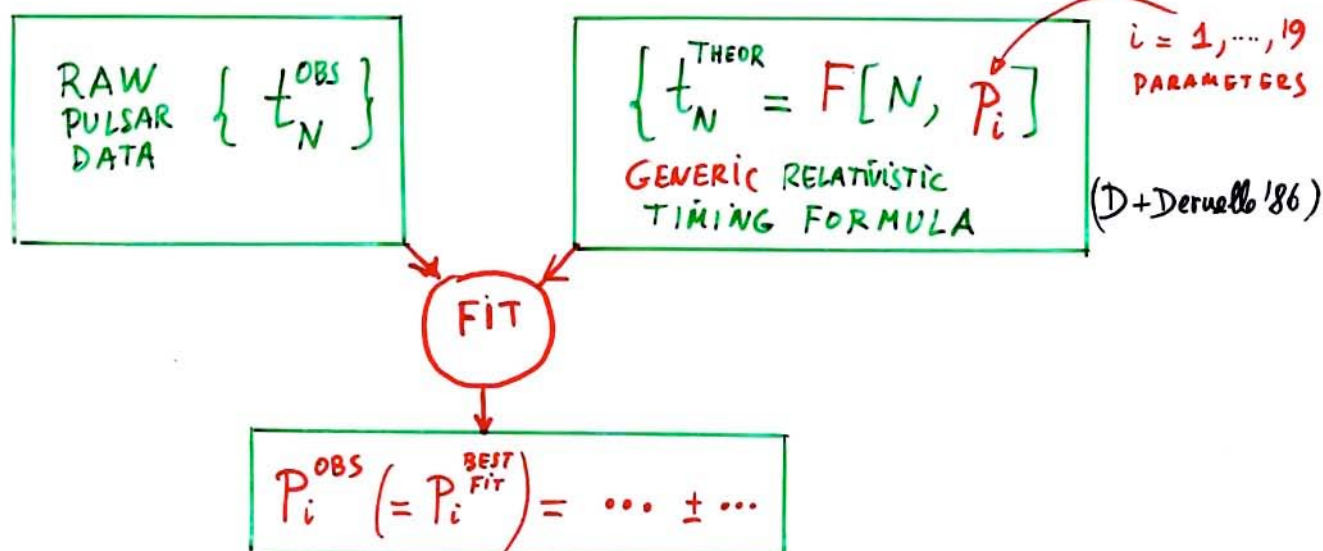
- ① MOTION OF TWO STRONGLY SELF-GRAVITATING BODIES
(T.D. & DERUELLE '81 , T.D. '82, '83)
- ② RELATIVISTIC TIMING OF A BINARY PULSAR
(BLANDFORD, TEUKOLSKY '76 , T.D. & DERUELLE '85, '86)
- ③ USE OF BINARY PULSARS AS PROBES OF RELATIVISTIC GRAVITY
(EARDLEY '75 , WILL, EARDLEY '77 , T.D. '88, T.D. & TAYLOR '92)

USING BINARY PULSAR MEASUREMENTS TO PROBE RELATIVISTIC GRAVITY

TWO COMPLEMENTARY APPROACHES

①. PHENOMENOLOGICAL ANALYSIS OF BINARY PULSAR DATA "PARAMETRIZED POST-KEPLERIAN FORMALISM" (PPK)

(Blandford + Teukolsky '76, D + Deruelle '86, D. '88, D + Taylor '92)



EACH RELATIVISTIC THEORY OF GRAVITY PREDICTS

$$P_i^{\text{THEOR}} = f_i^{\text{THEORY}}(m_1, m_2, (\lambda, \eta))$$

REDUNDANCY : $19 - 2(-2) = 15$ TESTS OF RELATIVISTIC GRAVITY

MOST PROBE STRONG-FIELD ASPECTS OF GRAVITY

N.B. EACH SUCH TEST IS A POTENTIAL KILLER OF G.R.

RELATIVISTIC TIMING FORMULA

Damour and Deruelle [36, 47] proved that it is possible to describe all of the independent $O(v^2/c^2)$ timing effects in a simple mathematical way common to a wide class of alternative theories. This made it possible to revert to a theory-independent analysis of timing data, and led to the possibility of working within a strong-field analog of the PPN formalism, the so-called [37] "parametrized post-Keplerian" approach. The part of the Damour-Deruelle phenomenological timing model describing orbital effects reads

$$t_b - t_0 = F[T; \{p^K\}; \{p^{PK}\}; \{q^{PK}\}] , \quad (2.1a)$$

where t_b denotes the solar-system barycentric (infinite frequency) arrival time, T the pulsar proper time (corrected for aberration, see below),

$$\{p^K\} = \{P_b, T_0, e_0, \omega_0, x_0\} \quad (2.1b)$$

is the set of Keplerian parameters,

$$\{p^{PK}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\} \quad (2.1c)$$

the set of *separately measurable* post-Keplerian parameters, and

$$\{q^{PK}\} = \{\delta_r, A, B, D\} \quad (2.1d)$$

the set of *not separately measurable* post-Keplerian parameters. The right hand side of Eq. (2.1a) is given by

$$F(T) = D^{-1} [T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)] , \quad (2.2a)$$

$$\Delta_R = x \sin \omega [\cos u - e(1 + \delta_r)] + x [1 - e^2(1 + \delta_\theta)^2]^{1/2} \cos \omega \sin u , \quad (2.2b)$$

$$\Delta_E = \gamma \sin u , \quad (2.2c)$$

$$\Delta_S = -2r \ln \{1 - e \cos u - s [\sin \omega (\cos u - e) + (1 - e^2)^{1/2} \cos \omega \sin u]\} , \quad (2.2d)$$

$$\Delta_A = A \{\sin[\omega + A_s(u)] + e \sin \omega\} + B \{\cos[\omega + A_s(u)] + e \cos \omega\} , \quad (2.2e)$$

where

$$x = x_0 + \dot{x}(T - T_0) , \quad (2.3a)$$

$$e = e_0 + \dot{e}(T - T_0) , \quad (2.3b)$$

and where $A_s(u)$ and ω are the following functions of u ,

$$A_s(u) = 2 \arctan \left[\left(\frac{1+e}{1-e} \right)^{1/2} \tan \frac{u}{2} \right] , \quad (2.3c)$$

$$\omega = \omega_0 + k A_s(u) , \quad (2.3d)$$

and u is the function of T defined by solving the Kepler equation

$$u - e \sin u = 2\pi \left[\left(\frac{T - T_0}{P_b} \right) - \frac{1}{2} \dot{P}_b \left(\frac{T - T_0}{P_b} \right)^2 \right] . \quad (2.3e)$$

BINARY PULSAR TESTS OF STRONG FIELD/RADIATIVE

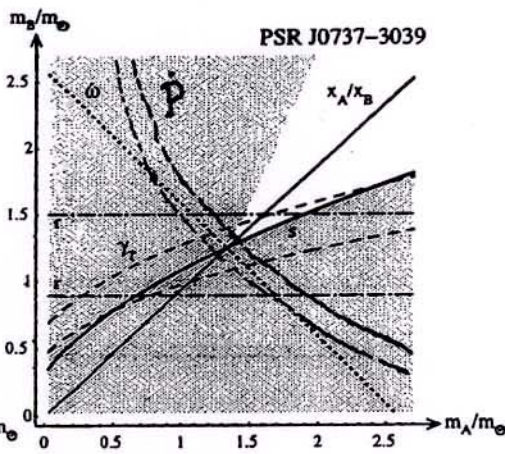
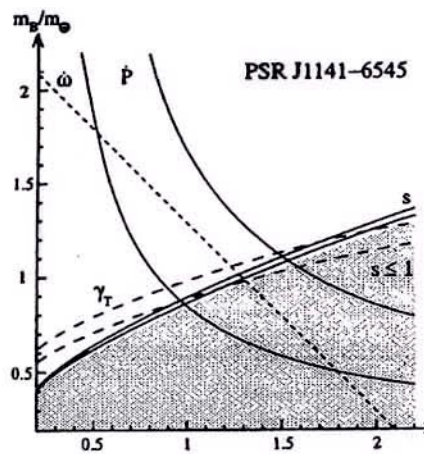
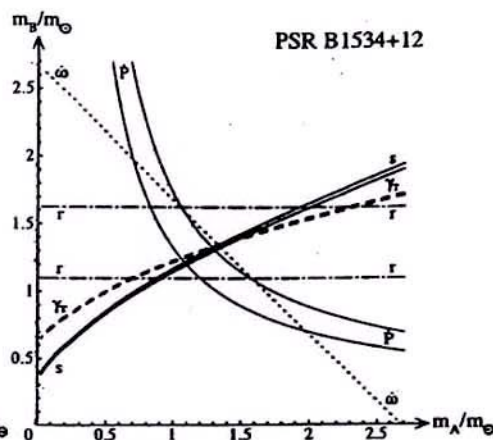
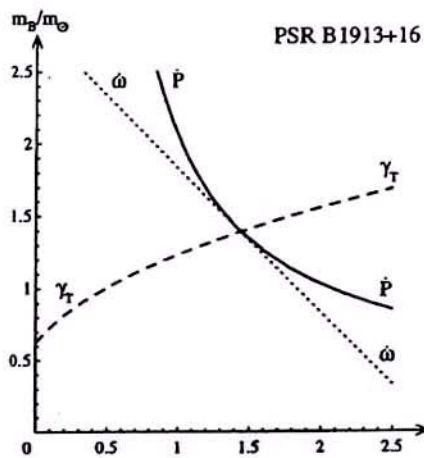
$$3 - 2 = 1$$

1 RADIATIVE + STRONG-FIELD $\sim 10^{-3}$ TEST

$$5 - 2 = 3$$

1 RAD. + STRONG FIELD

2 PURE STRONG-FIELD TESTS $\sim 10^{-2}$



$$4 - 2 = 2$$

1 RAD. + STRONG F

1 STRONG FIELD

$$6 - 2 = 4$$

3 STRONG FIELD TESTS

1 RAD. + STRONG FIELD

(V/c)⁵

EQUATIONS OF MOTION IN GENERAL RELATIVITY

66
0116

The problem of motion in Newtonian and Einsteinian gravity 183

accelerations. Then each body must satisfy the following equation of motion (Damour and Deruelle, 1981a; Damour, 1982):

$$a^i = A_0^i(\tilde{z} - \tilde{z}') + c^{-2} A_2^i(\tilde{z} - \tilde{z}', \tilde{v}, \tilde{v}') + c^{-4} A_4^i(\tilde{z} - \tilde{z}', \tilde{v}, \tilde{v}', \tilde{S}, \tilde{S}') \\ + c^{-5} A_5^i(\tilde{z} - \tilde{z}', \tilde{v} - \tilde{v}') + O(c^{-6}), \quad (154)$$

with

$$A_0^i = -Gm'R^{-2}N^i, \quad (155)$$

$$A_2^i = Gm'R^{-2} \{ N^i [-v^2 - 2v'^2 + 4(vv') + \frac{3}{2}(Nv')^2 + 5(Gm/R) + 4(Gm'/R)] \\ + (v^i - v'^i)[4(Nv) - 3(Nv')] \}, \quad (156)$$

$$A_4^i = B_4^i + C_4^i + D_4^i, \quad (157)$$

$$B_4^i = Gm'R^{-2} \{ N^i [-2v'^4 + 4v'^2(vv') - 2(vv')^2 + \frac{3}{2}v^2(Nv')^2 + \frac{9}{2}v'^2(Nv')^2 \\ - 6(vv')(Nv')^2 - \frac{15}{8}(Nv')^4 + (Gm/R)(-\frac{15}{4}v^2 + \frac{5}{4}v'^2 - \frac{5}{2}(vv') + \frac{39}{2}(Nv)^2 \\ - 39(Nv)(Nv') + \frac{17}{2}(Nv')^2) + (Gm'/R)(4v'^2 - 8(vv') + 2(Nv)^2 \\ - 4(Nv)(Nv') - 6(Nv')^2) \\ + (v^i - v'^i)[v^2(Nv') + 4v'^2(Nv) - 5v'^2(Nv') - 4(vv')(Nv) \\ + 4(vv')(Nv') - 6(Nv)(Nv')^2 + \frac{9}{2}(Nv')^3 \\ + (Gm/R)(-\frac{63}{4}(Nv) + \frac{55}{4}(Nv')) + (Gm'/R)(-2(Nv) - 2(Nv'))] \}, \quad (158)$$

$$C_4^i = G^3 m' R^{-4} N^i [-\frac{57}{4}m^2 - 9m'^2 - \frac{69}{2}mm'], \quad (159)$$

$$D_4^i = \left(\frac{S^{ik}}{m} + 2 \frac{S'^{ik}}{m'} \right) (v^i - v'^i) \left(\frac{Gm'}{R} \right)_{,kl} + \left(2 \frac{S^{kl}}{m} + 2 \frac{S'^{kl}}{m'} \right) (v^i - v'^i) \left(\frac{Gm'}{R} \right)_{,ik}, \quad (160)$$

and

$$A_5^i = \frac{4}{3} G^2 mm' R^{-3} \{ V^i [-V^2 + 2(Gm/R) - 8(Gm'/R)] \\ + N^i(Nv)[3V^2 - 6(Gm/R) + \frac{5}{3}(Gm'/R)] \}. \quad (161)$$

The two parameters m and m' appearing in eqs. (154)–(161) are the 'Schwarzschild masses' of the condensed bodies. They are two constants which appear in the external gravitational field, in which are hidden many internal structure effects (see the discussion of the 'effacement of internal structure' in Section 6.14). On the other hand, the spin tensors undergo a slow evolution (on the post-Newtonian time scale, i.e. β_e^{-2} times the orbital period) which is also obtained in the Einstein–Infeld–Hoffmann–Kerr-type approach (Damour, 1982, and references therein). Introducing, à la Schiff, a suitable spin-vector, $\tilde{S}$, associated with $S_{\mu\nu}$, the law of evolution ('spin precession') reads for the first body (see also references in Section 6.13.2)

$$\frac{d\tilde{S}}{dt} = \left[\frac{Gm'}{c^2 R^2} \tilde{N} \times \left(\frac{3}{2} \tilde{v} - 2\tilde{v}' \right) \right] \times \tilde{S} + O\left(\frac{1}{c^4}\right). \quad (162)$$

"DRESSED
MASSES"
IN CORPORATING
STRONG-SELF-FIELD
EFFECTS

GRAVITATIONAL
RADIATION
DAMPING

↑
DIRECT
EFFECT OF
PROPAGATION
OF GRAVITY
AT SPEED C

SOME HIGH-PRECISION BINARY-PULSAR TESTS

1913+16:

Weisberg, Taylor '04

Damour Taylor '91 145^{OBS} CORRECTION

$$\frac{\dot{P}_b^{OBS} - \dot{P}_b^{GALACTIC}}{\dot{P}_b^{GR}[k^{OBS}, \gamma_{TIMING}^{OBS}]} = 1.0013 \pm 0.0021$$

1534+12:

Taylor, Wolszczan, Damour, Weisberg '92
Stairs et al. '02

$$\frac{s^{OBS}}{s^{GR}[k^{OBS}, \gamma_{TIMING}^{OBS}]} = 1.000 \pm 0.007$$

0737-3039

Lyne et al. '04, Kramer et al '04

$$\frac{s^{OBS}}{s^{GR}[k^{OBS}, R^{OBS}]} = 0.9998^{+0.0006}_{-0.0011}$$

RADIATIVE AND STRONG-FIELD EINSTEIN GRAVITY OK_{AT}

10^{-3} LEVEL

SECOND APPROACH TO PROBING GRAVITY WITH BINARY PULSAR DATA:

② THEORY-DEPENDENT ANALYSIS OF PSR DATA

CHOOSE A CLASS OF SIMPLE ALTERNATIVES TO GR CONTAINING A SMALL NUMBER OF PARAMETERS, BUT SUFFICIENTLY MANY TO EXHIBIT INTERESTING EFFECTS

TENSOR-SCALAR GRAVITY:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{g} [R(g) - 2(\partial_\mu \varphi)^2] + S_{\text{MATTER}}[\varphi; \tilde{g}_{\mu\nu} = e^{2a(\varphi)} g_{\mu\nu}] - \int d^4x \sqrt{g} V(\varphi)$$

SCALAR FIELD φ
COUPLING FUNCTION $a(\varphi)$
POTENTIAL: FIXES φ_0

TWO-PARAMETER
COUPLING FUNCTION

$$a(\varphi) = \alpha_0 (\varphi - \varphi_0) + \frac{1}{2} \beta_0 (\varphi - \varphi_0)^2$$

• SIMPLE GENERALIZATION OF JORDAN-FIGERZ-BRANS-DICKE

$$a^{\text{JFB}}(\varphi) = \alpha_0 (\varphi - \varphi_0) \quad ; \quad \alpha_0^2 \equiv \frac{1}{2\omega + 3}$$

• MINIMAL THEORY LEADING TO PPN PARAMETERS
(Eddington, Nordtvedt, Will...)

$$\gamma^{\text{PPN}} - 1 = -2 \frac{\alpha_0^2}{1 + \alpha_0^2}$$

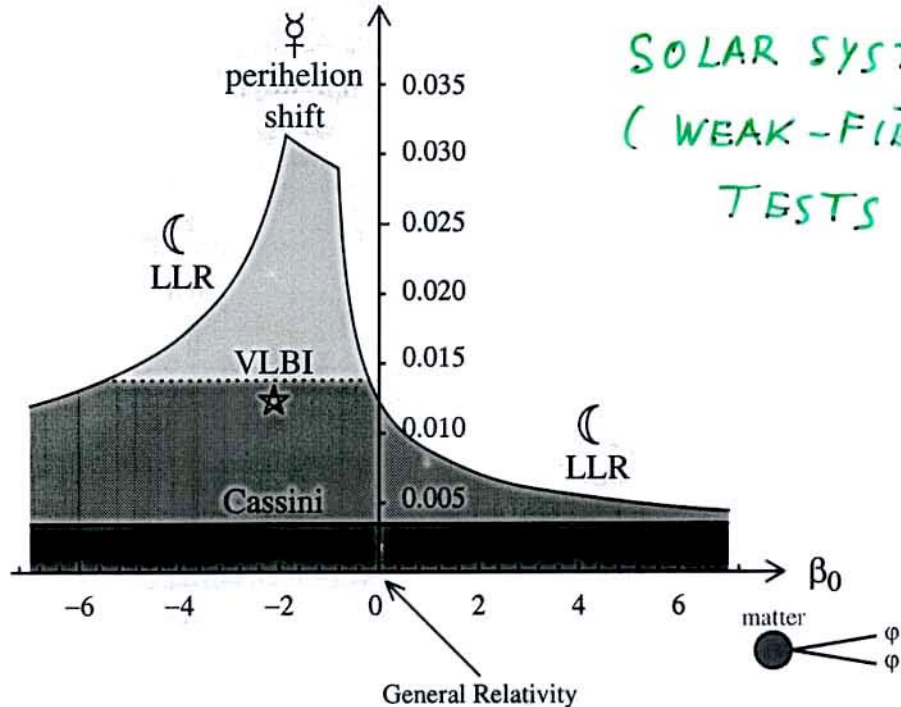
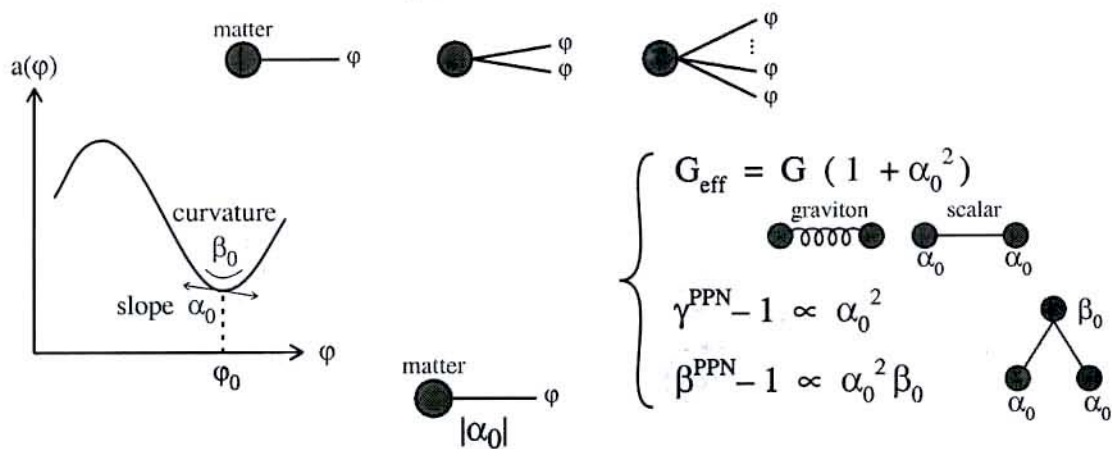
$$\beta^{\text{PPN}} - 1 = \frac{1}{2} \frac{\beta_0 \alpha_0^2}{(1 + \alpha_0^2)^2}$$

• FEATURES INTERESTING NON-PERTURBATIVE STRONG-FIELD EFFECTS

Tensor-scalar theories

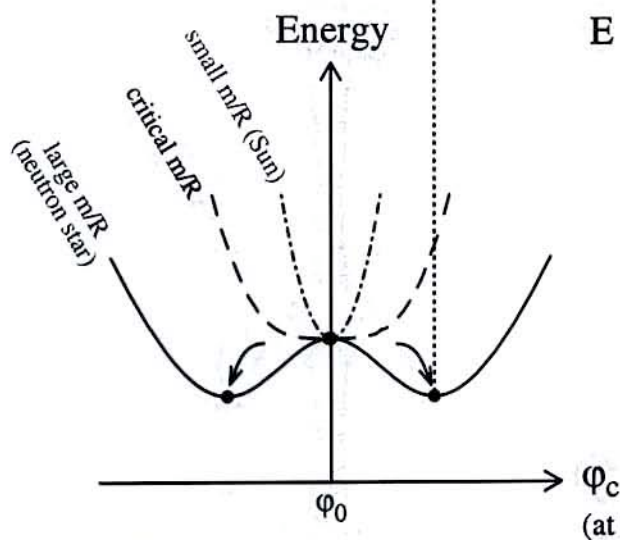
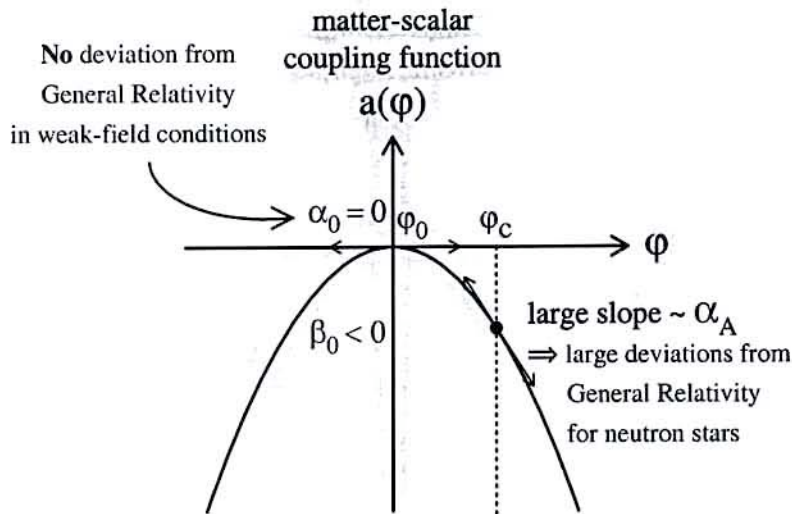
$$S = \frac{1}{16\pi G} \int \sqrt{-g} \left\{ \underset{\substack{\uparrow \\ \text{spin 2}}}{R} - 2 \underset{\substack{\uparrow \\ \text{spin 0}}}{(\partial_\mu \phi)^2} \right\} + S_{\text{matter}} \left[\text{matter}; \underset{\substack{\uparrow \\ \text{physical metric}}}{\tilde{g}_{\mu\nu}} \equiv e^{2a(\phi)} g_{\mu\nu} \right]$$

$$a(\phi) = \alpha_0 (\phi - \phi_0) + \frac{1}{2} \beta_0 (\phi - \phi_0)^2 + \dots$$



Vertical axis ($\beta_0 = 0$): Jordan-Fierz-Brans-Dicke theory $\alpha_0^2 = \frac{1}{2\omega_{\text{BD}} + 3}$

NON-PERTURBATIVE STRONG-FIELD EFFECTS T12

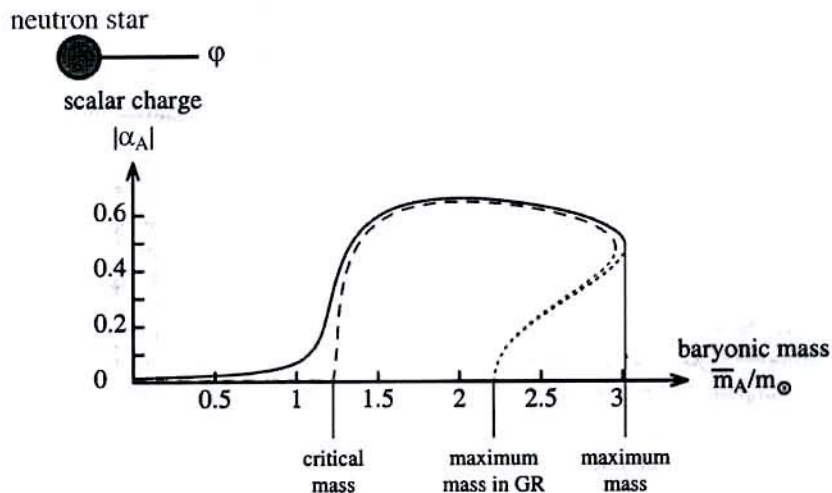


$$E \approx \int \left[\frac{1}{2} (\vec{\nabla} \varphi)^2 + \rho e^{\beta_0 \varphi^2/2} \right]$$

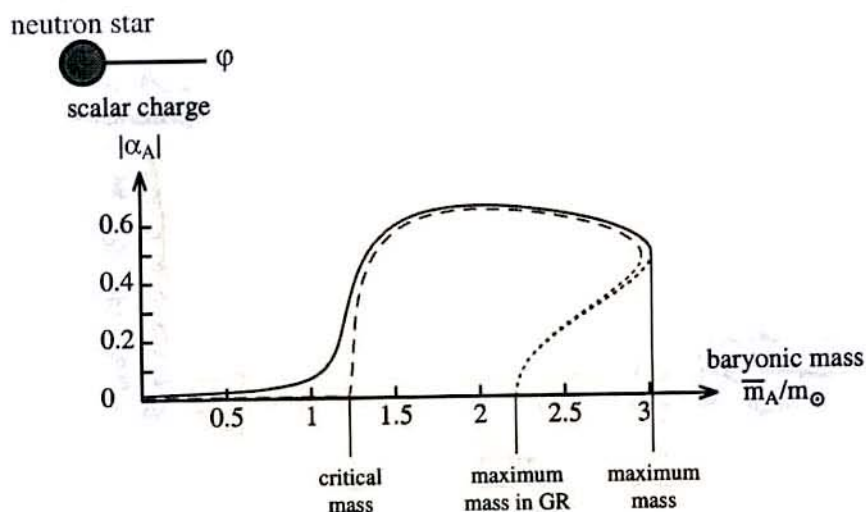
$\downarrow$
 $\frac{1}{2} R \varphi_c^2$
 parabola

$\downarrow$
 $m e^{\beta_0 \varphi_c^2/2}$
 Gaussian
 if $\beta_0 < 0$

“spontaneous scalarization” [T. Damour & G. Esposito-Farese 1993]

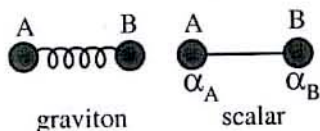


Strong-field effects

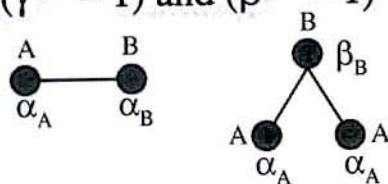


$$G_{AB}^{\text{eff}} = G (1 + \alpha_A \alpha_B)$$

depends on internal
structure of bodies A & B

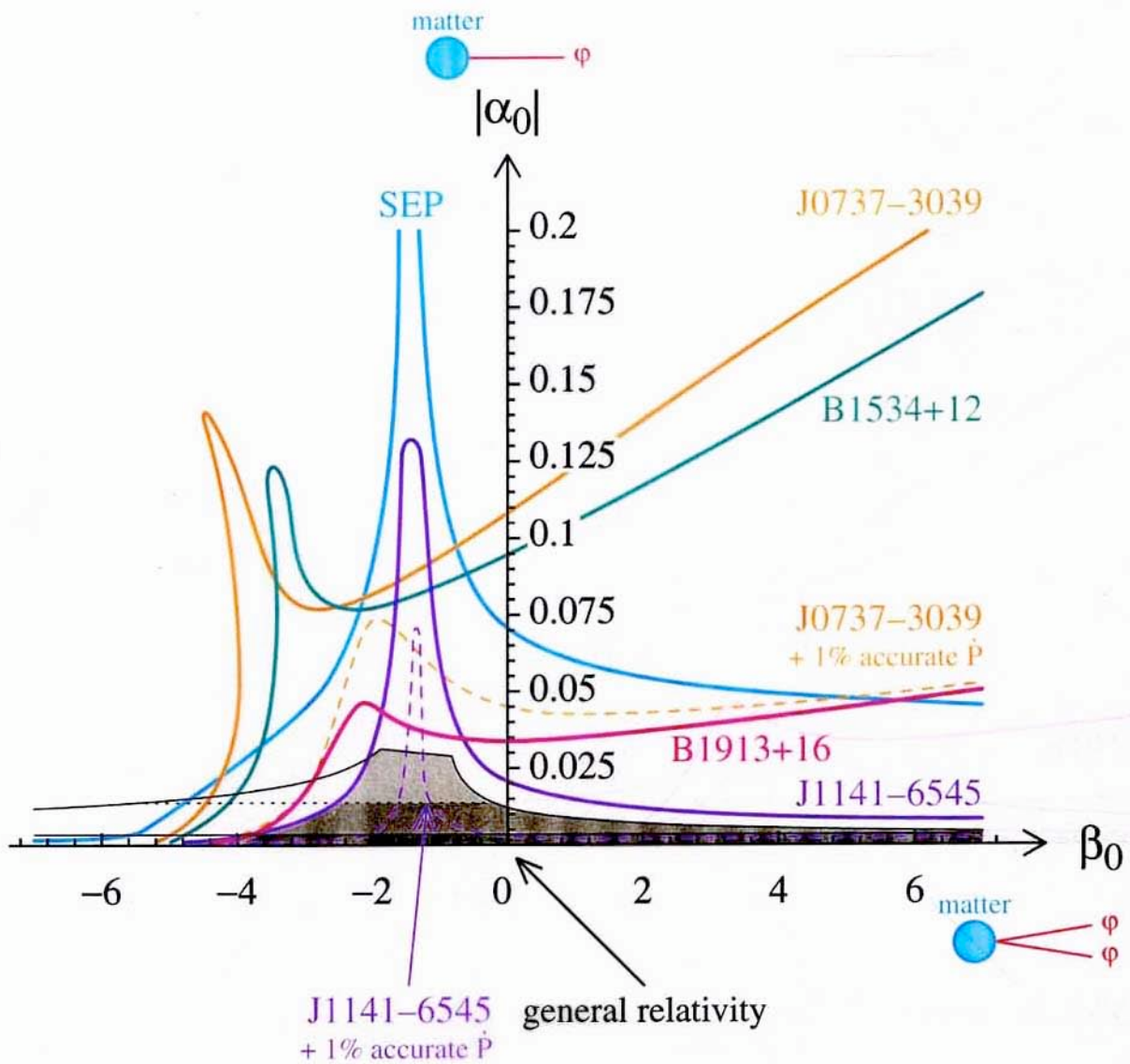


similarly for $(\gamma^{\text{PPN}} - 1)$ and $(\beta^{\text{PPN}} - 1) \Rightarrow$ all post-Newtonian effects



$$\begin{aligned} \text{Energy flux} = & \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 2} \\ & + \frac{\text{Monopole}}{c} \left(0 + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 0} \\ & \quad \quad \quad \uparrow \\ & \quad \quad \quad \propto (\alpha_A - \alpha_B)^2 \end{aligned}$$

Solar-system and best binary-pulsar
constraints on tensor-scalar theories
(updated April 2005)



WAS EINSTEIN 100% RIGHT?

SHOULD WE STOP TESTING

SPECIAL AND GENERAL RELATIVITY?

MOTIVATIONS FOR GOING BEYOND EINSTEIN

- RECONCILING GR WITH QUANTUM THEORY

$$G = \frac{\hbar}{M_P^2} = \frac{l_P^2}{\hbar} \quad : \quad l_P = \sqrt{\hbar G} \approx 10^{-33} \text{ cm}$$

- UNIFYING THE INTERACTIONS $(A_\mu^a, g_{\mu\nu})$

AND SIMPLIFYING THE MATTER SECTOR $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$

- UNDERSTANDING THE REMARKABLE HIERARCHIES :

COSMOLOGY $L_0 = c t_0 \sim 10^{28} \text{ cm} \gg \gg \text{ANY } L_{\text{PARTICLE}}$

COSMOLOG. CONSTANT $\rho_{\text{VAC}}^{\text{THY}} \sim \Lambda^4 + m^2 \Lambda^2 + m^4 + \dots \gg \gg \rho_{\text{VAC}}^{\text{OBS}} \sim (10^{-3} \text{ eV})^4$

PARTICLE PHYSICS $M_{\text{EW}} \sim 250 \text{ GeV} \ll M_{\text{GUT UNIF}} \sim 10^{16} \text{ GeV}$
OR $M_P \sim 10^{19} \text{ GeV}$

STRING THEORY

CV3

- ONLY CONCRETE PHYSICAL FRAMEWORK TO DISCUSS QUANTUM U.G.R.

- STARTING POINT:

$\left\{ \begin{array}{l} \bullet \rightarrow X^\mu(\tau) \text{ OR } X^\mu(\tau, \sigma) \\ \text{QUANTUM MECHANICS: } \hbar \\ \text{SPECIAL RELATIVITY: } c, \eta_{\mu\nu} \end{array} \right.$

$$\frac{S_{\text{STRING}}}{\hbar} = -\frac{1}{2\pi \ell_s^2} \iint d^2A \sqrt{(\dot{X}^\mu X'^\mu)^2 - (\dot{X}^\mu)^2 (X'^\mu)^2}$$



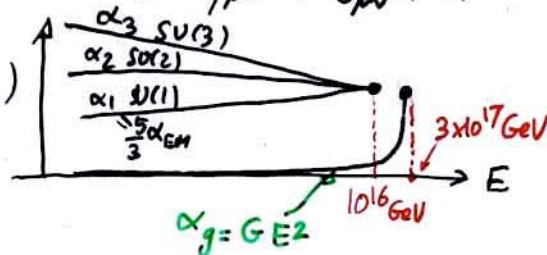
FUNDAMENTAL LENGTH: ℓ_s

- FINAL THEORY STILL UNDER CONSTRUCTION
BUT VERY RICH STRUCTURE, AND MANY DEEP BUILDING BLOCKS UNDERSTOOD

- $D=10$ (OR 11) $\Rightarrow$ INCLUDES KALUZA-KLEIN g_{MN}
 - g_{ab} SCALARS
 - $g_{a\mu}$ VECTORS
 - $g_{\mu\nu}$ TENSOR

- - $\{ \rightarrow A_\mu(x) \text{ GAUGE INTERACTIONS}$
 - $\{ \rightarrow h_{\mu\nu}(x) = g_{\mu\nu}(x) - \eta_{\mu\nu} \text{ EINSTEIN'S GRAVITY}$
- UNIFIES GAUGE THEORIES WITH GRAVITY
- "EXPLAINS WHY" $\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x) \text{ DYNAMICAL}$

- UNIFIES ALL INTERACTIONS (?)

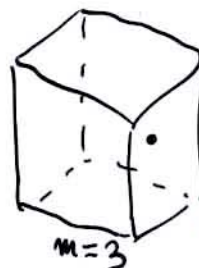


- STRINGS $\rightarrow$ "M-BRANES"

$m=0,1,2,3, \dots$

$\int X^\mu(\tau, \sigma)$
 $m=1$
STRING

$X^\mu(\tau, \sigma_1, \sigma_2)$
 $m=2$
MEMBRANE



3-brane

$X^\mu(\tau, \sigma_1, \sigma_2, \sigma_3)$
LIKE OUR
(3+1)-WORLD

- MORE GRAVITATIONAL-STRENGTH FIELDS

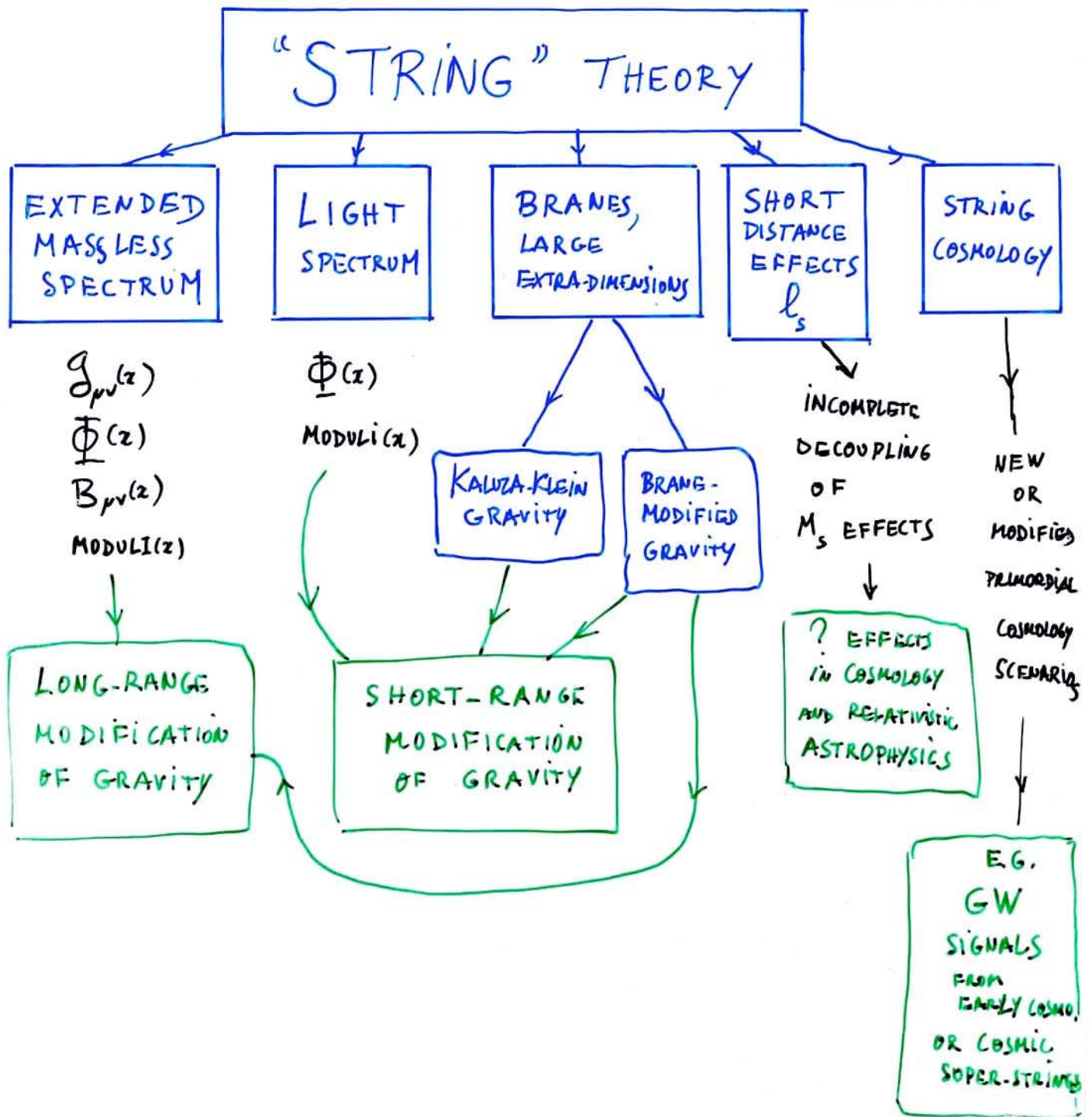
$\{ \rightarrow g_{\mu\nu}(x) + \Phi(x) + B_{\mu\nu}(x) + \dots$

DILATON
CRUCIAL RÔLE $g_s = e^{\Phi}$

LINKED TO "TORSION" AND, IN CERTAIN LIMIT
TO NON-COMMUTATIVITY OF SPACE
 $[x^\mu, x^\nu] = \theta^{\mu\nu}(g, B)$

STRING-INSPIRED PHENOMENOLOGY CV4

- NO CLEAR UNDERSTANDING OF HOW TO FIT OUR WORLD WITHIN STRING THEORY
- ⇒ DISCUSS PHENOMENOLOGICAL POSSIBILITIES; OPEN NEW EXPERIMENTAL OPPORTUNITIES



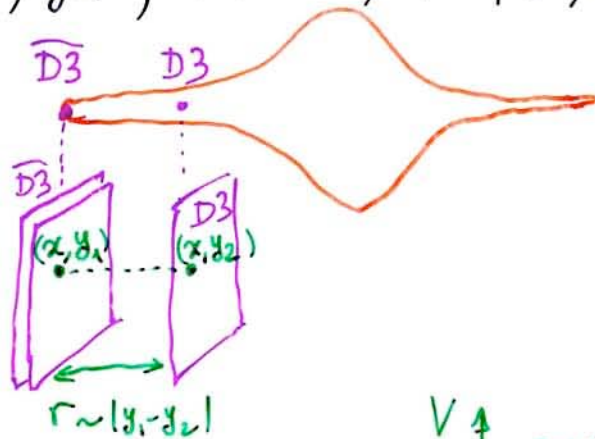
COSMIC SUPERSTRINGS?

Witten '85; ... Dvali, Tye; Tye, ...; KKLMMT; Copeland, Myers, Polchinski; Dvali, Vilenkin

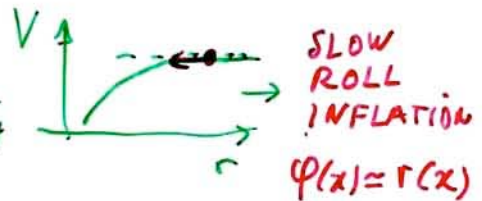
10 dim spacetime:

$$X^M = (x^\mu, y^a)$$

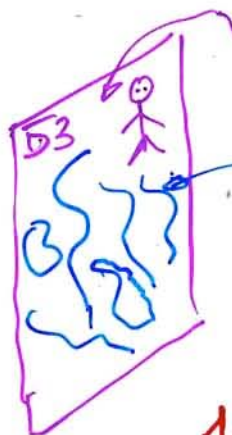
4 6 COMPACT



$$V(r) = A - \frac{B}{r^4}$$



HEAT OF
HOT
BIG BANG



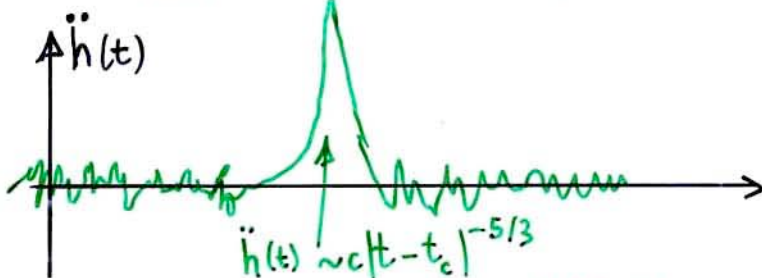
OUR WORLD

COSMOLOGICAL
NETWORK OF
MASSIVE (F OR D) STRINGS
WITH
STRING TENSION

$$10^{-11} \lesssim G\mu \lesssim 10^{-6} \text{ Tye}$$

$$G\mu \sim 10^{-8} - 10^{-9} \text{ KKLMMT, Copeland \& M.P.}$$

GRAVITATIONAL WAVE BURSTS



RECURRENT
CUSPS

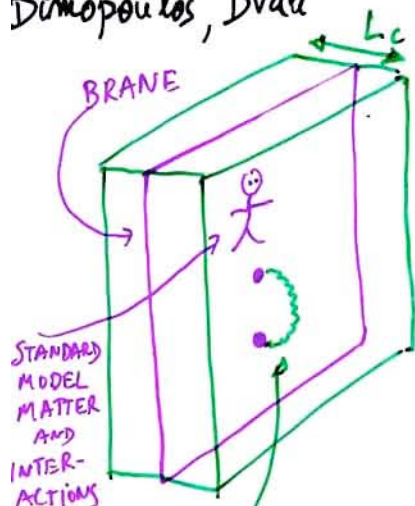


POTENTIALLY DETECTABLE IN LIGO/VIRGO/...; LISA; PULSAR TIMING
Damour, Vilenkin

BRANES AND GRAVITY

CV6

"LARGE" BUT COMPACT
EXTRA-DIMENSIONS
Antoniadis, Arkani-Hamed,
Dimopoulos, Dvali



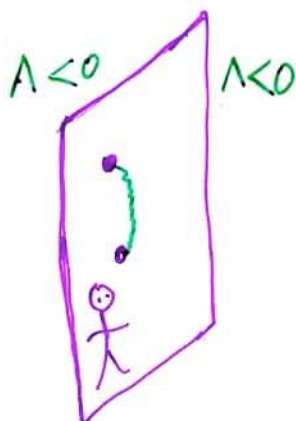
BULK GRAVITY

HIGHER-DIMENSIONAL
GRAVITY WHEN

$$r < L_c$$

AND (if $l_s \sim \text{TeV}$)
INTERESTING OBSERVABLE
EFFECTS IN LHC

INFINITE EXTRA-DIMENSIONS
BUT "MISMATCHED" GRAVITY
Randall, Sundrum

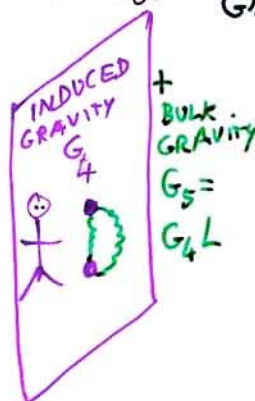


GRAVITY $\approx$
SURFACE WAVE

MODIFICATION OF
GRAVITY WHEN

$$r \lesssim \text{BULK CURVATURE RADIUS} \approx r_c$$

Dvali,
Gabadadze
Porrati



GRAVITY $\approx$
SURFACE $\oplus$ BULK
PROPAGATION

MODIFICATION OF
GRAVITY WHEN

$$r \gtrsim L \equiv \frac{G_5}{G_4}$$

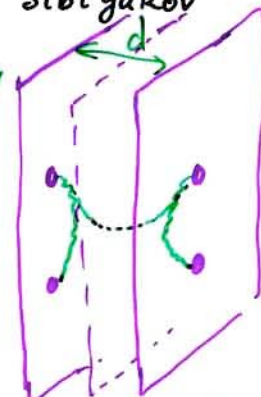
AND SMALL
MODIFICATIONS
FOR $r < L$

Dvali, Gruzinov, Zeldarriaga

$$U' = \frac{GM}{r} \left[1 - \frac{1}{L} \sqrt{\frac{r^3 c^2}{GM}} \right]$$

EFFECTS IN SOLAR-SYSTEM,
LUNAR RANGING...

MULTI-BRANES
Kogan, Mouslopoulas,
Papazoglou, Ross,
Santiago;
Gregory, Rubakov,
Sibiryakov



TUNNELLING
(EVANESCENT WAVE)
BETWEEN SEVERAL
GRAVITON WAVES

MULTI-GRAVITY

MODIFICATION OF
GRAVITY BOTH
WHEN

$$r \lesssim r_c$$

AND

$$r \gtrsim r_c e^{d/r_c}$$

BUT PROBLEMS
WITH
"PAULI-FIERZ"
TYPE
MASSIVE GRAVITY

INVERSE-SQUARE LAW TESTS

(Adelberger, Heckel, Nelson 103)

$$V(r) = -\frac{G_{m_1 m_2}}{r} \left[1 + \alpha e^{-\frac{r}{\lambda}} \right]$$

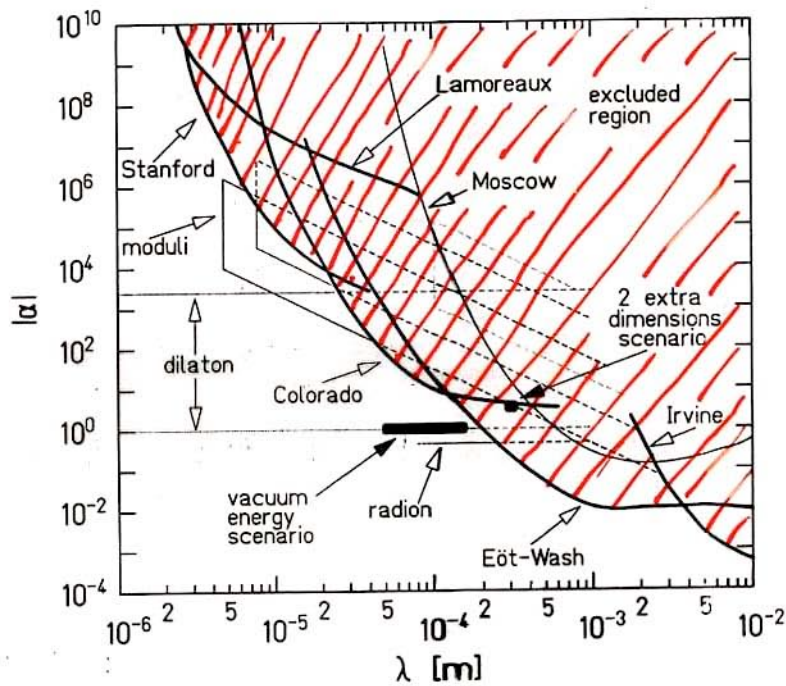


Figure 5: 95%-confidence-level constraints on **ISL**-violating Yukawa interactions with $1 \mu\text{m} < \lambda < 1 \text{ cm}$. The heavy curves **give** experimental upper limits (the Lamoreaux constraint was computed in Reference (151)). Theoretical expectations for extra dimensions (56), moduli (101), **dilaton** (102), and radion (83) are shown as well.

DILATON, MODULI, ... AND VIOLATIONS OF THE EQUIVALENCE PRINCIPLE

EQUIVALENCE PRINCIPLE : $S = \int d^4x \frac{\sqrt{g} R(g)}{16\pi G} + S_{\text{MATTER}}[\psi, A, H; g_{\mu\nu}, \alpha_a, m_a]$

ONLY ONE LONG-RANGE FIELD WITH GRAVIT-STRENGTH COUPLING TO MATTER

ALL COUPLING CONSTANTS ARE THE SAME AS IN SPECIAL RELAT.

? WHY $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$? WHY $\mu = \frac{m_e}{m_p} \approx \frac{1}{1836}$?

STANDARD MODEL OF PARTICLE PHYSICS: $\exists \sim 20$ ARBITRARY PARAMETERS

IN STRING THEORY, $\exists$ NO DIMENSIONLESS PARAMETERS
 $\exists$ ONLY A DIMENSIONFULL PARAMETER: l_s

ALL DIMENSIONLESS PARAMETERS MUST ARISE AS VACUUM EXPECTATION VALUES OF SOME FIELDS

EG. IN $D=10$, STRING THY CONTAINS A STRING COUPLING CONSTANT g_s

WHOSE VALUE:

$$g_s = \exp \langle \Phi(x) \rangle$$

DILATON = PARTNER OF GRAVITON

IN $D=4$, THERE APPEAR OTHER FIELDS DETERMINING THE VALUES OF THE 4-DIM "CONSTANTS": MODULI FIELDS,
 DETERMINING THE VOLUME AND SHAPE OF COMPACTIFIED DIMENSIONS

AND

$$\alpha = F_\alpha(g_s, \text{MODULI}, \dots)$$

$$G = l_s^2 F_G(g_s, \text{MODULI}, \dots)$$

.....

INTUITIVE MEANING OF $g_{\mu\nu}(x) + \Phi(x) + \dots$

	GEOMETRY	COUPLING CONSTANTS	
NEWTON	RIGID	RIGID	
EINSTEIN	SOFT	RIGID	EINSTEIN EQUIVALENCE PRINCIPLE
STRING Theory	SOFT	SOFT	
			VIOLATION OF THE EQUIVALENCE PRINCIPLE

$$g_{\text{geometry}} \sim g_{\text{gravitation}} \sim g_{\text{gauge coupling constant}} \sim G_{\text{gravitational coupling constant}}$$

$$g_{\mu\nu}(x) \sim g^2(x) \sim G(x)$$

BUT THEN ONE WOULD EXPECT:

- NON-UNIVERSALITY OF FREE FALL $\frac{\Delta a}{a} \sim 10^{-5}$

- COSMOLOGICAL VARIATION OF COUPLING CONSTANTS

$$\frac{\dot{\alpha}}{\alpha} \sim \frac{\dot{\rho}}{\rho} \sim H_0 \sim 10^{-10} \text{ yr}^{-1}$$

- MODIFICATION OF POST-NEWTONIAN GRAVITY

$$\gamma - 1 \sim \mathcal{O}(1)$$

CONSISTENCY OF DILATON+MODULI $\Phi(x)$ WITH PRESENT EXPERIMENTAL DATA?

① $m_\Phi \neq 0, V(\Phi) \neq 0$  IN LOW-ENERGY WORLD $\Rightarrow$ ONLY SHORT-RANGE EFFECTS $\propto \frac{e^{-m_\Phi r}}{r}$

RECENT EXPERIMENTS $\Rightarrow \lambda_\Phi = \frac{1}{m_\Phi} \leq 0.1 \text{ mm} \Rightarrow m_\Phi \geq 10^{-3} \text{ eV}$
 Hoyle...2001
 Chiaverini...2003
 Long...2003

THE VALUE OF m_Φ IS MODEL-DEPENDENT. SOME MODELS NEED TO FIX Φ EARLY ON (BEFORE INFLATION) $\Rightarrow m_\Phi \sim M_s \gg H_{\text{inf}}$

IN SOME MODELS m_Φ IS LINKED TO SUSY BREAKING: $V(\Phi) \sim M_{\text{susy}}^4 \mathcal{V}\left(\frac{\Phi}{M_P}\right)$
 $\Rightarrow m_\Phi \sim \frac{M_{\text{susy}}^2}{M_P} \sim \frac{(1 \text{ TeV})^2}{2.4 \times 10^{18} \text{ GeV}} \sim 10^{-3} \text{ eV}$
 Taylor, Veneziano '88
 Ferrara et al '94
 Antoniadis et al '97

$\Rightarrow$ POSSIBLE MODIFICATIONS OF CAVENISH EXPERIMENTS JUST BELOW 0.1 mm CURRENT DATA

② $m_\Phi = 0, V(\Phi) \approx 0$; BUT $\exists$ NON-TRIVIAL COUPLING FUNCTIONS $B_i(\Phi)$
 $\mathcal{L}_{\text{eff}} = B_R(\Phi) R(g) + B_\Phi(\Phi) (\nabla\Phi)^2 + B_F(\Phi) F_{\mu\nu}^2 + \dots$ $\downarrow$ $V_{\text{eff}}(\Phi)$ THROUGH PRESENCE OF MATTER
 IF $\exists \Phi_m$; $\partial B_i(\Phi_m)/\partial \Phi_m = 0$ Damour, Nordtvedt; Damour, Polyakov

$\exists$ MECHANISM OF NATURAL COSMOLOGICAL ATTRACTION: $\Phi \rightarrow \Phi_m$

AND Φ NEARLY DECOUPLES FROM MATTER WHEN $\Phi \approx \Phi_m$

$\Rightarrow$ NATURALLY SUPPRESSED MODIFICATIONS OF LONG-RANGE GRAVITY

③ BOTH A QUINTESSENCE-LIKE $V(\Phi) \neq 0$ AND COUPLING TO MATTER $B(\Phi)$

$\Rightarrow m_\Phi$ DEPENDS ON SURROUNDING MATTER DENSITY, SO THAT Φ IS SHORT-RANGED IN EARTH-BOUND EXPTS Khoury, Weltman, Brax, ...

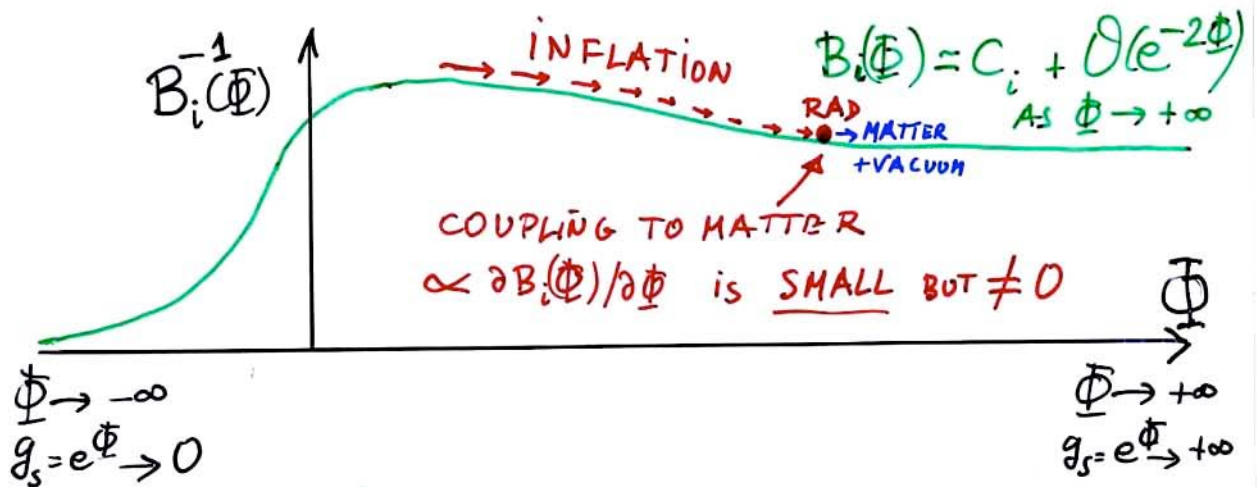
ATTRACTOR SCENARIO, WITH RUN-AWAY

Damour, Polyakov

; Damour, Piazza, Veneziano

$$\mathcal{L}_{\text{EFF}} = B_R(\Phi) R(g) + B_\Phi(\Phi) (\nabla \Phi)^2 + B_F(\Phi) F_{\mu\nu}^2 + B_\chi(\Phi) (\nabla \chi)^2 + B_V(\Phi) \chi^n$$

↑ DILATON COUPLING FUNCTIONS



OBSERVATIONAL CONSEQUENCES TODAY

RESIDUAL
COUPLING

$$\alpha_{\text{had}}^2(\Phi_{\text{end}}) \sim 10 \left(\frac{b_F}{b_{\lambda C}} \right)^2 \left(\frac{\delta \rho}{\rho} \right)^{\frac{8}{n+2}} \sim 2.5 \times 10^{-8} \quad V(\chi) \sim \chi^n \quad \text{IF } n=2$$

$$\Rightarrow \gamma_{\text{PPN}} - 1 \simeq -2\alpha_{\text{had}}^2 \sim -5 \times 10^{-8}$$

$$\frac{\Delta a}{a} \sim 5 \times 10^{-5} \alpha_{\text{had}}^2 \sim 10^{-12}$$

$$\frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} \sim \pm \sqrt{1 + q_0 - \frac{3\Omega_m}{2}} \sqrt{10^{12} \frac{\Delta a}{a}} \quad 10^{-16} \text{ yr}^{-1}$$

FUTURE EXPERIMENTS ON GRAVITY

- GRAVITY PROBE B

- COMPARISON OF ATOMIC CLOCKS

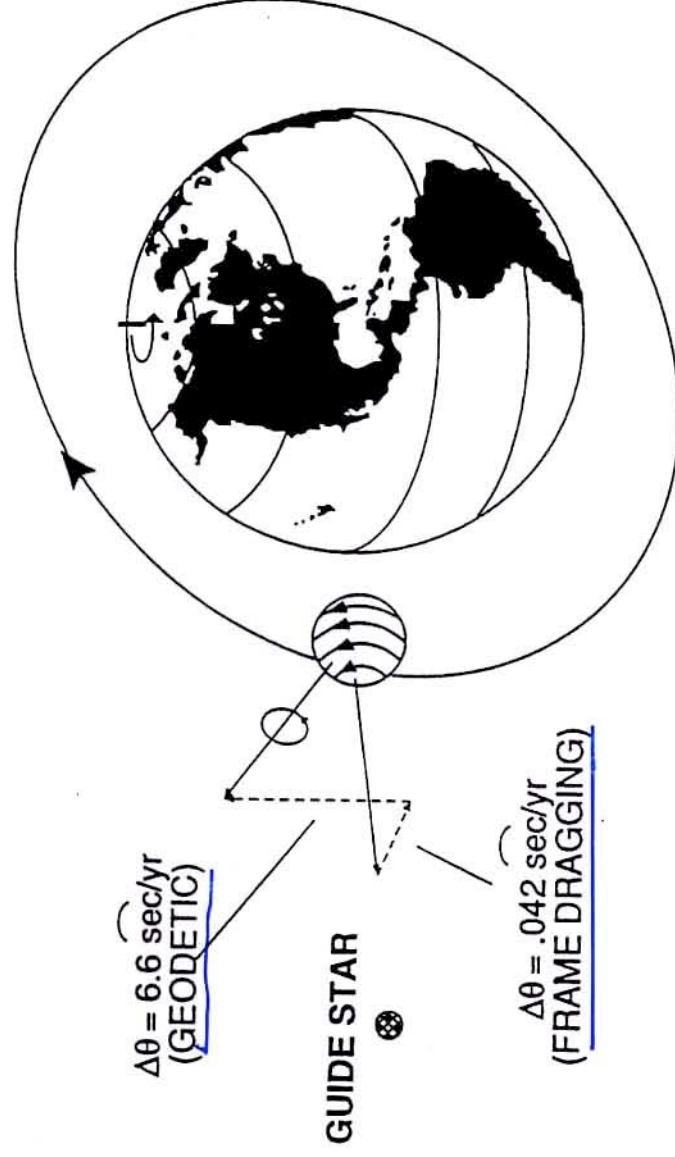
- EXPLORING SUB-MICRON DEVIATIONS FROM NEWTON'S LAW

- OLD AND NEW BINARY PULSARS
- IMPROVED SOLAR-SYSTEM TESTS
 - MORE
 - GAIA ESA GLOBAL ASTRONOMY 4-10 parsec $\gamma-1 \sim 2.5 \times 10^{-6}$
 $\beta-1 \sim 5 \times 10^{-6}$
 - LATOR $\gamma-1 \sim 10^{-7}$
 $\gamma-1 \sim 10^{-9}$

- GRAVITATIONAL WAVES
 - COALESCENCE OF BINARY BLACK HOLES
 - COALESCENCE OF BINARY NEUTRON STARS
 - GW BURSTS FROM CUSPS ON MASSIVE STRINGS
- LIGO/VIRGO/GEO
LISA

- IMPROVED (SATELLITE) TESTS OF THE EQUIVALENCE PRINCIPLE
 - MICROSCOPE (2007) $\frac{\Delta a}{a} \sim 10^{-15}$
ONERA/CNES
 - STEP $\frac{\Delta a}{a} \sim 10^{-18}$
STAMPFORD/NASA/ESA/CNES
- IMPROVED CMB MEASUREMENTS
PLANCK

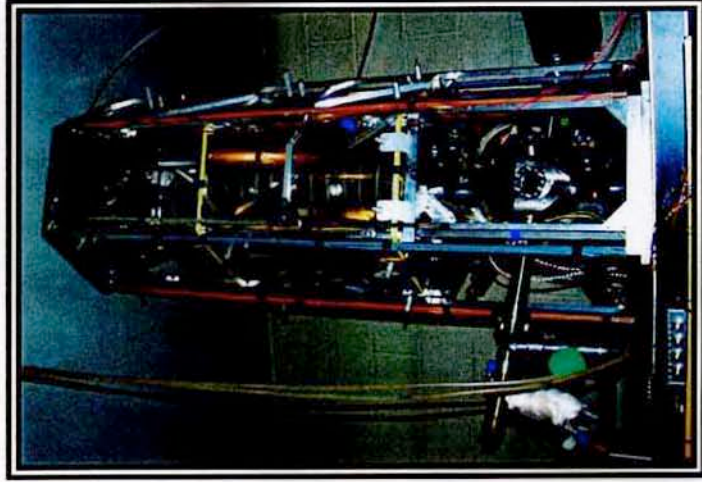
THE TWO GYRO EFFECTS PREDICTED BY SCHIFF





Fontaines Atomiques

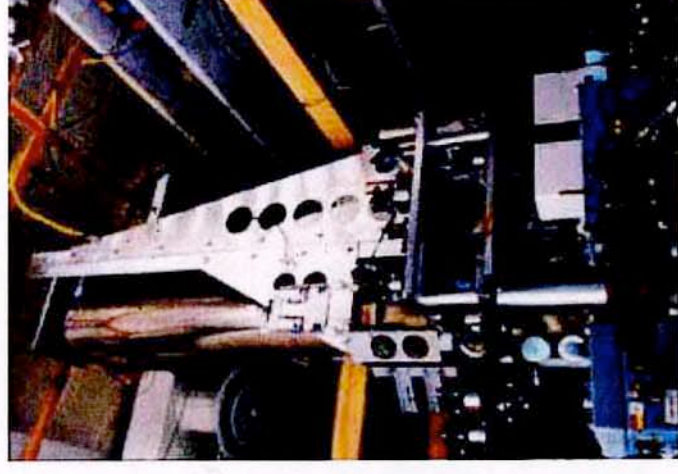
8 fountains in operation at SYRTE, PTB, NIST, USNO, Penn St, IEN. 5 with accuracy at $1 \cdot 10^{-15}$. More than 10 under construction.



BNM-SYRTE, FR



PTB, D



NIST, USA

LATOR PROJECT

(Turyshev, Shao, Nordkvist 103)

$$\gamma - 1 \sim 10^{-9}$$

7



The LATOR Mission: Relativistic Deflection of Light

JPL

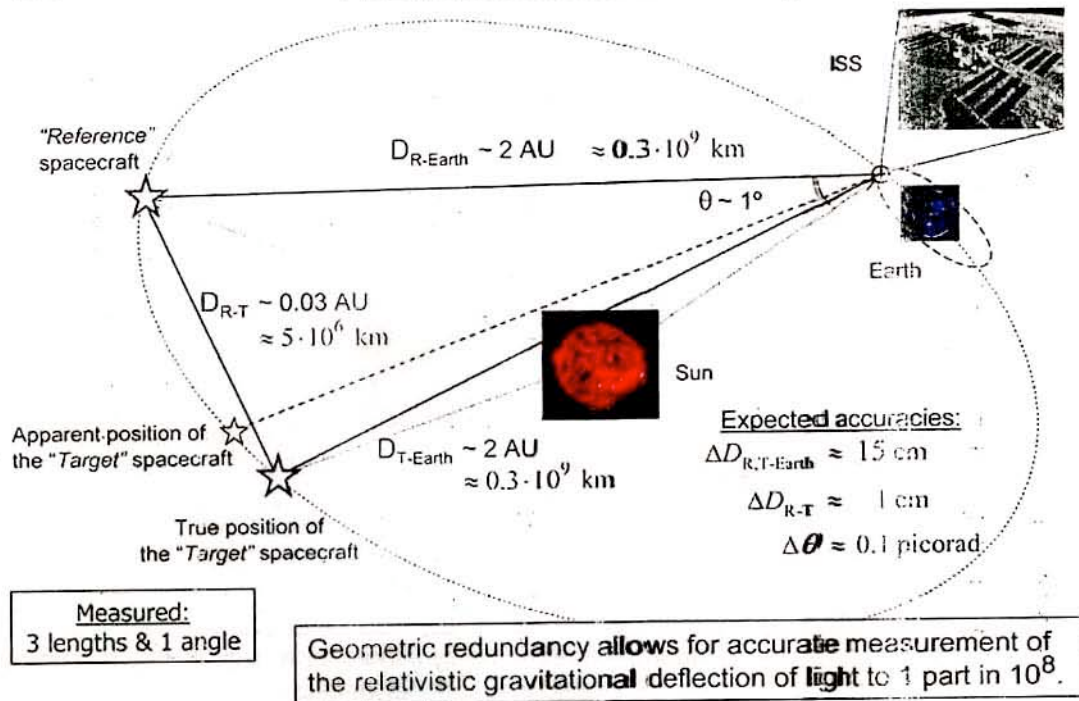


FIG. 2: Geometry of the LATOR experiment to measure deviations from the Euclidean geometry in the solar gravity field.

TABLE I: Comparable sizes of various light deflection effects in the solar gravity field.

Effect	Analytical Form	Value (μas)	Value (pm)
First Order	$2(1 + \gamma) \frac{M}{R}$	1.75×10^{16}	8.487×10^8
Second Order	$([2(1 + \gamma) - \beta + \frac{3}{4}\delta]\pi - 2(1 + \gamma)^2) \frac{M^2}{R^2}$	3.5	1702
Frame-Dragging	$\pm 2(1 + \gamma) \frac{J}{R^2}$	± 0.7	± 339
Solar Quadrupole	$2(1 + \gamma) J_2 \frac{M}{R}$	0.2	97

MICROSCOPE^(CNES) STEP^(NASA/ESA/CNES) ^{A26} SATELLITE TESTS OF THE EQUIVALENCE PRINCIPLE

24

The STEP scientific model payload

ESA/NASA/CNES

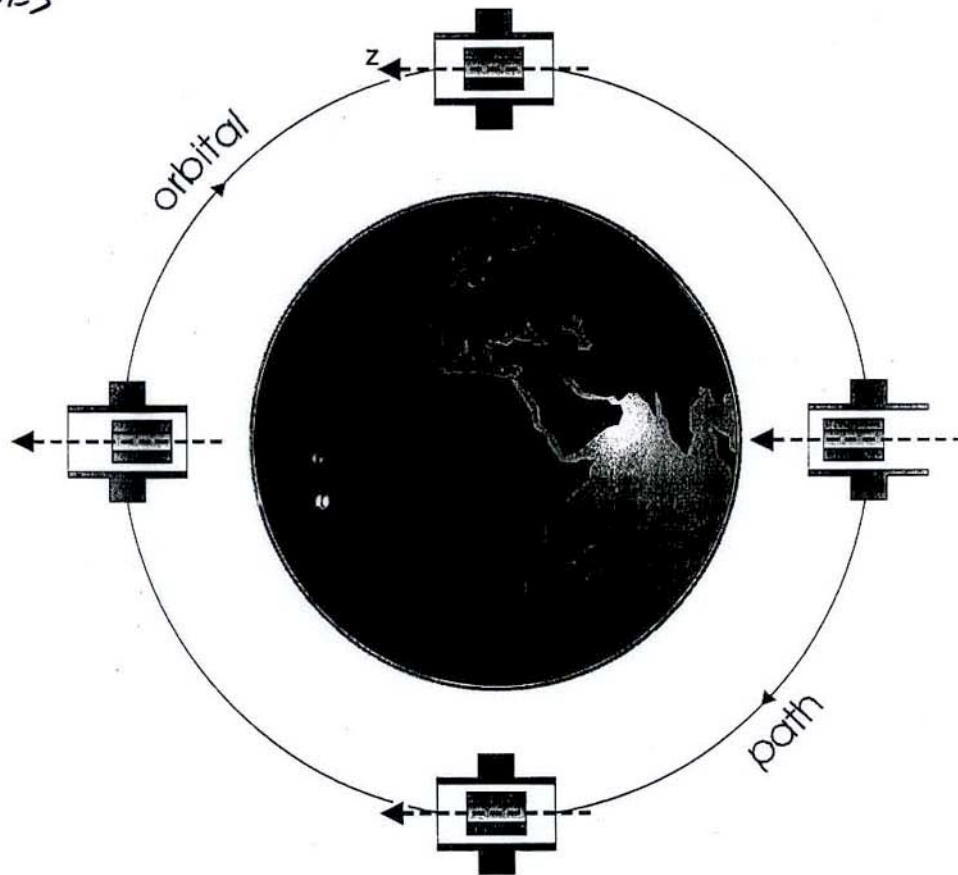


Fig. 3.2: Equivalence principle violation: The Figure shows the relative motion of free masses where the ratio of inertial mass to gravitational mass depends on the composition of the masses. These test masses are constrained by linear magnetic bearings and sensing circuits. Here, the Equivalence Principle violation signal appears at the orbital frequency. In the normal mode of operation the spacecraft would be spun about an axis perpendicular to the orbital plane at a non-integral multiple of the orbital frequency, shifting the EP signal frequency to the spin-frequency $\mp$ the orbital frequency (depending on the spin sense).

CONSTRAINTS ON $\alpha_0^2 \equiv \alpha_{\text{hadron}}^2$ FROM PRESENT AND FUTURE EXPERIMENTS [LONG-RANGE DEVIATIONS]

COMPOSITION-INDEPENDENT

LIGO/VIRGO

PRESENT SOLAR SYSTEM EXPTS
BINARY PULSAR EXPTS

PRESENT VLBI TESTS $\Delta\theta_0$

CASSINI
GPB

MORE

PARSEC ASTROMETRY
POINTS

GAIA

TIME DELAY FROM LASER LINKS
TO HELIOCENTRIC CLOCKS

SORT

LATOR

COMPOSITION-DEPENDENT

α_{had}^2

OKLO

* USING
 $(\frac{BD}{\sqrt{2}}) \sim 10^{17}$
EFFECTIVE

GROUND CLOCK (DIFFERENTIAL)*

GEOCENTRIC CLOCK (DIFFERENTIAL)*

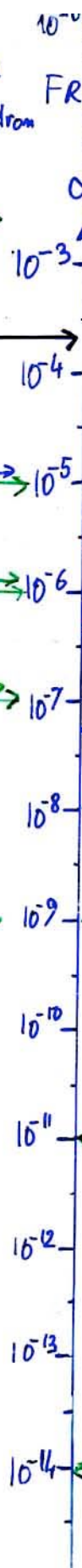
PRESENT EP EXPTS
WEBB ET AL. $(\Delta\alpha/\alpha)_{Z \sim 1} \sim 10^{-5}$

SPACETIME

ULTIMATE
HELIOCENTRIC CLOCK (DIFFERENTIAL)*

PSCOPE

STEP



CONCLUSIONS

THE NEW GRAVITY FRONTIER

- UP TO THE END OF THE 1980's, ONE CONSIDERED ONLY FEW, NATURAL MODIFICATIONS OF EINSTEIN'S GRAVITY (JORDAN-FIERZ-BRANS-DICKE), [OR QUITE ARTIFICIAL, UNMOTIVATED ALTERNATIVE THEORIES]
- RECENTLY, A BETTER UNDERSTANDING OF THE RICH STRUCTURE OF STRING THEORY (DILATON, ..., BRANES, ..., LARGE DIMENSIONS, ..., WARPED COMPACTIFICATIONS...) HAS MOTIVATED THE CONSIDERATION OF MANY NEW TYPES OF MODIFICATIONS OF GR
 - SHORT-RANGE MODIFICATIONS : $< 0.1 \text{ mm}$
 - LONG-RANGE MODIFICATIONS
- IN ADDITION, RECENT OBSERVATIONAL DISCOVERIES SUGGEST THAT OUR CURRENT THEORETICAL GRAVITY FRAMEWORK MIGHT BE INCOMPLETE OVER LONG DISTANCES / TIMES :
 - "DARK MATTER" IN GALAXIES, HALOS OF GAL. AND LSS
 - "ACCELERATED EXPANSION", AND "DARK ENERGY"

? + PIONEER 10, 11 "ANOMALOUS" ACCELERATION
 $a \approx 9 \times 10^{-8} \text{ cm/s}^2 \approx c H_0$, BUT CANNOT BE UNIVERSAL (? EP)
NO CONVINCING THEORETICAL MODEL

? + SOME CLAIMS OF VARIATION OF CONSTANTS (Webb...; Petitjean...)

- NO HARD PREDICTIONS OR WELL-DEFINED TARGETS, BUT ONLY CONSEQUENCES OF GENERAL STRUCTURES : EG. IN STRING THEORY ALL COUPLING CONSTANTS ARE FIELDS $\Rightarrow$ EXPECT SOME EQUIVALENCE PRINCIPLE VIOLATION AT SOME LEVEL.

- BUT, $\exists$ "EXISTENCE PROOFS" (MODELS) THAT NEW INTERACTIONS COULD MODIFY EINSTEIN/NEWTON GRAVITY JUST BELOW CURRENT TESTS.

EG RUN-AWAY DILATON $\Rightarrow$ CORRELATED EFFECTS

$$\frac{\delta \rho}{\rho} \sim 5 \times 10^{-5} ; \frac{\delta a}{a} \lesssim 10^{-12} ; \gamma - 1 \sim 10^{-7} ; \frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} \sim 10^{-16} \text{ yr}^{-1}$$